Title: Lecture - QFT I, PHYS 601

Speakers: Gang Xu

Collection/Series: Quantum Field Theory I (Core), PHYS 601, October 7 - November 6, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: November 05, 2024 - 9:00 AM

URL: https://pirsa.org/24110007

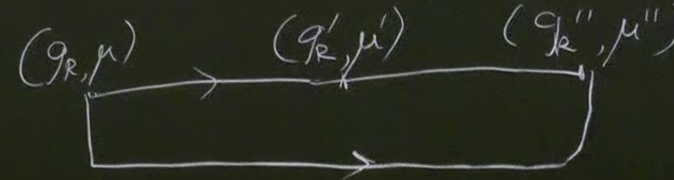
Renormalization

Renormalization group

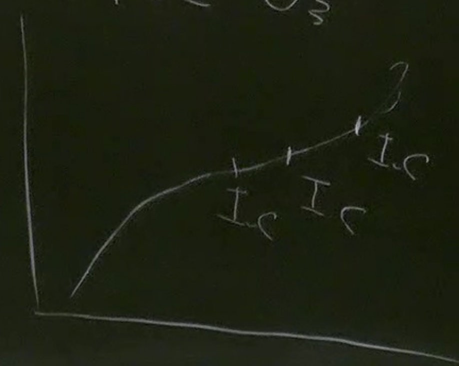
$$F(x, \mu, g_R) = g_R + g_R^2 \alpha \ln \frac{\mu}{x}$$

$$F(x, \mu', g_R')$$

$$F(x, \mu'', g_R')$$



$$U_1 U_2 = U_3$$

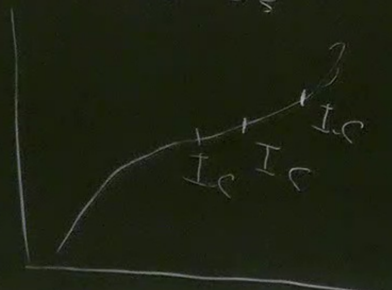


$$(g_R, \mu) \rightarrow (g_R', \mu') \rightarrow (g_R'', \mu'')$$

$$f\left(g_R, \frac{\mu}{\mu''} = x\right) = g_R'' = f\left(g_R' = f\left(g_R, \frac{\mu}{\mu'}\right), \frac{\mu'}{\mu''}\right)$$

$$g_R' = g_R + g_R^2 \alpha \ln \frac{\mu}{\mu'}$$

$$U_1 U_2 = U_3$$



$$g_R'' = g_R + g_R^2 \alpha \ln \frac{\mu}{\mu''}$$

$$g_R'' = g_R + g_R^2 \alpha \ln \frac{\mu}{\mu'} + g_R^2 \alpha \ln \frac{\mu'}{\mu''} = g_R + g_R^2 \alpha \ln \frac{\mu}{\cancel{\mu'}} \cdot \frac{\cancel{\mu'}}{\mu''} = g_R''$$

(second order)

dimension regularization

$d = 4 - \epsilon$

$\lambda \to$ charge density

x_2

x_1

an infinite line of charge

x

y

$$V(x) = \int_{-\infty}^{\infty} \frac{\lambda dy}{\sqrt{x^2+y^2}} = \ln|y + \sqrt{x^2+y^2}| \Big|_{-\infty}^{\infty} = \infty$$

test charge moves

$$V(x_1) = V(x_2)$$

the familiar cutoff scheme

$y \to y + c$ is broken

↳ different result

$$V(x) = \int_{-L}^{L} \frac{\lambda\, dy}{\sqrt{x^2+y^2}} = \ln \frac{L+\sqrt{x^2+L^2}}{-L+\sqrt{x^2+L^2}}$$

$$\lim_{L\to\infty} \left(V(x_1) - V(x_2) \right) = \ln \frac{x_2^2}{x_1^2}$$

$$\int dy \to \int dy^d = \underline{\int d\Omega_d \, y^{d-1} \, dy}$$

$d=1$

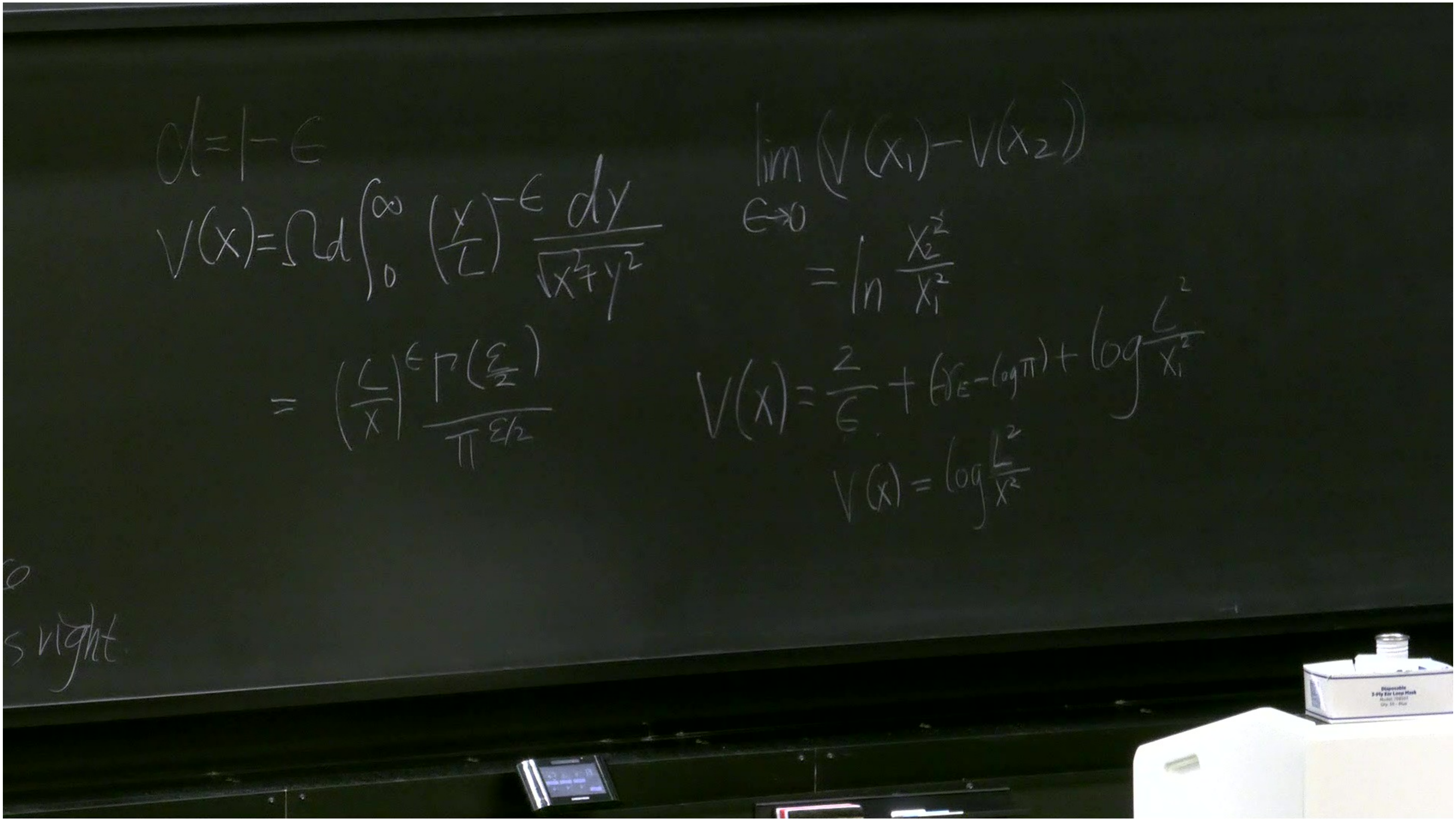
$V(x)$

$$\Omega_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$$

$$V(x) = \Omega_d \int_0^\infty \left(\frac{y}{L}\right)^{d-1} \frac{dy}{\sqrt{x^2+y^2}}$$

forced to introduce

so dimension is right



$d = 1 - \epsilon$
$V(x) = \Omega_d \int_0^\infty \left(\frac{y}{L}\right)^{-\epsilon} \frac{dy}{\sqrt{x^2+y^2}}$
$= \left(\frac{L}{x}\right)^{\epsilon} \frac{\Gamma\left(\frac{\epsilon}{2}\right)}{\pi^{\epsilon/2}}$
$\lim_{\epsilon \to 0} \left(V(x_1) - V(x_2)\right)$
$= \ln \frac{x_2^2}{x_1^2}$
$V(x) = \frac{2}{\epsilon} + (\gamma_E - \log \pi) + \log \frac{L^2}{x_1^2}$
$V(x) = \log \frac{L^2}{x^2}$
s right.

photon propagator

① $\mathcal{L}_{KG} \propto \phi\,(\partial^2+m^2)\,\phi$

$\downarrow$

operator — p^2+m^2

$\downarrow$

$\frac{i}{p^2-m^2}$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= \frac{1}{2}A_\nu\left(\partial^2\eta^{\mu\nu} - \partial^\mu\partial^\nu\right)A_\mu$$

$\downarrow$

zero mode $\left(-p^2\eta^{\mu\nu} + p^\mu p^\nu\right)p_\nu = 0$

g

gauge fix $\partial_\mu A^\mu = 0$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2$$

operator $-p^2 \eta^{\mu\nu} + p^\mu p^\nu \left(1 - \frac{1}{\xi}\right)$

$$\frac{-i}{p^2 + i\epsilon} \left(\eta^{\mu\nu} - (1-\xi) \frac{p^\mu p^\nu}{p^2} \right)$$

$\xi = 1$ $\quad \dfrac{-i\eta^{\mu\nu}}{p^2 + i\epsilon}$