

Title: METRICS and its use to probe fundamental physics with black-hole ringdown phase

Speakers: Adrian Chung

Collection/Series: Strong Gravity

Subject: Strong Gravity

Date: October 31, 2024 - 1:00 PM

URL: <https://pirsa.org/24100135>

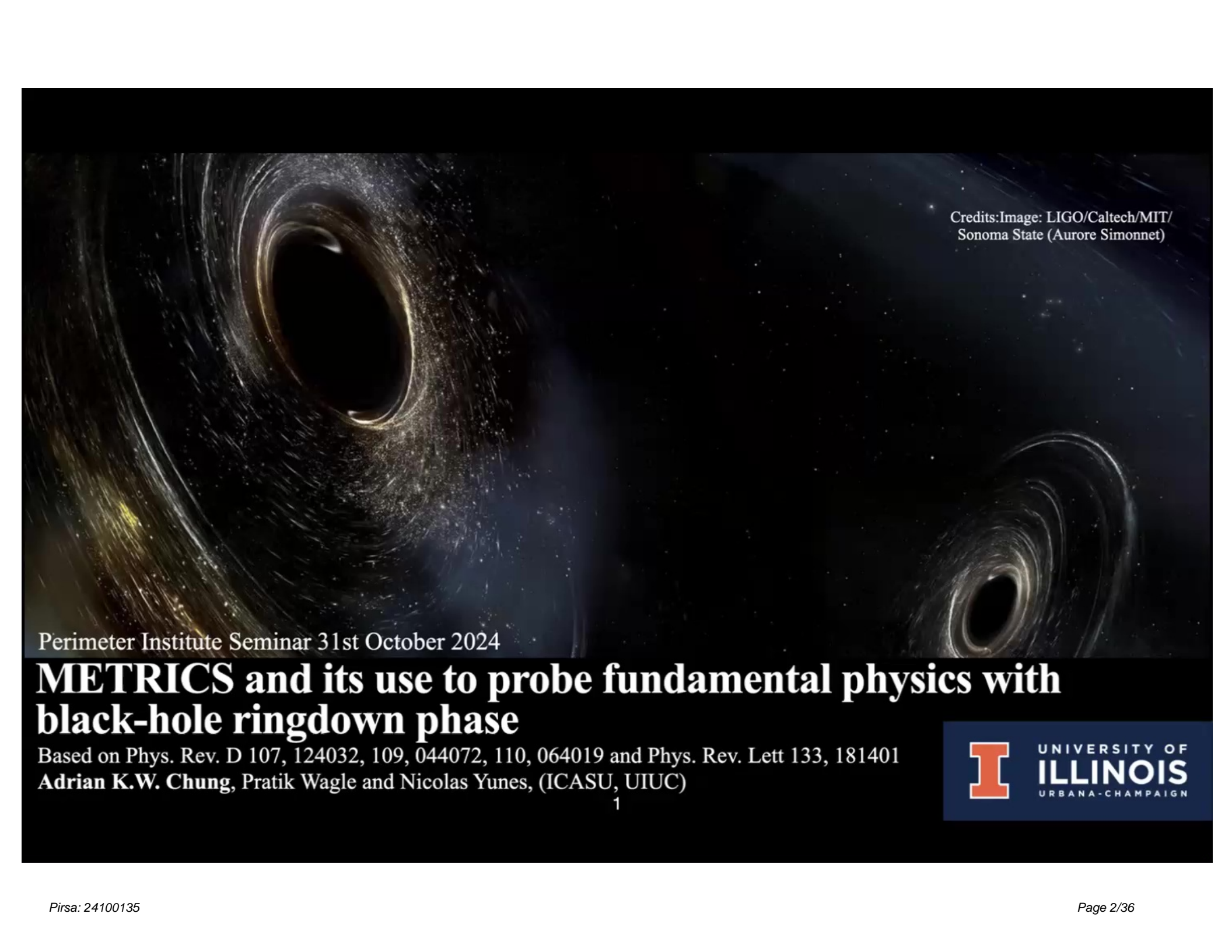
Abstract:

Quasinormal modes of a black hole are closely related to the dynamics of the spacetime near the horizon. In this connection, the black hole ringdown phase is a powerful probe into the nature of gravity. However, the challenge of computing quasinormal mode frequencies has meant that ringdown tests of gravity have largely remained model-independent. In this talk, I will introduce Metric pErTuRbations with speCtral methodS (METRICS) [1], a novel spectral scheme capable of accurately computing the quasinormal mode frequencies of black holes, including those with modifications beyond Einstein's theory or the presence of matter. I will demonstrate METRICS' accuracy in calculating quasinormal mode frequencies within general relativity, as a validation, and its application to Einstein-scalar-Gauss-Bonnet gravity [2, 3], an example of modified gravity theory to which METRICS has been applied. I will also present preliminary results from applying METRICS to dynamical Chern-Simons gravity. Finally, I will discuss potential future applications of METRICS beyond computing black hole quasinormal modes.

[1]: <https://arxiv.org/abs/2312.08435>

[2]: <https://arxiv.org/abs/2405.12280>

[3]: <https://arxiv.org/abs/2406.11986>



Credits: Image: LIGO/Caltech/MIT/
Sonoma State (Aurore Simonnet)

Perimeter Institute Seminar 31st October 2024

METRICS and its use to probe fundamental physics with black-hole ringdown phase

Based on Phys. Rev. D 107, 124032, 109, 044072, 110, 064019 and Phys. Rev. Lett 133, 181401
Adrian K.W. Chung, Pratik Wagle and Nicolas Yunes, (ICASU, UIUC)

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Metric pErTuRbations wIth speCtral methods

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Content of the talk

- What is METRICS? Why?
- METRICS in general relativity
- METRICS in modified gravity
- Future directions

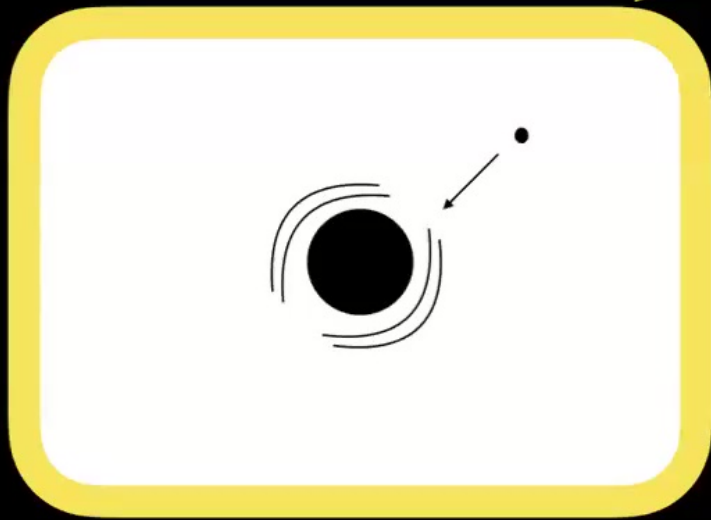
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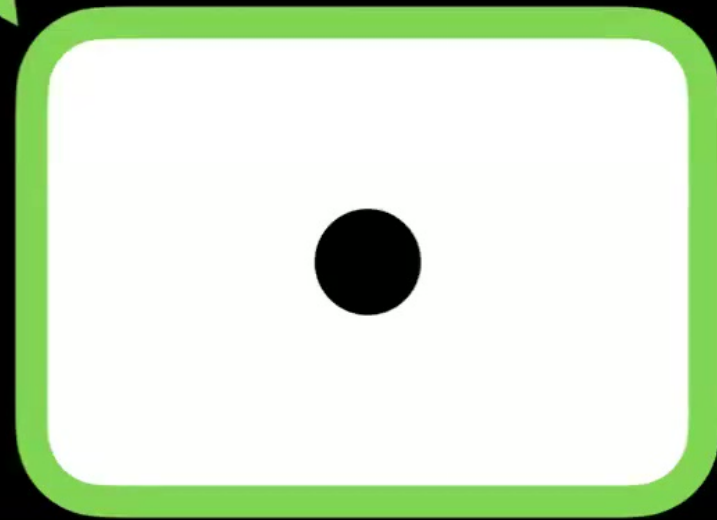


Black-hole metric perturbations

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$



Perturbed black hole



Unperturbed state

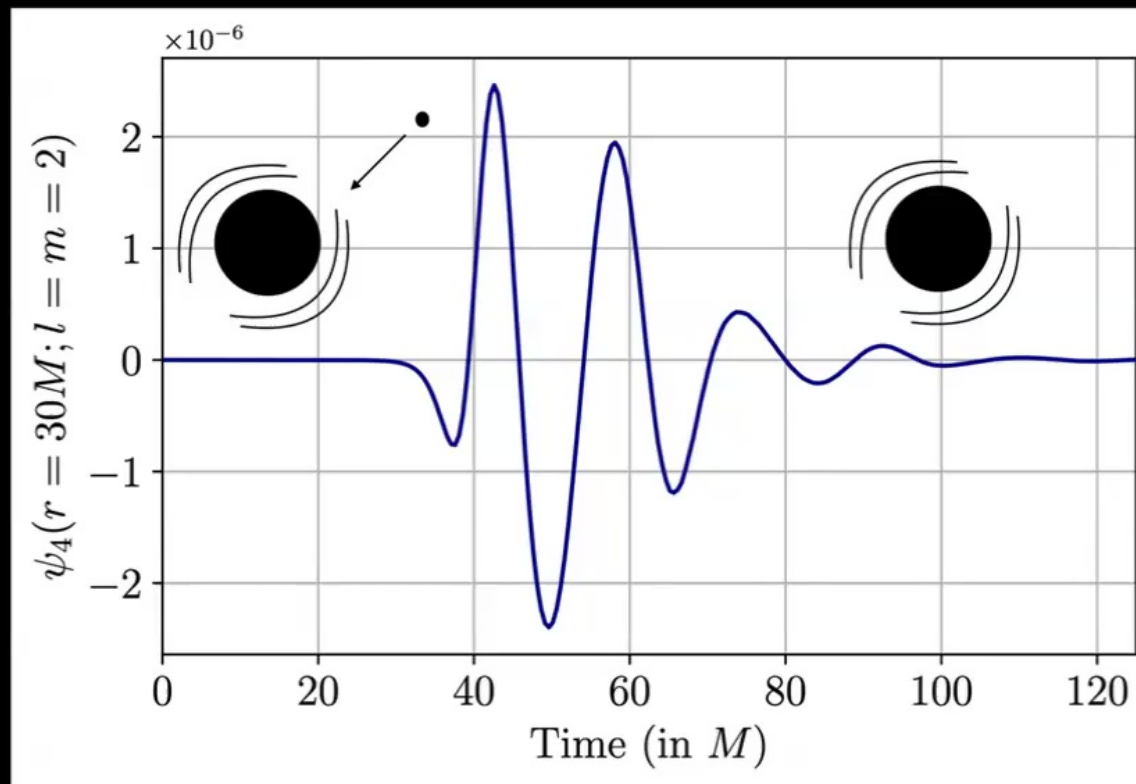
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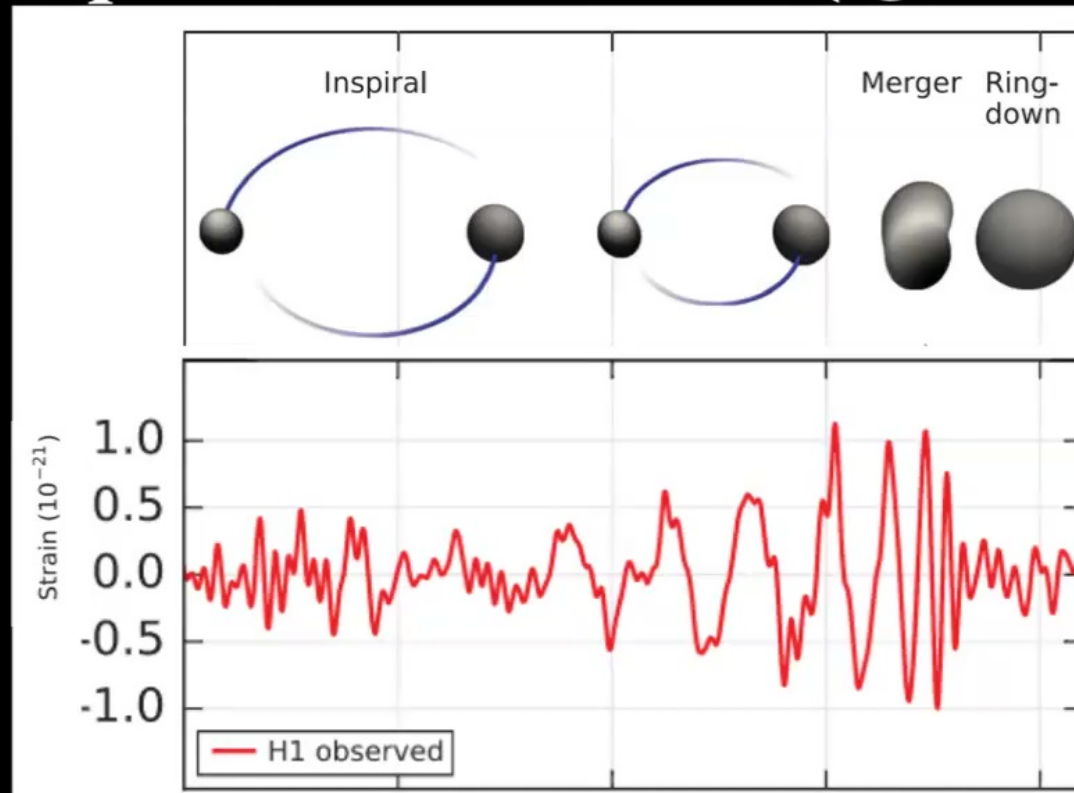
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Black-hole perturbations

Perturbations of a BH by the radial infall of a test mass. Mass ratio 20:1. Simulated using NRPy+

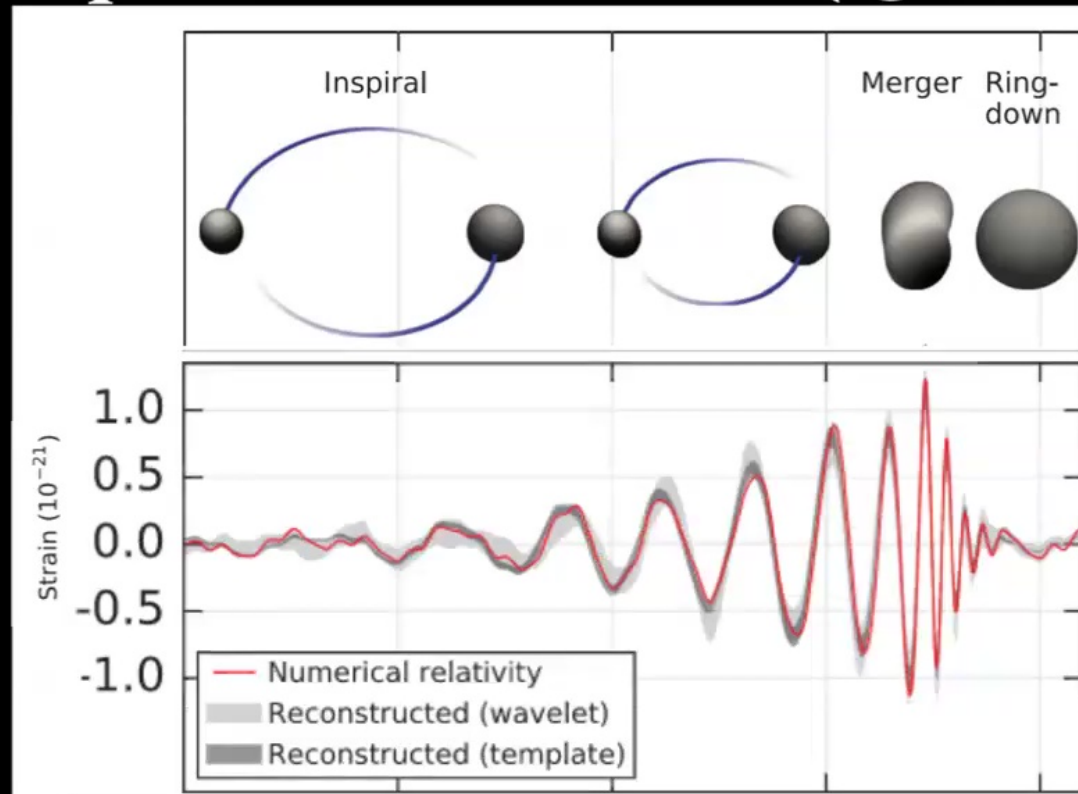


Black-hole quasinormal modes (QNMs)



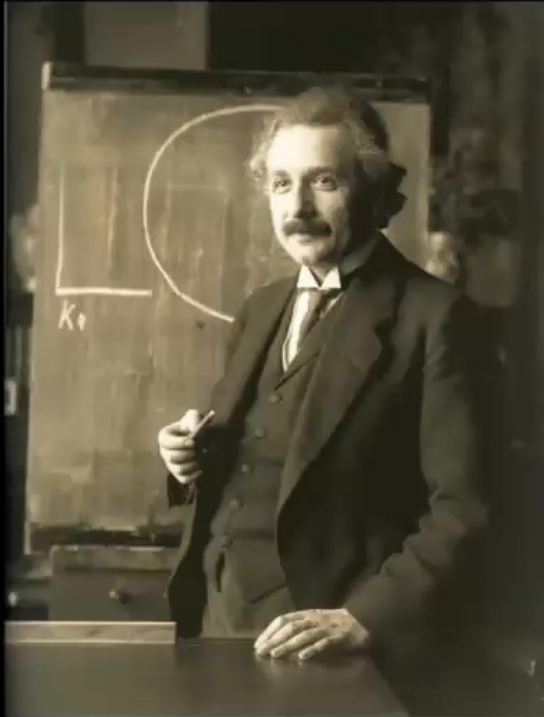
Modified from: B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. 116, 061102

Black-hole quasinormal modes (QNMs)

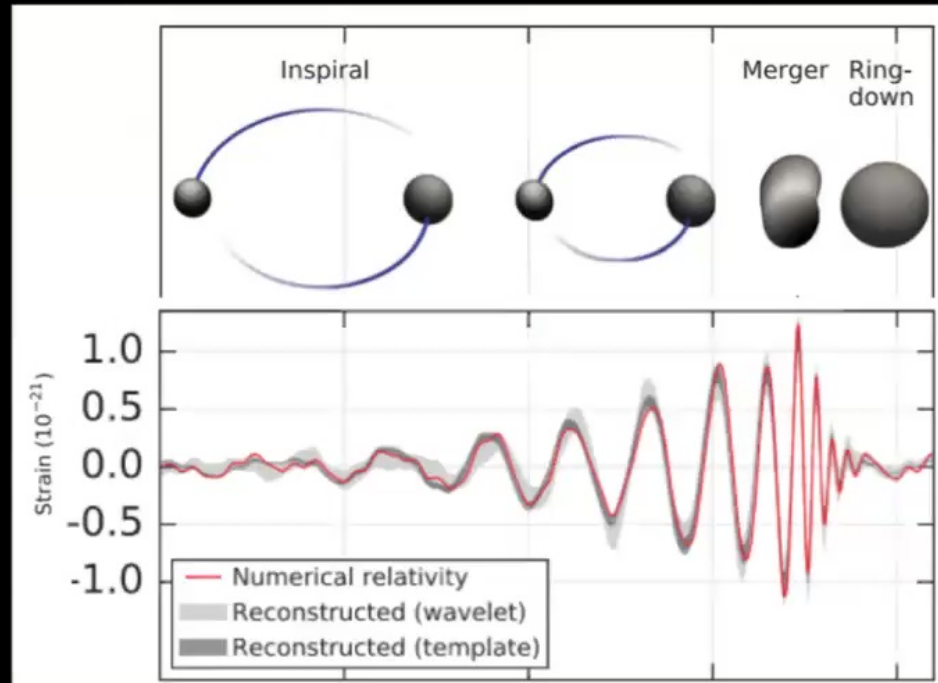


Modified from: B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. 116, 061102

Why should we care about black-hole QNMs?

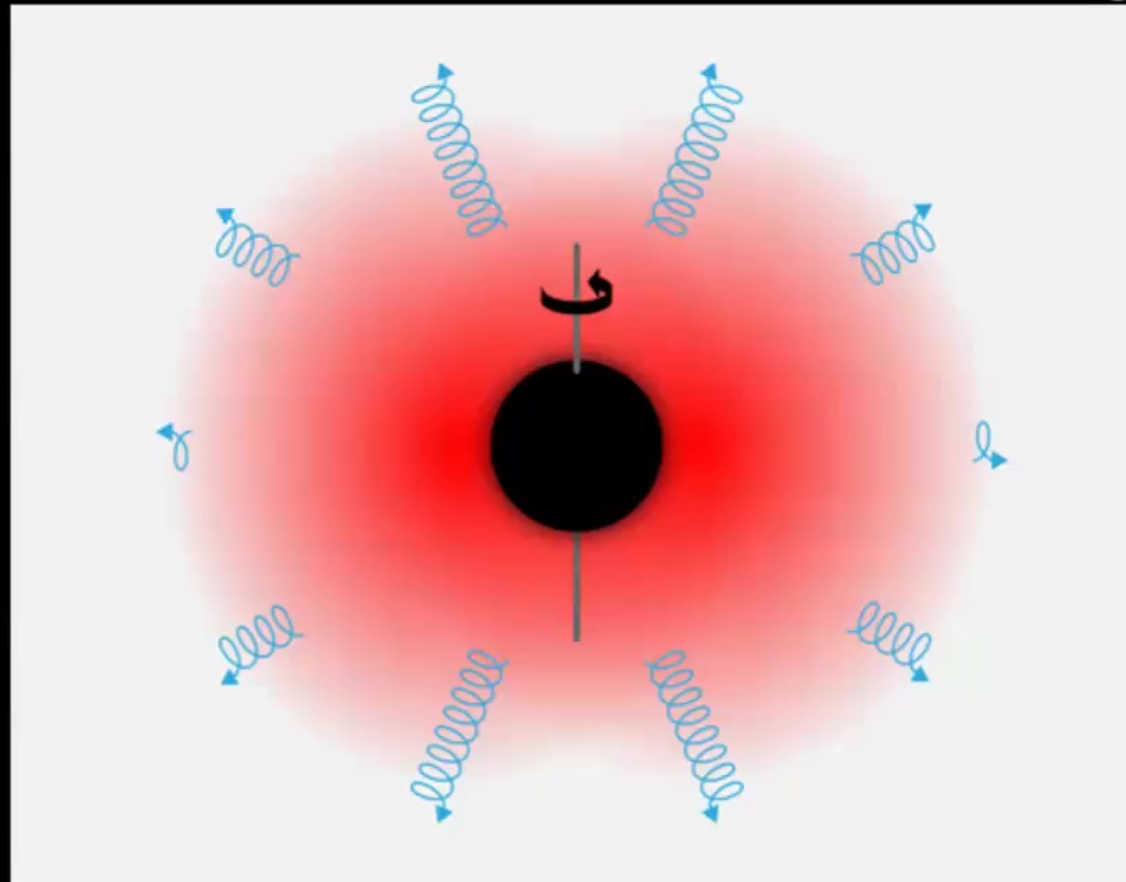


Picture credit: https://upload.wikimedia.org/wikipedia/commons/3/3e/Einstein_1921_by_F_Schmutzer_-_restoration.jpg



See, e.g., Chung et. al, Phys. Rev. D 99, 124023 (2019)

Why should we care about black-hole QNMs?

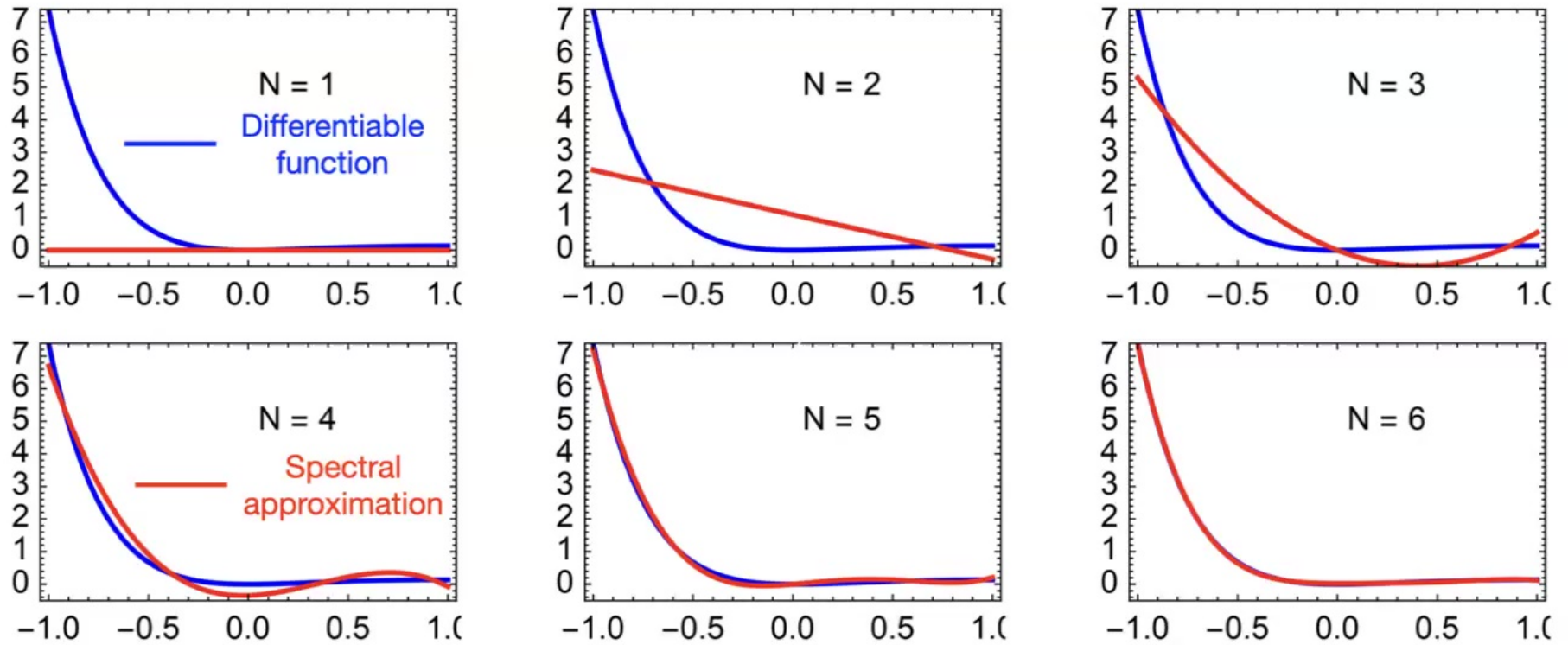


See, e.g., Chung et. al,
Phys. Rev. D 104,
084028 (2021)

Picture credit: <https://physics.aps.org/articles/v10/83>

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Spectral functions



Modified from Pedro G. S. Fernandes, David J. Mulryne, arXiv: 2212.07293



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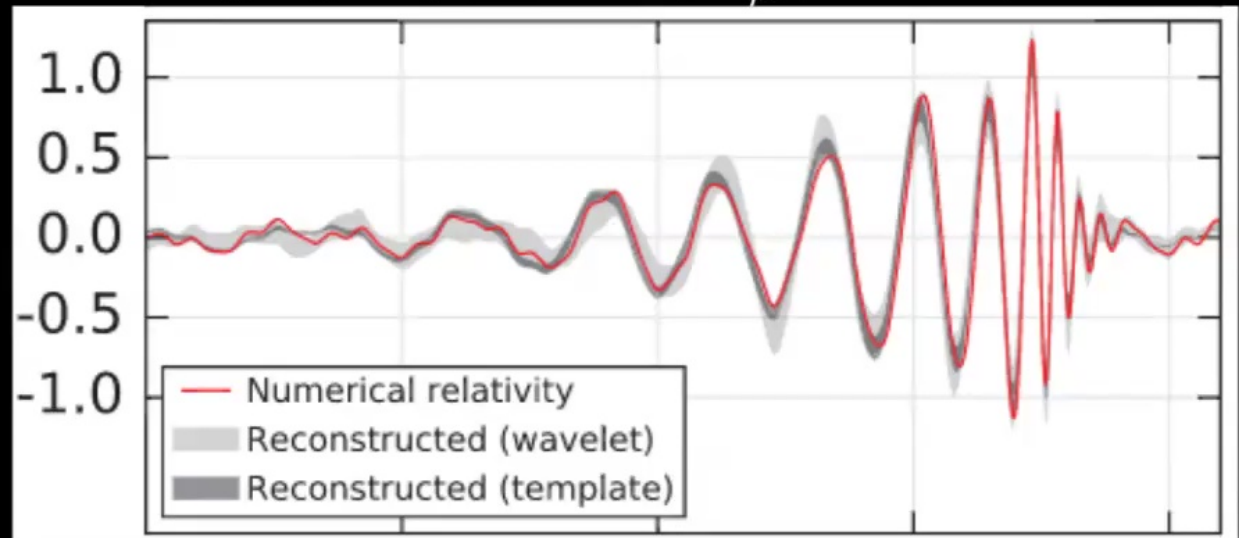


Sketch (in GR)

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

Boundary conditions:

- Purely ingoing at the horizon: $h_{\mu\nu}(r \rightarrow r_H) \propto e^{-i(\omega - m\Omega_H)r_*}$
- Purely outgoing at spatial infinity: $h_{\mu\nu}(r \rightarrow +\infty) \propto e^{i\omega r}$



PRD 107, 124032
(2023) & 109,
044072 (2024)

Sketch (in GR)

Construct an asymptotic factor $A(r)$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

$$h_{\mu\nu} = A(r)e^{-i\omega t + im\phi} \sum_{\ell=0}^N \sum_{n=0}^N a_{\mu\nu}(\ell, n) \times (\text{spectral function})_{\ell n}$$

$$R_{\mu}^{\nu} = 0 \Rightarrow [R_{\mu}^{\nu}]^{(1)} = A(r)e^{-i\omega t + im\phi} \sum_{\ell=0}^N \sum_{n=0}^N b_{\mu\nu}(\ell, n; \omega) \times (\text{spectral function})_{\ell n} = 0$$

Orthogonality

$$b_{\mu\nu} = \underbrace{\mathcal{D}_{\mu\nu}^{\gamma\delta}(\omega)}_{\text{quadratic in } \omega} \quad a_{\gamma\delta} = 0$$

PRD 107, 124032
(2023) & 109,
044072 (2024)

Some technical details

- Kerr metric in the Boyer-Lindquist coordinates
- The Regge-Wheeler gauge is used
- At most using 30 spectral bases
- Newton-Raphson method for solving for the eigenvalues

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Linearized Einstein equations

How long are the equations?

The screenshot shows a software interface with several equations and their outputs. The equations are labeled Eq12, Eq13, Eq14, Eq22, and Eq23. The outputs are displayed in a scrollable area. A red circle highlights the number '19430' in the output for Eq14, indicating the length of the equation. The interface includes buttons for 'large output', 'show less', 'show more', 'show all', and 'set size limit...'.

Eq12 Output: $-4 b m r \omega Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] G_5[r, x] h_1[r] - 2 b m r^2 \omega Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] G_5[r, x] h_1[r] - 4 b m r p \omega Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] G_5[r, x] h_1[r] + 2 b m r r p \omega Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] G_5[r, x] h_1[r] + 2 m r p^2 \omega Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] G_5[r, x] h_1[r] - 8 b m r r x^2 \omega Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] G_5[r, x] h_1[r] + 6 b r r p x^2 G_1[r, x] G_2[r, x] G_3[r, x] G_4[r, x] G_5[r, x]^2 h_1[r] Y[x] G_5^{(2,0)}[r, x] - 6 b r^2 x^2 G_1[r, x] G_2[r, x] G_3[r, x] G_4[r, x] G_5[r, x]^2 h_1[r] Y[x] G_5^{(2,0)}[r, x] - 2 b r r p x^2 G_1[r, x] G_2[r, x] G_3[r, x]^2 G_4[r, x] G_5[r, x] h_1[r] Y[x] G_5^{(2,0)}[r, x]$

Eq13 Output: $2 b^2 m^4 r^4 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 4 b^2 m^3 r^3 p Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] + 2 b^2 m^2 r^2 p^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 4 b^2 m^2 r^2 p^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 4 b^2 m^2 r^2 p^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x] + 4 b^2 m^4 Y[x] G_2[r, x] G_3[r, x]^2 G_4[r, x]^2 h_1[r] G_5^{(2,0)}[r, x] - 8 b^2 m^3 r p Y[x] G_2[r, x] G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x] + 4 b^2 m^2 r^2 p^2 Y[x] G_2[r, x] G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x]$

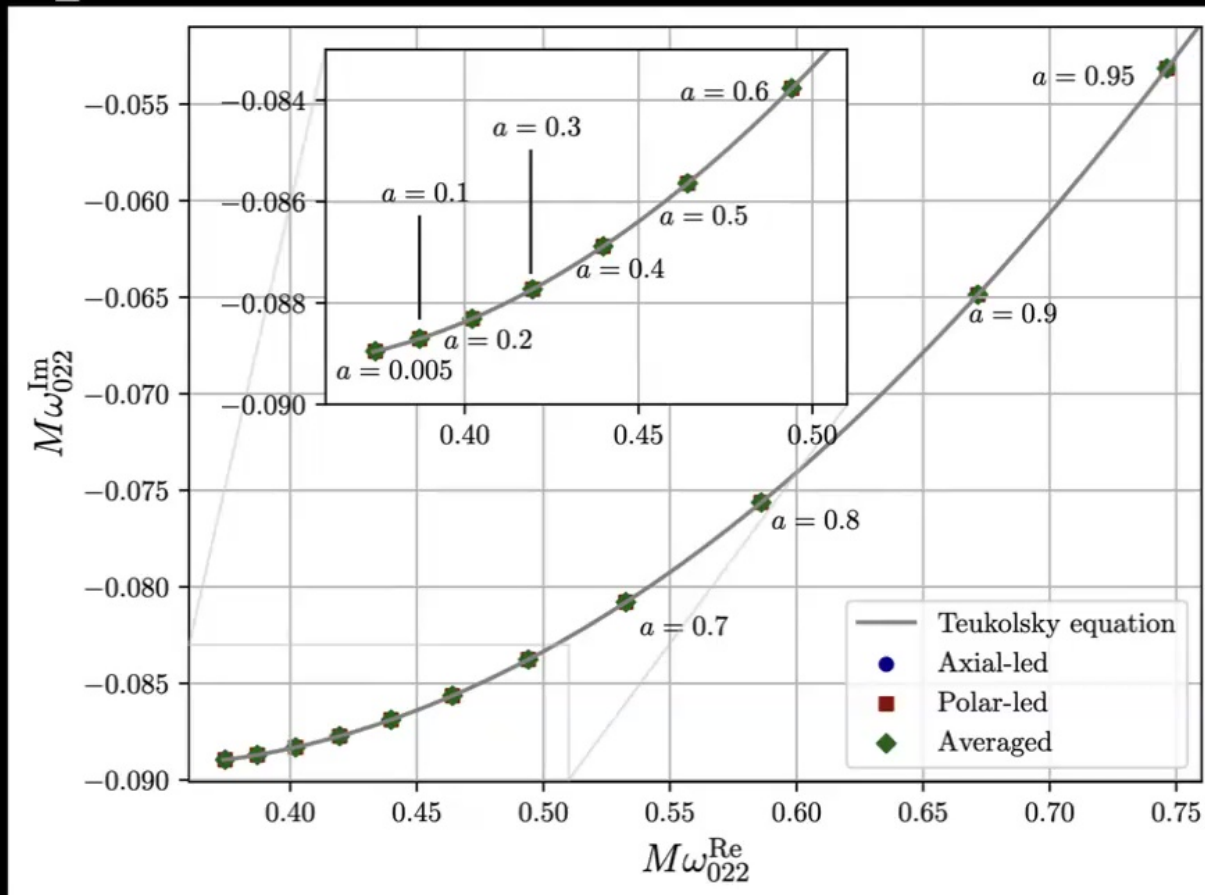
Eq14 Output: $-2 b^2 m^4 r^4 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 4 b^2 m^3 r^3 p Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 4 b^2 m^2 r^2 p^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 4 b^2 m^2 r^2 p^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x] + 4 b^2 m^4 Y[x] G_2[r, x] G_3[r, x]^2 G_4[r, x]^2 h_1[r] G_5^{(2,0)}[r, x] - 8 b^2 m^3 r p Y[x] G_2[r, x] G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x] + 4 b^2 m^2 r^2 p^2 Y[x] G_2[r, x] G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x]$

Eq22 Output: $2 b m^2 r^2 Y[x] G_1[r, x] + 4 b m^2 r r p^2 Y[x] + 16 b m r r p^2 Y[x] G_2[r, x] + 16 b^2 r^3 r p x^6 G_1[r, x]$

Eq23 Output: $-4 b m r \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 2 b m r^2 \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] + 4 b m r r p \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 2 b m r r p \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] + 2 m r p^2 \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] - 2 b m r \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x] - 2 b m r r p \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x] - 2 b m r r p \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x] - 2 b m r r p \omega^2 Y[x] G_1[r, x]^2 G_2[r, x]^2 G_3[r, x]^2 G_4[r, x] h_1[r] G_5^{(2,0)}[r, x]$

Numerical results

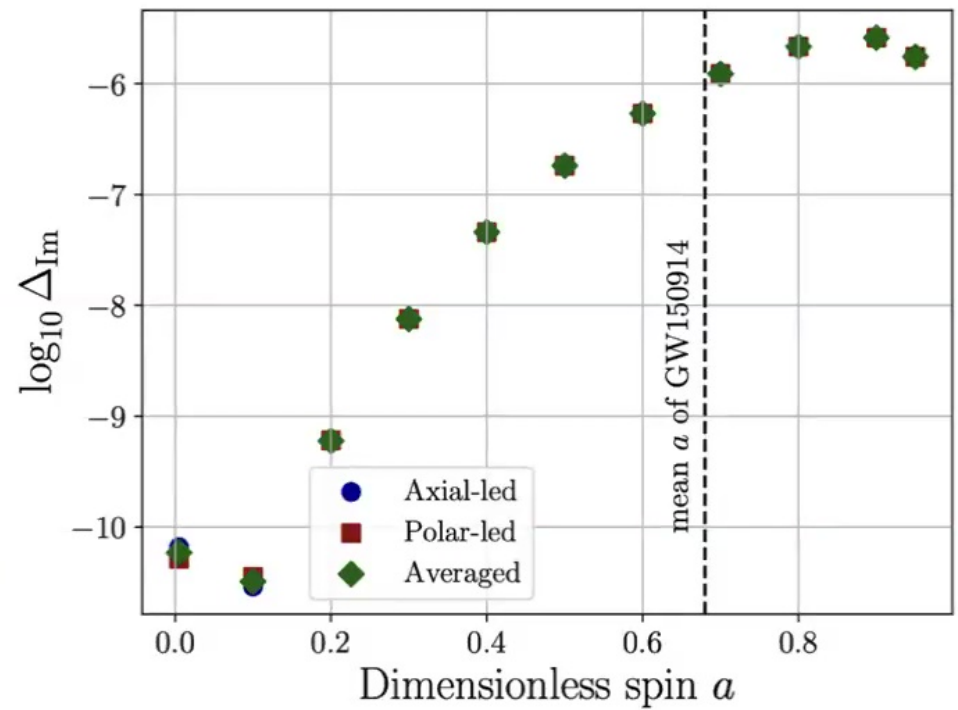
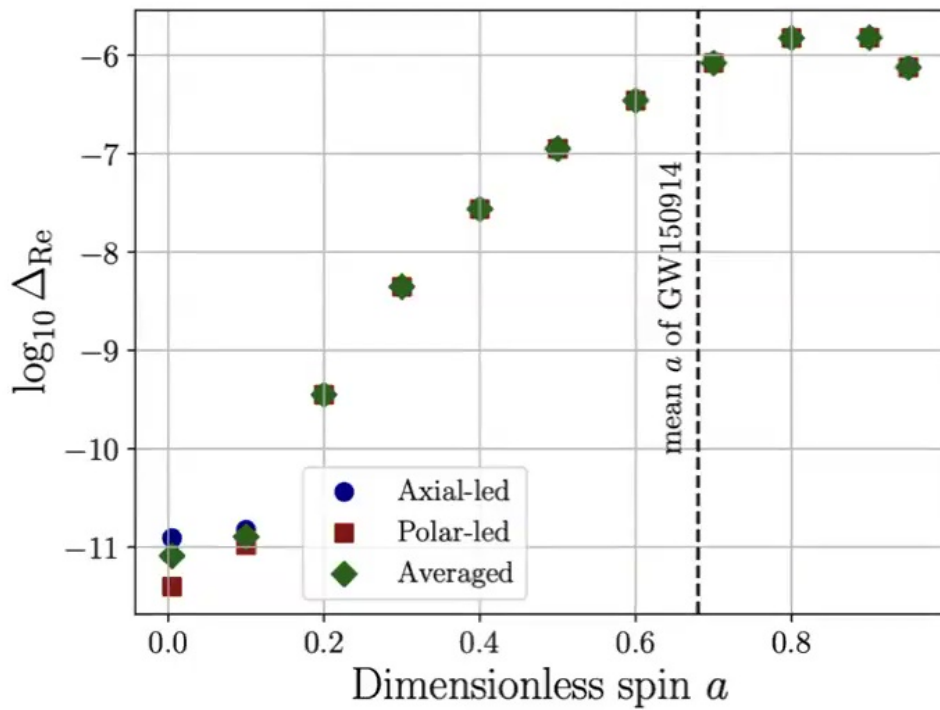
Complex plane



Numerical results

Relative error

$$\Delta_{\text{Re/Im}} = \left| \frac{\omega_{\text{Re/Im}}(\text{Spec}) - \omega_{\text{Re/Im}}(\text{Teuk})}{\omega_{\text{Re/Im}}(\text{Teuk})} \right|$$



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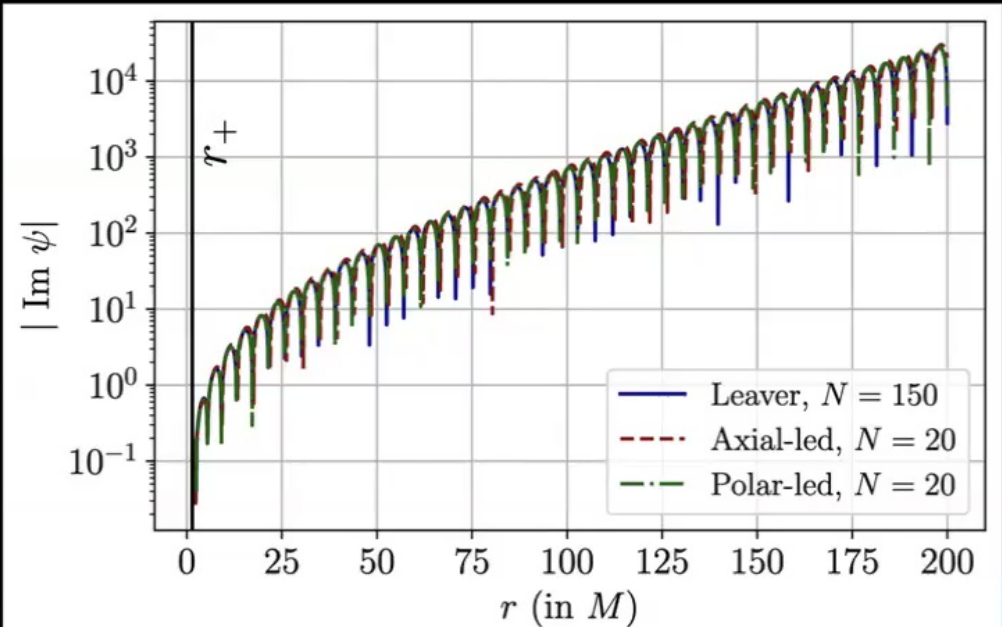
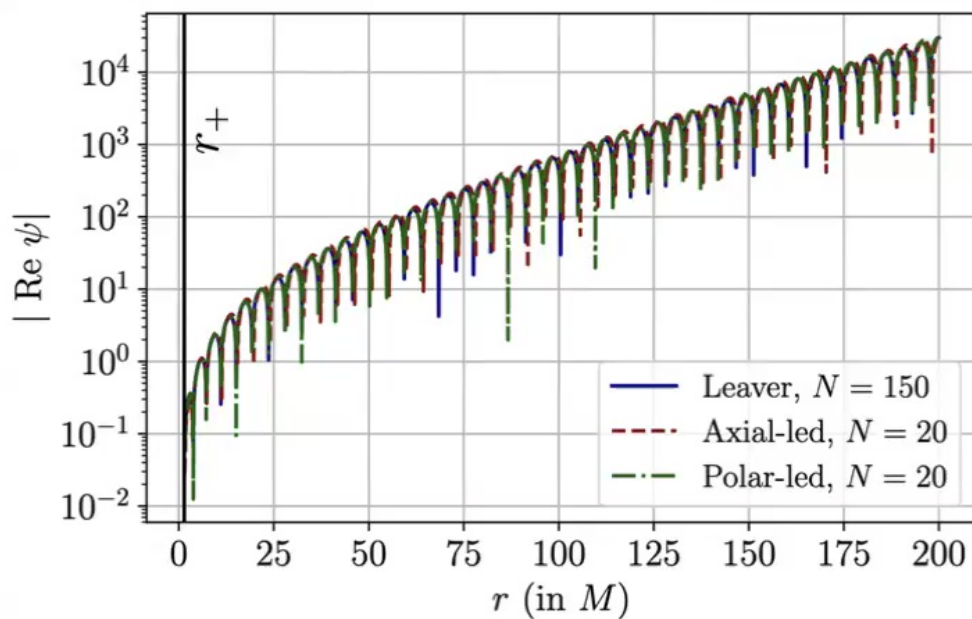


Numerical results

Metric reconstruction

- We compute the Teukolsky perturbation function for a Kerr BH of $a = 0.9$,

$$\psi = (r - iMa \cos \theta)^4 \psi_4$$



New lessons learnt about Kerr BH perturbations

- The Regge-Wheeler gauge is applicable for rapidly rotating Kerr BHs
- The associated Legendre polynomials can just be applied fine

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General relativity

In the unit of $c = G = 1$

$$R_{\mu}^{\nu} = 0$$

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Modified gravity

In the unit of $c = G = M = 1$, and write $\zeta = \alpha^2$

$$R_{\mu}^{\nu} + \zeta \left(\mathcal{A}_{\mu}^{\nu} - \frac{1}{2} \nabla_{\mu} \vartheta \nabla^{\nu} \vartheta \right) = 0,$$

$$\square \vartheta + \mathcal{A}_{\vartheta} = 0,$$

E.g. for dynamical Chern-Simons (dCS) gravity [e.g. Alexander & Yunes 2009],

$$\mathcal{A}^{\mu\nu} \equiv \left(\nabla_{\sigma} \vartheta \right) \epsilon^{\sigma\delta\alpha(\mu} \nabla_{\alpha} R^{\nu)}_{\delta} + \left(\nabla_{\sigma} \nabla_{\delta} \vartheta \right) *R^{\delta(\mu\nu)\sigma},$$

$$\mathcal{A}_{\vartheta} = \frac{1}{4} R_{\nu\mu\rho\sigma} *R^{\mu\nu\rho\sigma} \quad \text{where} \quad *R^{\mu\nu\rho\sigma} = \frac{1}{2} \epsilon^{\rho\sigma\alpha\beta} R^{\mu\nu}_{\alpha\beta}$$

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In the unit of $c = G = M = 1$, and write $\zeta = \alpha^2$

$$R_{\mu}^{\nu} + \zeta \left(\mathcal{A}_{\mu}^{\nu} - \frac{1}{2} \nabla_{\mu} \vartheta \nabla^{\nu} \vartheta \right) = 0,$$

$$\square \vartheta + \mathcal{A}_{\vartheta} = 0,$$

We can solve for $g_{\mu\nu}$ as

$$g_{\mu\nu} = g_{\mu\nu}^{(\text{GR})} + \zeta g_{\mu\nu}^{(1)} \text{ [e.g. Yunes \& Pretorius PRD 2009, Cano \& Ruipérez JHEP 2019]}$$

E.g. for Einstein scalar Gauss Bonnet (EsGB) gravity [Ripley & Pretorius CQG 2019]

$$\mathcal{A}_{\mu}^{\nu} \equiv \delta_{\mu\lambda\gamma\delta}^{\nu\sigma\alpha\beta} R^{\gamma\delta}_{\alpha\beta} \nabla^{\lambda} \nabla_{\sigma} \vartheta - \frac{1}{2} \delta_{\mu}^{\nu} \delta_{\eta\lambda\gamma\delta}^{\eta\sigma\alpha\beta} R^{\gamma\delta}_{\alpha\beta} \nabla^{\lambda} \nabla_{\sigma} \vartheta,$$

$$\mathcal{A}_{\vartheta} = R^2 - 4R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

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Sketch (modified gravity)

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu} \quad \longleftarrow \text{Construct an asymptotic factor } A(r)$$

$$\vartheta = \Phi + \vartheta^{(1)}$$

$$h_{\mu\nu}^{(1)} = A(r) e^{-i\omega t + im\phi} \sum_{\ell=0}^N \sum_{n=0}^N a_{\mu\nu / \vartheta}(\ell, n) \times (\text{spectral function})_{\ell n}$$

E.g. in GR
 $E_{\mu}^{\nu} := R_{\mu}^{\nu} = 0$
 $E_{\vartheta} := \square \vartheta = 0$

$$\begin{aligned} [E_{\mu}^{\nu}]^{(1)} &= A(r) e^{-i\omega t + im\phi} \sum_{\ell=0}^N \sum_{n=0}^N b_{\mu\nu / \vartheta}(\ell, n; \omega) \times (\text{spectral function})_{\ell n} = 0 \\ [E_{\vartheta}]^{(1)} & \end{aligned}$$

Orthogonality

$$b_{\mu\nu / \vartheta}(\ell, n; \omega) = 0$$

quadratic / higher degree in ω

Phys. Rev. D 107, 124032, 109, 044072, 110,
064019 & Phys. Rev. Lett 133, 181401

The length of linearized field equations just grows out of control

```

-((10-3*sqrt(11))
(65094 627 693 476 844213 721 090 952 857600 h1[z, x] - 19626 768 720 649 131242 786 347 347456 000 sqrt(11) h1[z, x] - 32422 395 515 663038 004 741 400 955 852 800 n^2 h1[z, x] - 9775 720 066 359616 327 949 889 403392 000 sqrt(11) n^2 h1[z, x] +
97593 021 278 304 798996 291 059 796 275 200 z h1[z, x] - 29425 403 067 627 975 505 822 362 957 312 000 sqrt(11) z h1[z, x] - 32371 336 451 461176 003 831 174 102630 400 n^2 z h1[z, x] - 9 760 325 179 259230 558 050 693 733888 000 sqrt(11) n^2 z h1[z, x] -
65182 448 216 473 969380 320 839 140 096000 z^2 h1[z, x] - 19653 247 604 619 516863 413 618 039296 000 sqrt(11) z^2 h1[z, x] + 64 045 021 614 150364 522 785 476 613683 200 n^2 z^2 h1[z, x] +
19551 509 656 056 620423 757 311 199 744000 sqrt(11) n^2 z^2 h1[z, x] - 130 250 793 602 645 892 809 383 227 583897 600 z^3 h1[z, x] - 39 272 091 911 454535 461 499 856 699904 000 sqrt(11) z^3 h1[z, x] -
64681 837 443 594 221551 763687 249 305600 n^2 z^3 h1[z, x] + 19502 307 777 378 430651 569 246 027264 000 sqrt(11) n^2 z^3 h1[z, x] - 59 998 224 939 010205 194 633 163 468800 z^4 h1[z, x] -
18090 145 473 640 076791 090 467 840 000 sqrt(11) z^4 h1[z, x] - 32 432 825 414 105319 022 787 324 217241 600 n^2 z^4 h1[z, x] - 9 778 864 799 062 758 000 034 454 602 752 000 sqrt(11) n^2 z^4 h1[z, x] +
32586 810 106 297 632080 587 874 635 161600 z^5 h1[z, x] + 9 825 292 930 650 055 370 129 712 485504 000 sqrt(11) z^5 h1[z, x] - 32 249 627 335 958 083 913 854 835 625 574 400 n^2 z^5 h1[z, x] -
9 723 628 500 196 522 927 163 953 228 800000 sqrt(11) n^2 z^5 h1[z, x] + 153 097 721 023 627 672 193 393 956 966400 z^6 h1[z, x] + 46 160 659 717 625 329 116 203 764 736000 sqrt(11) z^6 h1[z, x] +
20168 000 121 973 111 823 132 133 273 600 n^2 z^6 h1[z, x] + 6080 800 834 220 613 689 667 392 512 000 sqrt(11) n^2 z^6 h1[z, x] + 70 942 909 987 870 430 520 503 184 588 800 z^7 h1[z, x] + 21 390 092 178 702 069 879 794 260 992 000 sqrt(11) z^7 h1[z, x] -
60911 853 001 494 397945 865 987 686 480 n^2 z^7 h1[z, x] - 18 365 614 680 203 662 785 402 389 504 000 sqrt(11) n^2 z^7 h1[z, x] - 5 278 940 874 214 541 645 005 837 670 400 z^8 h1[z, x] - 1 591 660 568 938941 398 954 203 648 000 sqrt(11) z^8 h1[z, x] -
9968 636 214 257 222 719 193 290 291 200 n^2 z^8 h1[z, x] - 3005 656 908 5 728255 768 758 784 000 sqrt(11) n^2 z^8 h1[z, x] - 19 380 035 292 887 672 961 869 312 000 z^9 h1[z, x] + 5 821 591 682 313476 332 776 960 000 sqrt(11) z^9 h1[z, x] +
38196 806 234 349 557360 368 025 600 n^2 z^9 h1[z, x] - 6 1775 977 626 987 920 000 sqrt(11) z^9 h1[z, x] - 6 1775 977 626 987 920 000 sqrt(11) z^9 h1[z, x] - 6 1775 977 626 987 920 000 sqrt(11) z^9 h1[z, x] +
1759 479 024 217 202 640 000 sqrt(11) z^9 h1[z, x] - 1759 479 024 217 202 640 000 sqrt(11) z^9 h1[z, x] - 1759 479 024 217 202 640 000 sqrt(11) z^9 h1[z, x] - 1759 479 024 217 202 640 000 sqrt(11) z^9 h1[z, x] +
5299 063 093 711 789 200 000 z^5 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 1597 727 638 387 445 400 000 sqrt(11) z^5 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 11 579 222 465 546 057 600 000 z^6 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] -
3491 266 934 751 839040 000 sqrt(11) z^6 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 29 679 846 393 295 152 000 000 z^7 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 8 948 810 392 903 909 440 000 sqrt(11) z^7 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] -
27134 423 791 416 098 400 000 z^8 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 8181 336 601 692 726 480 000 sqrt(11) z^8 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 102 676 692 640 631 200 000 z^9 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] +
30958 187 654 874 480 000 sqrt(11) z^9 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 27 246 404 468 088 436 800 000 z^10 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 8 215 100 046 082 931 040 000 sqrt(11) z^10 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] +
29620 673 648 880 974 400 000 z^11 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 8930 969 139 173 234 400 000 sqrt(11) z^11 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 11 447 100 030 201 316 000 000 z^12 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] +
3451 430 521 622 157 360 000 sqrt(11) z^12 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 5 378 464 248 028 226 400 000 z^13 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 1 621 667 987 106 420 880 000 sqrt(11) z^13 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] -
9506 573 091 872 380 000 000 z^14 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 2 866 339 635 257 229 120 000 sqrt(11) z^14 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 5 807 154 578 208 252 800 000 z^15 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] -
1 750 922 985 046 486 000 000 sqrt(11) z^15 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 1 978 615 324 460 905 200 000 z^16 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 596 574 966 880 375 560 000 sqrt(11) z^16 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] -
373548 562 707 702 000 000 z^17 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 112 629 129 407 090 280 000 sqrt(11) z^17 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 30 327 340 951 024 800 000 z^18 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] -
9144 037 347 611 760 000 sqrt(11) z^18 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 118 029 369 122 400 000 z^19 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] + 35 587 193 783 760 000 sqrt(11) z^19 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] -
149977 393 200 000 z^20 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 45219 885 480 000 sqrt(11) z^20 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 63 282 000 000 z^21 c x^12 h5(0,1)[z, x] H4(2,0)[z, x] - 19 056 120 000 sqrt(11) z^21 c x^12 h5(0,1)[z, x] H4(2,0)[z, x])

```

```

... 500 360 ...
z, x] H4(2,0)[z, x]

```

~ 10⁶ terms !

Based on Phys. Rev. D 107, 124032, 109, 044072, 110, 064019 and Phys. Rev. Lett 133, 181401
Adrian K.W. Chung, Pratik Wagle and Nicolas Yunes, (ICASU, UIUC)



Some technical details

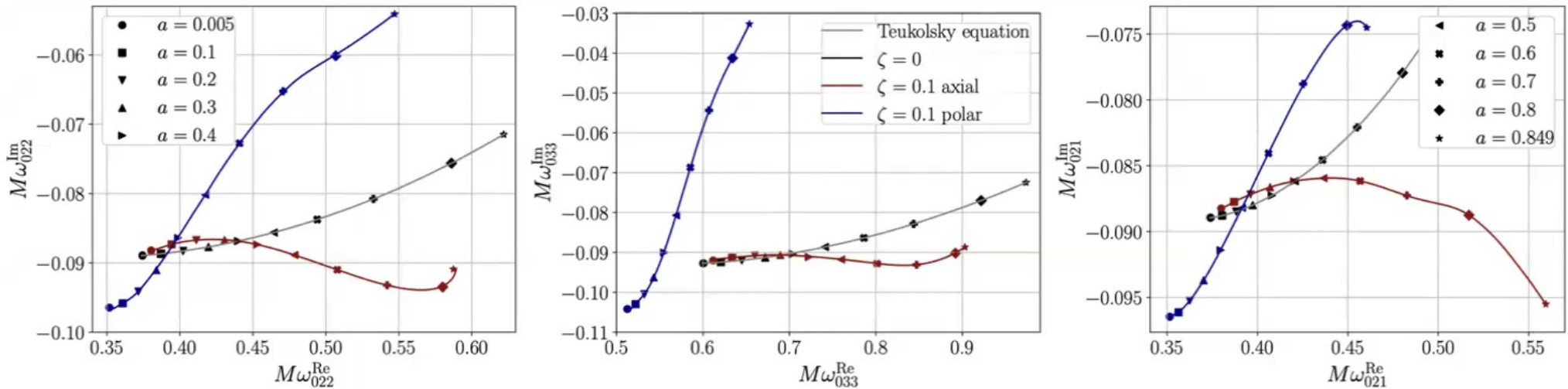
- We solve ω as $\omega^{(\text{GR})} + \zeta \omega^{(1)}$
- The Regge-Wheeler gauge is used
- At most using 25 spectral bases
- Newton-Raphson method for solving for the eigenvalues
- Metric modifications up to 40th order in a are used for the computation

Based on Phys. Rev. D 107, 124032, 109, 044072, 110, 064019 and Phys. Rev. Lett 133, 181401
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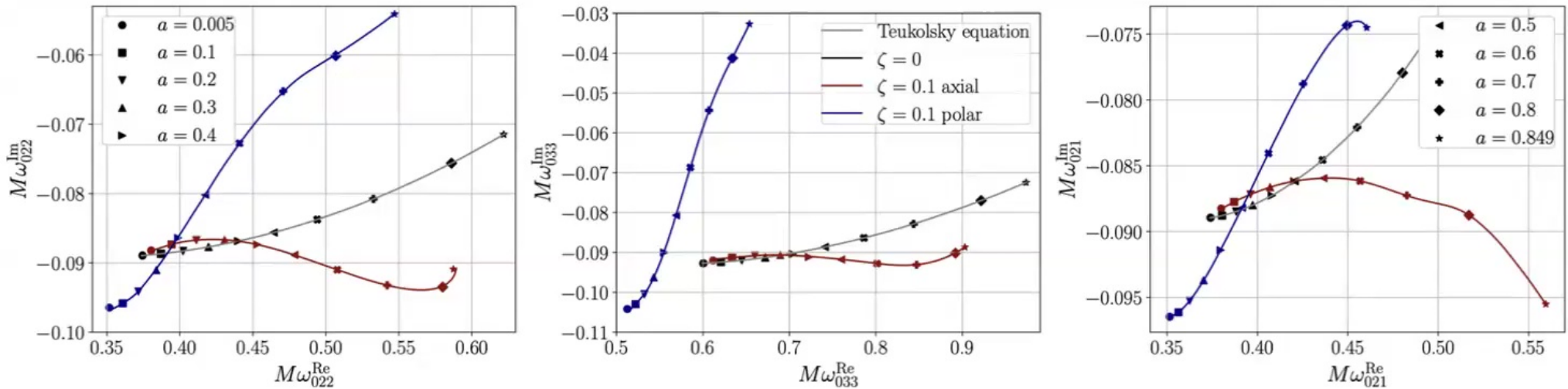


Quasinormal-mode spectra in scalar-Gauss-Bonnet gravity



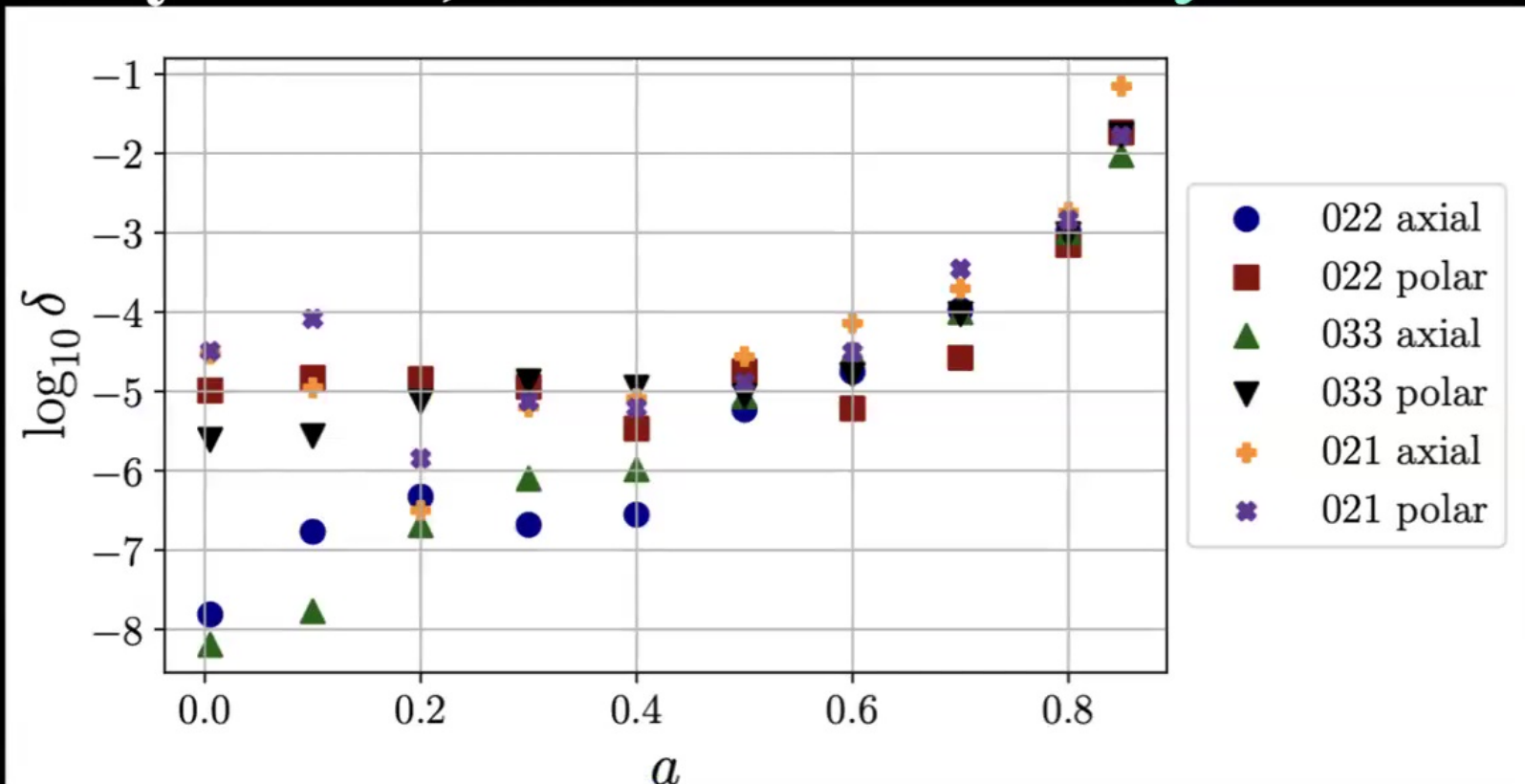
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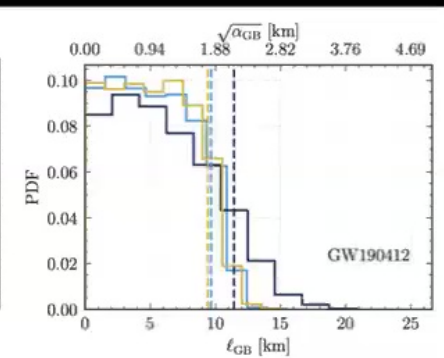
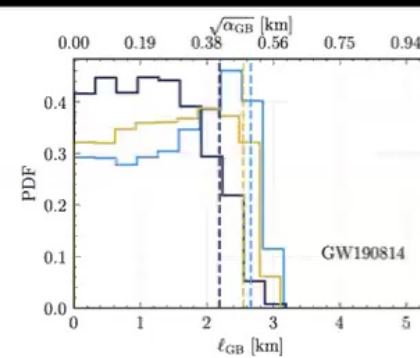
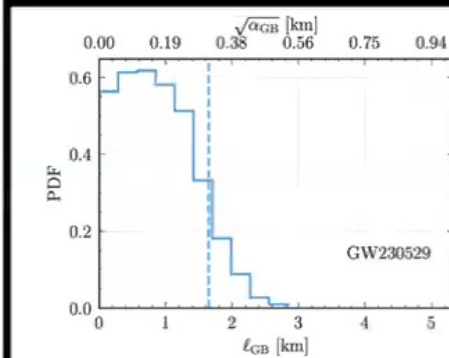
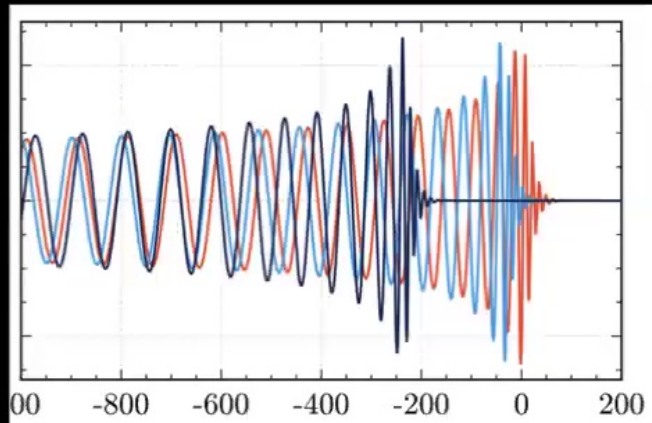
Accuracy of $\omega^{(1)}$, where $\omega = \omega^{(\text{GR})} + \zeta\omega^{(1)}$



Adrian K.W. Chung (akwchung@illinois.edu), and Nicolas Yunes, (ICASU, UIUC)

Application and extensions of our METRICS - EsGB results

- Gain insight into numerical-relativity simulations in EsGB gravity [Corman & East 2024]
- Model specific test of EdGB gravity with LIGO data [Julié, Pompili & Buonanno 2024]



- Applied METRICS to EdGB with strong coupling [Blázquez-Salcedo, Khoo, Kleihaus & Kunz 2024]

Based on Phys. Rev. D 107, 124032, 109, 044072, 110, 064019 and Phys. Rev. Lett 133, 181401
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Preliminary results in dynamical Chern-Simons gravity

- Lagrangian $\mathcal{L} = R + \zeta \vartheta R_{\alpha\beta\gamma\delta} \tilde{R}^{\alpha\beta\gamma\delta} - \frac{1}{2} \zeta \nabla_\mu \vartheta \nabla^\mu \vartheta$
- $\omega^{(1)}$ at $a = 0.00498$, uncertainty $< 10^{-4}$

nlm	$\omega_A^{(1)}$	$10^4 \times \omega_P^{(1)}$	$\omega_S^{(1)}$
022	$0.24958 + 0.12606i$	$0.2+2.9i$	$-0.58281 - 0.07866i$
033	$0.92112 + 0.16491i$	$0.6-4.5i$	$-1.41725 - 0.11278i$

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Possible future applications of METRICS

- QNMs of other modified gravity theories \longrightarrow PE
- Results cross check with modified Teukolsky formalism?
[c.f. e.g. D. Li et al PRX 2022, P. Wagle et al PRD 2024, etc]
- Waveform modeling of EMRIs, and self force calculations
[c.f. e.g. P. Bourg et al arXiv:2403.12634, A. Pound and B. Wardell arXiv: 2101.04592, etc]
- Environmental effects/model-specific search for scalar charge?
[c.f. e.g. Enrico Barausse, Vitor Cardoso, Paolo Pani PRD 2014, A. Maselli et al PRL 2021, Nat Ast. 2022]
- Rapidly rotating neutron star seismology?

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Summary

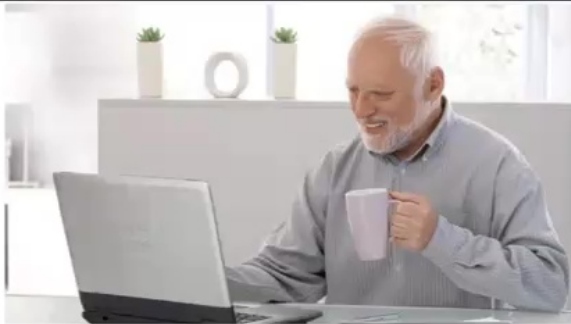
We develop METRICS which

- Can accurately compute BH QNM frequency,
- Can rapidly reconstruct metric perturbations,
- Can easily be adapted to a general/beyond GR black hole

Phys. Rev. D 107, 124032, 109, 044072, 110,
064019 & Phys. Rev. Lett 133, 181401

Based on Phys. Rev. D 107, 124032, 109, 044072, 110, 064019 and Phys. Rev. Lett 133, 181401
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METRICS makes black-hole perturbations easier!



Start your research

We will be linearizing the following field equations [11],

$$\begin{aligned}
 R^\mu{}_\nu + \frac{\alpha}{\kappa_g} C^\mu{}_\nu - \frac{1}{2\kappa_g} [\bar{T}^{\theta}]^\mu{}_\nu &= 0, \\
 \square \tilde{\vartheta} + \frac{\alpha}{4} R_{\nu\mu\rho\sigma} {}^* R^{\mu\nu\rho\sigma} &= 0,
 \end{aligned}
 \tag{33}$$

where $\square = \nabla_\mu \nabla^\mu$ is the d'Alembert operator,

$$\begin{aligned}
 C^\mu{}_\nu &\equiv (\nabla_\sigma \tilde{\vartheta}) \epsilon^{\sigma\delta\alpha(\mu} \nabla_\alpha R_{\nu)\delta} + (\nabla_\sigma \nabla_\delta \tilde{\vartheta}) {}^* R^{\delta(\mu}{}_{\nu)\sigma}, \\
 [\bar{T}^{\theta}]^\mu{}_\nu &\equiv (\nabla^\mu \tilde{\vartheta}) (\nabla_\nu \tilde{\vartheta}).
 \end{aligned}$$

Google a gravity theory

Spectral method for metric perturbations of black holes:
Kerr background case in general relativity
Adrian Ka-Wai Chung,^{1,*} Pratik Wagle,^{1,2,†} and Nicolás Yunes³
¹Illinois Center for Advanced Studies of the Universe & Department of Physics,
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA
²Max Planck Institute for Gravitational Physics (Albert Einstein Institute), D-14478 Potsdam, Germany
(Date: December 15, 2023)

We present a novel approach, Metric μ E-Turbations with spectral methods (METRICS), to calculate the gravitational metric perturbations and the quasinormal-mode frequencies of rotating black holes of any spin without decoupling the linearized field equations. We demonstrate the method by applying it to perturbations of Kerr black holes of any spin, simultaneously solving all ten linearized Einstein equations in the Regge-Wheeler gauge through purely algebraic methods and computing the fundamental (co-rotating) quadrupole mode frequency at various spins. We moreover show that the METRICS approach is accurate and precise, yielding (i) quasinormal mode frequencies that agree with Leaver's, continuous-fraction solution with a relative fractional error smaller than 10^{-5} for all dimensionless spins below up to 0.95, and (ii) metric perturbations that match the Teukolsky-Saunders exact solution with $\mathcal{O}(10^{-6})$ accuracy.

Apply METRICS

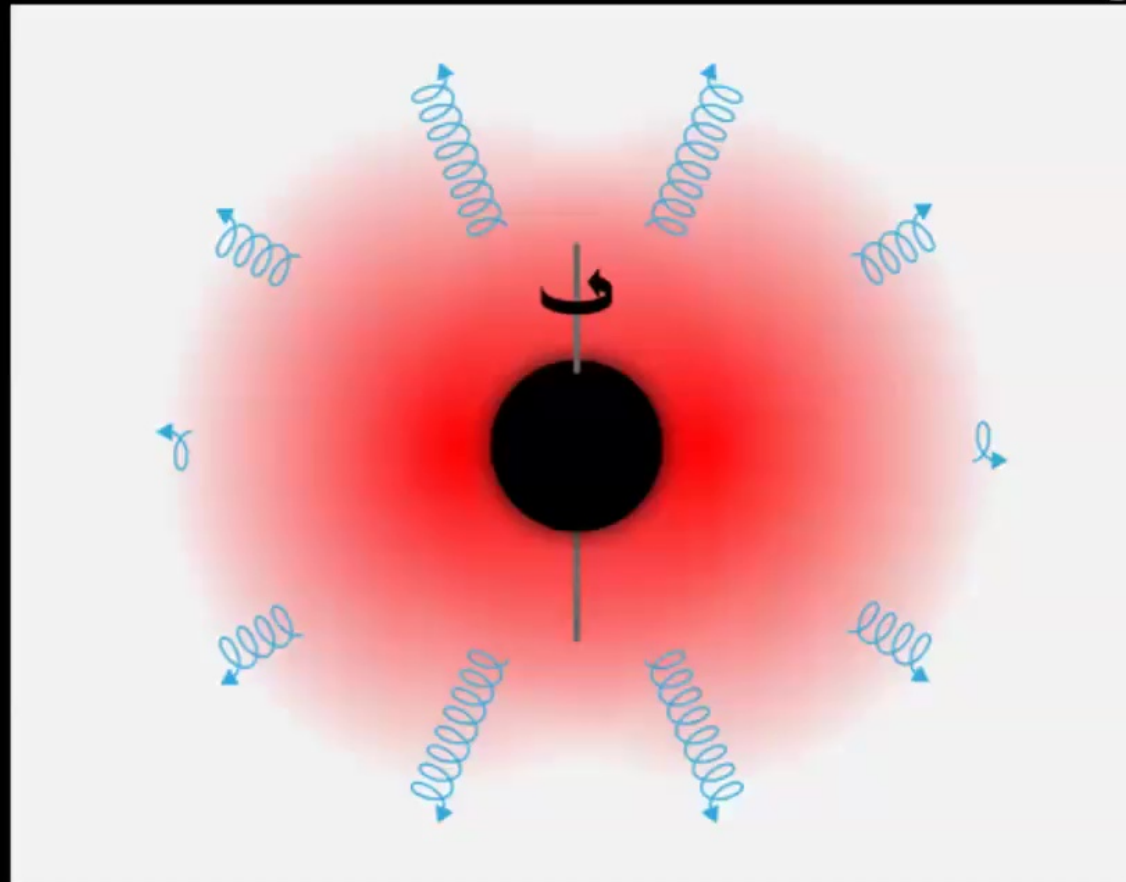


Get QNM frequencies sometime later

Based on Phys. Rev. D 107, 124032, 109, 044072, 110, 064019 and Phys. Rev. Lett 133, 181401
Adrian K.W. Chung, Pratik Wagle and Nicolas Yunes, (ICASU, UIUC)



Why should we care about black-hole QNMs?



See, e.g., Chung et. al,
Phys. Rev. D 104,
084028 (2021)

Picture credit: <https://physics.aps.org/articles/v10/83>

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