Title: Lecture - QFT II, PHYS 603

Speakers: Francois David

Collection/Series: Quantum Field Theory II (Core), PHYS 603, November 12 - December 11, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: November 13, 2024 - 9:00 AM

URL: https://pirsa.org/24110009

Room 255 François DAVID

- Path & Functional Integral Quantization ← QT
  Scalar Field. Dirac Field Perturbation Theory
- Non-Abelian Gauge Theories ; Gauge Fixing
  U(1) sym., SU(2), ...
- Renormalization - Group ← QT, QFT I, S. Mech
  UV singularities

Not Treated

Full Renormalization
(all orders in PT)

S-Matrix, Amplitudes
LSZ...

IR singularities, Asymptotic Symmetries
at ∞ (BSM)

Bootstrap, Integrability, Holography

Not Treated

- Full Renormalization (all orders in PT)
- S-Matrix, Amplitudes, LSZ...
- IR singularities, Asymptotic Symmetries at ∞ (BSM)
- Bootstrap, Integrability, Holography

1

- Free scalar Field (KG)
- Minkowski $\leftrightarrow$ Euclidean

"East Coast Metric"

$(-+++)$ signature $\quad \mathbb{M}^{1,3 \text{ or } (d-1)}$

vector spacetime

$x = (t, \vec{x}) = (x^\mu) \qquad x^0 = t, \ (x^i) = \vec{x}$

$ds^2 = -dt^2 + d\vec{x}^2 \qquad \eta_{\mu\nu} = \begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$

$p = (p^\mu) \qquad p^2 = -E^2 + \vec{p}^{\,2}$ momentum vector $\quad p^0 = E$

$p^2 + m^2 = 0$ mass shell rel

$(+ - - -)$ signature West Coast metric

$$x^2 = t^2 - \vec{x}^2$$
$$p^2 = E^2 - \vec{p}^2$$

Euclidean metric $\mathbb{E}^d = \mathbb{R}^d$

$(+ + + +)$ $\quad x^2 = x^{0\,2} + \vec{x}^2$

Quantum system at Finite Temperature

Harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{m}{2}\omega^2$$

evolution operator

$$U(t) = e^{-\frac{it}{\hbar}H}$$

$$\langle q_F | U(t) | q_I \rangle = \int D$$

$t = t_F - t_I$

$q(t_I$

$q(t_F$

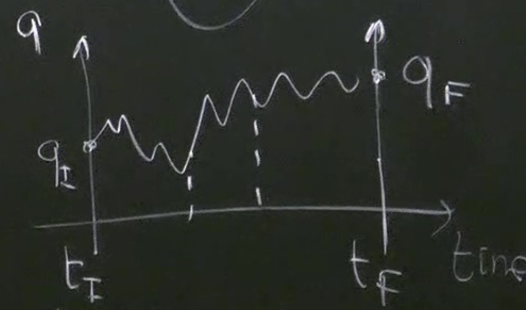
Harmonic oscillator 1 dim space $\longrightarrow q$

$$H = \frac{p^2}{2m} + \frac{m}{2}\omega^2 q^2$$

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

evolution operator

$$U(t) = e^{-\frac{it}{\hbar}H}$$



$$\langle q_F | U(t) | q_I \rangle = \int D[q]\, e^{\frac{i}{\hbar} S[q]}$$

classical action

$t = t_F - t_I$

$q(t_I) = q_I$

$q(t_F) = q_F$

$S[q] = \int_{t_I}$

H. Oscill $\leftarrow$

$\sqrt{m}\, q =$

$S[\phi] = \int d$

$\longrightarrow q$

$$S[q] = \int_{t_I}^{t_F} dt \left(\frac{m}{2}\dot{q}^2 - \frac{m\omega^2}{2} q^2\right)$$ ← CM course

$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$

H. Oscill $\longleftrightarrow$ 1+0 dim QFT

(2)

q_F

$\sqrt{m}\, q = \phi \qquad \omega = \text{Mass}$

1+0 dim.
KG Field

$$S[\phi] = \int dt \left(\frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{M^2}{2}\phi^2\right)$$

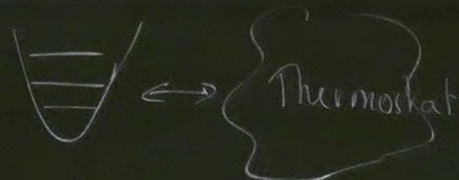
time

them

UV singularities

| Bootstrap,

(3)



equilibrium

Temperature T_{emp}

$\beta = \frac{1}{k_B \cdot T}$

density matrix

$$\rho = \frac{\exp(-\beta H)}{Z}$$

Z ← partition function

$\mathrm{Tr}\,\rho = 1$

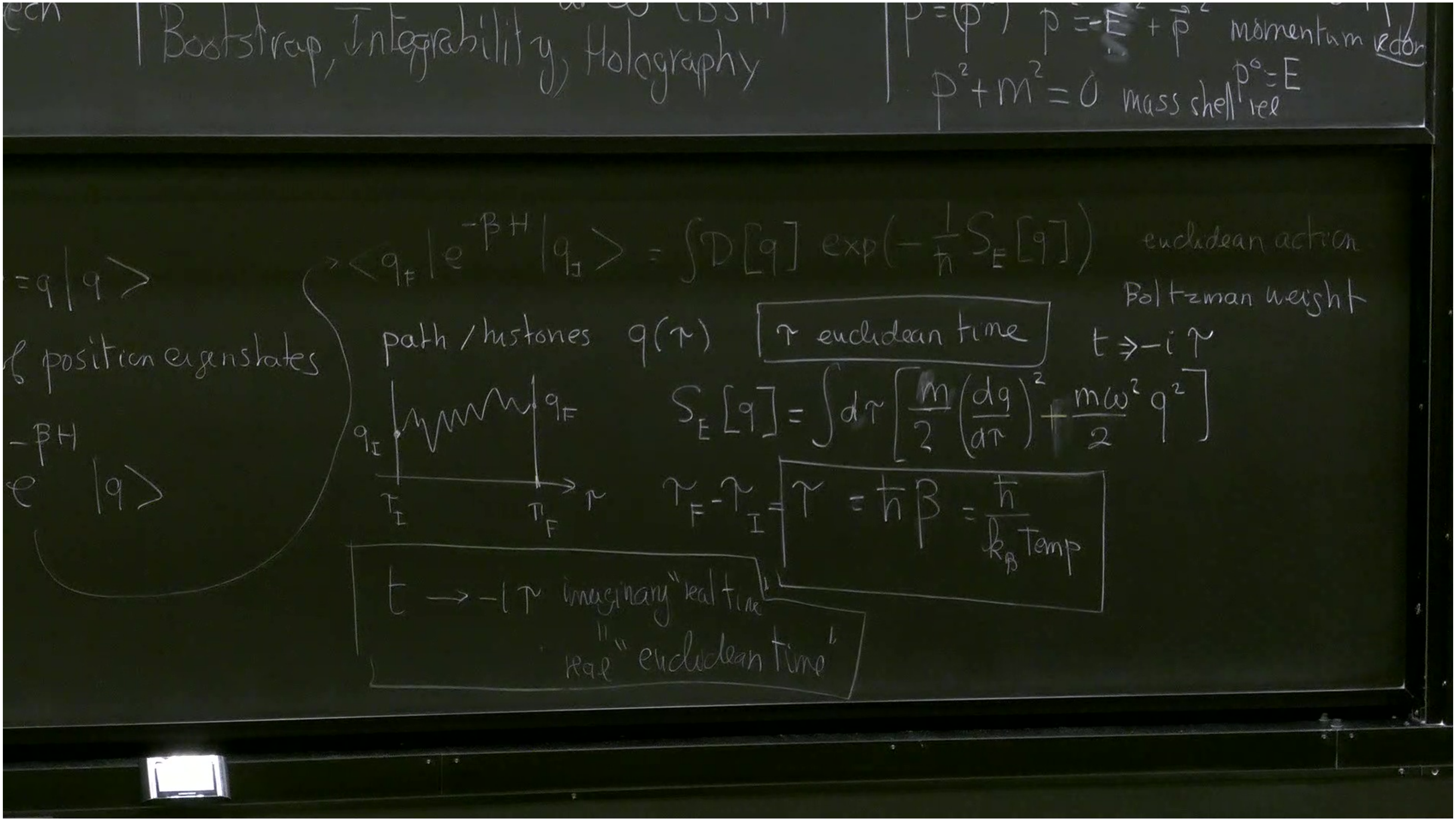
$Q|q\rangle = q|q\rangle$

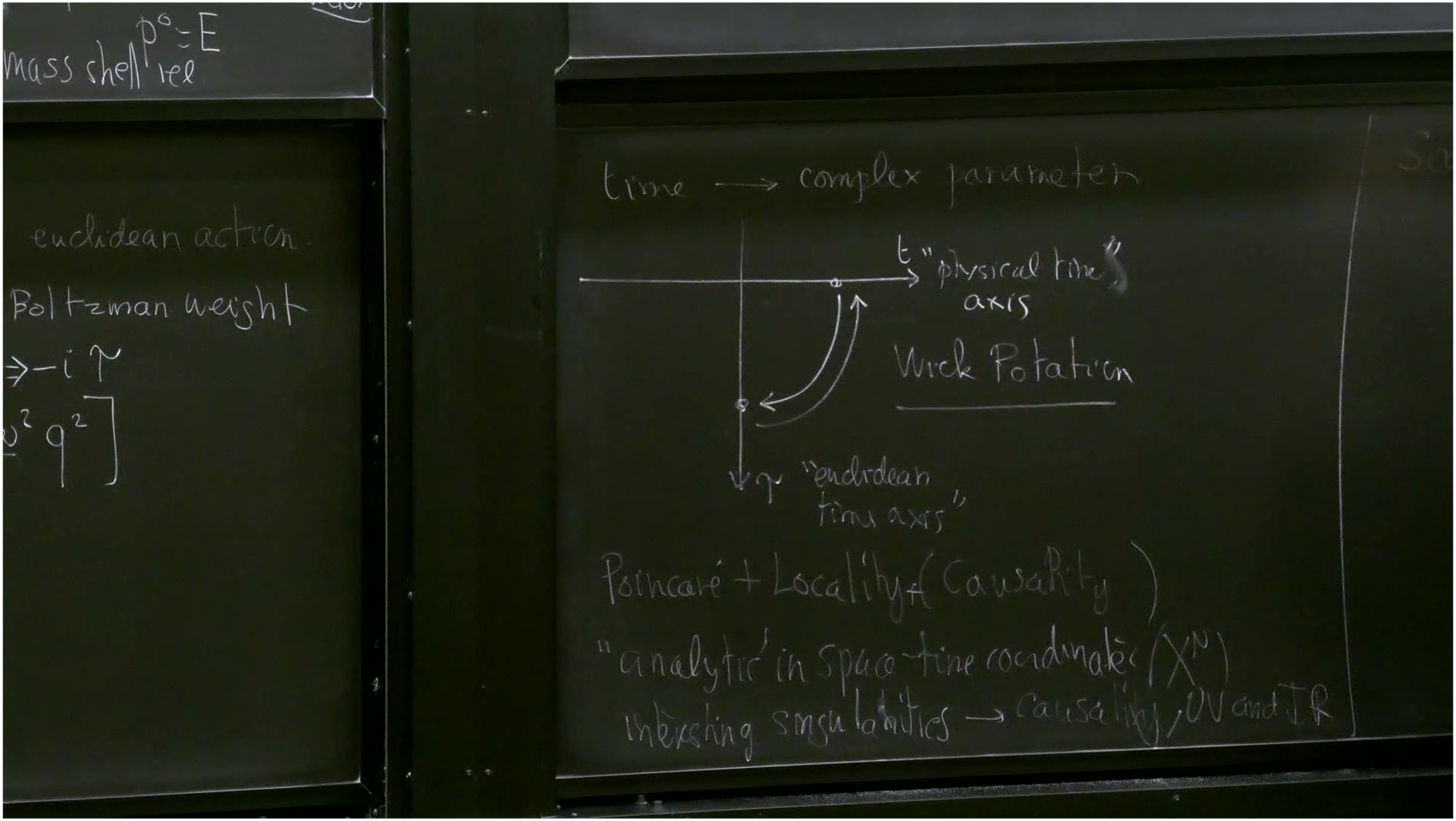
basis of position eigenstates

$$Z = \mathrm{Tr}\left[\exp(-\beta H)\right] = \int dq \, \langle q| e^{-\beta H} |q\rangle$$

$$\exp(-\beta H) \sim \exp\left(\frac{it}{\hbar} H\right) = U(t)$$

$\beta\hbar \longleftrightarrow \frac{1}{i} t$ "imaginary time"


Bootstrap, Integrability, Holography
$p = (p^\mu)$ $p^2 = -E^2 + \vec{p}^{\,2}$ momentum vector
$p^2 + m^2 = 0$ mass shell rel $p^0 = E$
$\langle q_F | e^{-\beta H} | q_I \rangle = \int \mathcal{D}[q] \exp\left(-\frac{1}{\hbar} S_E[q]\right)$
euclidean action
Boltzman weight
$= q|q\rangle$
of position eigenstates
path / histories $q(\tau)$
$\boxed{\tau \text{ euclidean time}}$
$t \Rightarrow -i\tau$
$S_E[q] = \int d\tau \left[\frac{m}{2}\left(\frac{dq}{d\tau}\right)^2 + \frac{m\omega^2}{2} q^2\right]$
q_I q_F
τ_I τ_F τ
$e^{-\beta H} |q\rangle$
$\tau_F - \tau_I = \boxed{\tau = \hbar\beta = \frac{\hbar}{k_B \, \text{Temp}}}$
$t \to -i\tau$ imaginary "real time"
"real" "euclidean time"


mass shell $p^0 \simeq E$
rel
euclidean action
Boltzman weight
$\Rightarrow -i\tau$
q^2]
time → complex parameter
t "physical time" axis
Wick Rotation
τ "euclidean time axis"
Poincaré + Locality + (Causality)
"analytic" in space-time coordinates (X^μ)
interesting singularities → causality, UV and IR

parameter

'physical time' axis

...ck Rotation

...s''

(Causality)

...ime coordinates (X^μ)

...s → Causality, UV and IR

t real

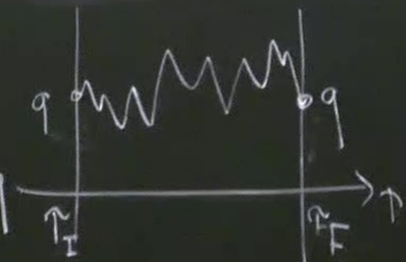
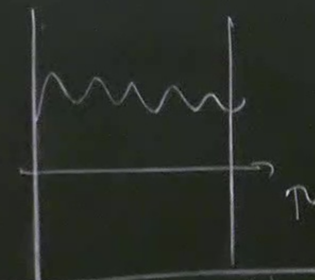
$\|U(t)\| = 1$

$= \sqrt{\mathrm{Max} \frac{\langle \psi | U^\dagger U | \psi \rangle}{\langle \psi \psi \rangle}}$

Spectrum of H bounded from below

$\|U(t)\| < \infty$ only if $\mathrm{Im}\, t < 0$

$$Z = \int dq \langle q | e^{-\beta H} | q \rangle$$

$\int dq$  $=$ 

Partition function

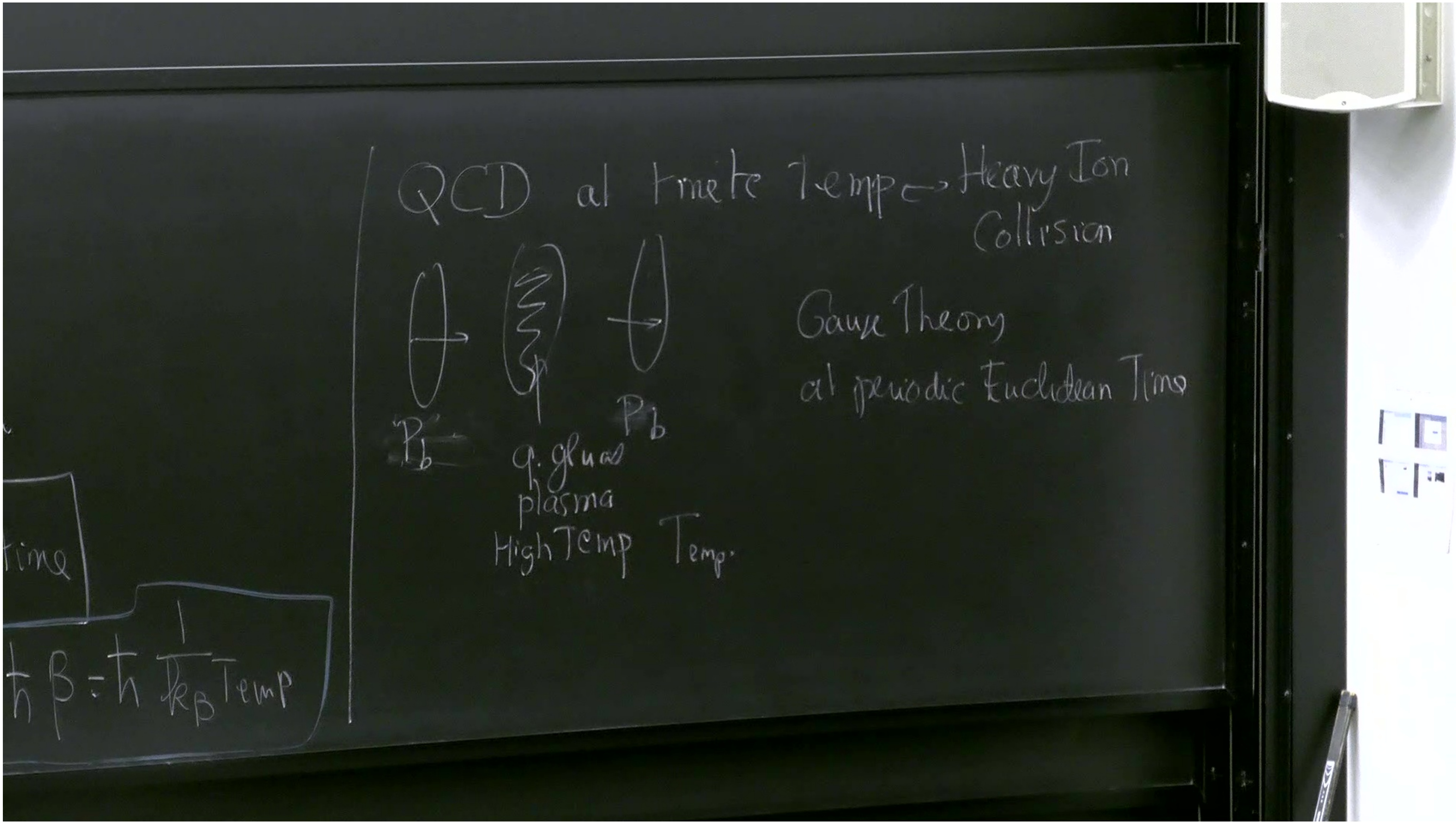
Temperature Temp

$\Updownarrow$

periodic Euclidean time

periodic path in euclidean time

$$\tau = \hbar \beta = \hbar \frac{1}{k_B \mathrm{Temp}}$$


QCD at finite Temp ⟷ Heavy Ion Collision
Pb
Pb
q. gluon plasma
High Temp
Temp.
Gauge Theory
at periodic Euclidean Time
time
ħβ = ħ 1/(k_β Temp)

5 KG Scalar (massive) Free field spin = 0, ϕ real

$\phi(x)$ $\quad x \in \mathbb{M}^{1,d-1}$ or $\mathbb{R}^d$ $\quad x = (t, \vec{x})$

$(-\Delta + m^2)\phi = 0$, $\quad \Delta = -\partial_t^2 + \vec{\nabla}^2$

KG equation real time t

$$\boxed{S[\phi] = \int dt \int d^{d-1}\vec{x} \; \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}\left((\vec{\nabla}\phi)^2 + m^2\phi^2\right)}$$

$(-+++)$ $\quad = \int d^d x \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2\right)$

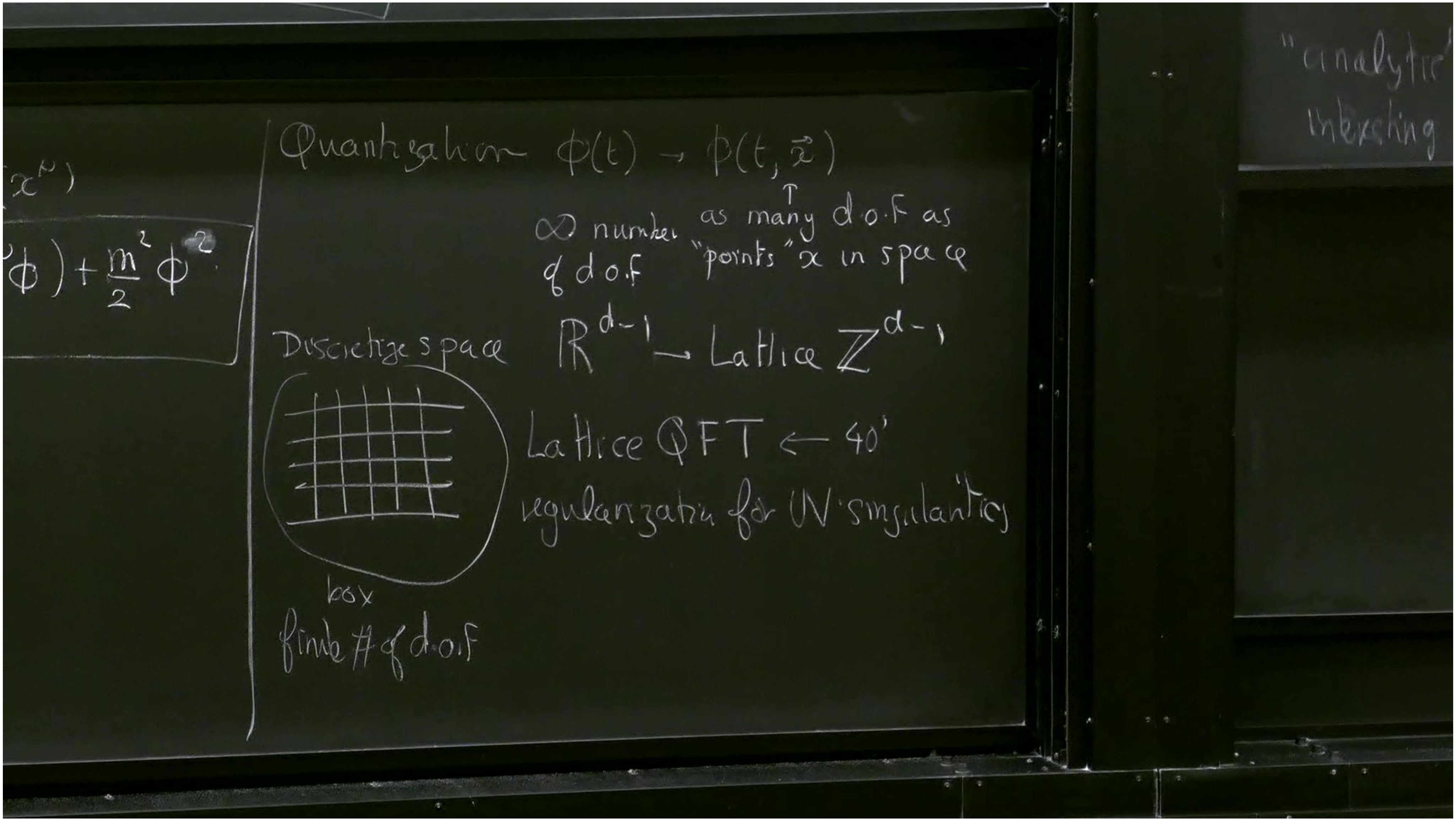
Wick Rotation

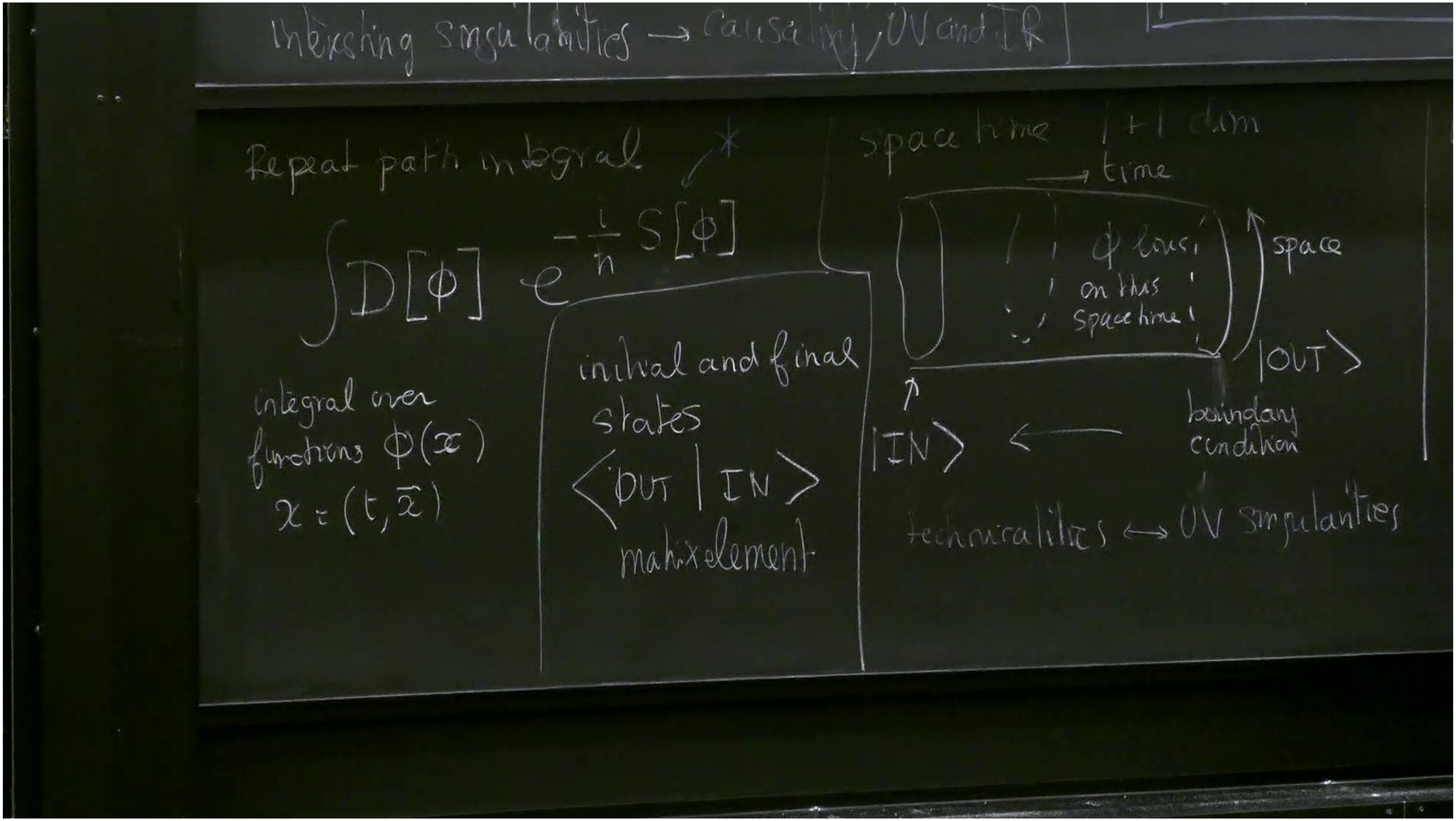
$t \to -i\tau$

$\vec{x} \to \vec{x}$

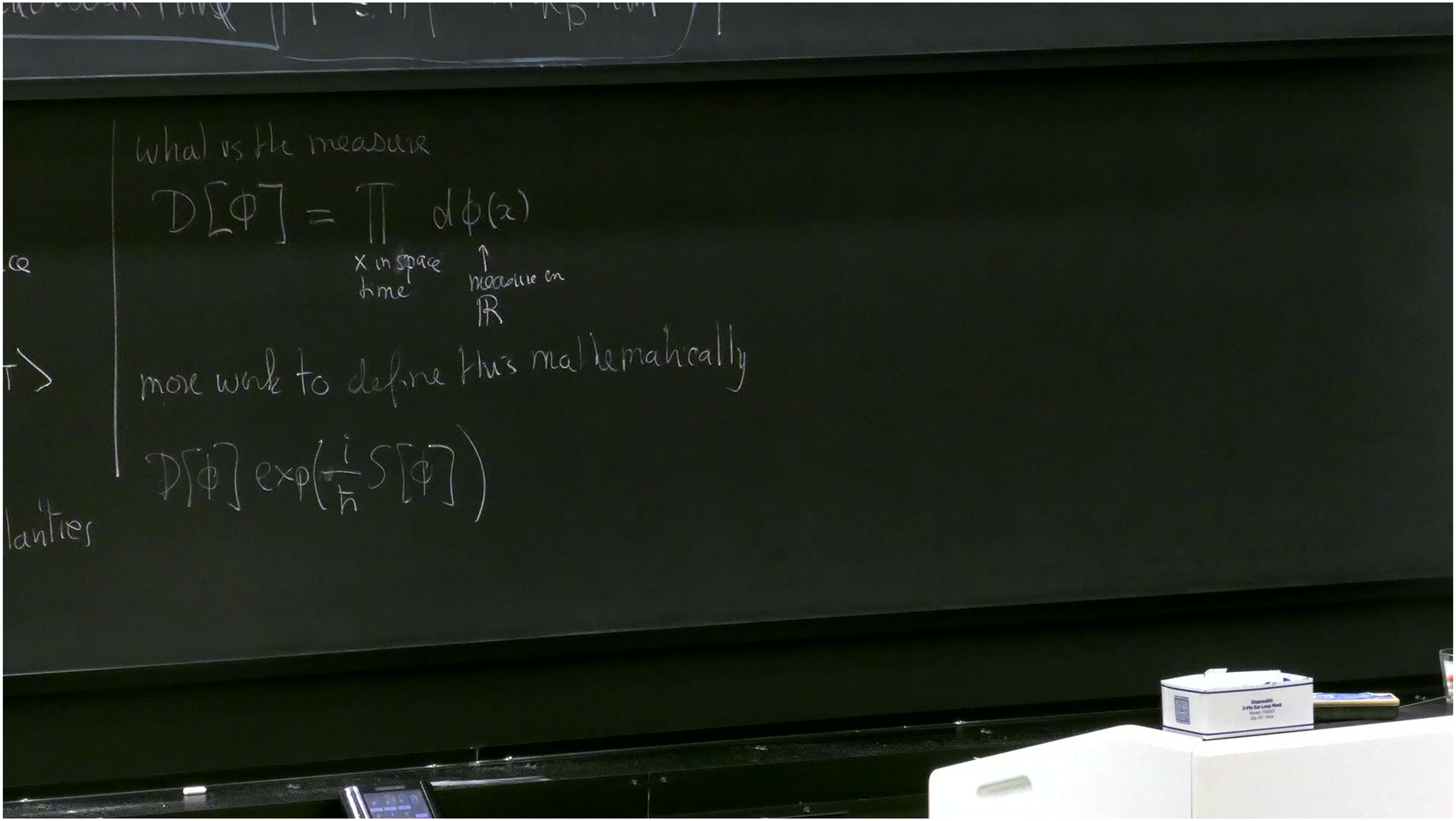
end

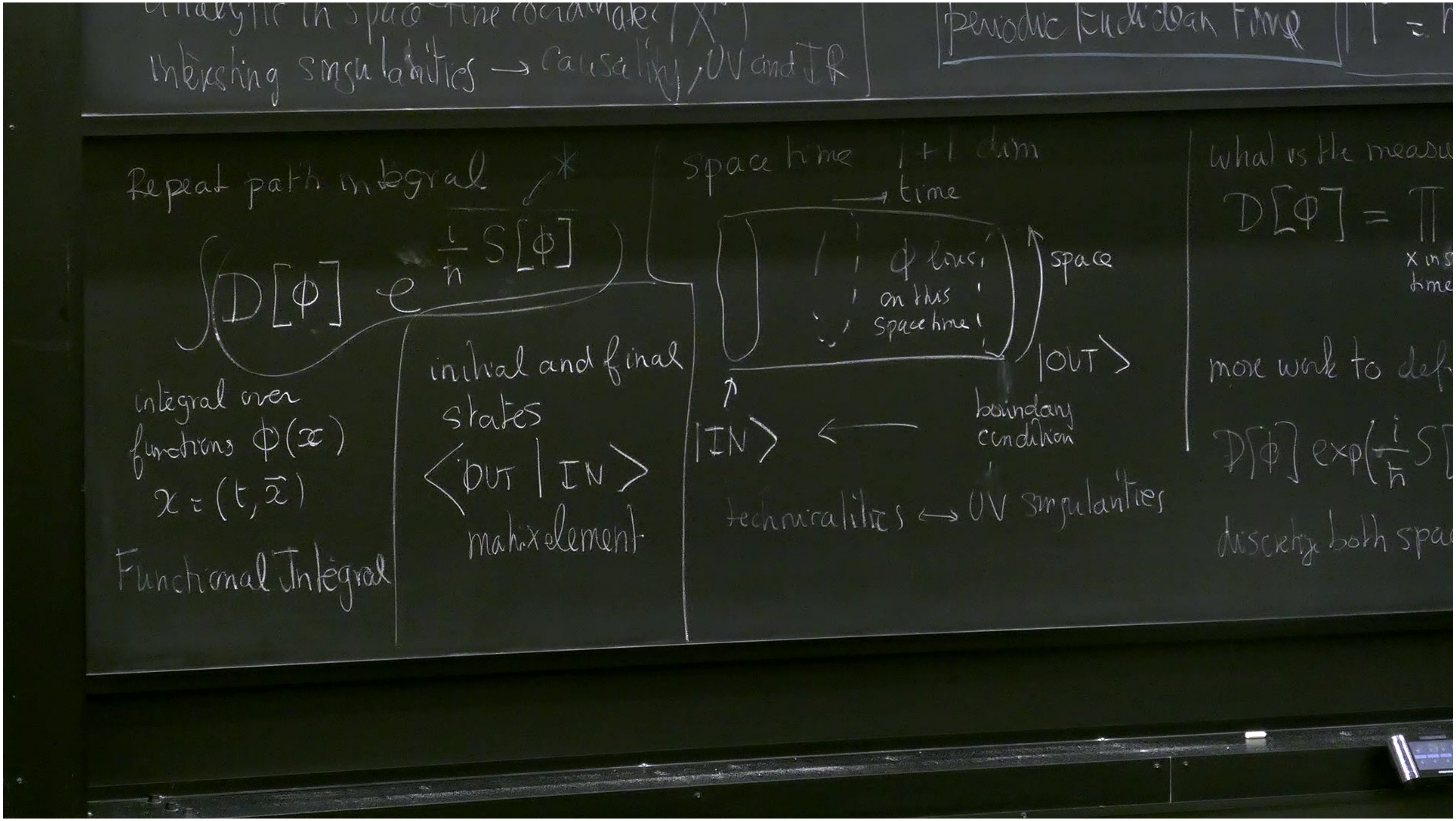
S_E

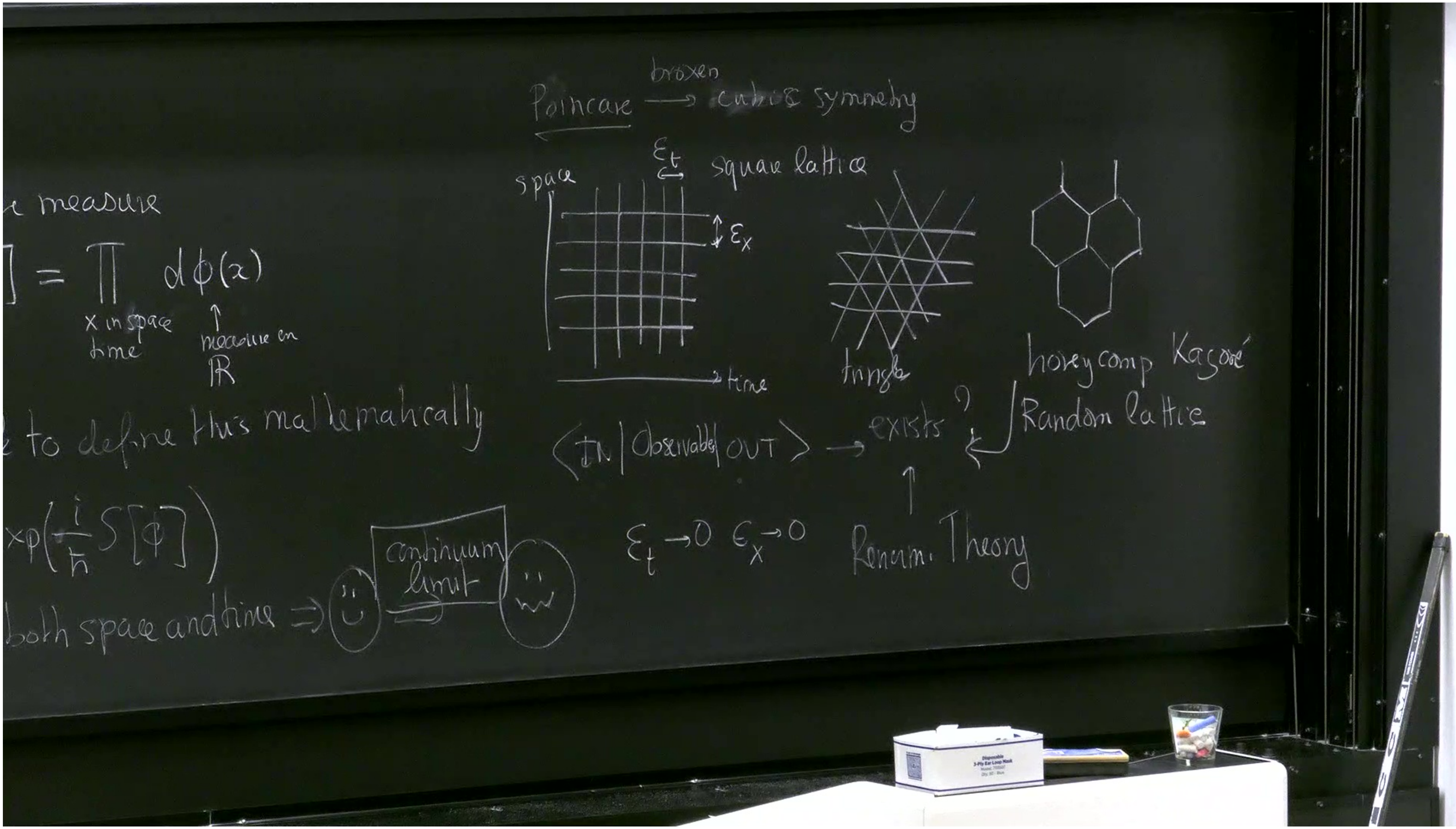

Quantization $\phi(t) \to \phi(t, \vec{x})$
∞ number of d.o.f
as many d.o.f as "points" x in space
Discretize space $\mathbb{R}^{d-1} \to$ Lattice $\mathbb{Z}^{d-1}$
box
finite # of d.o.f
Lattice QFT ← 40'
regularization for UV singularities
"analytic"
interesting



Interesting singularities → causality, UV and IR
Repeat path integral
$\int D[\phi]\, e^{-\frac{i}{\hbar} S[\phi]}$
integral over functions $\phi(x)$
$x = (t, \vec{x})$
initial and final states
$\langle OUT | IN \rangle$
matrix element
space time 1+1 dim
time
space
ϕ lives on this space time
$|OUT\rangle$
$|IN\rangle$
boundary condition
technicalities ↔ UV singularities


what is the measure
$D[\phi] = \prod d\phi(x)$
x in space time
measure on $\mathbb{R}$
more work to define this mathematically
$D[\phi] \exp\left(\frac{i}{\hbar} S[\phi]\right)$

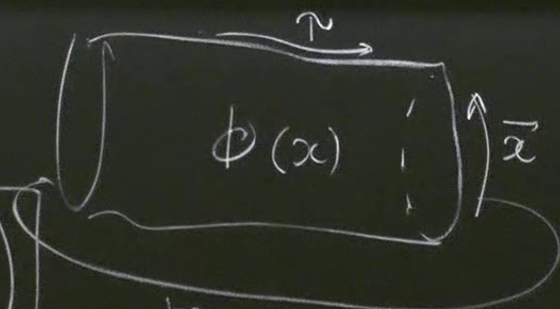

interesting singularities → causality, UV and IR
periodic Euclidean time
Repeat path integral
$\int D[\phi]\, e^{\frac{i}{\hbar} S[\phi]}$
integral over functions $\phi(x)$
$x = (t, \vec{x})$
Functional Integral
initial and final states
$\langle OUT \mid IN \rangle$
matrix element
space time 1+1 dim
time
space
φ lives on this space time
$|OUT\rangle$
$|IN\rangle$
boundary condition
technicalities ⟷ UV singularities
what is the measu
$D[\phi] = \prod$
x in s time
more work to def
$D[\phi]\, \exp(\frac{i}{\hbar} S$
discretize both spa


broxen
Poincare → symmetry
space
ε_t square lattice
ε_x
time
measure
$= \prod d\phi(x)$
x in space time
measure on $\mathbb{R}$
to define this mathematically
triangle
honey comp
Kagomé
Random lattice
$\langle IN \mid Observable \mid OUT \rangle \rightarrow$ exists ?
$\varepsilon_t \rightarrow 0$ $\varepsilon_x \rightarrow 0$
Renorm. Theory
$xp(\frac{i}{\hbar} S[\phi])$
continuum limit
both space and time $\Rightarrow$

Euclidean QFT KG

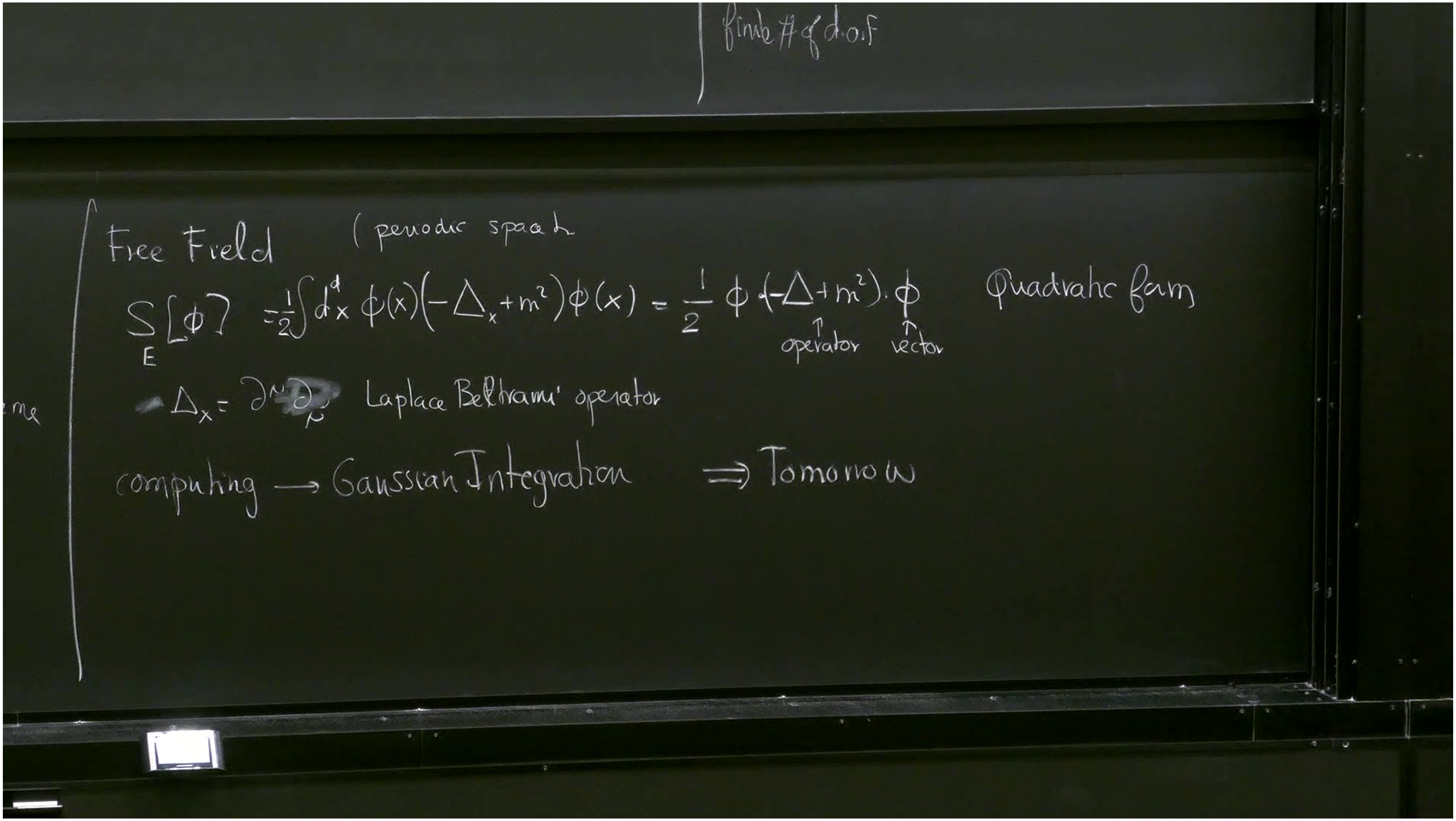
$$\boxed{\int D_E[\phi] \exp\left(-\beta S_E[\phi]\right)}$$

$$D_E[\phi] = \prod_{x\in\mathbb{R}^d} d\phi(x)$$



periodic in Euclidean time for finite Temp

$$\tau = \frac{\hbar}{k_B T}$$


finite # of d.o.f
Free Field (periodic space
$S_E[\phi] = \frac{1}{2}\int d^dx\, \phi(x)(-\Delta_x + m^2)\phi(x) = \frac{1}{2}\phi(-\Delta + m^2)\cdot\phi$
Quadratic form
operator
vector
$\Delta_x = \partial^\mu \partial_\mu$
Laplace Beltrami operator
computing → Gaussian Integration
⇒ Tomorrow