Title: Lecture - QFT II, PHYS 603

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Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: November 29, 2024 - 9:00 AM

URL: https://pirsa.org/24110017

Gauge Theories; Feynman Diagram

BRST Sym. Gauge Fixed.

SU(2) A gauge Field, A_μ^a

Gauge Fix. $F[A]=0$ $F[A]=\partial\cdot A$

Lorentz. $\partial^\mu A_\mu^a = 0$

Ghost Fields $(\bar{c}, c)$ $c^a(x)$, $\bar{c}_a(x)$

Fermions, Spin 0, Grassmann

Auxiliary Field B

$$S[F[A]] = \int dB \, e^{iB\cdot F[A]}$$

Auxiliary Field $B_a(x)$

$$iB\cdot F[A] = i\int d^4x \, B_a(x)\, \partial^\mu A_\mu^a(x)$$

Gauge Fixed Action

$$S[A,c,\bar{c},B] = S_{YM}[A] +$$

iagram

$$S[F[A]] = \int dB\, e^{iB\cdot F[A]}$$

Auxiliary Field $B_a(x)$

$$iB\cdot F[A] = i\int d^4x\, B_a(x)\,\partial^\mu A^a_\mu(x)$$

$_a(x)$

Gauge Fixed Action

$$S[A,c,\bar{c},B] = S_{YM}[A] + \underbrace{\partial\bar{c}\cdot Dc + B\partial A}_{\text{gauge Fixing term}}$$

contains A

gauge constraint

$B\, e^{i B\cdot F[A]}$

(x)

$(x)\, \partial^\mu A^a_\mu(x)$

$$A] + \partial\bar{c}\cdot Dc + \boxed{B\partial A}$$

contains A gauge constraint

gauge fixing term

ξ - gauge (Feynman, ...) $\xi > 0$ parameter

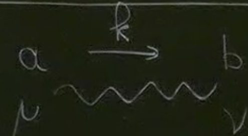
$$\delta[\partial A] \longrightarrow \exp\left(\boxed{-\frac{i}{2\xi}\partial A\cdot\partial A}\right)$$

$$\partial A \partial A = \int d^4x\, \partial^\mu A^a_\mu \cdot \partial^\nu A^a_\nu$$

Fuzzy Gauge Fixing

$$B\cdot A \longrightarrow \frac{1}{2\xi}(\partial A)^2$$

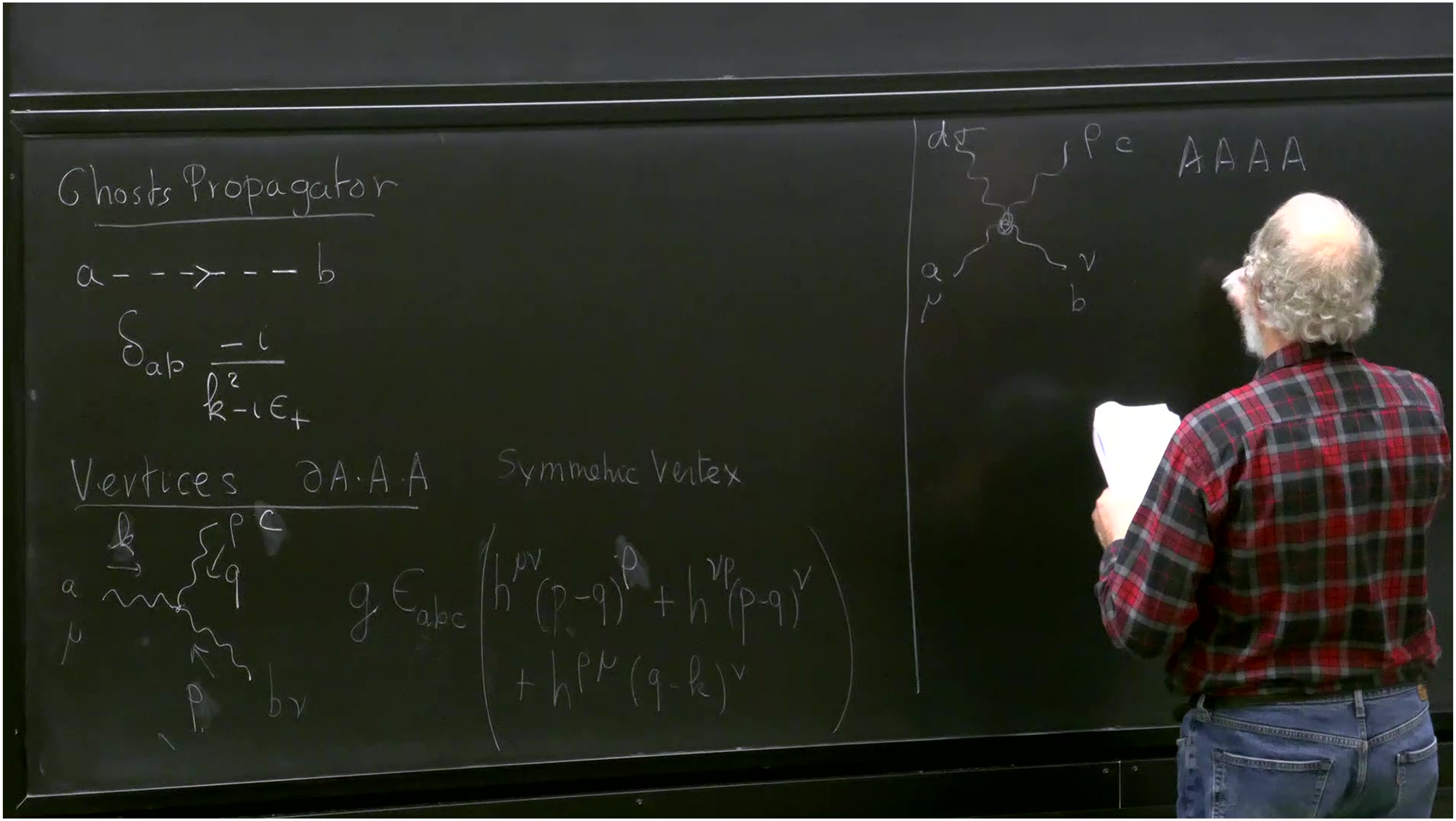
A ∼∼ A

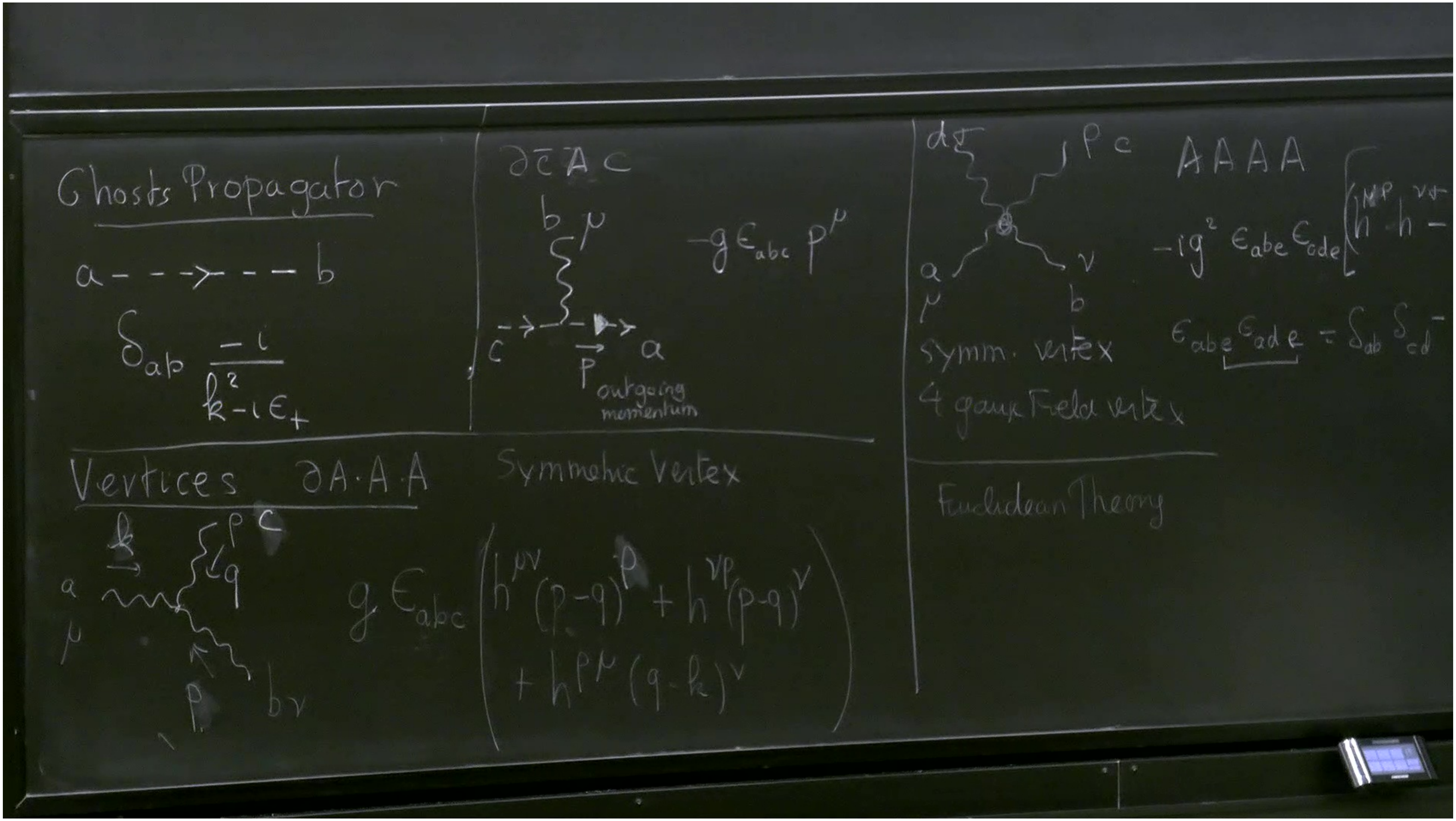


$$\frac{-i}{k^2 - i\epsilon_+}\,\delta^{ab}\left(\eta_{\mu\nu} + (\xi-1)\frac{k_\mu k_\nu}{k^2}\right)$$

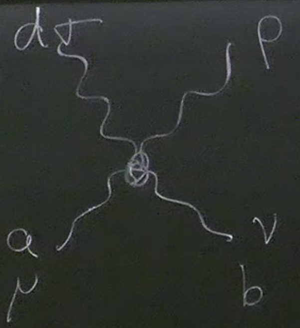
Propagator like in QED $(-+++)$

gauge parameter


Ghosts Propagator
a - - -> - - - b
δab -i / (k² - i ε+)
Vertices ∂A·A·A
Symmetric Vertex
g εabc (h^μν (p-q)^ρ + h^νρ (p-q)^ν + h^ρμ (q-k)^ν)
AAAA


Ghosts Propagator
a - - -> - - - b
δab −i / (k² − iε+)
∂c̄AC
b μ
c
a
P outgoing momentum
−g εabc p^μ
symm. vertex
4 gauge field vertex
AAAA
−ig² εabe εcde
εabe εade = δab δcd
Vertices ∂A·A·A
k
ρ c
q
a
μ
p
b ν
g εabc
Symmetric Vertex
h^μν (p−q)^ρ + h^νρ (p−q)^ν + h^ρμ (q−k)^ν
Euclidean Theory

p^μ



AAAA

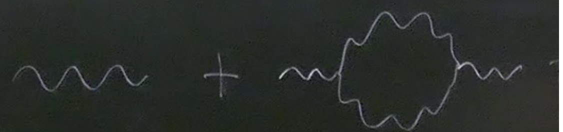
$$-ig^2\,\epsilon_{abe}\epsilon_{cde}\left[\left(h^{\mu\rho}h^{\nu\sigma}-h^{\mu\sigma}h^{\nu\rho}\right)+''+''\right]$$

symm. vertex

$$\underbrace{\epsilon_{abe}\,\epsilon_{ade}}=\delta_{ab}\delta_{cd}-\delta_{ad}\delta_{bc}$$

4 gauge field vertex

Euclidean Theory: Exercise



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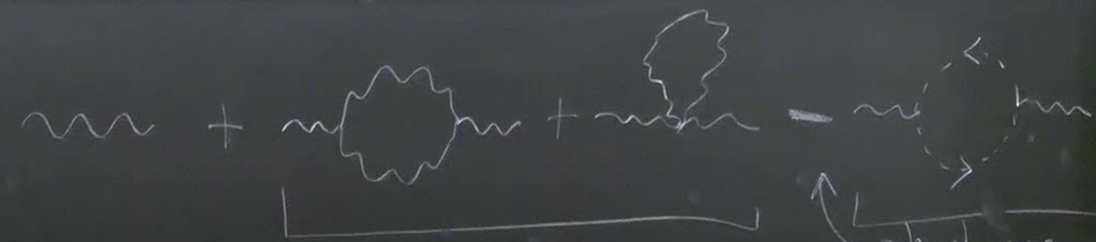
$$\epsilon_{abe}\epsilon_{cde}\left[\left(h^{\mu\rho}h^{\nu\sigma} - h^{\mu\sigma}h^{\nu\rho}\right) + \text{''} + \text{''}\right]$$

$$\epsilon_{ade} = \delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc}$$

Exercise



ghost are Fermions

not satisfied

not satisfied

$$\underset{k}{\overset{\mu}{\sim}}\ \text{(2PI)}\ \underset{k}{\overset{\mu}{\sim}} = \Sigma_\mu(p)$$

satisfied

$$\partial^\mu \Sigma_\mu(p) = 0$$

on shell

* Ward Identities

* Renormalizable

$-g\,\epsilon_{abc}\,p^{\mu}$

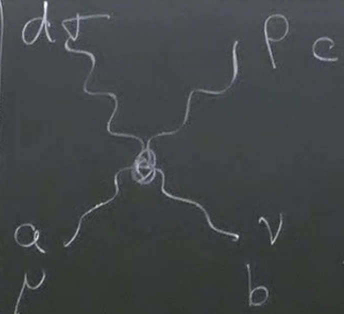
a

going
mentum

Vertex

$+ h^{\nu\rho}(p-q)^{\nu}$

$(q-k)^{\nu}$



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$$-ig^2\,\epsilon_{abe}\,\epsilon_{cde}\left[\left(h^{\mu\rho}h^{\nu\sigma} - h^{\mu\sigma}h^{\nu\rho}\right) + \text{''} + \text{''}\right]$$

symm. vertex

4 gauge Field vertex

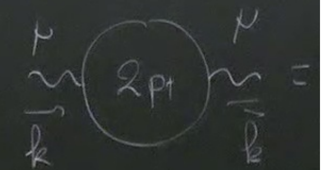
$$\epsilon_{abe}\,\underbrace{\epsilon_{ade}} = \delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc}$$

Euclidean Theory : Exercise

Gauge vectors + Higgs Field

$\underline{Z_{\pm}, W}$ ← composite Fields ← Higgs Mechanism

$\sim\sim$ +



$\partial^{\mu}\sum_{\mu}(p) =$

on shell

* Ward Identie
* Renormaliz

BRST Symmetry (Constrained Systems) → HE, String Th, Gravity

Disordered system

....

Becchi - Rouet - Stora + Tyutin

Gauge Local Symmetry SU(2)

Poincaré Symmetry

$\xrightarrow{\text{Gauge Fixing}}$

Rigid SU(2)

Poincaré

U(1) ghost number conservation

BRST Symmetry (Constrained Systems) → HE, String Th, Gravity

Disordered system

...

Becchi - Rouet - Stora + Tyutin

Gauge Local Symmetry SU(2)

Poincaré Symmetry

Rigid SU(2)

Poincaré

$\xrightarrow{\text{Gauge Fixing}}$ U(1) ghost number

Rigid Conservation

$A \to 0 \quad c \to +1, \ \bar{c} \to -1$

+ BRST

, Gravity

system

BRST is a supersymmetry (not a spacetime SS like in SUSY)

How do fields transforms?

Gauge Transf $A_\mu \to A_\mu + \underbrace{\epsilon}_{\text{infini parameter}} D_\mu \underbrace{\alpha}_{\text{arbitrary gauge transf}}$

replace it by

$$A_\mu \to A_\mu + \underbrace{\epsilon}_{\text{grassmann}} \cdot D_\mu \underbrace{\mathbb{C}}_{\text{not arbitrary but the ghost field}}$$

generator of BRST

$\boxed{Q}$ $\boxed{Q A_\mu = D_\mu \mathbb{C}}$ $\boxed{Q A^a_\mu = \partial_\mu \mathbb{C}^a + \epsilon^a{}_{bc} A^b_\mu \mathbb{C}^c}$

$$\boxed{Q\,\mathbb{C} = \frac{i}{2}\{\mathbb{C},\mathbb{C}\} = \frac{i}{2}\,\mathbb{C}\cdot\mathbb{C}}$$

2×2 matrix

$$\boxed{Q\,\mathbb{C}^a = \frac{i}{2}F^a{}_{bc}\,\mathbb{C}^b\mathbb{C}^c = \frac{i}{2}\epsilon^{abc}\underbrace{\mathbb{C}^b\mathbb{C}^c}_{\text{anticommute}}}$$

$\neq 0$

$$Q\,\bar{\mathbb{C}} = -i B$$

$$Q\,\bar{\mathbb{C}}_a = -i B_a$$

$$Q B = 0$$

$$Q B_a = 0$$

ξ-gauge, no B

$$\boxed{Q\,\bar{\mathbb{C}} = \frac{2}{\xi}\,\partial A}$$

$$\boxed{\begin{aligned} Q(A\cdot B) &= QA\cdot B + A\,QB \quad \text{if } A \text{ is bosonic} \quad \#A \text{ even} \\ &= QA\cdot B - A\,QB \quad \text{if } A \text{ is fermionic} \quad \#A \text{ is odd}\end{aligned}}$$

$$\left.\begin{aligned} Q\, S_{YM}[A] &= 0 \\ Q\, S_{\text{Gauge fixing}}[A, c, \bar{c}, B] &= 0 \end{aligned}\right]$$ BRST invariants

$$\boxed{Q^2 = 0}$$ most important property

$\boxed{\text{Gauge Fixing} \longrightarrow \text{BRST}}$

Q does not depend on the choice of gauge fixing condition

$F[A] = \partial A \Longrightarrow$ more general $F[A]$

$A \to 0 \quad c \to +1,\ \bar{c} \to -1$

$\leftarrow$ BRST

For a general $F[A]=0$ $\quad S_{G\,Fix}[A, c, \bar{c}, B] = Q\left[\bar{c} \cdot F[A]\right]$

$\boxed{Q^2 = 0} \implies Q \cdot S_{GF} = 0$ any $F[A]$

* v.e.v Φ some observable $\langle \Phi \rangle$ if $\Phi = Q\,\$ \Rightarrow \langle \Phi \rangle = 0$

* Physical Gauge Invariant observables ?

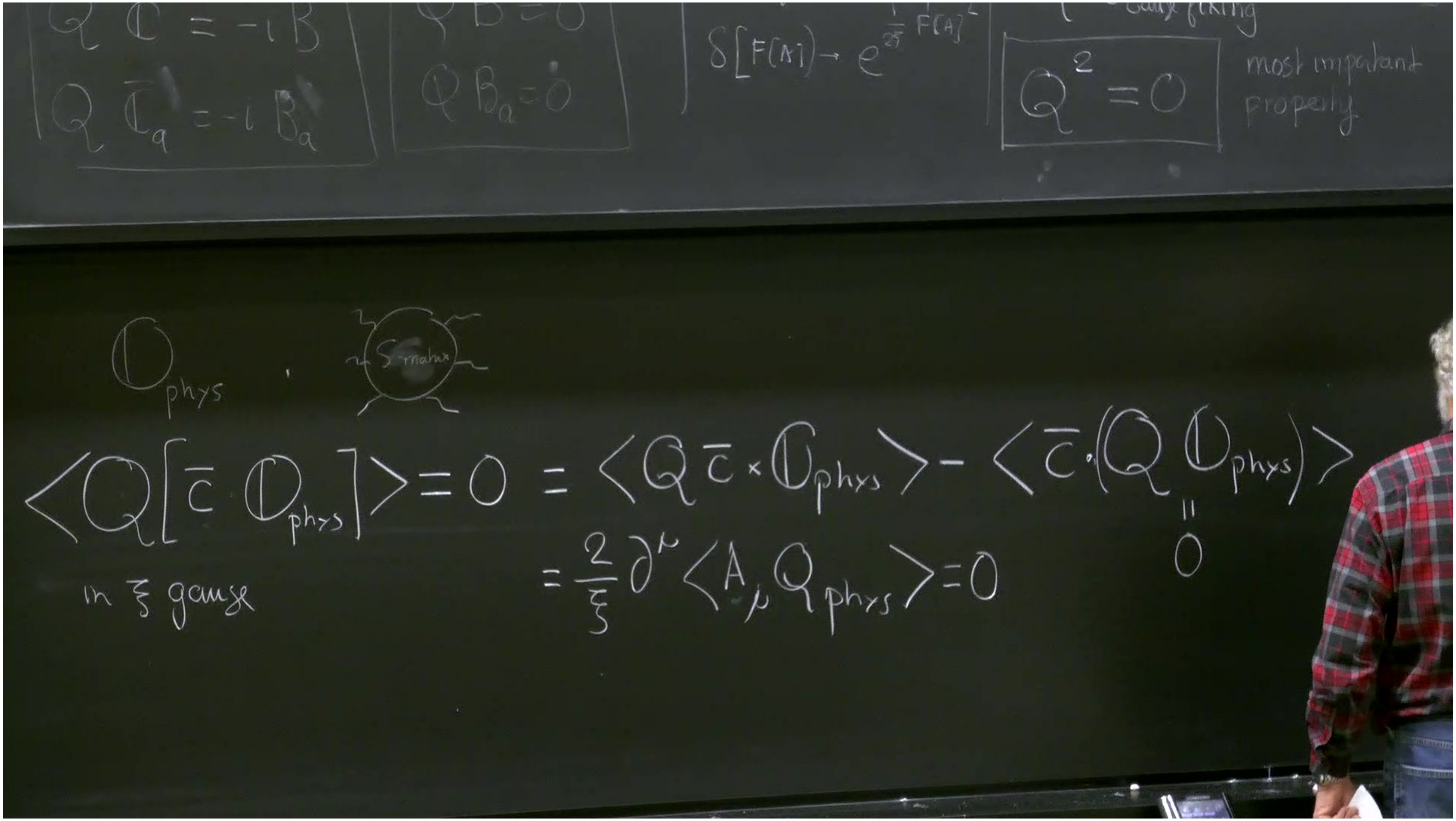
$$\boxed{\Phi \text{ is physical iff } Q\,\Phi = 0}$$

$\boxed{Q}$ $\boxed{Q A_\mu = D_\mu c}$ $\boxed{Q A^a_\mu = \partial_\mu c^a + \epsilon^a_{bc} A^b_\mu c^c}$

$O_{phys} \to O'_{physical} = O_{phys} + Q\,S$ phys observable modulo Q transformation

$\boxed{\langle O_{phys} \rangle = \langle O'_{phys} \rangle}$ and independant of $F[A]$ gauge fixing choice

$\langle S_{YM} \rangle \neq 0$ $\langle S_{G\,Fixing} \rangle = 0$


Q C = -i B
Q C̄_a = -i B_a
Q B = 0
Q B_a = 0
δ[F(A)] → e^{1/2ξ F(A)²}
gauge fixing
Q² = 0
most important property
O_phys
S-matrix
⟨Q[C̄ · O_phys]⟩ = 0 = ⟨Q C̄ × O_phys⟩ − ⟨C̄ · (Q O_phys)⟩
= 2/ξ ∂^μ ⟨A_μ O_phys⟩ = 0
in ξ gauge

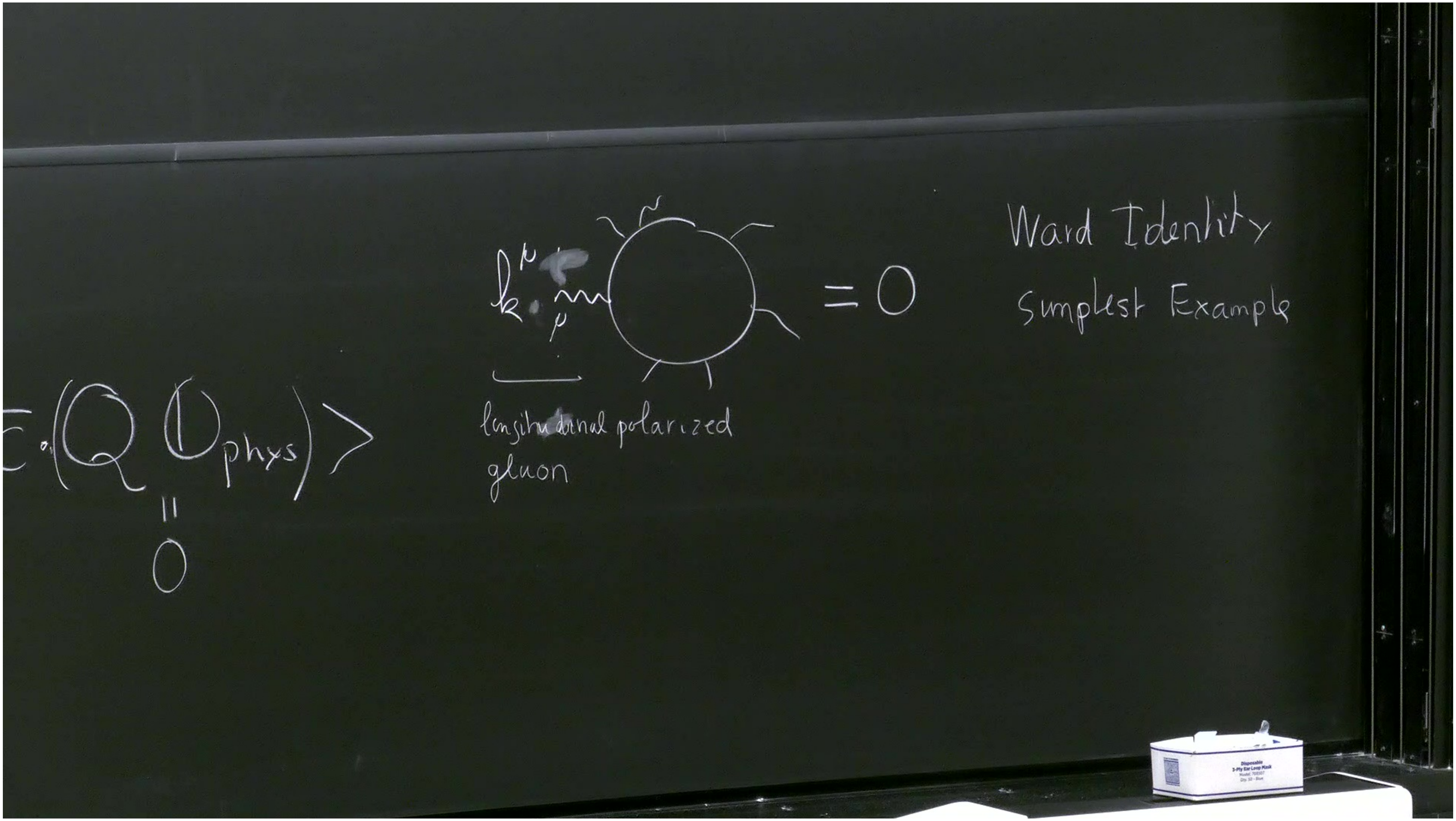
$\mathcal{O}_{phys}$:

$$\langle Q[\bar{c}\,\mathcal{O}_{phys}]\rangle = 0 = \langle Q\bar{c} \times \mathcal{O}_{phys}\rangle - \langle \bar{c}\cdot(\underbrace{Q\,\mathcal{O}_{ph}}_{=0}$$

in ξ gauge

$$= \frac{2}{\xi}\partial^{\mu}\langle A_{\mu}\mathcal{O}_{phys}\rangle = 0$$

Ward-Takahashi identities → Zinn-Justin Identies ⇐ more complicated form
+ Lee


Ward Identity
Simplest Example
= 0
longitudinal polarized
gluon
phys