Title: Black hole ringdown nonlinearities

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Collection/Series: Training Programs (TEOSP)

Subject: Other

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How does this third order effect in ringdown change a linearised mode fit?

Single linear mode

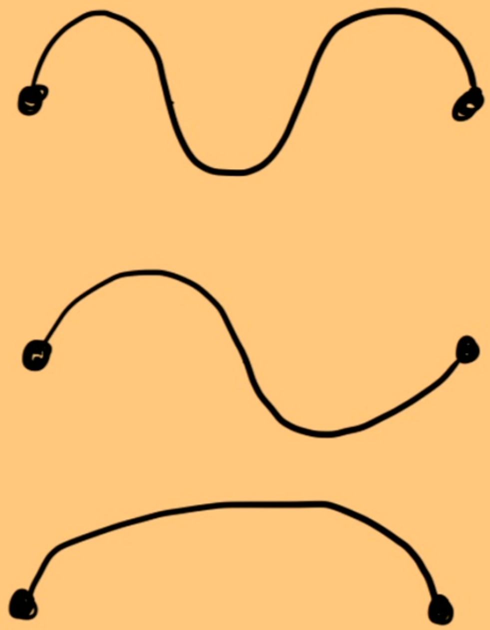
220 pure

220 + 220R + ...

Multiple linear modes

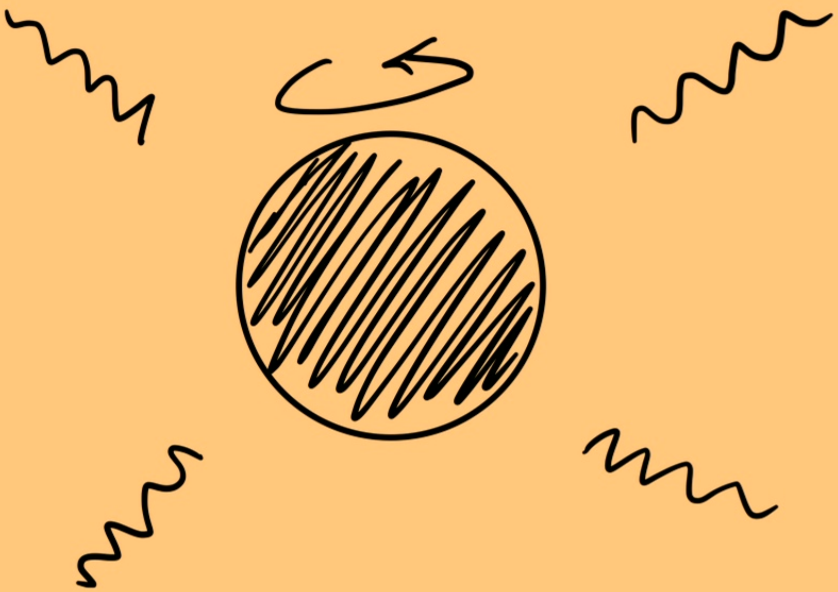
+ more?

Guitar string – Normal-Modes



ω_n

Black hole – Quasi-Normal-Modes



$$T(M, a, s)\psi^{(1)} = 0$$

$$\mathcal{A} \times e^{im\varphi - i\omega v} R(r)\, S(\vartheta)$$

$$\omega_{\ell mn}$$

[Teukolsky, 1973]

$$\omega_{\ell mn}(a, M) \in \mathbb{C}$$

Measure single complex frequency (e.g. 220)

$$\Re\left(\omega_{220}\right) + \Im\left(\omega_{220}\right) \overset{GR}{\rightarrow} M, a$$

$$\overset{GR}{\rightarrow} \omega_{\ell mn}(a, M)$$

Also measure third frequency component (e.g. real part 221)

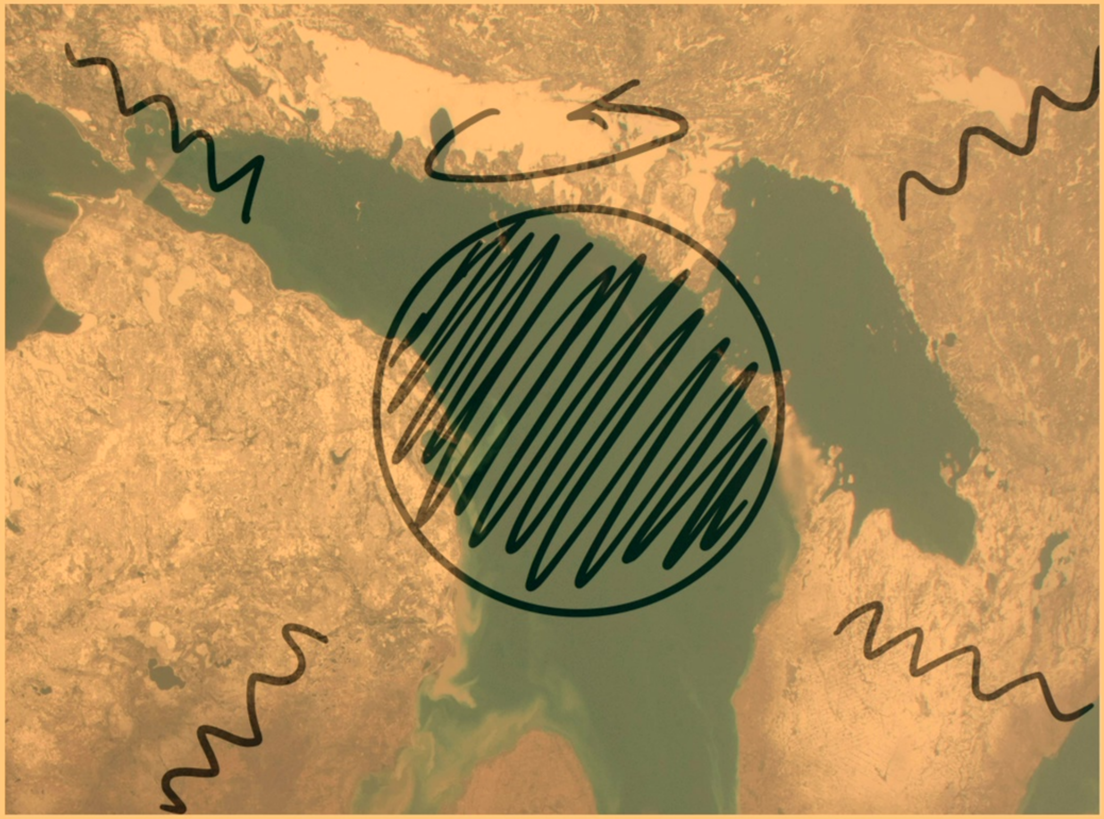
$$\Re\left(\omega_{221}\right) \overset{?}{=} \Re\left(\omega_{221}(a, M)\right)$$

[E. Berti et al. 2018, V. Cardoso & P. Pani 2019, O. Dreyer et al. 2003]

Longest lived linear perturbation

$$\tau \approx 6 \times 10^{-5} \frac{M_{\rm BH}}{M_\odot} s$$

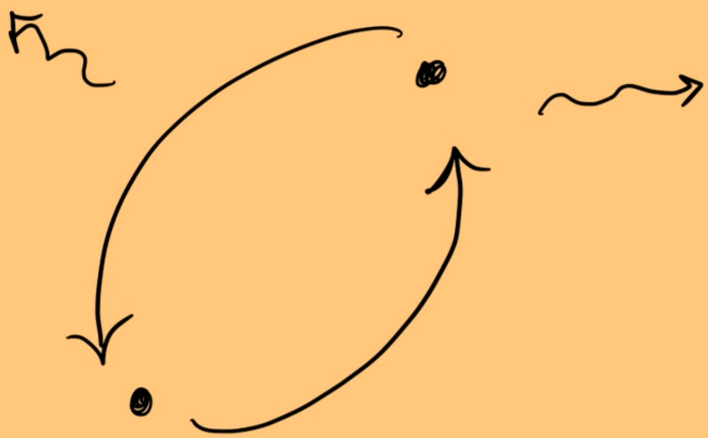
$$\approx 2\text{ms}$$



[credit to G. Carullo for analogy]

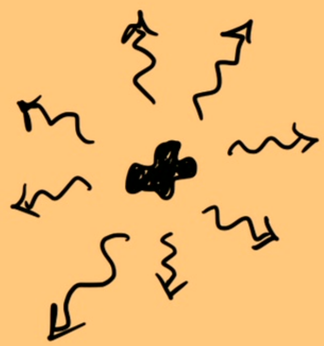
Binary black hole mergers

INSPIRAL



~ Perturbation Theory

MERGER

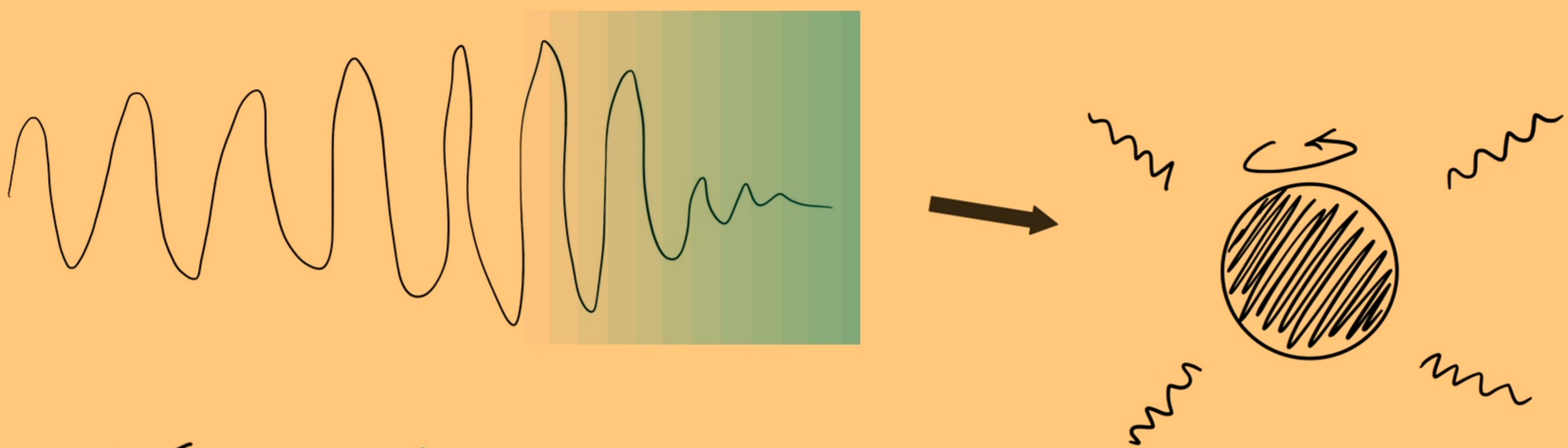


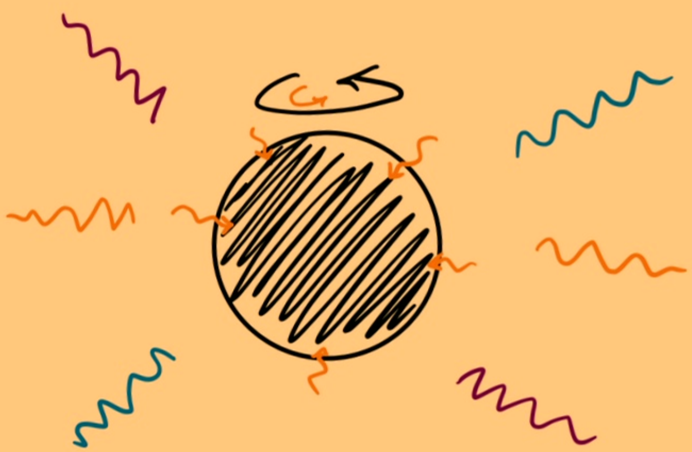
Not Perturbative

RINGDOWN

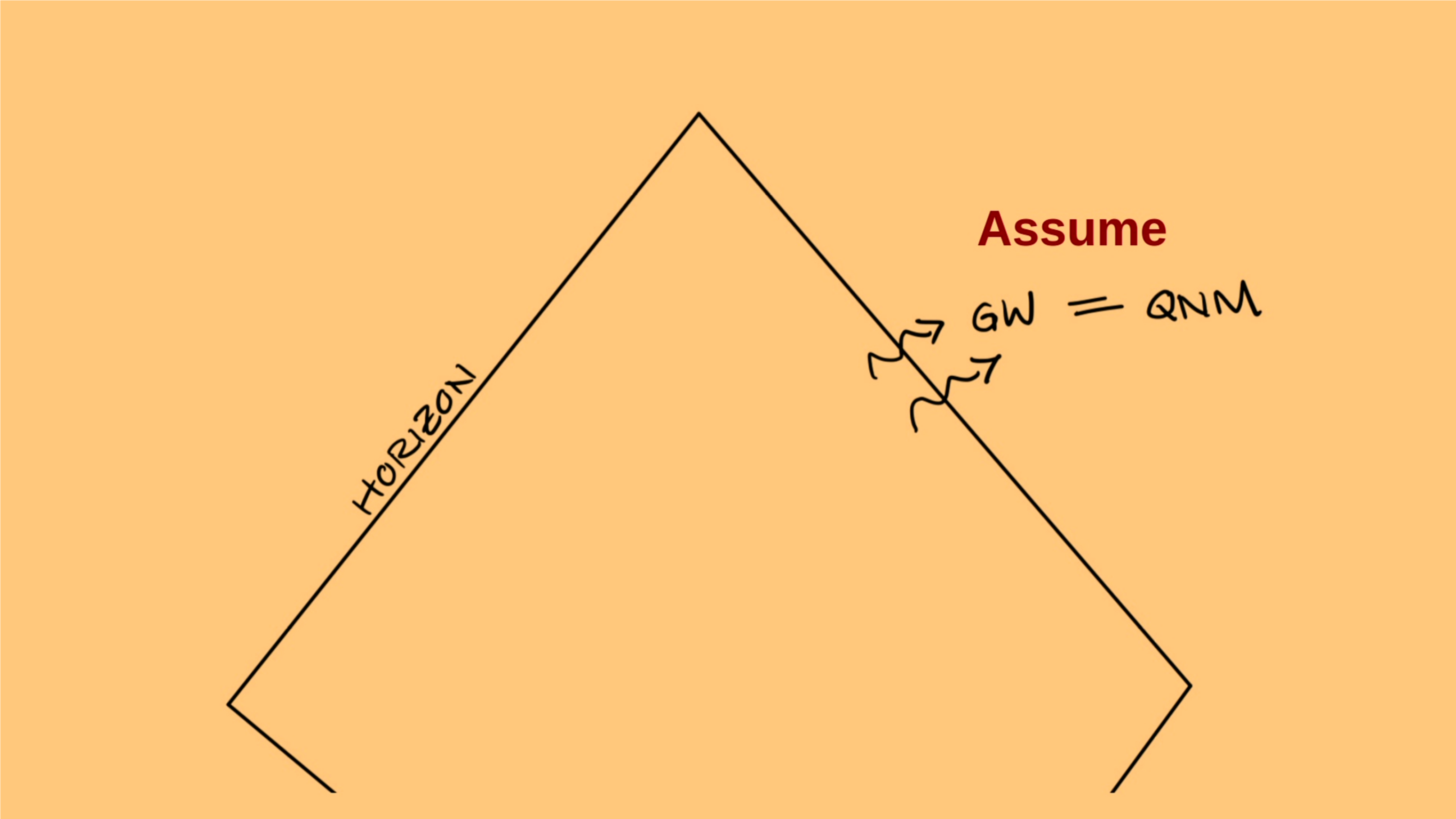
~ Perturbation Theory

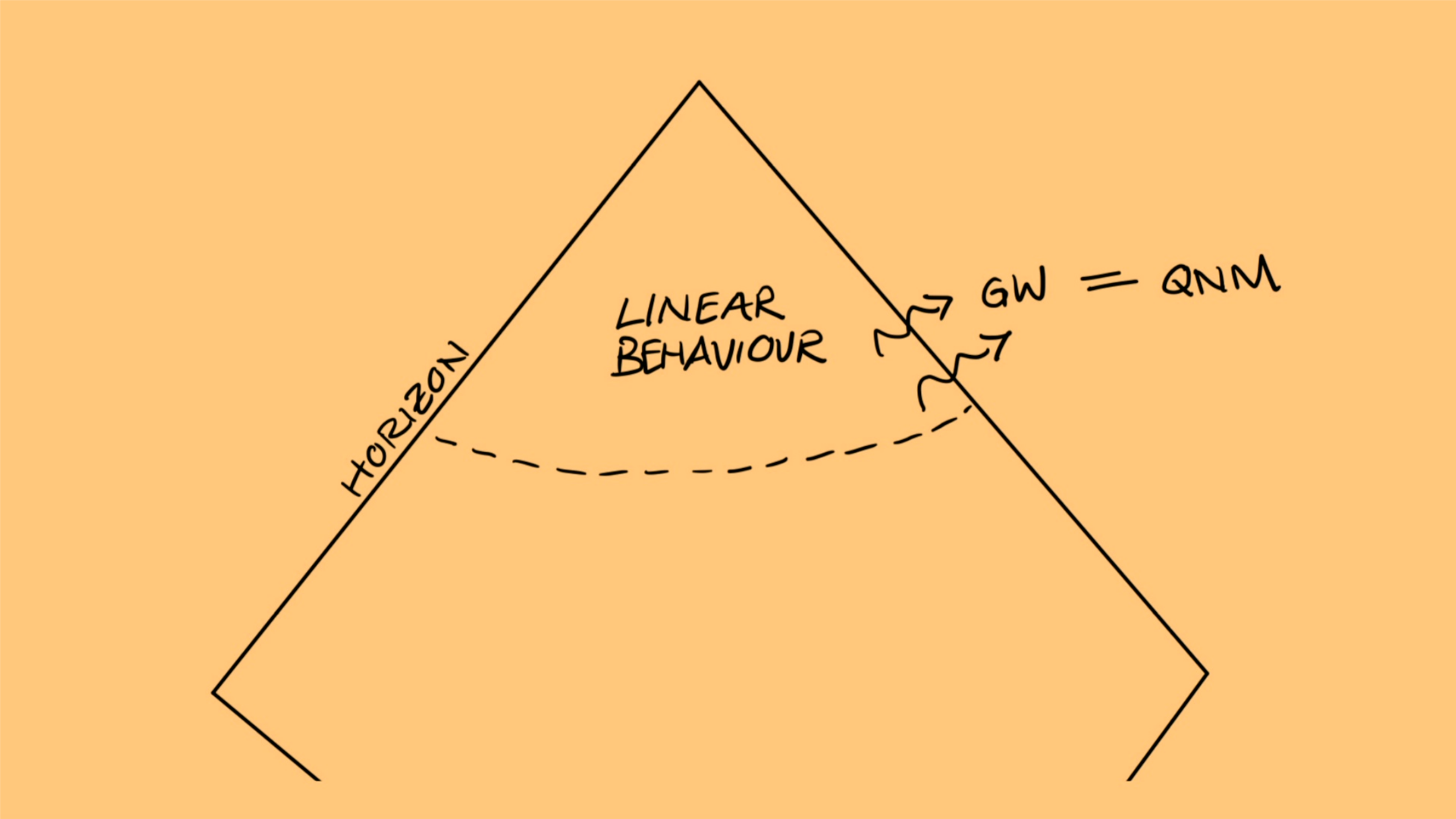
Ringdown ~ Linearised perturbations on Kerr?





Want to quantify non-linear effects


Assume
GW = QNM
HORIZON

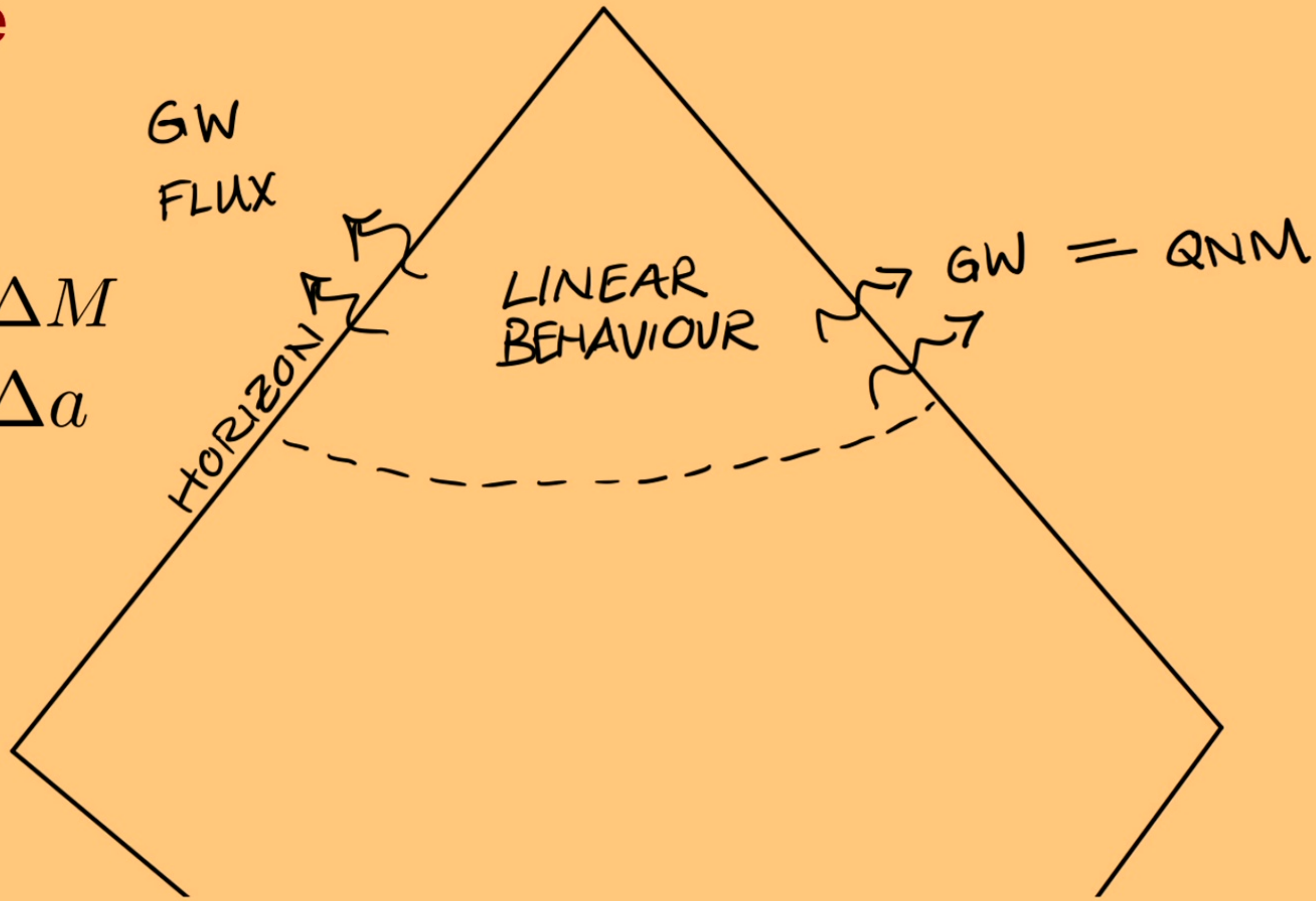

LINEAR
BEHAVIOUR
GW = QNM
HORIZON

Can calculate

$$\Delta M$$

$$\Delta a$$

$$\propto \mathcal{A}^2$$



Our set up

$(\ell mn) = (220)$

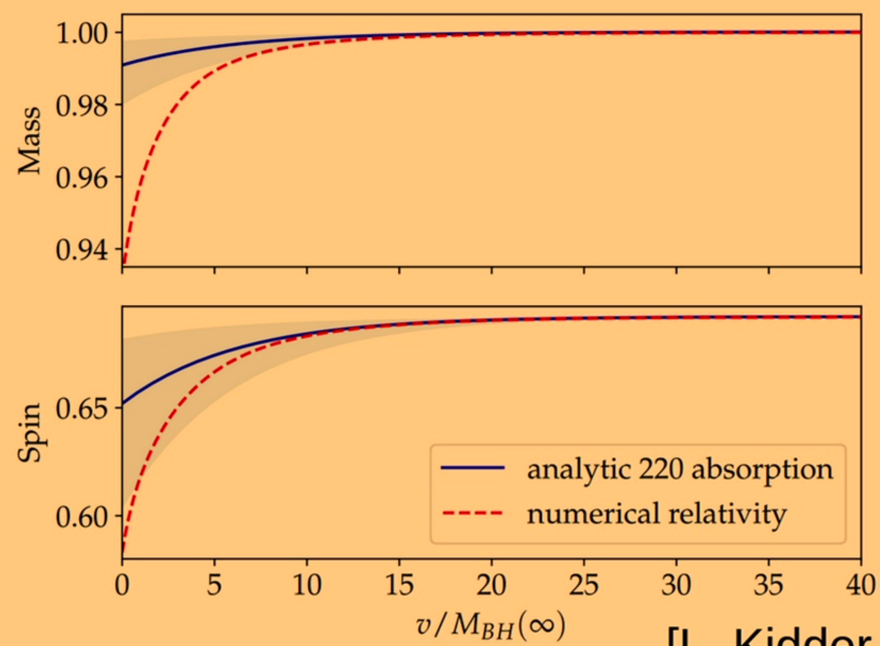
Amplitude from NR (equal mass, quasi-circular, remnant spin $a/M = 0.692$)

[Giesler et al. 2019]

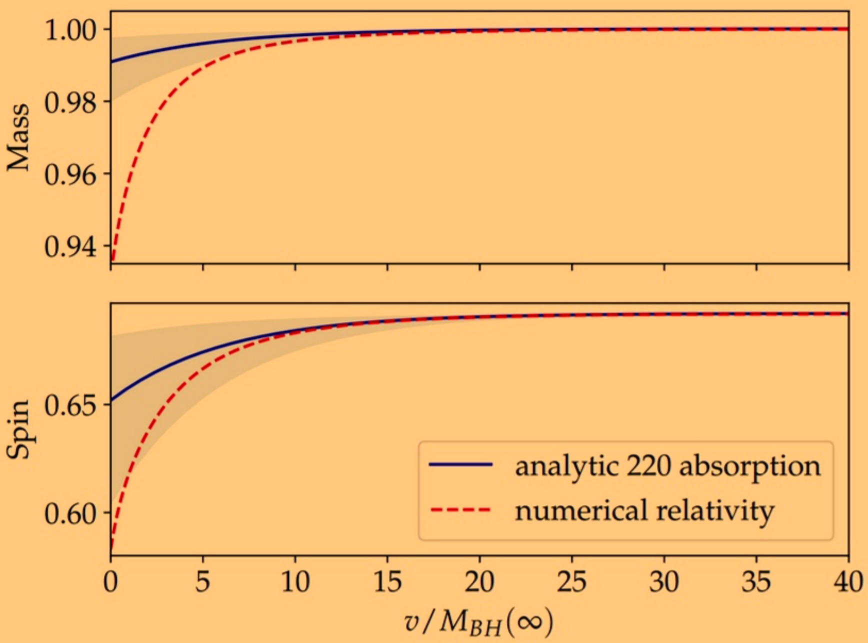
$\longrightarrow$

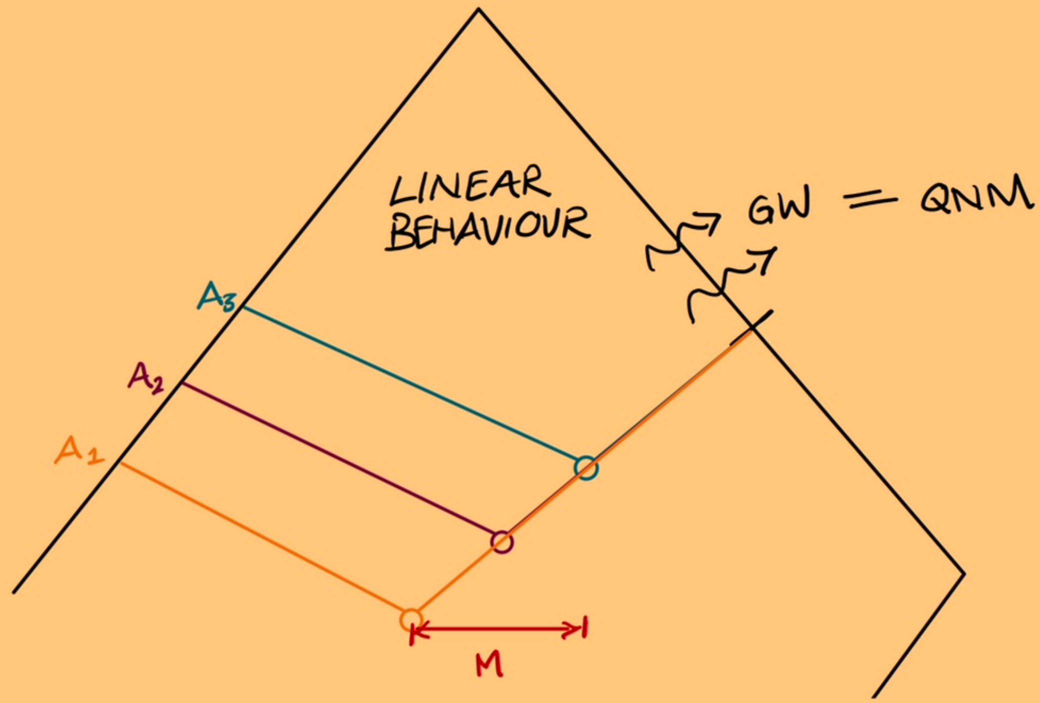
$$\Delta M/M = (0.92\%)$$

$$\Delta a/a = (6.34\%)$$

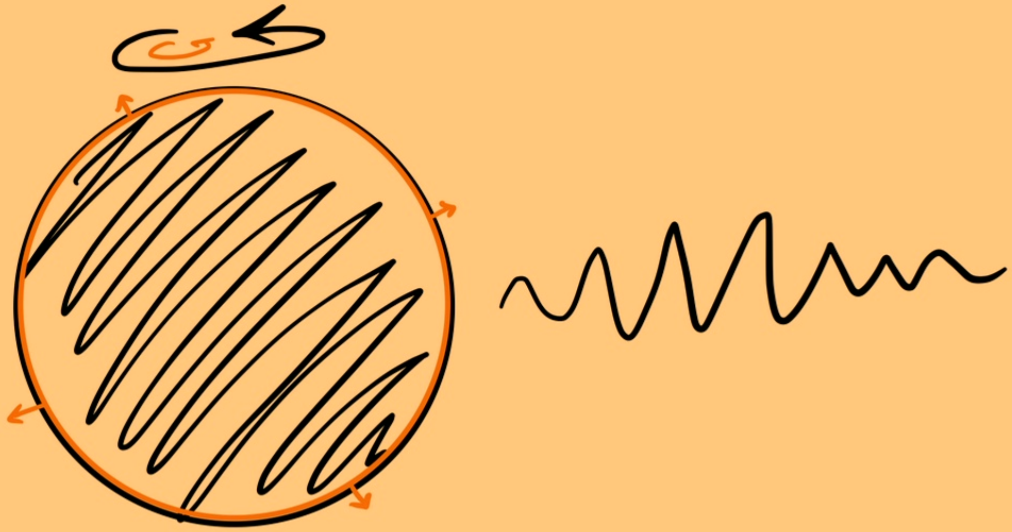


[L. Kidder et al. 2018]


1.00
0.98
0.96
0.94
Mass
0.65
0.60
Spin
analytic 220 absorption
numerical relativity
0
5
10
15
20
25
30
35
40
$v/M_{BH}(\infty)$


LINEAR
BEHAVIOUR
GW = QNM
A3
A2
A1
M

Time evolution in linear code

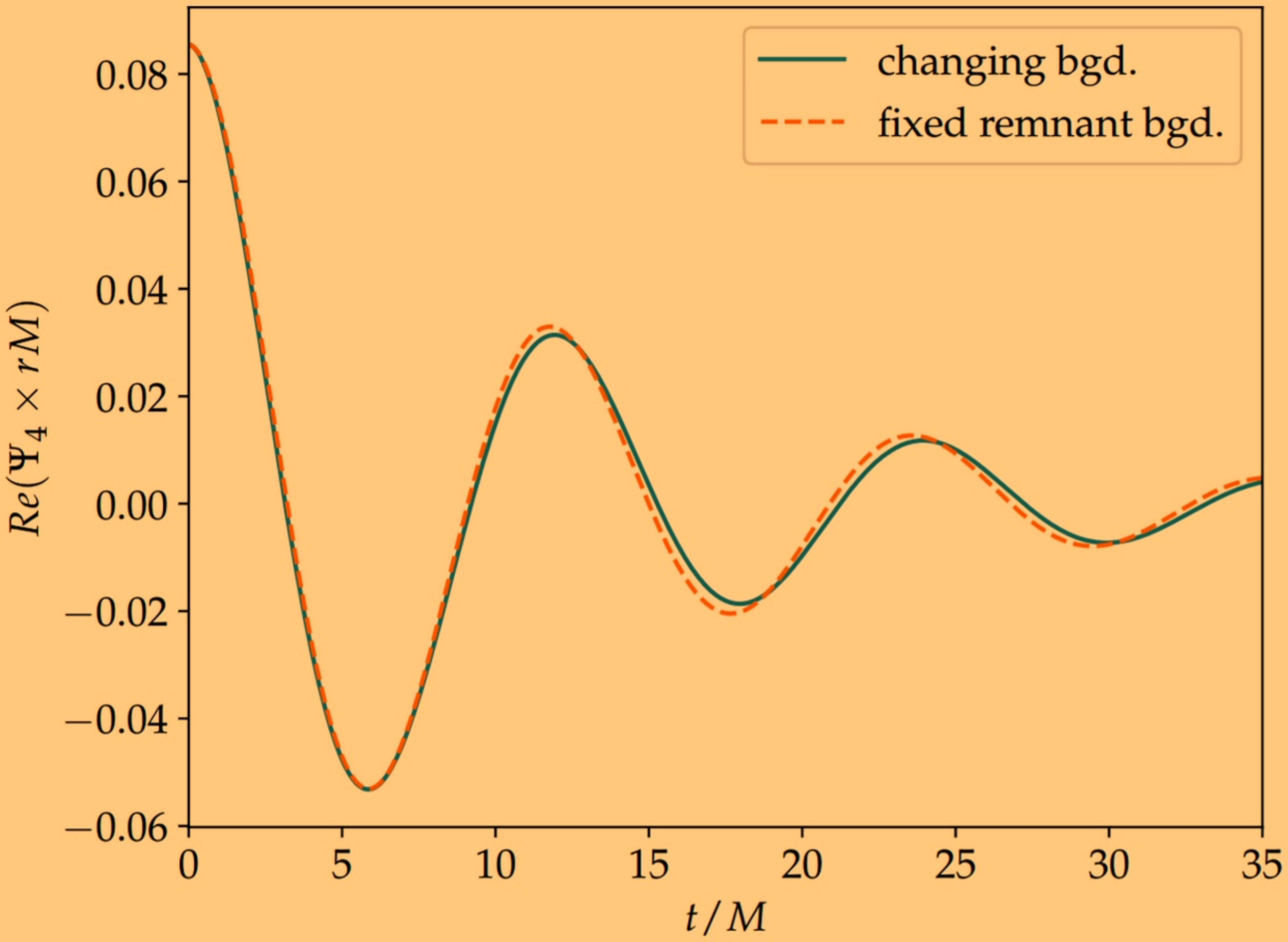


$$\omega_{220}(a/M)$$

$$\mathcal{A}_{(220)}$$

$$M_{\rm BH}(t) = M - \Delta M \exp\left[2\Im\omega_{220}t\right],$$
$$a_{\rm BH}(t) = a - \Delta a \exp\left[2\Im\omega_{220}t\right].$$

Using methods from
[H. Zhu et al., J. Ripley]

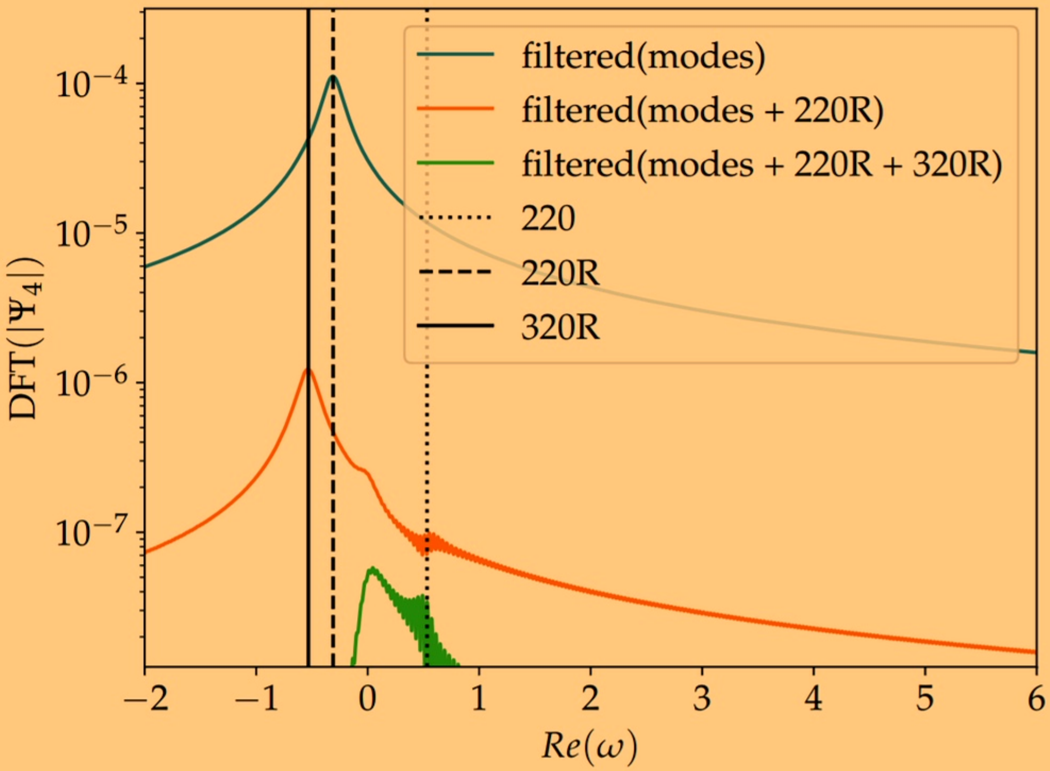


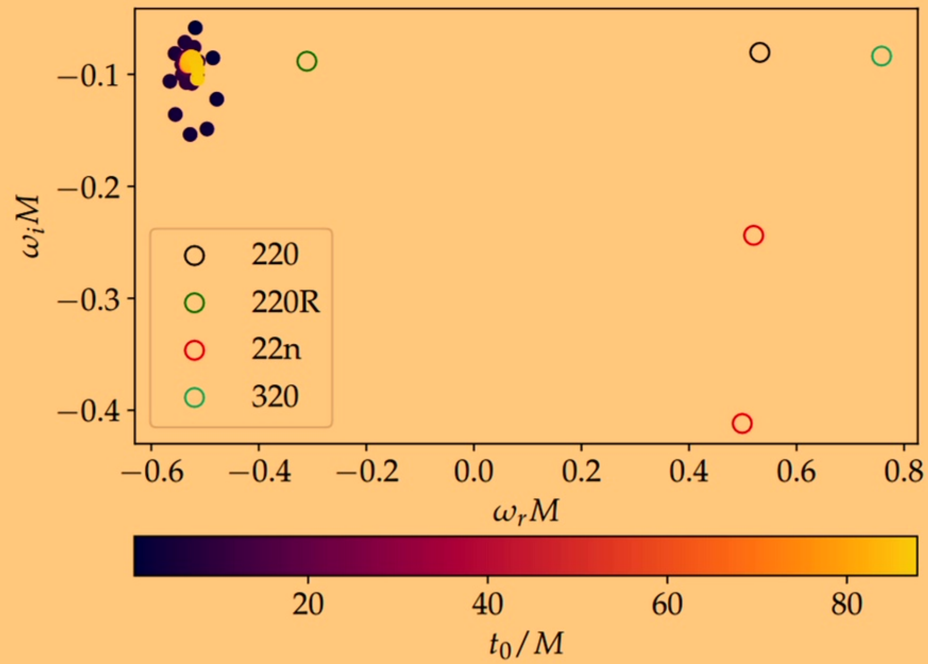
~ how to describe this difference?

~ superposition of excited modes?

~ non-linear mode?

Identifying frequencies





[S. Ma et al. 2022, S. Ma et al. 2023]

Linearised mode fit?

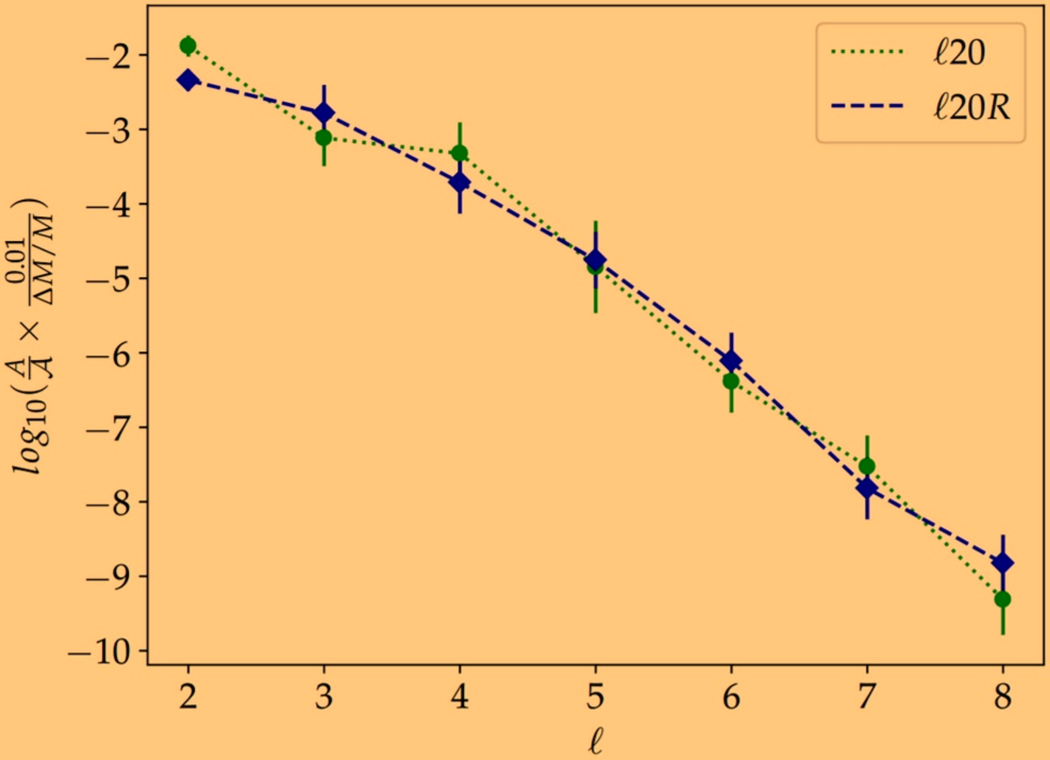
– Absorption induced mode excitation

$$\sum_{1}^{N} A_N \exp\left[-i\omega_N t\right]$$

$$\omega_{\ell m n}(a, M)$$

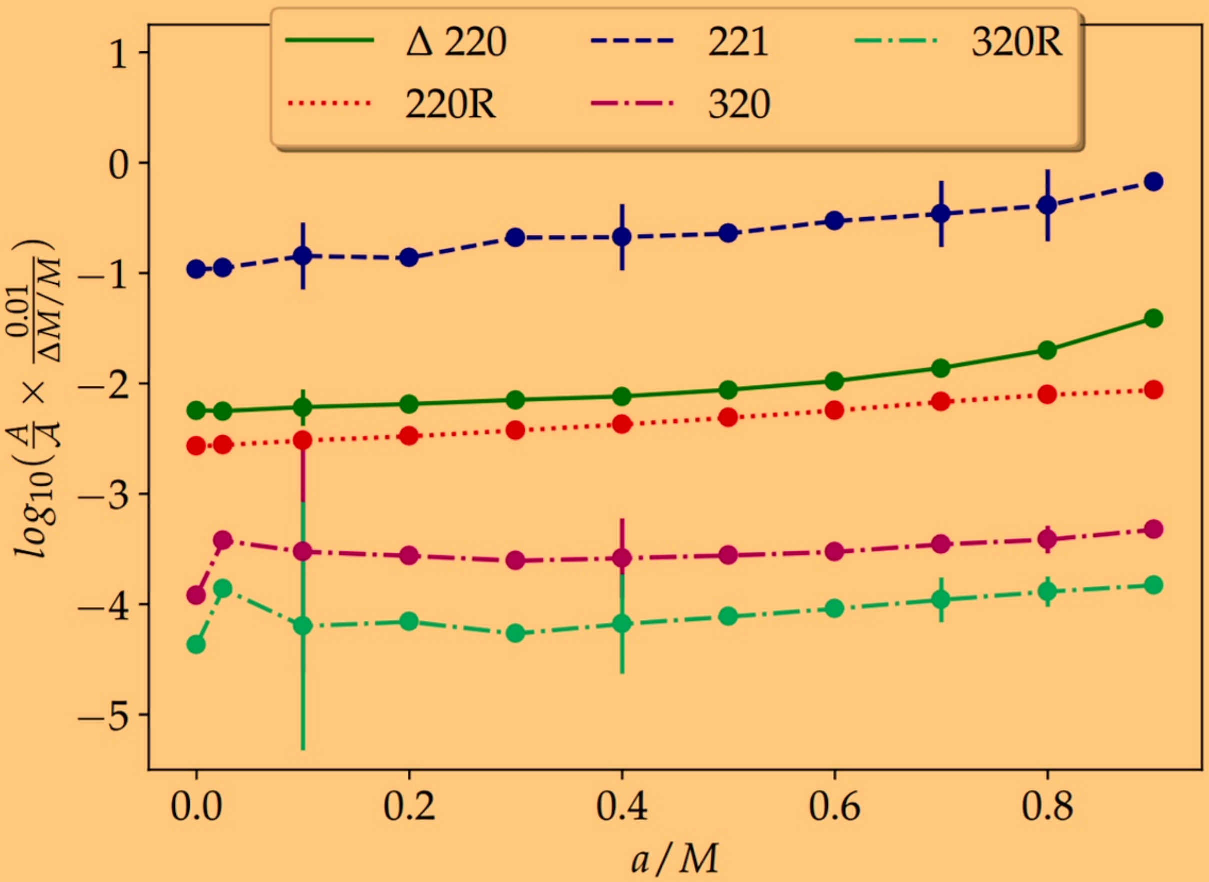
[L. Sberna et al. 2021]

Linearised mode fit?



$$\left|A_N^{t_{\mathrm{ref}}}\right| = \left|A_N^{\mathrm{fit}}(t_0)\right| \exp\left(-\Im\omega_N(t_0 - t_{\mathrm{ref}})\right)$$

$$\propto \mathcal{A}^3$$



$$\propto \mathcal{A}^3$$

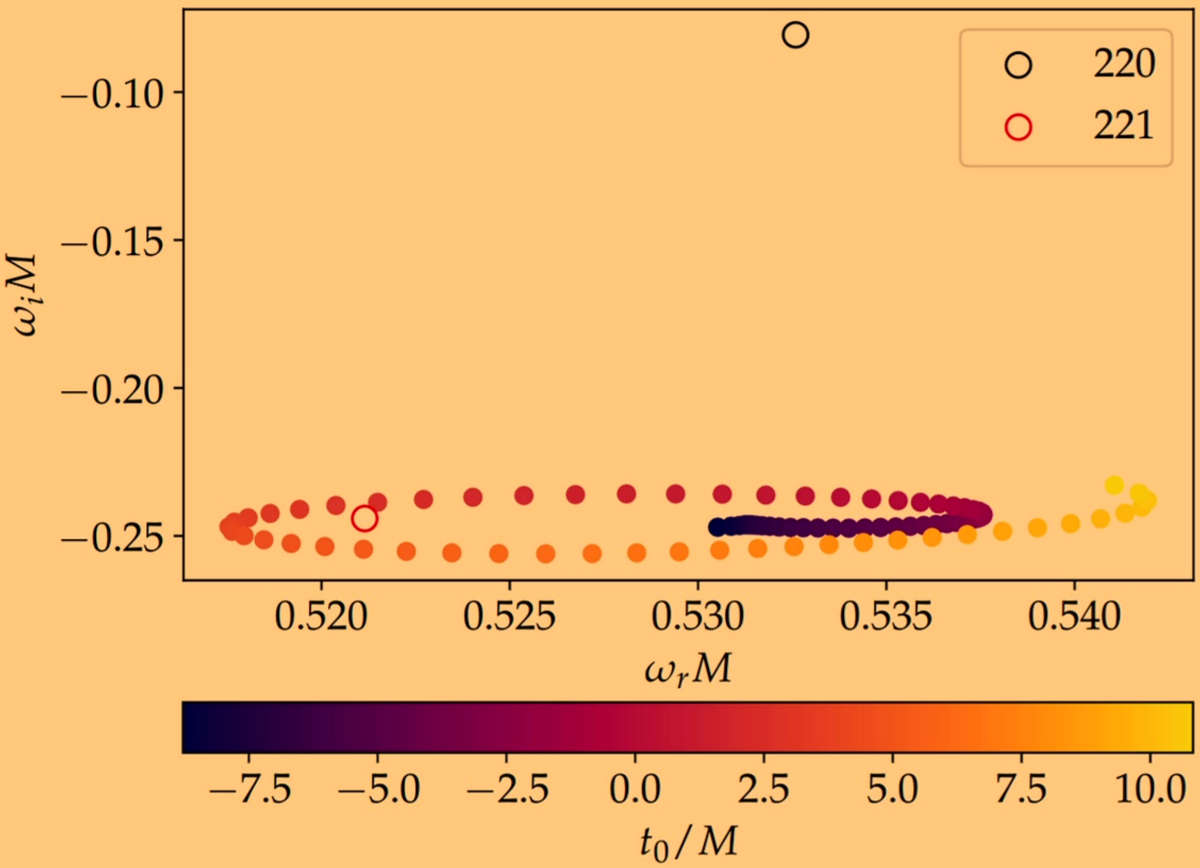
$$\left|A_N^{t_{\mathrm{ref}}}\right| = \left|A_N^{\mathrm{fit}}(t_0)\right| \exp\left(-\Im\omega_N(t_0 - t_{\mathrm{ref}})\right)$$

Non-linear modes?

$$M_{\rm BH}(t) = M - \Delta M \exp\left[2\Im\omega_{220} t\right],$$
$$a_{\rm BH}(t) = a - \Delta a \exp\left[2\Im\omega_{220} t\right].$$

$$\sum_{1}^{N} A_N \exp\left[-i\omega_N t\right]$$

Non-mode content

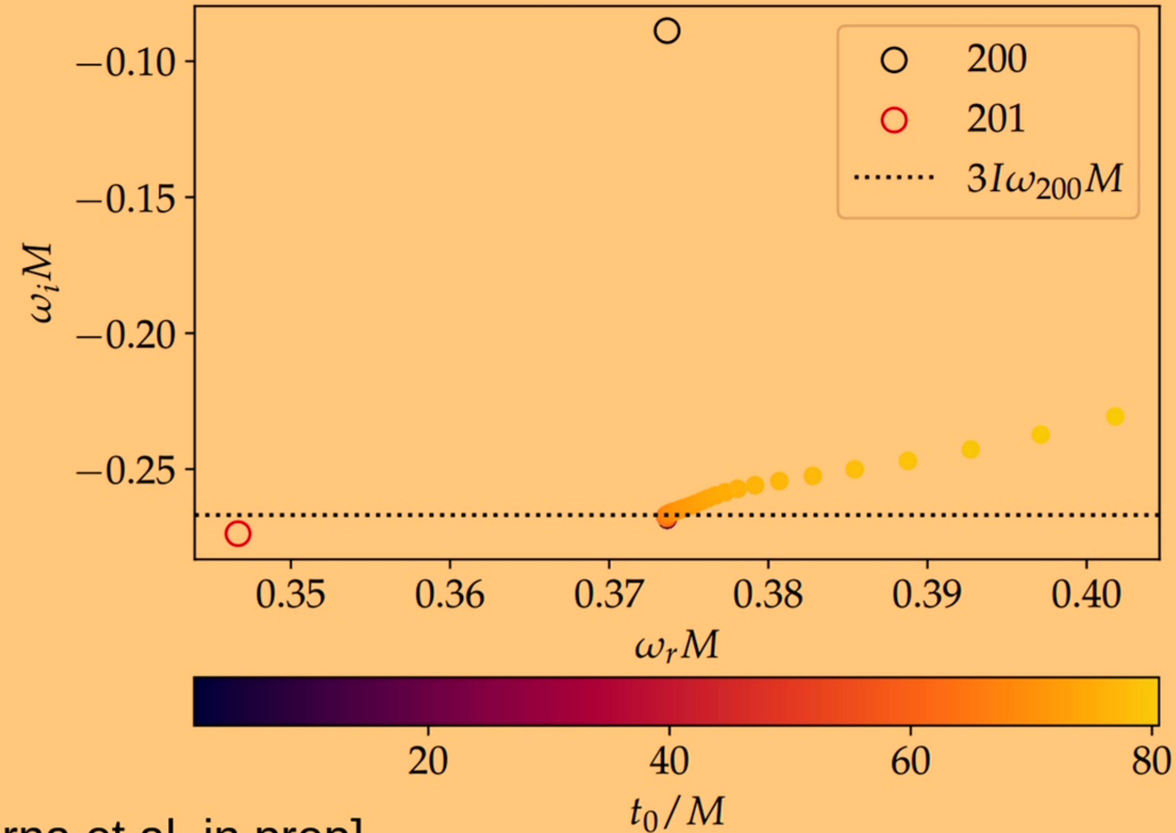


$$M_{\rm BH}(t) = M - \Delta M \exp\left[2\Im\omega_{220}t\right],$$
$$a_{\rm BH}(t) = a - \Delta a \exp\left[2\Im\omega_{220}t\right].$$

Pure changing frequency after filtering out fundamental mode

$$\phi(t) = \int_{t_0}^{t} \omega_{200}(t')dt' = \int_{t_0}^{t} \frac{m_1 \omega_{200}}{m(t')} dt'$$

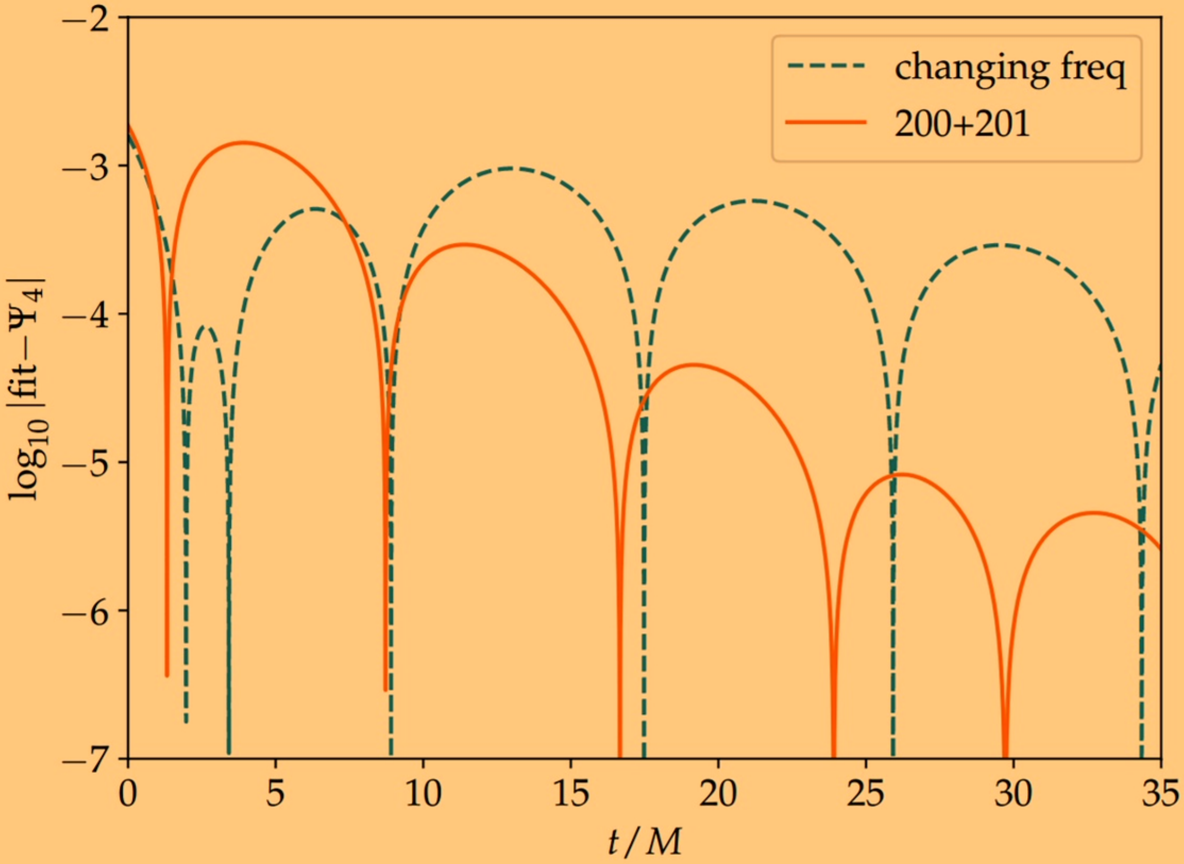
$$\Psi = \tilde{A}\left[1 + \tilde{Q}\frac{\delta m(t)}{m_2 - m_1}\right] \exp\left(i\phi(t)\right)$$



[J. Redondo Yuste et al. 2024, L. Sberna et al. in prep]

Changing frequency vs overtone fit

Residual



Conclusion

- Want to measure frequencies $\omega_{\ell mn}$ but it's unclear when we should fit these to gravitational wave data.

- A changing background can look like exciting other modes including retrogrades, higher l modes, overtones

- A component with ~ overtone decay can be generated by a changing background, and may interfere with overtone fitting. It can be described using changing frequencies.

- This third order effect can be comparable to second order mode doubling

Questions

Did this experiment approximate
the third order effect well?

What about other non-linear effects?