Title: How much entanglement is needed for quantum error correction?

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**Subject:** Other

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Abstract:

It is commonly believed that logical states of quantum error-correcting codes have to be highly entangled such that codes capable of correcting more errors require more entanglement to encode a qubit. Here we show that this belief may or may not be true depending on a particular code. To this end, we characterize a tradeoff between the code distance d quantifying the number of correctable errors, and geometric entanglement of logical states quantifying their maximal overlap with product states or more general "topologically trivial" states. The maximum overlap is shown to be exponentially small in d for three families of codes: (1) low-density parity check (LDPC) codes with commuting check operators, (2) stabilizer codes, and (3) codes with a constant encoding rate. Equivalently, the geometric entanglement of any logical state of these codes grows at least linearly with d. On the opposite side, we also show that this distance-entanglement tradeoff does not hold in general. For any constant d and k (number of logical qubits), we show there exists a family of codes such that the geometric entanglement of some logical states approaches zero in the limit of large code length. This work was done by the collaboration with Sergey Bravyi, Zhi Li, and Beni Yoshida. (https://arxiv.org/abs/2405.01332)

How much entanglement is needed
for quantum error correction?

(with Sergey Bravyi, Zhi Li, and Beni Yoshida)

① Motivation

QEC

Entanglement Theory

$\exists$ Trade-off relation?

d
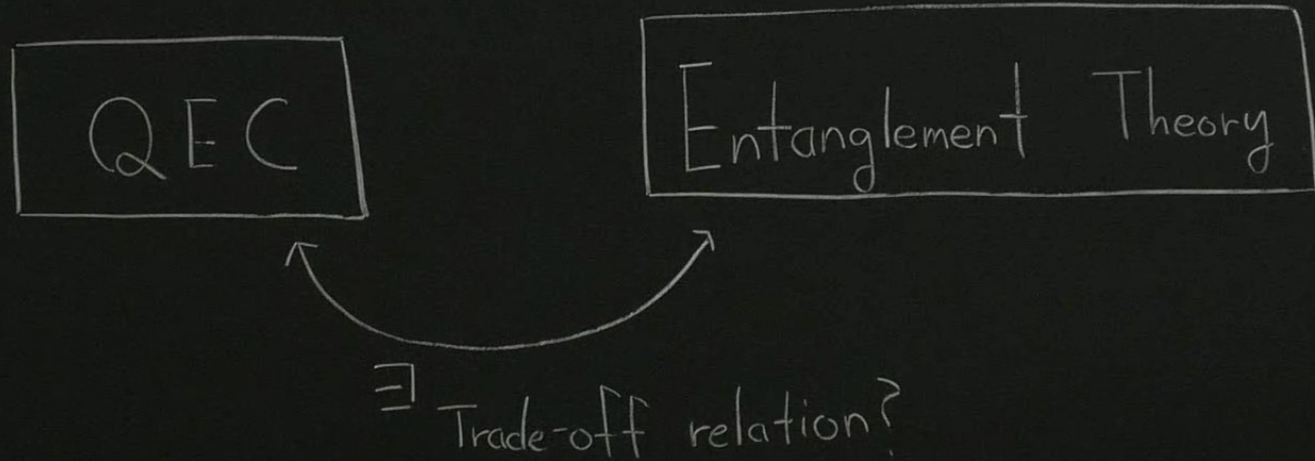
correction?

nd Beni Yoshida)

③

| Entanglement Theory |

→

relation?

② QEC.

$[[n, k, d]]$

$C = (\mathbb{C}^2)^{\otimes k} \subseteq (\mathbb{C}^2)^{\otimes n}$

$\forall |\psi_1\rangle, |\psi_2\rangle \in C, \quad |A| < d$

$\rho_A = \text{Tr}_{A^c}(|\psi_1\rangle\langle\psi_1|) = \text{Tr}_{A^c}(|\psi_2\rangle\langle\psi_2|)$

trade-off between d & $E(|\psi\rangle)$

① Stabilizer code

$$P_n = \{I, X, Y, Z\}^{\otimes n}$$

$S \subset P_n$, Abelian.

$$C_S = \{|\psi\rangle \mid s|\psi\rangle = (+1)|\psi\rangle, \; s \in S\}$$

Ex) $S = \{Z_1 Z_2, Z_2 Z_3, \dots Z_{n-1} Z_n\}$

$\Rightarrow C_S = \text{span} \{|0\rangle^{\otimes n}, |1\rangle^{\otimes n}\}$

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② LDPC code

$$L = \{\Pi_a\}, \quad \begin{array}{l} \text{supp}(\Pi_a) < S \\ n_L(q_i) < S \end{array} \quad (S < \infty)$$

$$C_L = \{|\psi\rangle \mid \Pi_a|\psi\rangle = (+1)|\psi\rangle, \quad \forall \Pi_a \in L\}$$

$$[[n, k = O(n), d = O(n)]]$$

② LDPC code

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$n_L(q_i) < S$

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$[[n, k = O(n), d = O(n)]]$

$|\psi\rangle,$

$, 2n\}$

③ Geometric Entanglement.

$E_0(|\psi\rangle) = -\max_{|prod\rangle} \log|\langle\psi|prod\rangle|^2$

$E_h(|\psi\rangle) = -\max_{\text{depth}(U)=h} \log|\langle\psi|U|prod\rangle|^2$

$E_h^c = -\max_{\substack{U\ \text{clifford,}\\ \text{depth}(U)=h}} \log|\langle\psi|U^c|prod\rangle|^2$

[4] Trade-off Results

Thm 1 (Stabilizer)

$$E_0(|\psi\rangle) \geq d-1$$
$$E_h^c(|\psi\rangle) \geq \frac{d}{2^h} - 1$$

Thm 2 (LDPC)

if $d > 5^4 \, 2^{5h}$, $\quad E_h(|\psi\rangle) > \alpha d \quad (\alpha > 0)$

$$E_h < n$$

$$E_h(|\psi\rangle) > \alpha d^2 \quad (\alpha > 0)$$

$|\psi\rangle$ has anyons