Title: METRICS and its use to probe fundamental physics with black-hole ringdown phase

Speakers: Adrian Chung

Collection/Series: Strong Gravity

Subject: Strong Gravity

Date: October 31, 2024 - 1:00 PM

URL: https://pirsa.org/24100135

Abstract:

Quasinormal modes of a black hole are closely related to the dynamics of the spacetime near the horizon. In this connection, the black hole ringdown phase is a powerful probe into the nature of gravity. However, the challenge of computing quasinormal mode frequencies has meant that ringdown tests of gravity have largely remained model-independent. In this talk, I will introduce Metric pErTuRbations wIth speCtral methodS (METRICS) [1], a novel spectral scheme capable of accurately computing the quasinormal mode frequencies of black holes, including those with modifications beyond Einstein's theory or the presence of matter. I will demonstrate METRICS' accuracy in calculating quasinormal mode frequencies within general relativity, as a validation, and its application to Einstein-scalar-Gauss-Bonnet gravity [2, 3], an example of modified gravity theory to which METRICS has been applied. I will also present preliminary results from applying METRICS to dynamical Chern-Simons gravity. Finally, I will discuss potential future applications of METRICS beyond computing black hole quasinormal modes.

[1]: https://arxiv.org/abs/2312.08435
[2]: https://arxiv.org/abs/2405.12280
[3]: https://arxiv.org/abs/2406.11986

Perimeter Institute Seminar 31st October 2024

METRICS and its use to probe fundamental physics with black-hole ringdown phase

Based on Phys. Rev. D 107, 124032, 109, 044072, 110, 064019 and Phys. Rev. Lett 133, 181401

Adrian K.W. Chung, Pratik Wagle and Nicolas Yunes, (ICASU, UIUC)

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1

Metric pErTuRbations wIth speCtral methodS

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Content of the talk

- What is METRICS? Why?
- METRICS in general relativity
- METRICS in modified gravity
- Future directions

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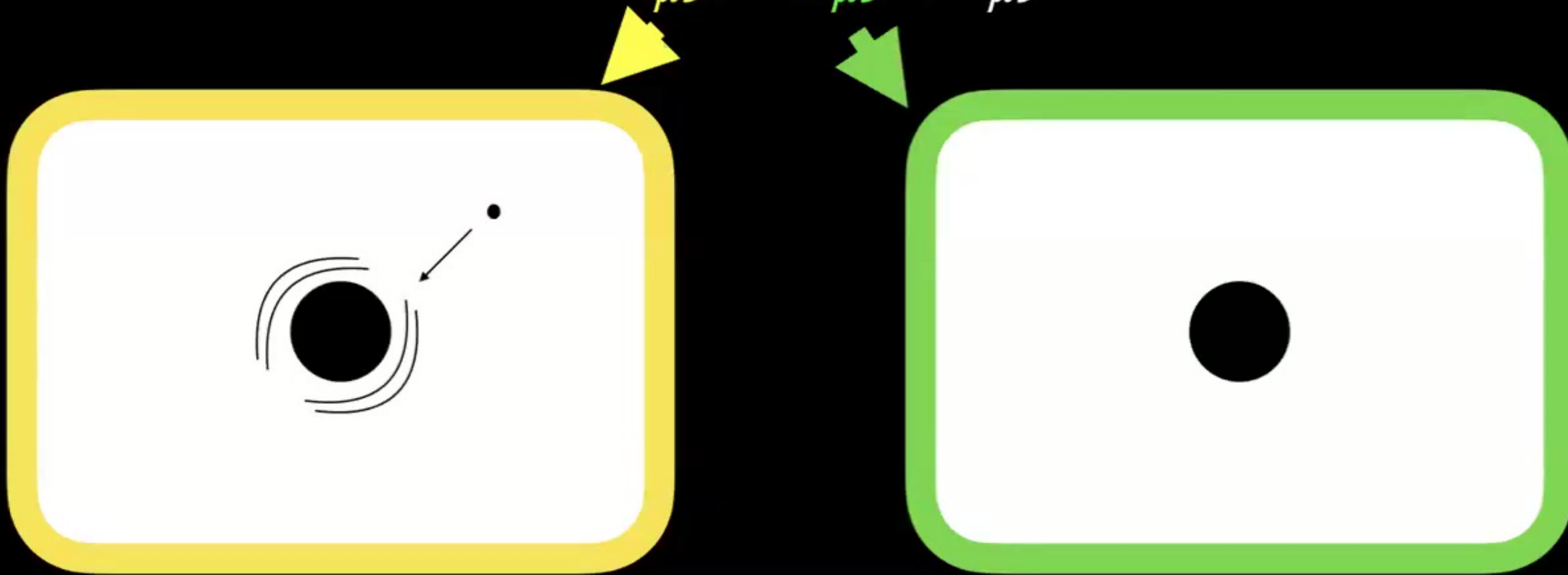
Black-hole metric perturbations

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$



Perturbed black hole

Unperturbed state

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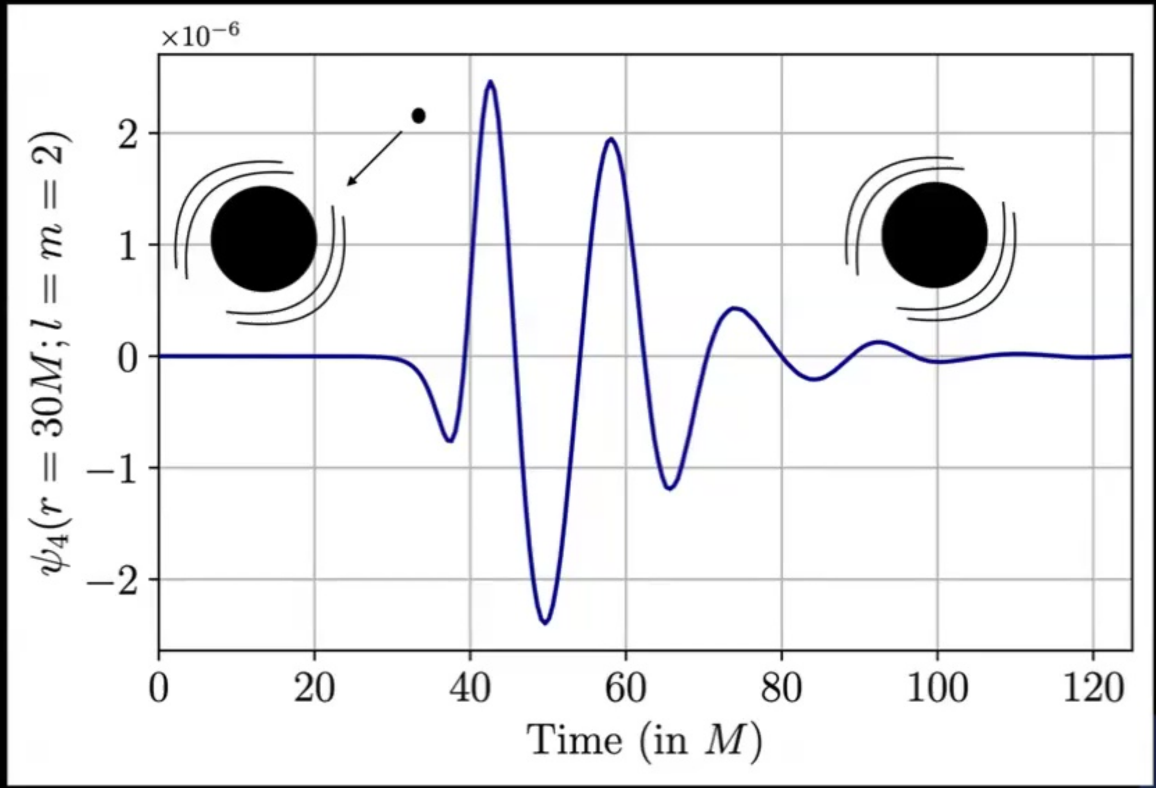
4

Black-hole perturbations

Perturbations of a BH by the radial infall of a test mass. Mass ratio 20:1. Simulated using NRPy+



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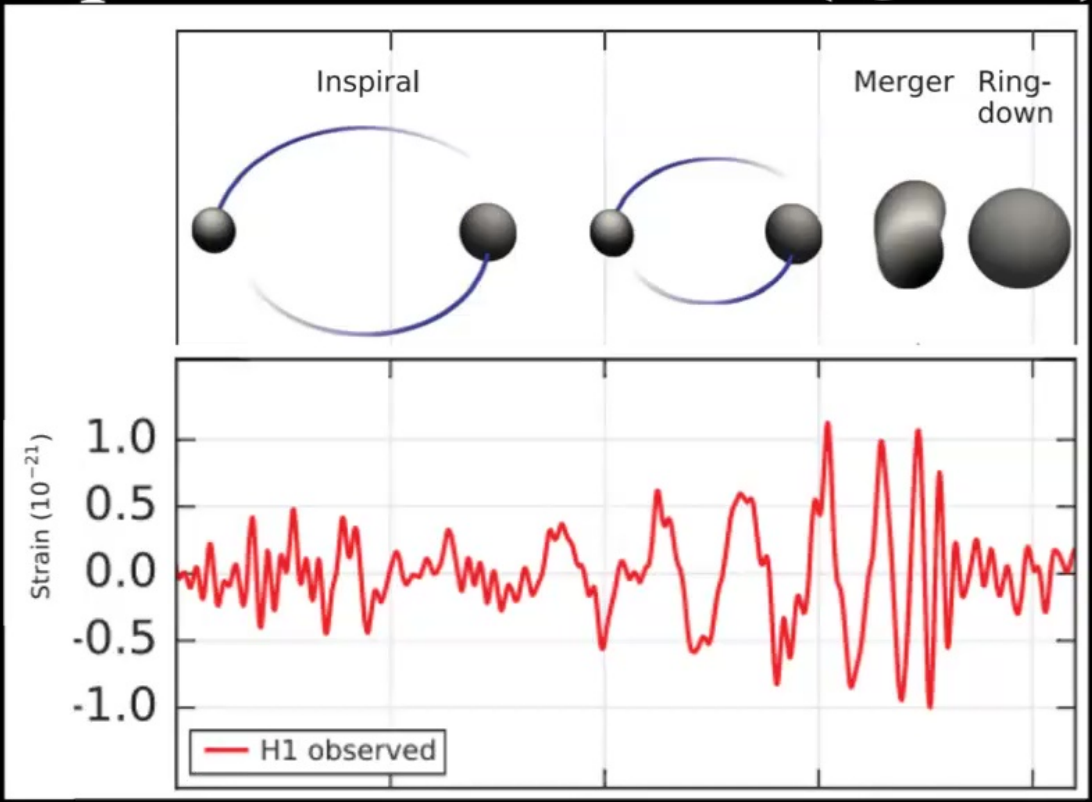
Black-hole quasinormal modes (QNMs)



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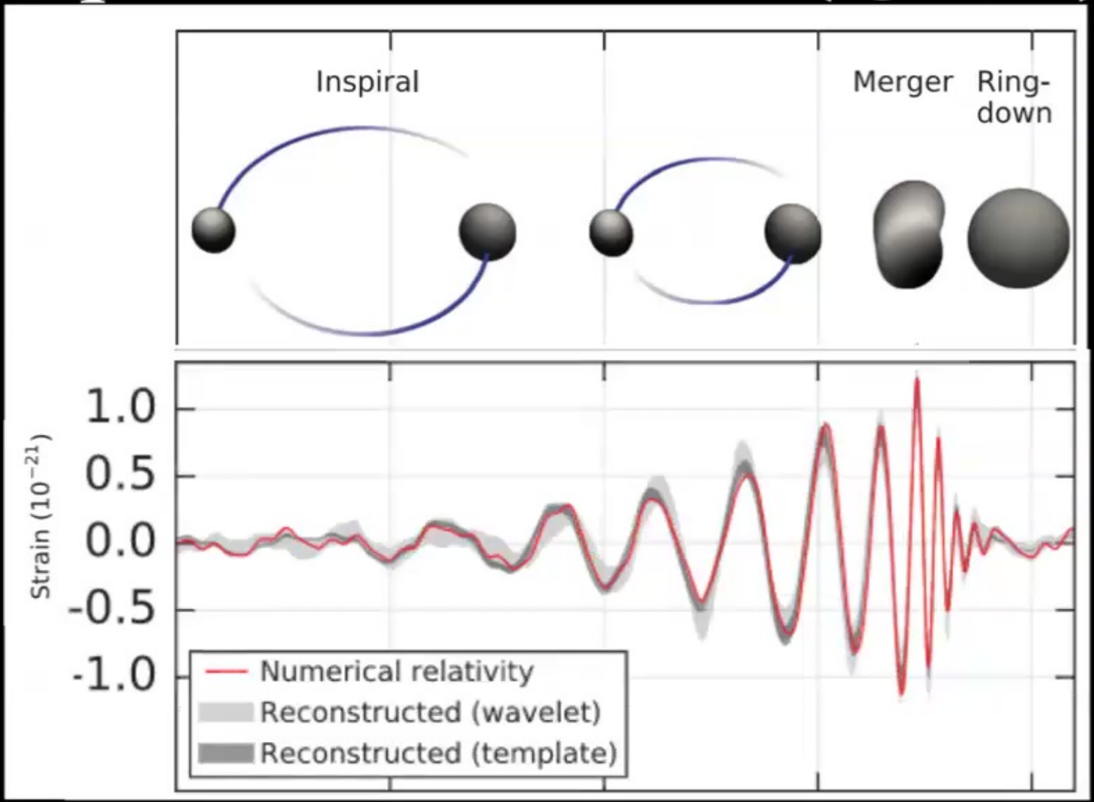
Black-hole quasinormal modes (QNMs)



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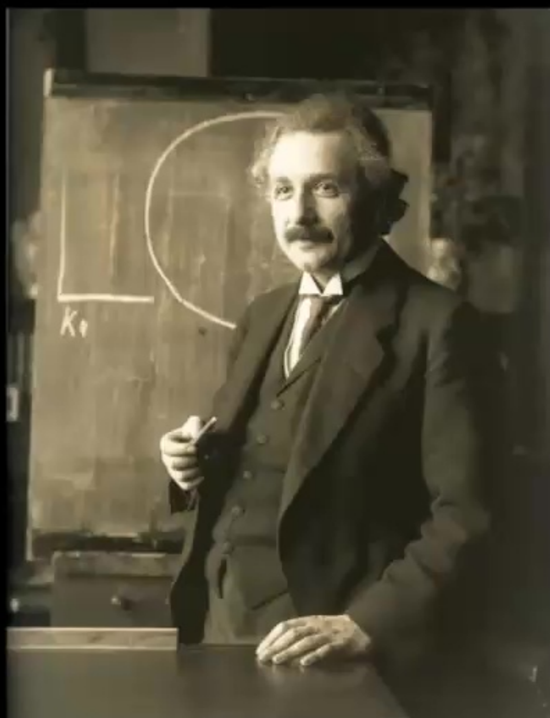
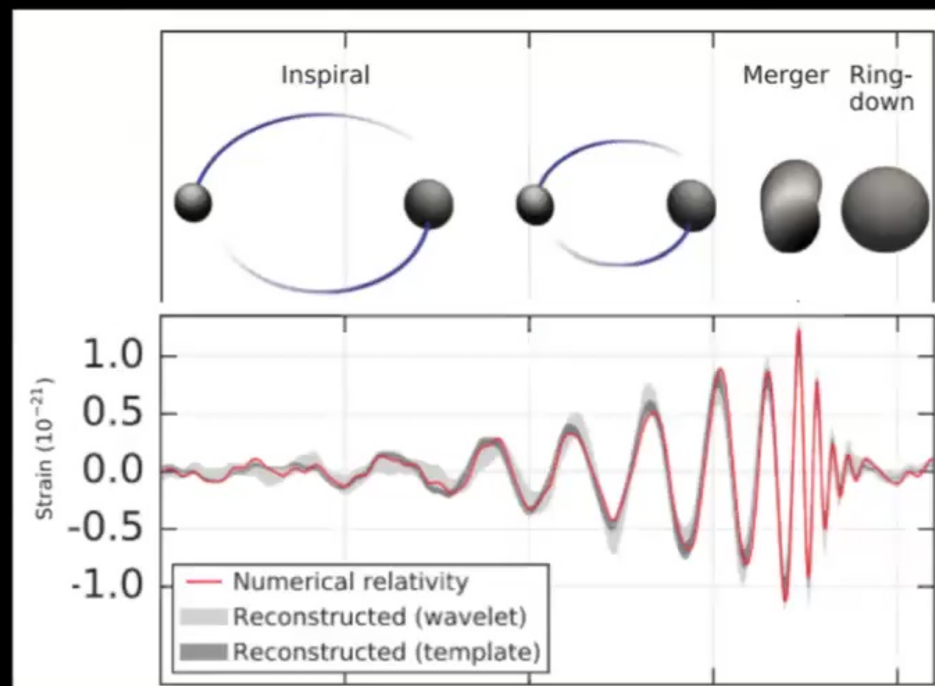
Why should we care about black-hole QNMs?



Picture credit: https://upload.wikimedia.org/wikipedia/commons/3/3e/Einstein_1921_by_F_Schmutzer_-_restoration.jpg



See, e.g., Chung et. al, Phys. Rev. D 99, 124023 (2019)

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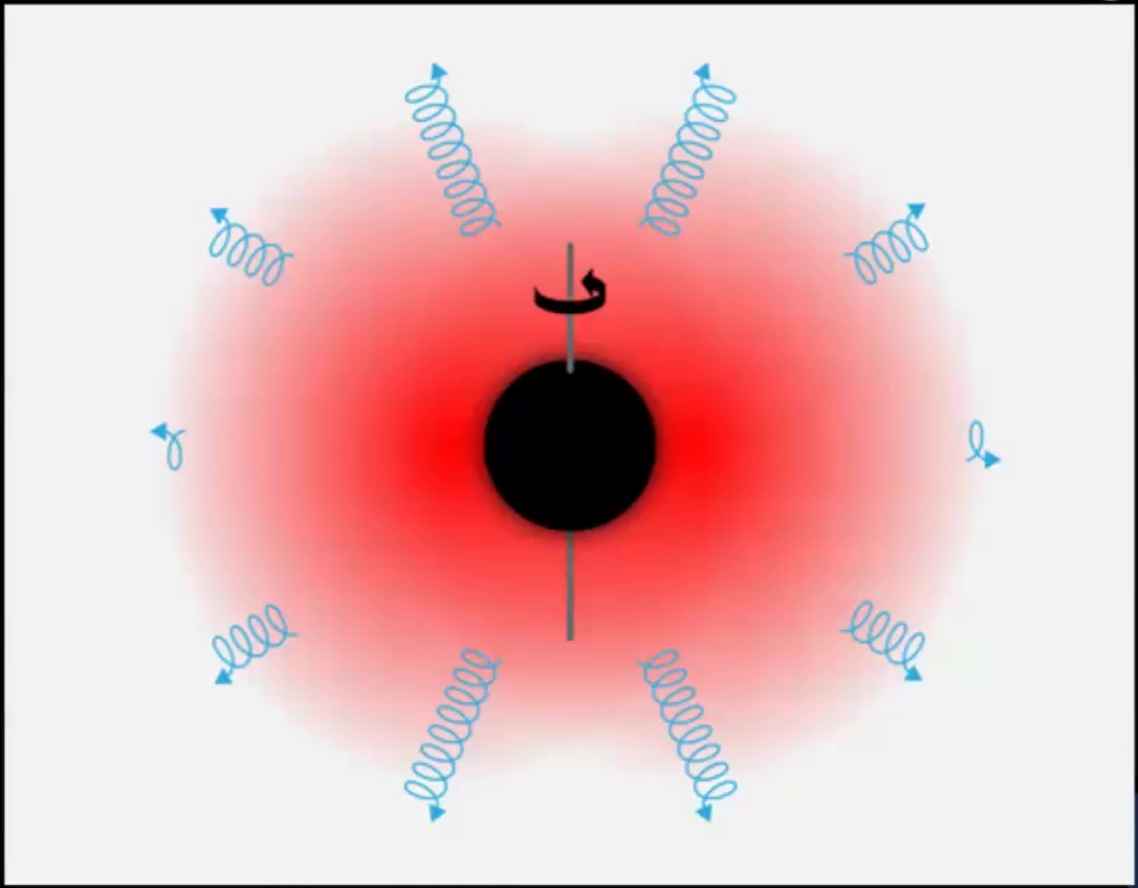
8

Why should we care about black-hole QNMs?



Picture credit: https://physics.aps.org/articles/v10/83

See, e.g., Chung et. al, Phys. Rev. D 104, 084028 (2021)

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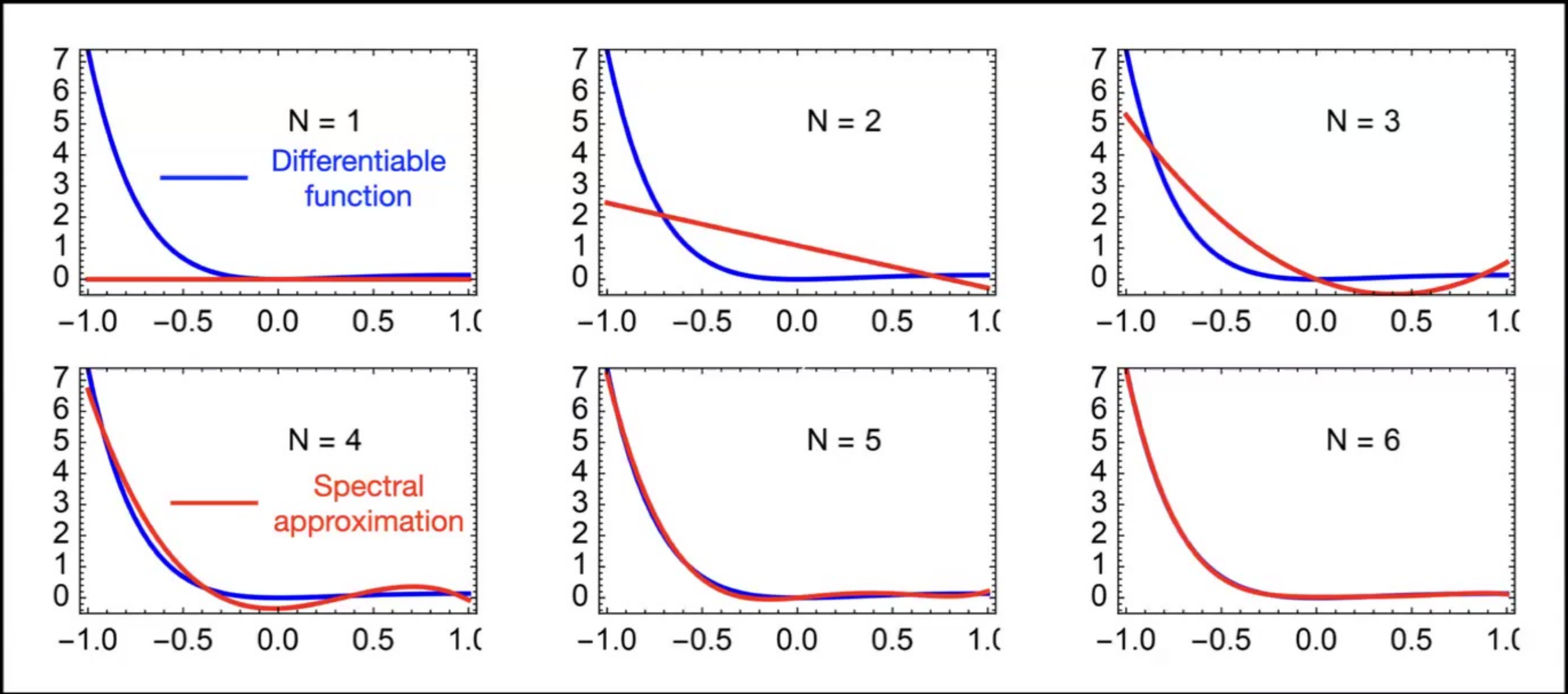
Spectral functions



Modified from Pedro G. S. Fernandes, David J. Mulryne, arXiv: 2212.07293

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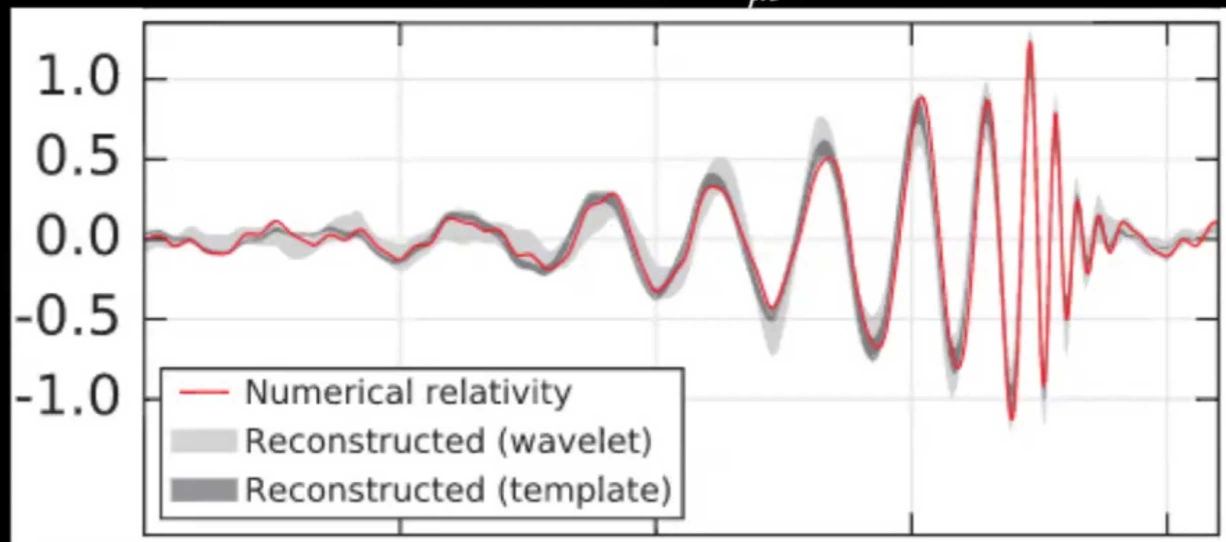
Sketch (in GR)

$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$

Boundary conditions:

- Purely ingoing at the horizon: $h_{\mu\nu}(r \to r_H) \propto e^{-i(\omega - m\Omega_H)r_*}$
- Purely outgoing at spatial infinity: $h_{\mu\nu}(r \to +\infty) \propto e^{i\omega r}$



PRD 107, 124032 (2023) & 109, 044072 (2024)

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Sketch (in GR)

Construct an asymptotic factor $A(r)$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = A(r)e^{-i\omega t + im\phi} \sum_{\ell=0}^{N} \sum_{n=0}^{N} a_{\mu\nu}(\ell, n) \times (\text{spectral function})_{\ell n}$$

$$R_\mu{}^\nu = 0 \Rightarrow [R_\mu{}^\nu]^{(1)} = A(r)e^{-i\omega t + im\phi} \sum_{\ell=0}^{N} \sum_{n=0}^{N} b_{\mu\nu}(\ell, n; \omega) \times (\text{spectral function})_{\ell n} = 0$$

Orthogonality

$$b_{\mu\nu} = \underbrace{\mathscr{D}_{\mu\nu}{}^{\gamma\delta}(\omega)}_{\text{quadratic in } \omega} a_{\gamma\delta} = 0$$

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12

Some technical details

- Kerr metric in the Boyer-Lindquist coordinates
- The Regge-Wheeler gauge is used
- At most using 30 spectral bases
- Newton-Raphson method for solving for the eigenvalues

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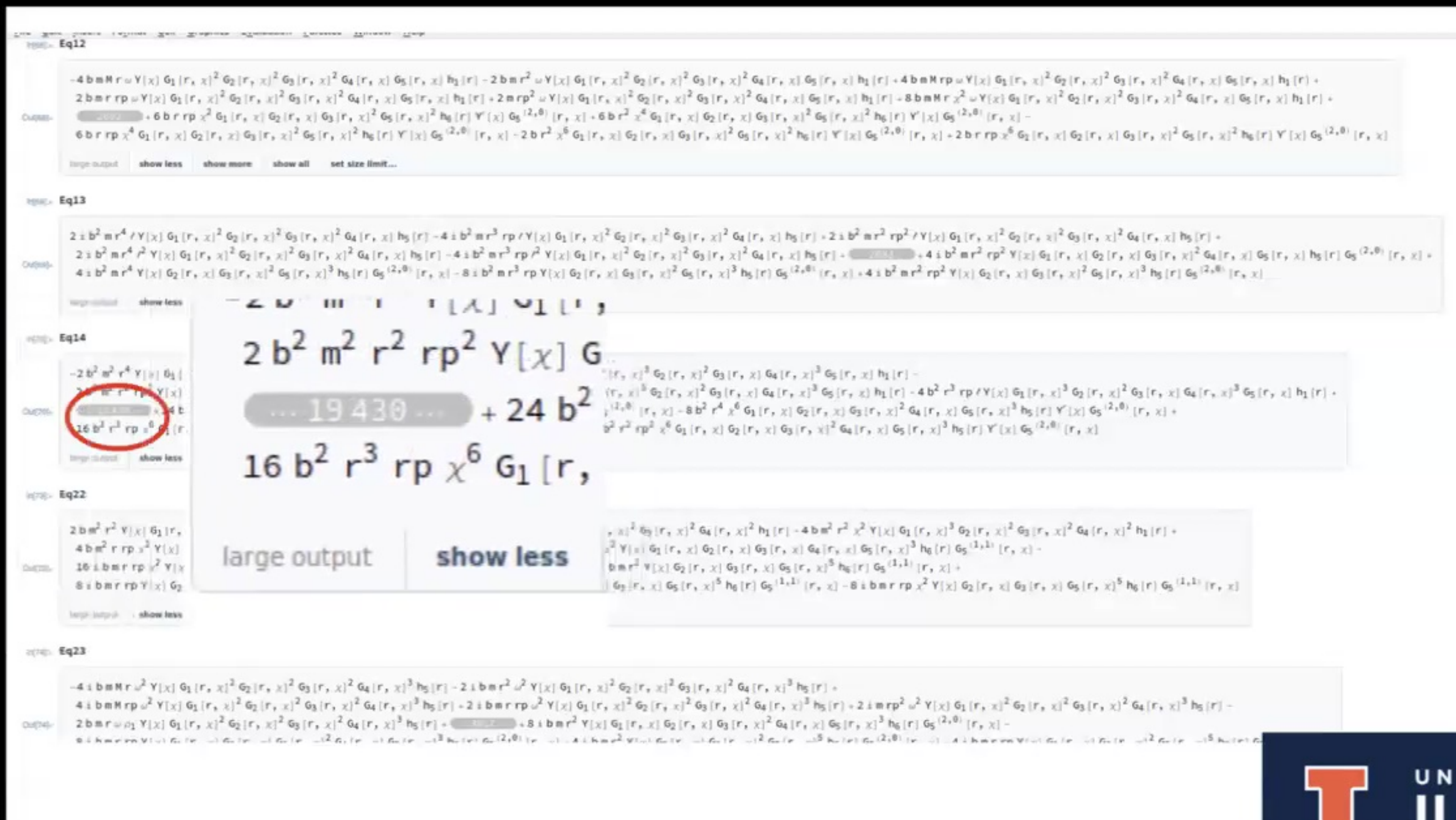
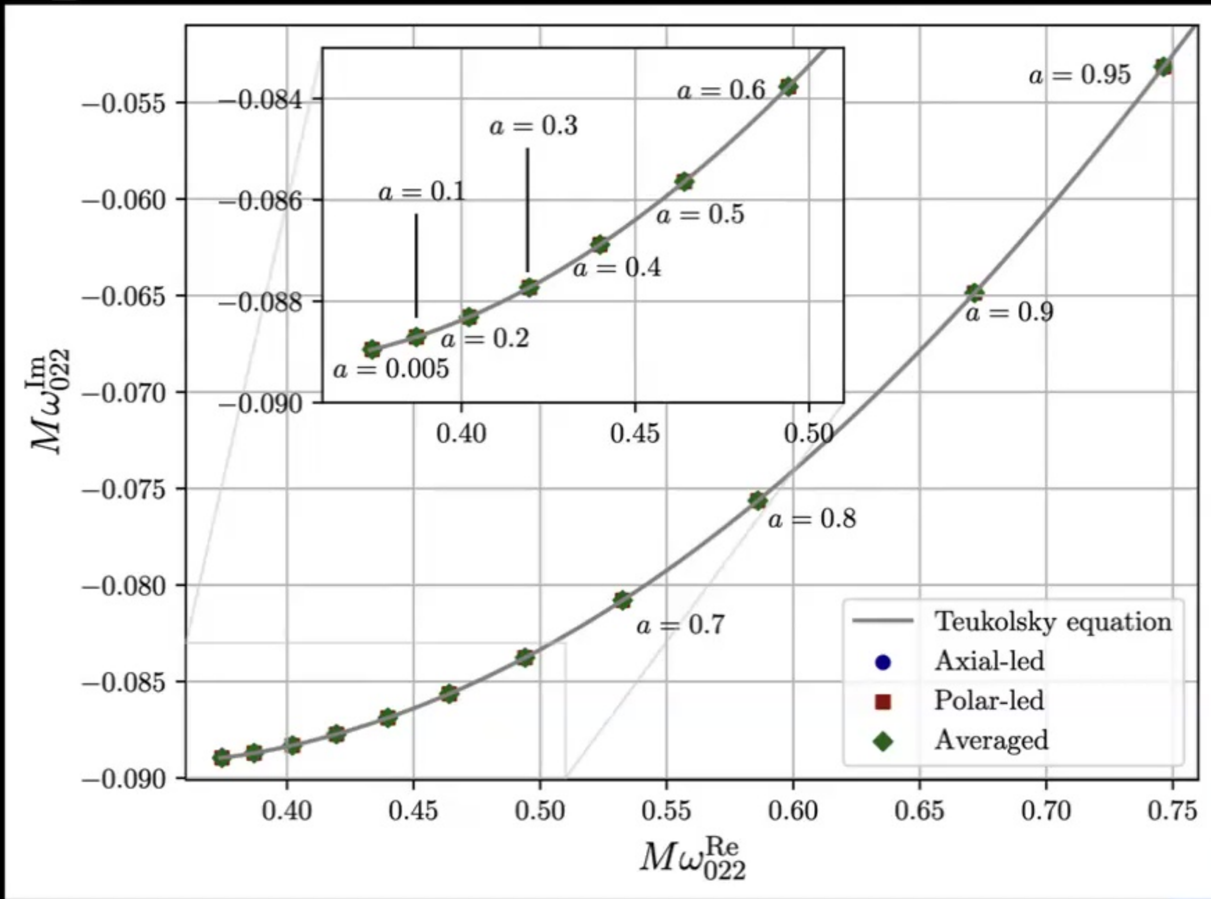
Linearized Einstein equations
How long are the equations?



14

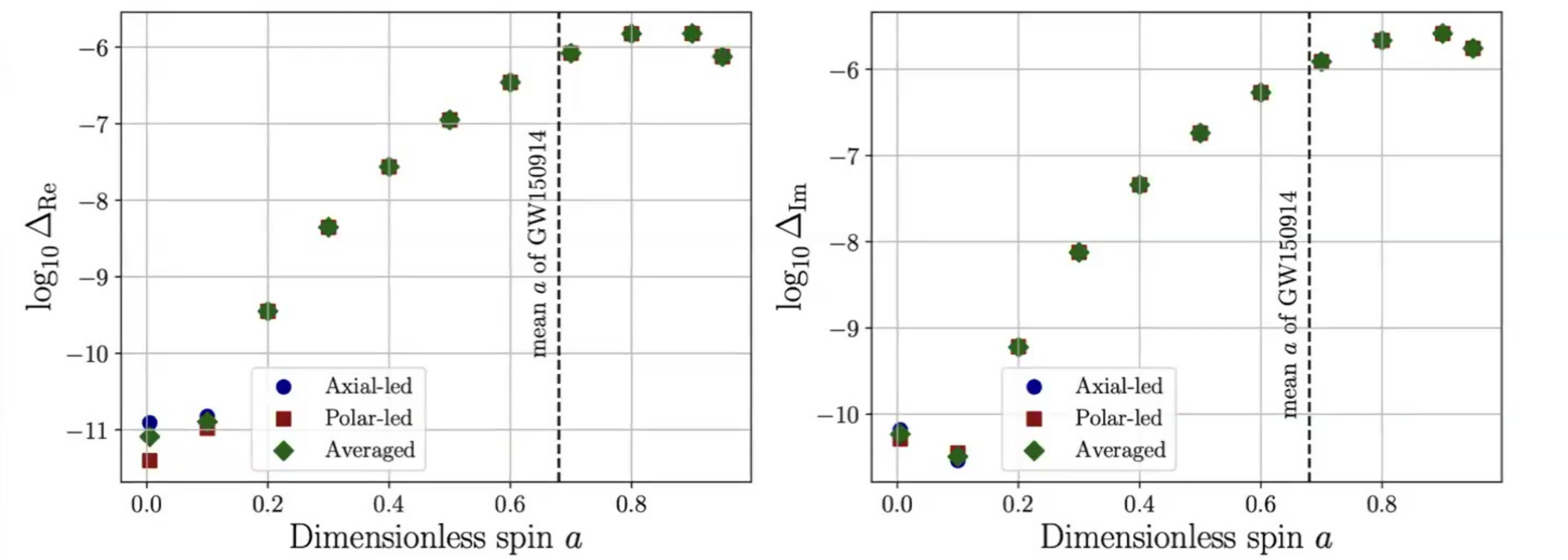
Numerical results
Complex plane



15

Numerical results

Relative error $\Delta_{Re/Im} = \left| \frac{\omega_{Re/Im}(Spec) - \omega_{Re/Im}(Teuk)}{\omega_{Re/Im}(Teuk)} \right|$



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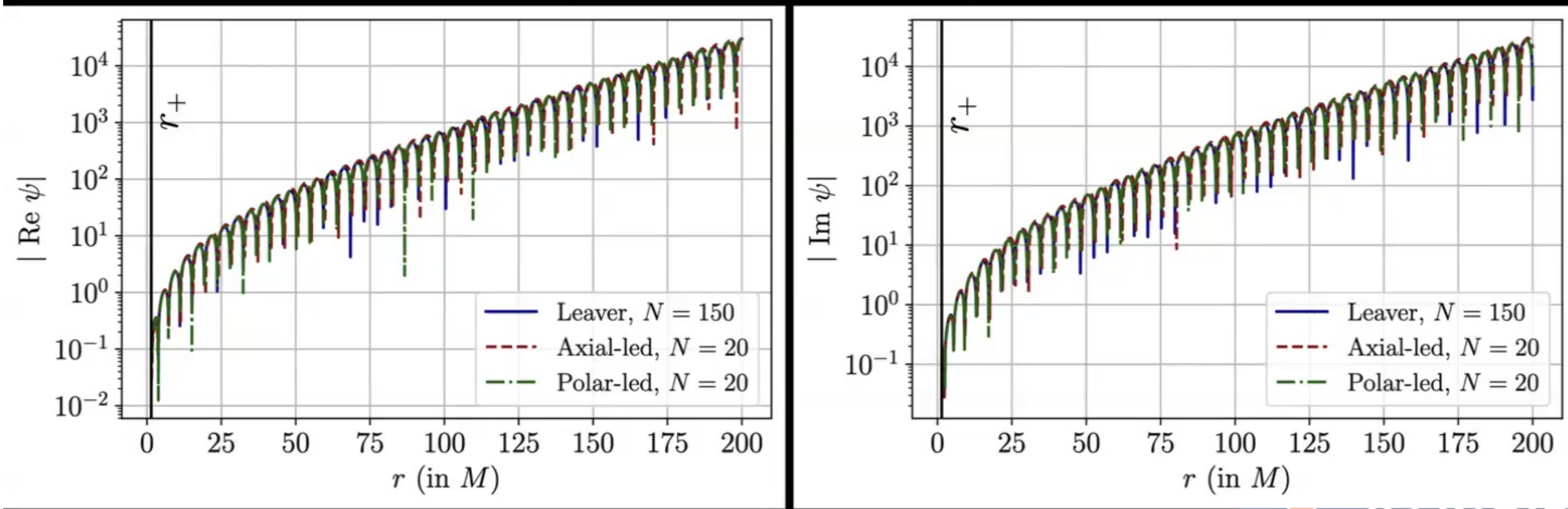
Numerical results
Metric reconstruction

- We compute the Teukolsky perturbation function for a Kerr BH of $a = 0.9$,

$$\psi = (r - iMa\cos\theta)^4\psi_4$$



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New lessons learnt about Kerr BH perturbations

- The Regge-Wheeler gauge is applicable for rapidly rotating Kerr BHs
- The associated Legendre polynomials can just be applied fine

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General relativity

In the unit of $c = G = 1$

$$R_{\mu}{}^{\nu} = 0$$

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Modified gravity

In the unit of $c = G = M = 1$, and write $\zeta = \alpha^2$

$$R_\mu{}^\nu + \zeta\left(\mathscr{A}_\mu{}^\nu - \frac{1}{2}\nabla_\mu\vartheta\,\nabla^\nu\vartheta\right) = 0,$$

$$\square\vartheta + \mathscr{A}_\vartheta = 0,$$

E.g. for dynamical Chern-Simons (dCS) gravity **[e.g. Alexander & Yunes 2009],**

$$\mathscr{A}^{\mu\nu} \equiv \left(\nabla_\sigma\vartheta\right)\epsilon^{\sigma\delta\alpha(\mu}\nabla_\alpha R^{\nu)}{}_\delta + \left(\nabla_\sigma\nabla_\delta\vartheta\right){}^*R^{\delta(\mu\nu)\sigma},$$

$$\mathscr{A}_\vartheta = \frac{1}{4}R_{\nu\mu\rho\sigma}{}^*R^{\mu\nu\rho\sigma} \quad \text{where} \quad {}^*R^{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon^{\rho\sigma\alpha\beta}R^{\mu\nu}{}_{\alpha\beta}$$

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In the unit of $c = G = M = 1$, and write $\zeta = \alpha^2$

$$R_\mu{}^\nu + \zeta \left(\mathscr{A}_\mu{}^\nu - \frac{1}{2} \nabla_\mu \vartheta \nabla^\nu \vartheta \right) = 0,$$

$$\Box \vartheta + \mathscr{A}_\vartheta = 0,$$

We can solve for $g_{\mu\nu}$ as $g_{\mu\nu} = g_{\mu\nu}^{(GR)} + \zeta g_{\mu\nu}^{(1)}$ [e.g. Yunes & Pretorius PRD 2009, Cano & Ruipérez JHEP 2019]

E.g. for Einstein scalar Gauss Bonnet (EsGB) gravity [Ripley & Pretorius CQG 2019]

$$\mathscr{A}_\mu{}^\nu \equiv \delta^{\nu\sigma\alpha\beta}_{\mu\lambda\gamma\delta} R^{\gamma\delta}{}_{\alpha\beta} \nabla^\lambda \nabla_\sigma \vartheta - \frac{1}{2} \delta_\mu{}^\nu \delta^{\eta\sigma\alpha\beta}_{\eta\lambda\gamma\delta} R^{\gamma\delta}{}_{\alpha\beta} \nabla^\lambda \nabla_\sigma \vartheta,$$

$$\mathscr{A}_\vartheta = R^2 - 4 R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

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Sketch (modified gravity)

$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$ ⟵ Construct an asymptotic factor $A(r)$

$\vartheta = \Phi + \vartheta^{(1)}$

↓

$$\begin{matrix} h_{\mu\nu} \\ \vartheta^{(1)} \end{matrix} = A(r) e^{-i\omega t + im\phi} \sum_{\ell=0}^{N} \sum_{n=0}^{N} a_{\mu\nu\,/\,\vartheta}(\ell, n) \times (\text{spectral function})_{\ell n}$$

↓

$$\begin{matrix} \left[E_\mu{}^\nu\right]^{(1)} \\ \left[E_\vartheta\right]^{(1)} \end{matrix} = A(r) e^{-i\omega t + im\phi} \sum_{\ell=0}^{N} \sum_{n=0}^{N} b_{\mu\nu\,/\,\vartheta}(\ell, n; \omega) \times (\text{spectral function})_{\ell n} = 0$$

E.g. in GR

$E_\mu{}^\nu := R_\mu{}^\nu = 0$

$E_\vartheta := \Box \vartheta = 0$

Orthogonality

↓

$$\underbrace{b_{\mu\nu\,/\,\vartheta}(\ell, n; \omega) = 0}_{\text{quadratic / higher degree in } \omega}$$

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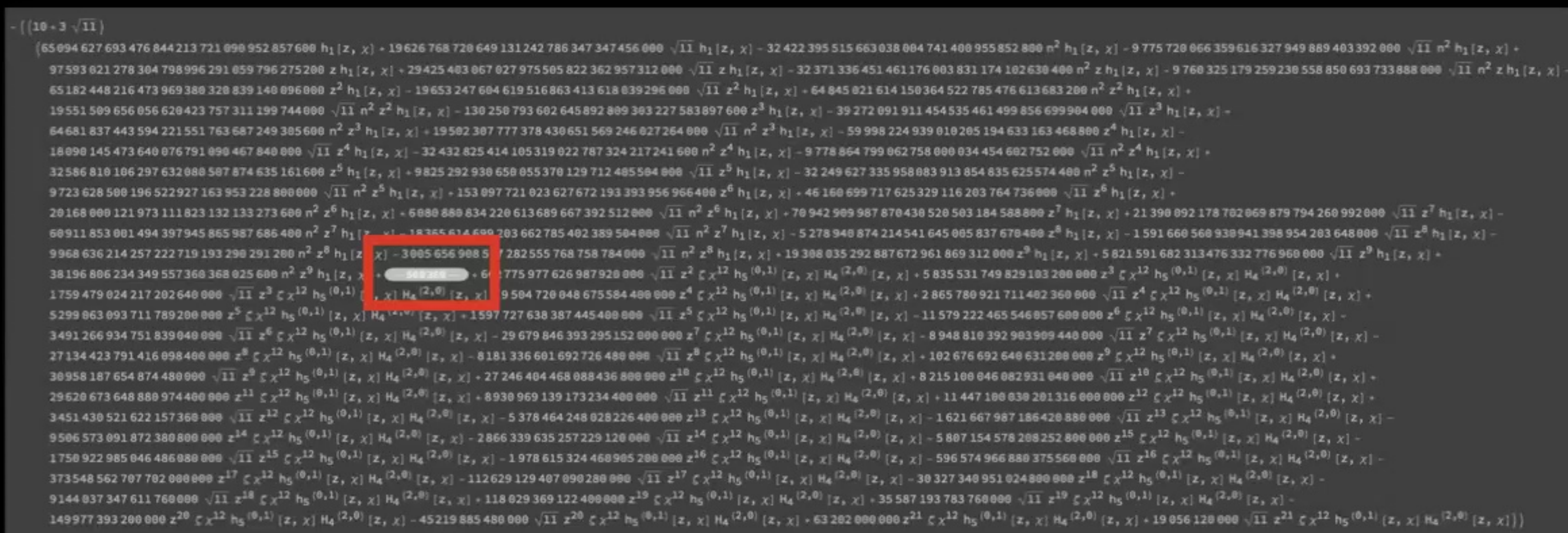
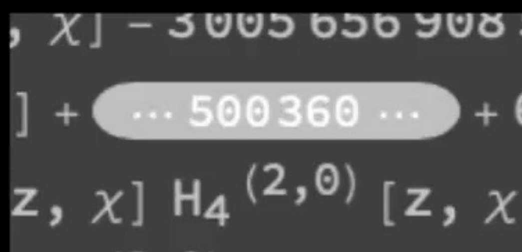
The length of linearized field equations just grows out of control





$\sim 10^6$ terms !

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Some technical details

- We solve ω as $\omega^{(GR)} + \zeta\omega^{(1)}$
- The Regge-Wheeler gauge is used
- At most using 25 spectral bases
- Newton-Raphson method for solving for the eigenvalues
- Metric modifications up to **40th order** in a are used for the computation

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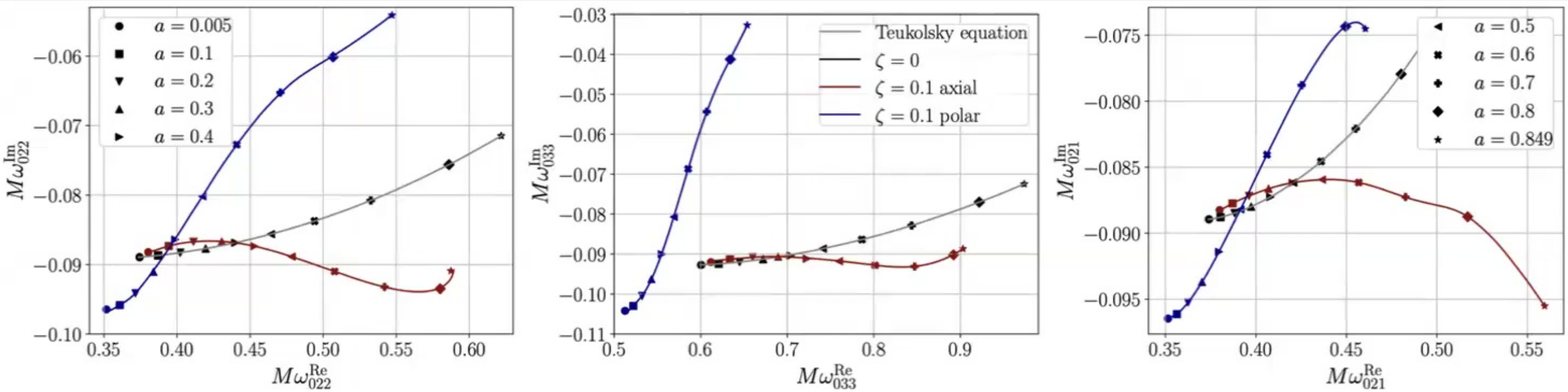
Quasinormal-mode spectra in scalar-Gauss-Bonnet gravity



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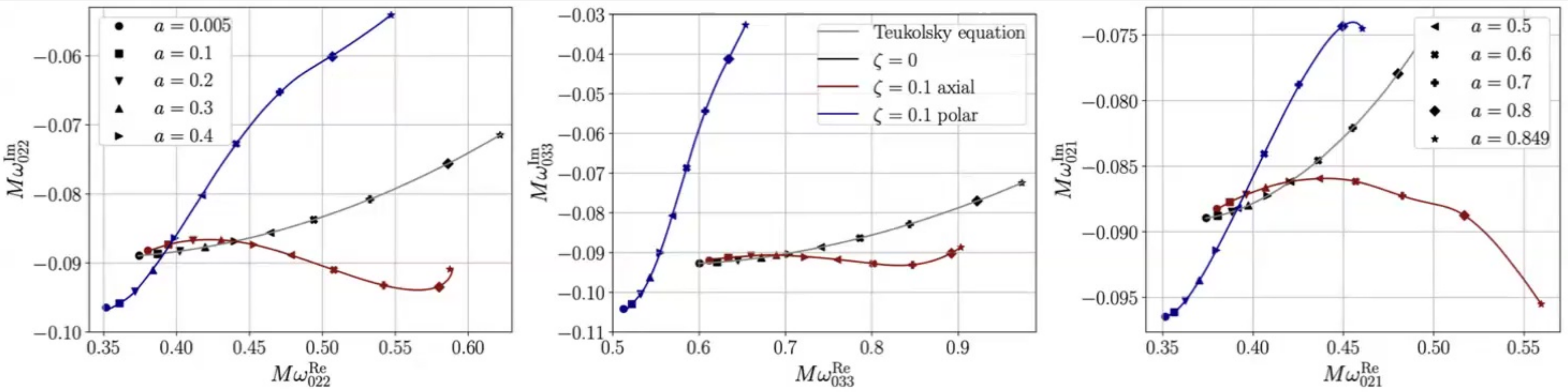

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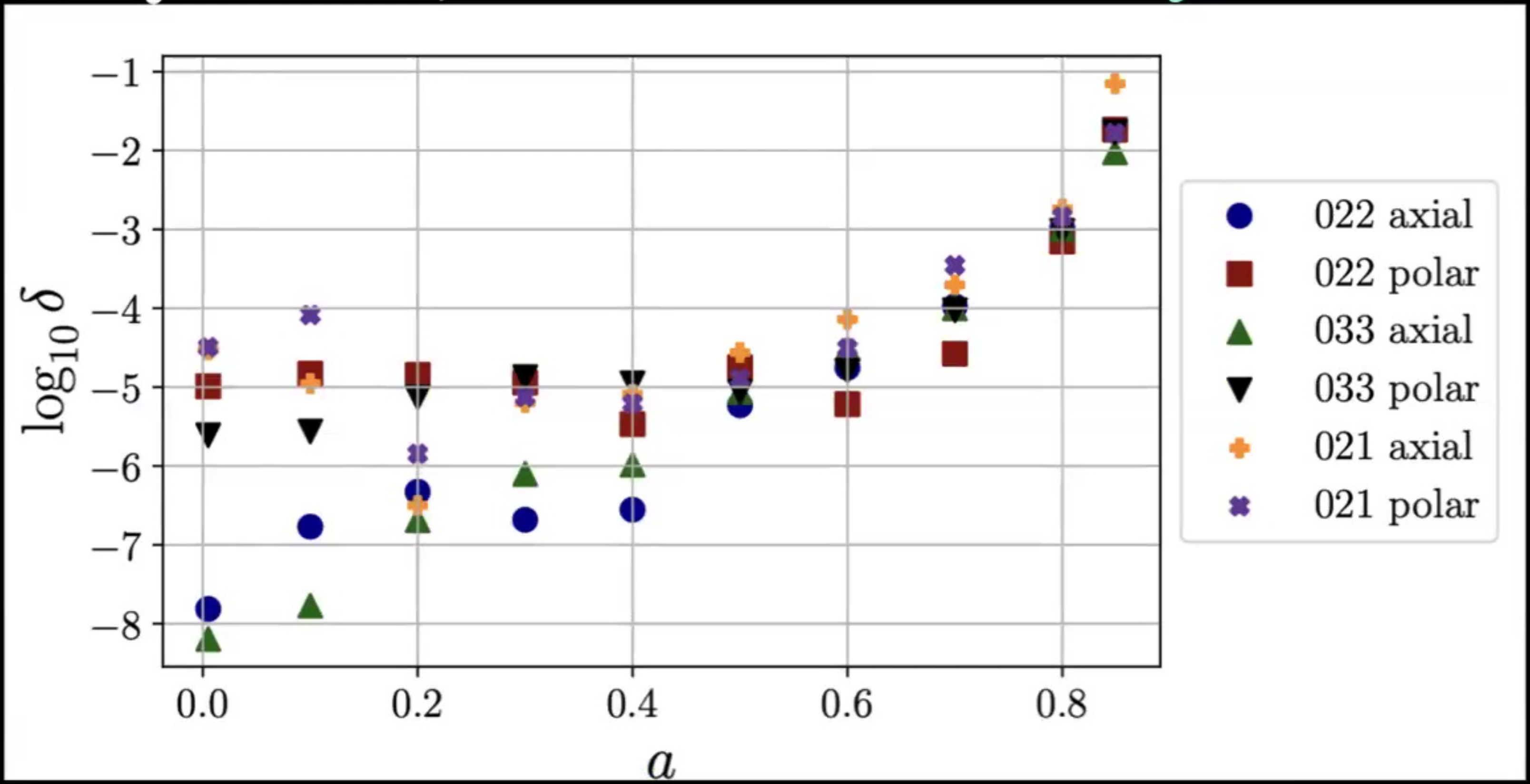
Accuracy of $\omega^{(1)}$, where $\omega = \omega^{(\mathrm{GR})} + \zeta\omega^{(1)}$



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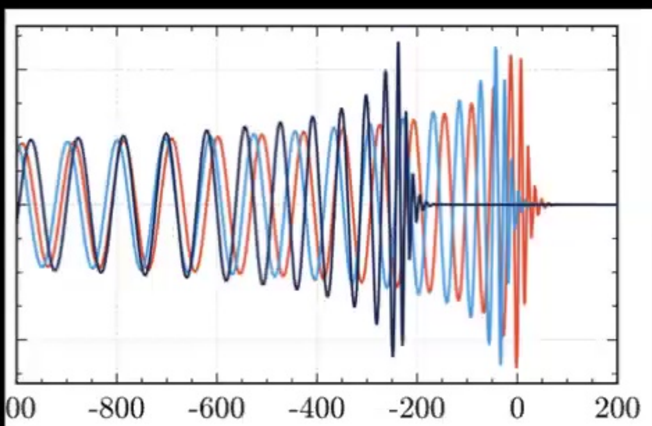
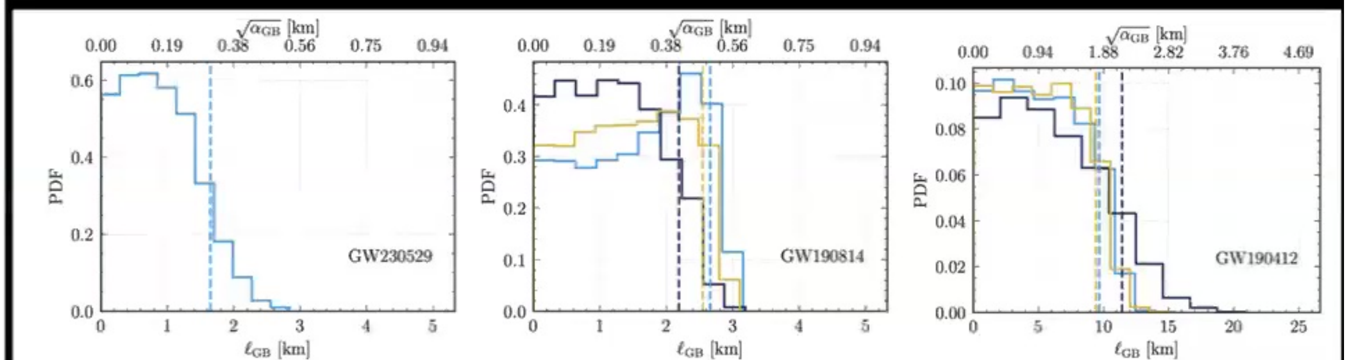
Application and extensions of our METRICS - EsGB results

- Gain insight into numerical-relativity simulations in EsGB gravity [Corman & East 2024]
- Model specific test of EdGB gravity with LIGO data [Julié, Pompili & Buonanno 2024]





- Applied METRICS to EdGB with strong coupling [Blázquez-Salcedo, Khoo, Kleihaus & Kunz 2024]

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Preliminary results in dynamical Chern-Simons gravity

- Lagrangian $\mathscr{L} = R + \zeta \vartheta R_{\alpha\beta\gamma\delta}\tilde{R}^{\alpha\beta\gamma\delta} - \frac{1}{2}\zeta \nabla_\mu \vartheta \nabla^\mu \vartheta$
- $\omega^{(1)}$ at $a = 0.00498$, uncertainty $< 10^{-4}$

nlm	$\omega_A^{(1)}$	$10^4 \times \omega_P^{(1)}$	$\omega_S^{(1)}$
022	$0.24958 + 0.12606i$	$0.2+2.9i$	$-0.58281 - 0.07866i$
033	$0.92112 + 0.16491i$	$0.6-4.5i$	$-1.41725 - 0.11278i$

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Possible future applications of METRICS

- QNMs of other modified gravity theories —> PE
- Results cross check with modified Teukolsky formalism?
  [c.f. e.g. D. Li et al PRX 2022, P. Wagle et al PRD 2024, etc ……]
- Waveform modeling of EMRIs, and self force calculations
  [c.f. e.g. P. Bourg et al arXiv:2403.12634, A. Pound and B. Wardell arXiv: 2101.04592, etc ……]
- Environmental effects/model-**<u>specific</u>** search for scalar charge?
  [c.f. e.g. Enrico Barausse, Vitor Cardoso, Paolo Pani PRD 2014, A. Maselli et al PRL 2021, Nat Ast. 2022]
- Rapidly rotating neutron star seismology?

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Summary

We develop METRICS which

- Can accurately compute BH QNM frequency,
- Can rapidly reconstruct metric perturbations,
- Can easily be adapted to a general/beyond GR black hole

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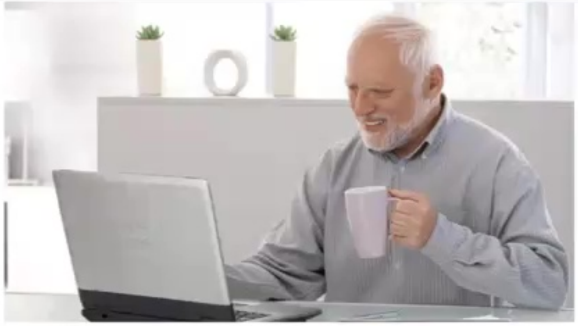
METRICS makes black-hole perturbations easier!



Start your research

We will be linearizing the following field equations [11],

$$R^{\mu}{}_{\nu} + \frac{\alpha}{\kappa_g} C^{\mu}{}_{\nu} - \frac{1}{2\kappa_g} \left[\bar{T}^{\vartheta}\right]^{\mu}{}_{\nu} = 0, \qquad (33)$$
$$\Box\tilde{\vartheta} + \frac{\alpha}{4} R_{\nu\mu\rho\sigma} {}^{*}R^{\mu\nu\rho\sigma} = 0,$$

where $\Box = \nabla_{\mu}\nabla^{\mu}$ is the d'Alembert operator,

$$C^{\mu}{}_{\nu} \equiv \left(\nabla_{\sigma}\tilde{\vartheta}\right) \epsilon^{\sigma\delta\alpha(\mu}\nabla_{\alpha}R_{\nu)\delta} + \left(\nabla_{\sigma}\nabla_{\delta}\tilde{\vartheta}\right) {}^{*}R^{\delta(\mu}{}_{\nu)}{}^{\sigma},$$
$$\left[\bar{T}^{\vartheta}\right]^{\mu}{}_{\nu} \equiv \left(\nabla^{\mu}\tilde{\vartheta}\right)\left(\nabla_{\nu}\tilde{\vartheta}\right).$$

Google a gravity theory

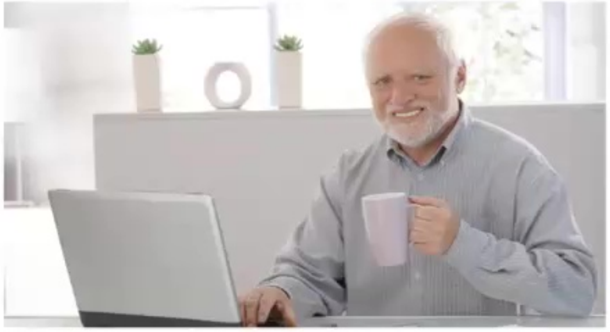
Spectral method for metric perturbations of black holes: Kerr background case in general relativity

Adrian Ka-Wai Chung,[1,*] Pratik Wagle,[1,2,†] and Nicolás Yunes[1]

[1] *Illinois Center for Advanced Studies of the Universe & Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[2] *Max Planck Institute for Gravitational Physics (Albert Einstein Institute), D-14476 Potsdam, Germany*
(Dated: December 15, 2023)

We present a novel approach, *Metric pErTuRbations with speCtral methodS* (METRICS), to calculate the gravitational metric perturbations and the quasinormal-mode frequencies of rotating black holes of any spin without decoupling the linearized field equations. We demonstrate the method by applying it to perturbations of Kerr black holes of any spin, simultaneously solving all ten linearized Einstein equations in the Regge-Wheeler gauge through purely algebraic methods and computing the fundamental (co-rotating) quadrupole mode frequency at various spins. We moreover show that the METRICS approach is accurate and precise, yielding (i) quasinormal mode frequencies that agree with Leaver's, continuous-fraction solution with a relative fractional error smaller than 10^{-5} for all dimensionless spins below up to 0.95, and (ii) metric perturbations that

Apply METRICS



Get QNM frequencies sometime later

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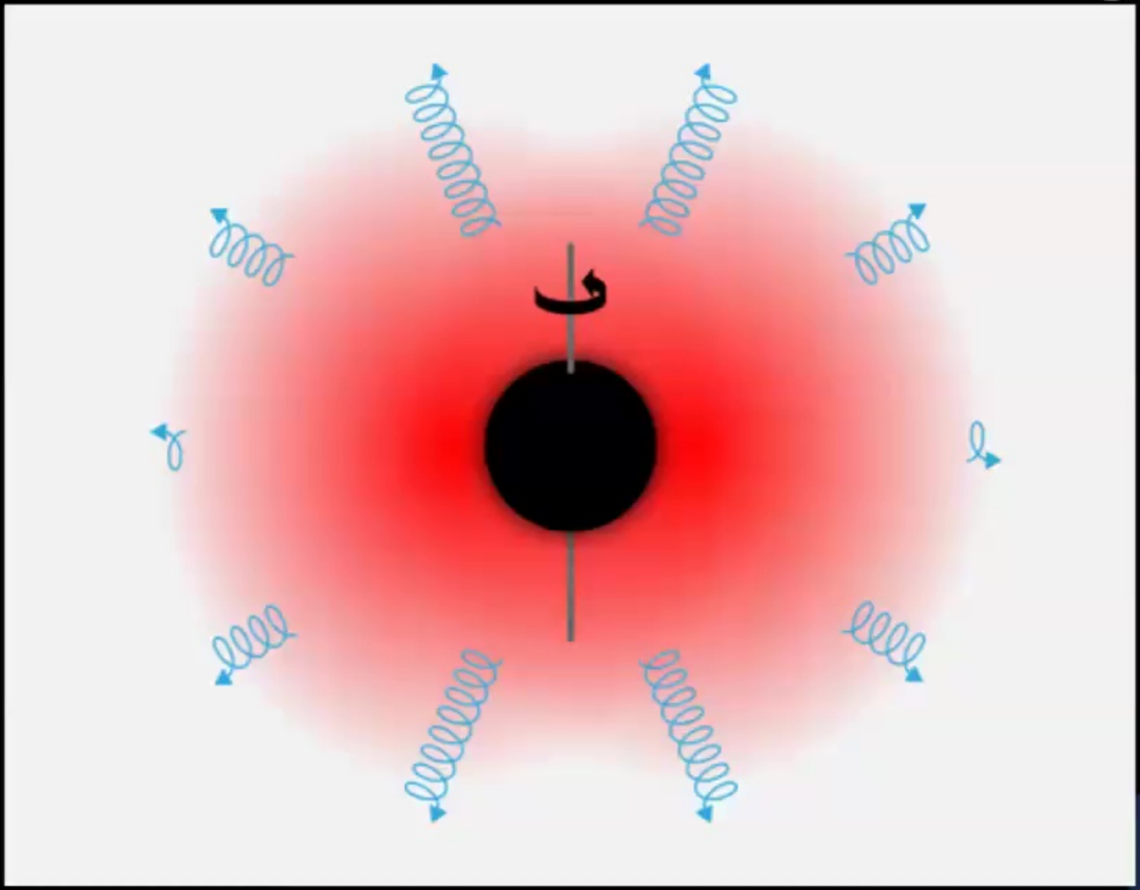
Why should we care about black-hole QNMs?



Picture credit: https://physics.aps.org/articles/v10/83

See, e.g., Chung et. al, Phys. Rev. D 104, 084028 (2021)

9

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