Title: Lecture - QFT I, PHYS 601

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Collection/Series: Quantum Field Theory I (Core), PHYS 601, October 7 - November 6, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

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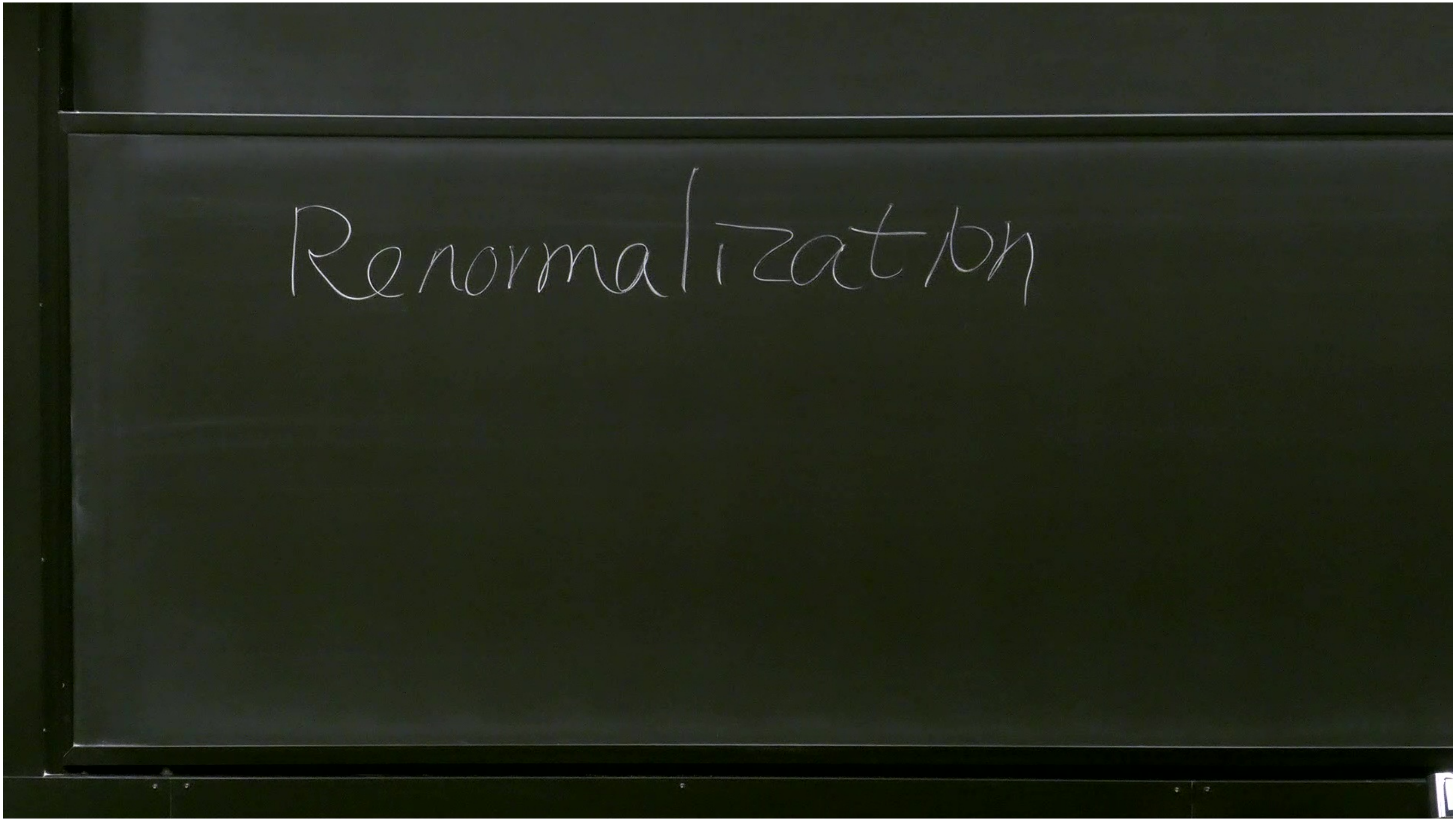
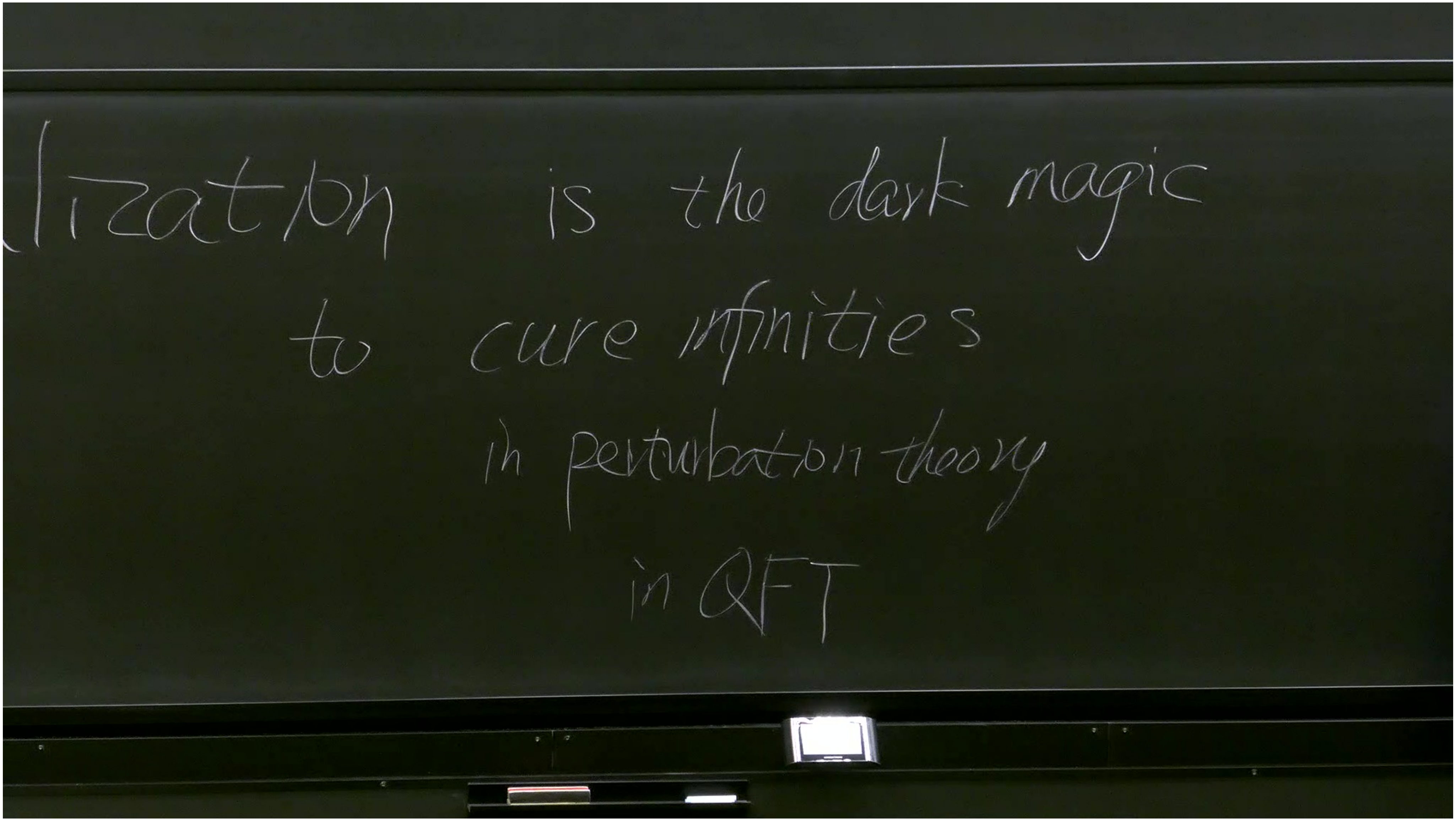

Renormalization


lization is the dark magic
to cure infinities
in perturbation theory
in QFT

erk magic

es X

theory X

X

definition

physical problem < scales / "bare" paramete[r]

replace "bare"/unphysical parameter

by "renormalized" parameter

"dressed" physical (depend on scale)

History. 1877 Boussinesq

1923 Debye-Hückel theory of electron gas.

goal is to compute $\phi(r)$!

system is neutral

N free electrons

spatially uniform positive charge

Coulomb's law $\phi = \frac{e}{r}$

Poisson eq $\nabla^2\phi = -4\pi\rho(r)$

$$\rho(r) = e\underline{n(r)} - e n_\infty$$

$$n(r) = p(r)\, N = \frac{e^{-\frac{W(r)}{kT}}}{Z} N$$

$$n(r\to\infty) = \frac{N}{Z} \qquad (\text{use } W(r) = \phi(r)\, e)$$

$$n(r) = n_\infty e^{-\frac{W(r)}{kT}}$$

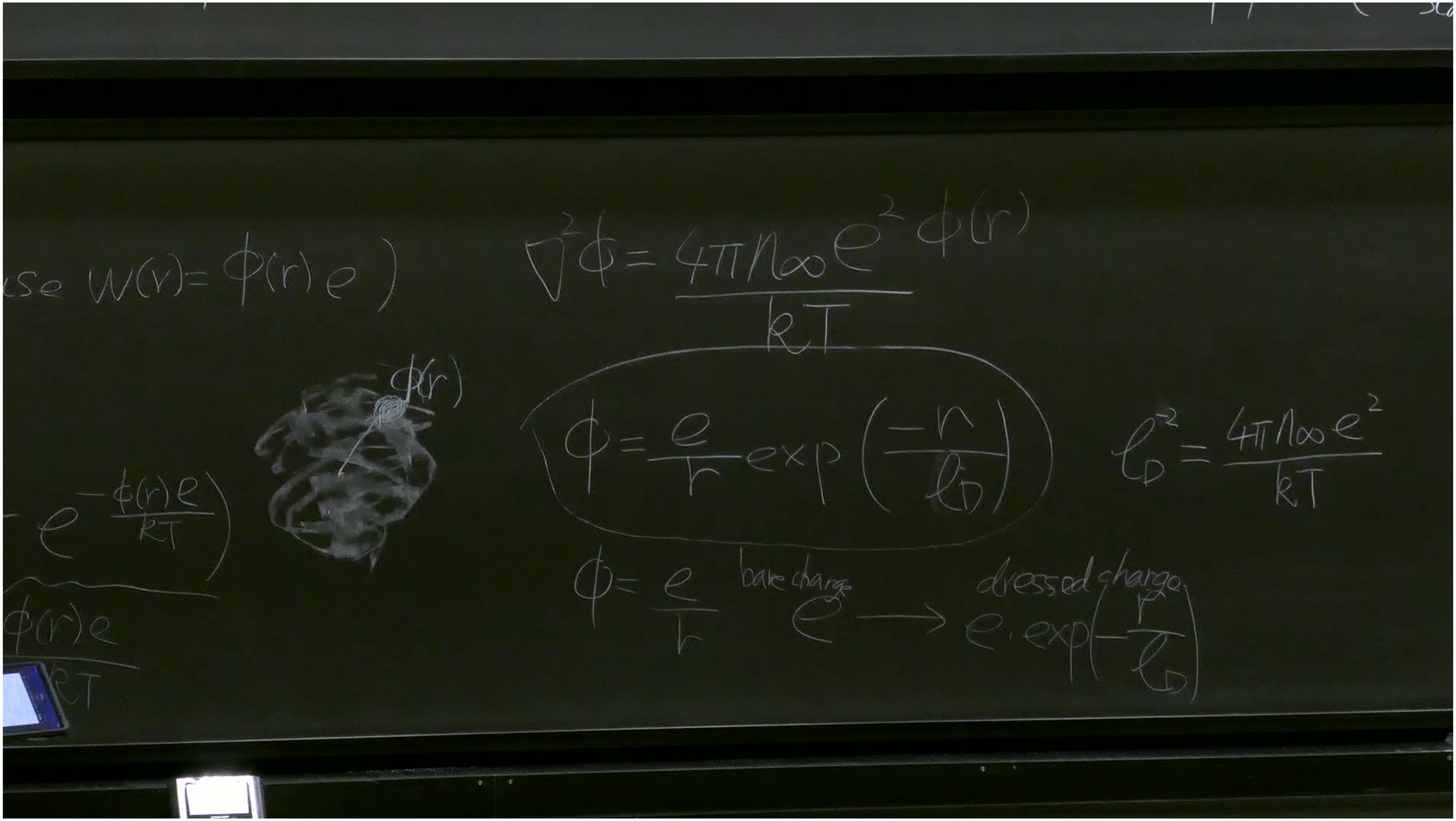
$$\rho(r) = -n_\infty e\Big(\underbrace{1 - e^{-\frac{\phi(r)e}{kT}}}_{\frac{\phi(r)e}{kT}}\Big)$$

$(r) = \phi(r) e\ )$

$$\nabla^2 \phi = \frac{4\pi n_\infty e^2}{kT} \phi(r)$$

$$\phi = \frac{e}{r} \exp\left(\frac{-r}{\ell_D}\right) \qquad \ell_D^{-2} = \frac{4\pi n_\infty e^2}{kT}$$

$\frac{r) e}{kT} \Big)$


se $W(r) = \phi(r) e$)
$\nabla^2 \phi = \frac{4\pi n_\infty e^2}{kT} \phi(r)$
$\phi(r)$
$\phi = \frac{e}{r} \exp\left(-\frac{r}{\ell_D}\right)$
$\ell_D^{-2} = \frac{4\pi n_\infty e^2}{kT}$
$e^{-\frac{\phi(r) e}{kT}}$)
$\frac{\phi(r) e}{kT}$
$\phi = \frac{e}{r}$
bare charge
dressed charge
$e \longrightarrow e \cdot \exp\left(-\frac{r}{\ell_D}\right)$

Quantum Mechanics 2-state system

$$H = H_0 + H_I$$

$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & \omega \end{pmatrix} \qquad H_I = g \begin{pmatrix} -1 & \Lambda \\ \Lambda & 0 \end{pmatrix}$$

cutoff

regularize

$$H_0 + H_I = \begin{pmatrix} -g & g\Lambda \\ g\Lambda & \omega \end{pmatrix} \qquad (E+g)(E-\omega) - (g\Lambda)^2 = 0$$

$$E_\pm = \frac{1}{2}(\omega - g \ldots$$

$$\Delta E = E_+ - E_- = \ldots$$

cs 2-state system

cutoff

$$H_I = g\begin{pmatrix}-1 & \Lambda \\ \Lambda & 0\end{pmatrix}$$

① regularize

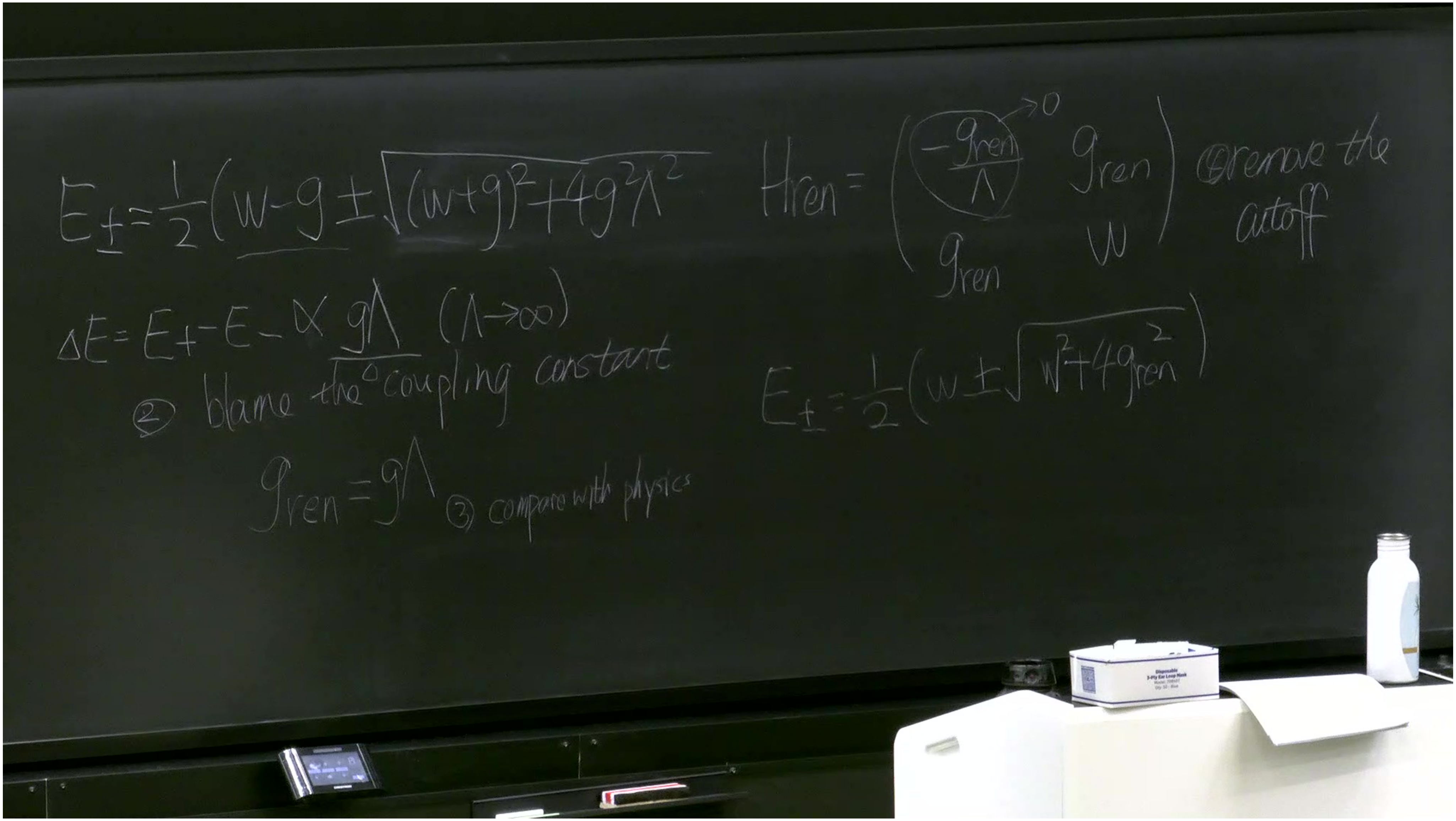
$$(E+g)(E-w)-(g\Lambda)^2=0$$

$$E_\pm = \frac{1}{2}\left(w-g \pm \sqrt{(w+g)^2+4g^2\Lambda^2}\right)$$

$$\Delta E = E_+ - E_- \propto g\Lambda \quad (\Lambda \to \infty)$$

② blame the coupling constant

$$g_{ren} = g\Lambda$$



$E_\pm = \frac{1}{2}\left(w - g \pm \sqrt{(w+g)^2 + 4g^2\Lambda^2}\right.$
$H_{ren} = \begin{pmatrix} \frac{-g_{ren}}{\Lambda} \to 0 & g_{ren} \\ g_{ren} & w \end{pmatrix}$
① remove the cutoff
$\Delta E = E_+ - E_- \propto g\Lambda \quad (\Lambda \to \infty)$
② blame the coupling constant
$E_\pm = \frac{1}{2}\left(w \pm \sqrt{w^2 + 4g_{ren}^2}\right)$
$g_{ren} = g\Lambda$
③ compare with physics

4 steps of renormalization

① regularize (with Λ $\quad d = 4-\epsilon$)

② blame the coupling constant

$$\underset{\text{bare}}{g} = g_R + \delta_2 g_R^2 + \dots$$

$\uparrow$ hide ∞ here

③ consistency check

④ remove cutoff

theory is finite