Title: Towards Anderson localisation of light by cold atoms

Speakers: Robin Kaiser

Collection/Series: Quantum Matter

Subject: Condensed Matter

Date: October 29, 2024 - 3:30 PM

URL: https://pirsa.org/24100132

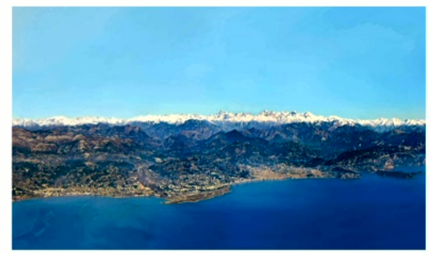
Abstract:

The quest for Anderson localization of light is at the center of many experimental and theoretical activities. Cold atoms have emerged as interesting quantum system to study coherent transport properties of light. Initial experiments have established that dilute samples with large optical thickness allow studying weak localization of light, which has been well described by a mesoscopic model. Recent experiments on light scattering with cold atoms have shown that Dicke super- or subradiance occurs in the same samples, a feature not captured by the traditional mesoscopic models. The use of a long range microscopic coupled dipole model allows to capture both the mesoscopic features of light scattering and Dicke super- and subradiance in the single photon limit. I will review experimental and theoretical state of the art on the possibility of Anderson localization of light by cold atoms.

Towards Anderson localisation of light by cold atoms

Robin KAISER

Nice, France



Quantum Seminar
Perimeter Institute Waterloo, Canada
October 29th 2024

Cold Atoms @ INPHYNI :

M. Hugbart

W. Guerin

G. Labeyrie

R. Saint-Jalm

A. Glicenstein, D. Benedicto, A. Apoorva, N. Matias
A. Kastberg, R. Caldani, M. Morisse, S. Asselie, J.M. Nazon, M. Biscassi, A. Gabteni, S. Tolila, A. Deo

Collaborations :
R. Bachelard, A. Cidrim, F. Pinheiro, P. Courteille, R. Teixeira,
L. Celardo, V. Viggiano, P. Facchi, S. Pascazio, L. Slodicka, A. Picozzi,
J. P. Rivet, F. Vakili, J. Chabé, C. Courde, O. Lai,

Multiple Scattering of Light in Atomic samples : Disorder vs cooperative effects

Anderson Localization

Multiple Scattering

Dicke States

"Local"

"Global"

Interferences

3

The Anderson Paper

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

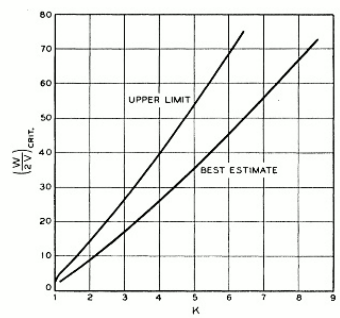
$$i\dot{a}_j = E_j a_j + \sum_{k\neq j} V_{jk} a_k.$$

'diagonal disorder' : W

Our fundamental theorem may be restated as: if $V(r_{jk})$ falls off at large distances faster than $1/r^3$, and if the average value of V is less than a certain critical V_c of the order of magnitude of W; then there is actually no transport at all, in the sense that even as $t\to\infty$ the amplitude of the wave function around site n falls off rapidly with distance, the amplitude on site n itself remaining finite.

'short range hoping'

Localized



Extended

4

K: connectivity / number of neighbours

Dimension $\rightarrow$

Ioffe-Regel paper

Mott papers

NON-CRYSTALLINE, AMORPHOUS, AND LIQUID ELECTRONIC SEMICONDUCTORS

A. F. IOFFE, Dr. Phys., Academician
Director, Institute for Semiconductors, Leningrad, U.S.S.R.
and
A. R. REGEL, Dr. Phys.
Vice-director, Institute for Semiconductors, Leningrad, U.S.S.R.

Reprinted from
PROGRESS IN SEMICONDUCTORS-4
LONDON : HEYWOOD & COMPANY LTD.
1960

7 . 10^{-7} cm. It appears that for all semiconductors with mobility

$$\mu < 100 \text{ cm}^2\,\text{V}^{-1}\,\text{sec}^{-1}, \qquad L < \lambda$$

and for

$$\mu < 5 \text{ cm}^2\,\text{V}^{-1}\,\text{sec}^{-1}, \qquad L < a$$

However, the free forward motion with a mean velocity v occurs only over distances L. It is clear, therefore, that for all semiconductors with mobility less than 100 cm^2 V^{-1} sec^{-1}, the concept of velocity of the charge carriers loses its meaning.

Ioffe-Regel criterion for localization …

" $k\ell \approx 1$ "

N. F. Mott and R. S. Allgaier: Localized States in Disordered Lattices 343

phys. stat. sol. 21, 343 (1967)
Subject classification: 13.4; 2

Cavendish Laboratory, University of Cambridge (a), and U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland 20910 (b)

Localized States in Disordered Lattices

By
N. F. Mott (a) and R. S. Allgaier (b)

We have shown that localized states may be produced:
a) by random electric fields as in Anderson's [9] and in Miller and Abrahams' [5] work on impurity-band conduction and
b) by fluctuations in density, which produce a wavy bottom to the conduction band since the energies of the extremities of a band are functions of volume.

an argument suggesting that it may be roughly right is the following. We suppose that a mean free path L such that $k\,L < 1$ is impossible, and that an interaction so strong that perturbation theory gives L below $1/k$ means in fact that ψ is localized. We then calculate L from Ziman's formalism and find what value

Regime of density fluctuations :
Mean free path important

" $W > \Omega$ "

5

Thouless papers

Link between boundary conditions (transport properties) and eigenstate statistics

J. Phys. C: Solid State Phys., Vol. 5, 1972. Printed in Great Britain

Numerical studies of localization in disordered systems

J T EDWARDS and D J THOULESS
Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT

The calculations of the effect of boundary conditions on the energy levels provide clear evidence for a transition from extended to localized states in the square lattice at a value of W/V of the order of 5 or 6, which is less than that predicted by Anderson (1958) by a factor of five. This method provides a sensitive test of localization, but does not disagree with other tests of localization.

The three dimensional results suggest a critical value of W/V larger by a factor of two or more, but the transition is less clear than in the two dimensional case. There may exist an intermediate region in which the states are not localized, but not adequately described in terms of weakly coupled plane waves.

or

$$\frac{V}{W} > \frac{N\Delta E}{2W}$$

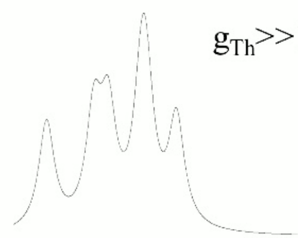
$$N\Delta E < 2V$$

$g_{\mathrm{Th}} \ll 1$

Level width

$$g_{\mathrm{Th}} = \frac{\Delta E}{\delta W}$$

Level spacing



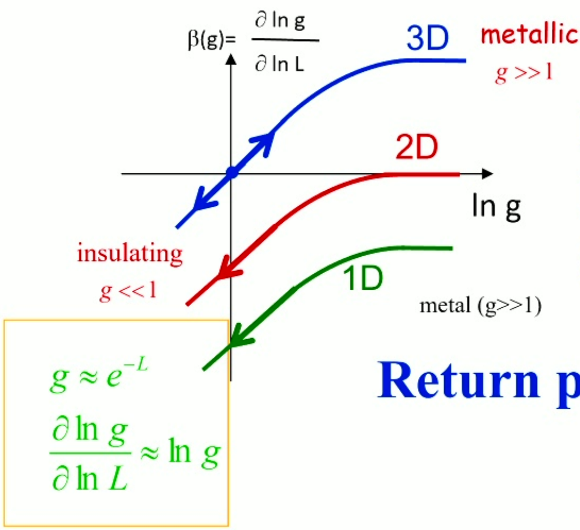
$g_{\mathrm{Th}} \gg 1$

6

Anderson Localization of non interacting waves in 1,2 and 3D

- **Scaling theory of localization :** Abrahams et al., PRL **42**, 673 (1979)

g : dimensionless conductance



In 3D : threshold for disorder
In 1&2D : all states are localized
(in infinite system, ε disorder)

Return probability needed for interference

$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|)$$

- **No microscopic theory**
  self consistent theory of localization,
  numerical simulations of toy systems

$W_{cr} \sim 16.5$

7

Localisation of light : theory in the 80’

John

VOLUME 53, NUMBER 22 PHYSICAL REVIEW LETTERS 26 NOVEMBER 1984

Electromagnetic Absorption in a Disordered Medium near a Photon Mobility Edge

Sajeev John[(a)]
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 30 July 1984)

A frequency regime in which electromagnetic waves in a strongly disordered medium undergo Anderson localization in $d = 3$ dimensions is suggested. In the presence of weak dissipation in $d = 2 + \epsilon$ it is shown that the renormalized energy absorption coefficient increases as the photon frequency ω approaches a mobility edge ω^* from the conducting side as $\alpha \sim (\omega^* - \omega)^{-(d-2)\nu/2}$, $\nu = 1/\epsilon$. This mobility edge occurs at a frequency compatible with the Ioffe-Regel condition.

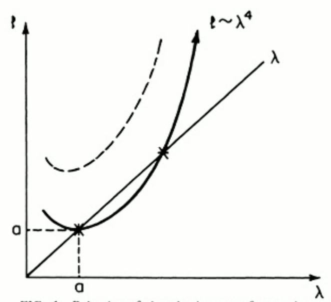


FIG. 1. Behavior of the elastic mean free path as a function of wavelength. In the long-wavelength Rayleigh-scattering limit $l \sim \lambda^4$. In the short-wavelength limit, $l \gtrsim a$, the correlation length. For a strongly scattering disordered medium (solid curve) there may exist a range of wavelengths for which $2\pi l/\lambda \simeq 1$, exhibiting weak localization. This would not occur in a dilute impurity limit (dashed curve).

Anderson

PHILOSOPHICAL MAGAZINE B, 1985, VOL. 52, NO. 3, 505–509

The question of classical localization
A theory of white paint?

By PHILIP W. ANDERSON
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544, U.S.A.

[*Received 26 January 1985 and accepted 4 March 1985*]

ABSTRACT
The expected behaviour of localizing media for classical wave propagation is analysed. Some possible examples in electromagnetic and acoustic phenomena are given.

† Many of the ideas in the present paper were also suggested in this excellent paper, but there is at least one major difference: we find that absorption (as measured by the reflection coefficient) *decreases* near the mobility edge, in contradiction to this reference.

Sornette

EUROPHYSICS LETTERS 1 October 1988
Europhys. Lett., 7 (3), pp. 269-274 (1988)

Strong Localization of Waves by Internal Resonances.

D. SORNETTE (*)(**) and B. SOUILLARD (**)

Electromagnetic waves in the presence of electromagnetic resonators. – An analogous discussion can be pursued for classical electromagnetic resonators; we do not detail it here and simply adress the case of quantum resonators. Exemples are systems of atoms or molecules excited near a resonant frequency corresponding to the *virtual* transition between two levels, … . Such systems exhibit a characteristic resonant frequency ω_0 and the corresponding resonant wavelength λ_0 can be very large and commonly $\lambda_0 \approx 1\,\mu$m. In

EUROPHYSICS LETTERS 1 July 1991
Europhys. Lett., 15 (5), pp. 535-540 (1991)

Effect of Resonant Scattering on Localization of Waves.

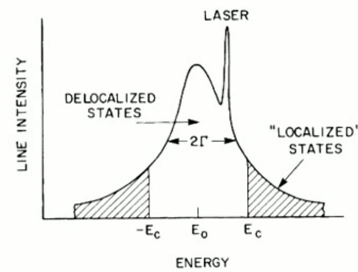
B. A. VAN TIGGELEN (*), A. LAGENDIJK (*)(**), A. TIP (*) and G. F. REITER (***)

overlapping must be considerable. Our results disagree with the work of Sornette and Souillard [18], who maximalized the amount of scattering, but did not distinguish between the *individual* and *collective* amount of scattering.

Localisation of light : Early Experiments

Anderson Localization and Mobility Edges in Ruby

J. Koo, L. R. Walker, and S. Geschwind
Phys. Rev. Lett. **35**, 1669 – Published 15 December 1975



Trap (pairs) emission as probe

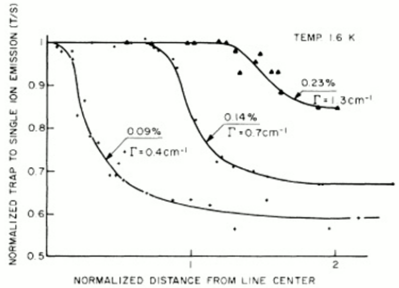


FIG. 2. Normalized trap (N_2 line) to single-ion emission (R_1 line) in ruby as a function of laser excitation in different regions of the R_1 line. The observed breaks suggest mobility edges separating delocalized states in the central region from the localized states beyond the break.

Energy Transfer and Anderson Localization in Ruby

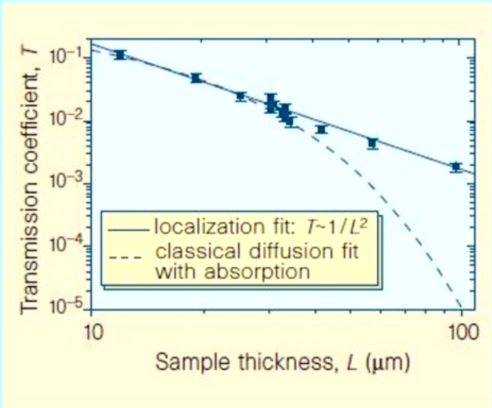
S. Chu, H. M. Gibbs, S. L. McCall, and A. Passner
Phys. Rev. Lett. **45**, 1715 – Published 24 November 1980

We have directly observed resonant nonradiative energy transfer in ruby,[7] and find that the ion-ion transfer is much slower than previously believed. This fact eliminates the possibility of observing an Anderson transition in ruby as reported by Koo, Walker, and Geschwind.[3] The conflict

Anderson Localization of Light in 3D :

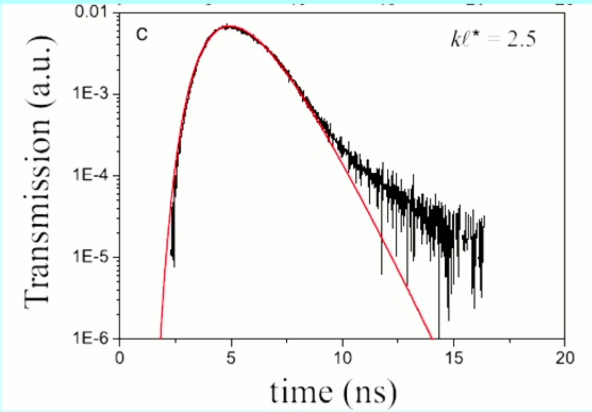
phase transition ⇒ strong scattering required

Semi-conductor powder



D.Wiersma et al., Nature 1997

White Paint



C.Aegerter et al., EPL 2006

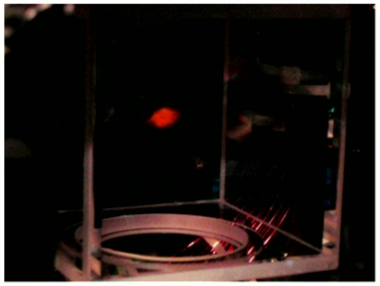
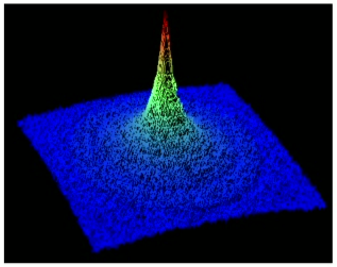
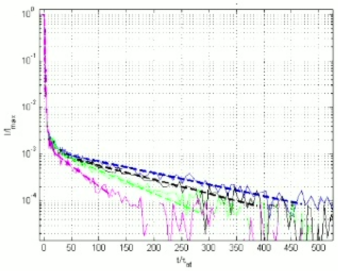
F. Scheffold et al., Nature 398, 206(1999)
T. v. der Beek et al., PRB 85 115401 (2012)

F. Scheffold et al., Nat. Photon. 7, 934 (2013)
T Sperling et al., New J. Phys. 18, 013039 (2016)

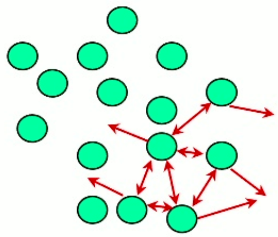
=> Not observed so far

10

Using atoms as scattering medium
Previous work


+

+


Photons ...



Random walk :

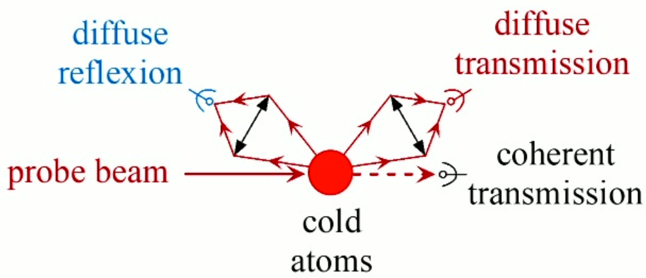
Diffusion

coefficient $D_0 \approx \ell^2 / \tau$

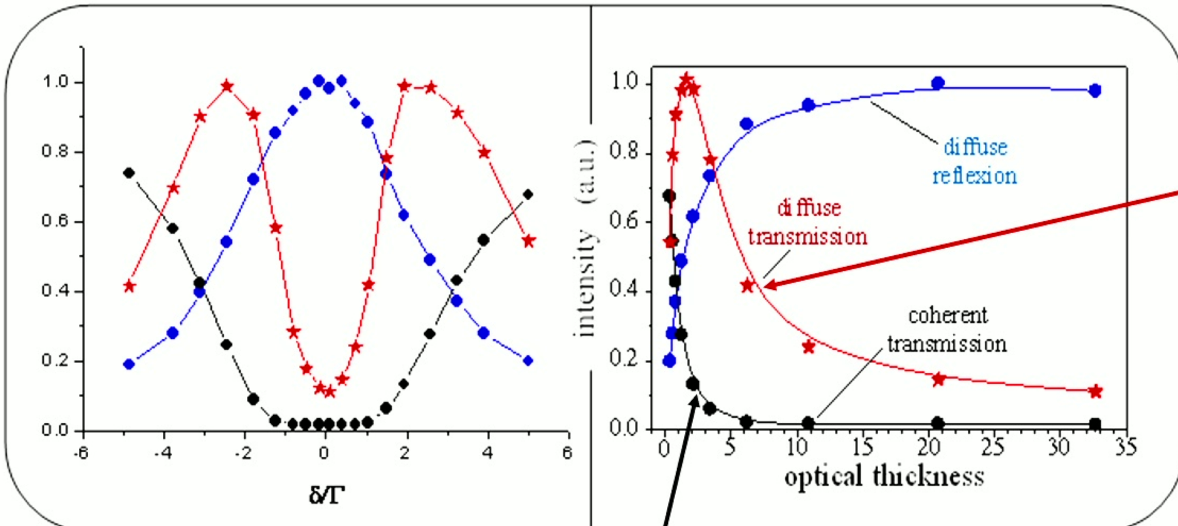
$\ell = 1/n\,\sigma$

12

Scattering Experiments with Cold Atomic



MOT parameters :
- MOT diameter ≈ few mm
- $N \approx$ few 10^{11}
- $b \approx 200$
- velocities ≈ 0.1m/s



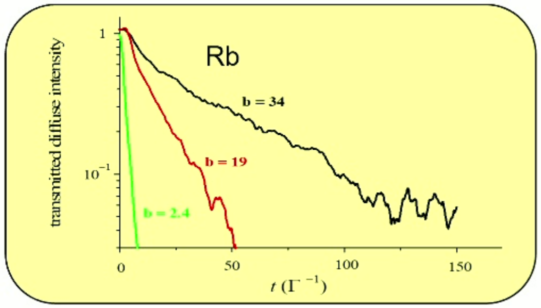
Ohm's law for photons

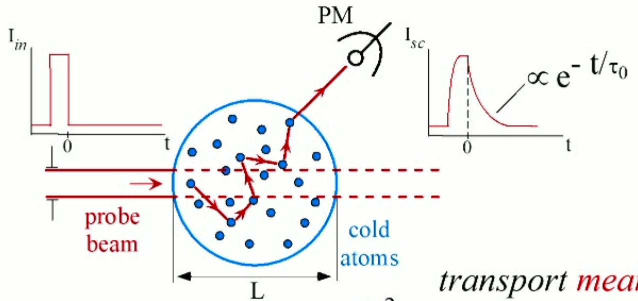
$\boxed{T_{diff} \propto 1/L}$

13

Beer-Lambert law : $T_{coh} \propto e^{-L/\ell}$

Time Resolved Experiments





$$D = \frac{1}{3}\frac{l_{tr}^2}{\tau_{tr}}$$

transport mean-free path

transport time

$$= \frac{1}{3} l_{tr} v_{tr}$$

transport velocity

$$\frac{v_{tr}}{c_0} = \frac{\ell_{tr}}{c_0 \tau_{tr}} \approx 3 \cdot 10^{-5}$$

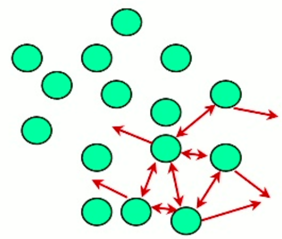
$$\tau_0 \approx \frac{L^2}{\pi^2 D} \Rightarrow \boxed{D \approx 0.66 m^2/s}$$

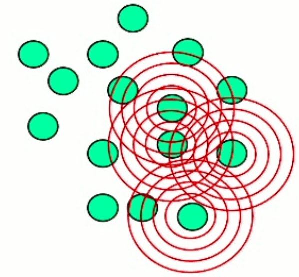
NO interference effect !
$\neq$ Localization

14

Phys. Rev. Lett. **91**, 223904 (2003)

Photons ... ... are waves





Random walk : **Diffusion** coefficient $D_0 \approx \ell^2 / \tau$

$\ell = 1/n\,\sigma$

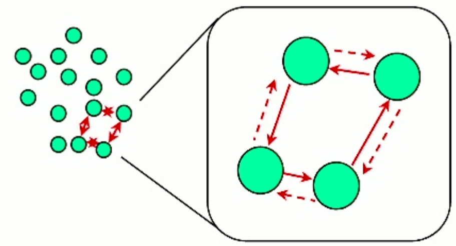
Interference correction to Diffusion coefficient

$D \approx D_0\,[1 - 1/(k\ell)^2]$

Strong Localization (D=0) :

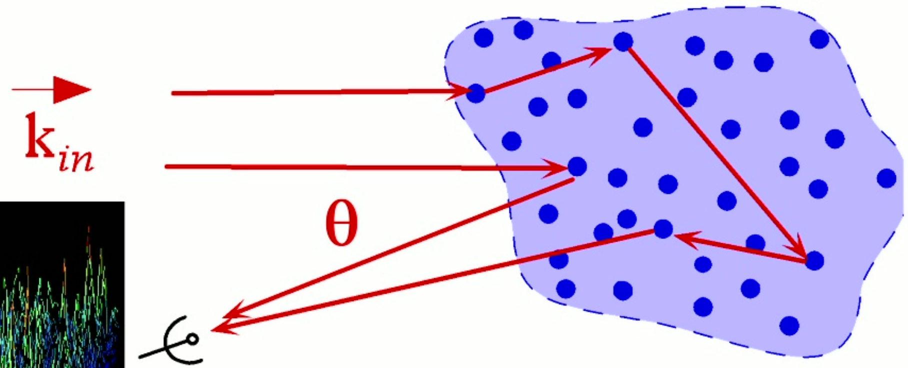
Ioffe-Regel criterium : $k\ell \approx 1$

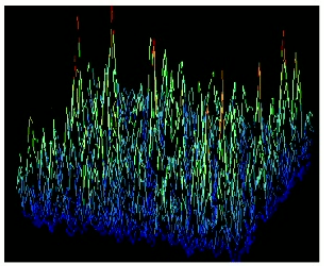
(near field scattering $\ell \approx$ ⍰)



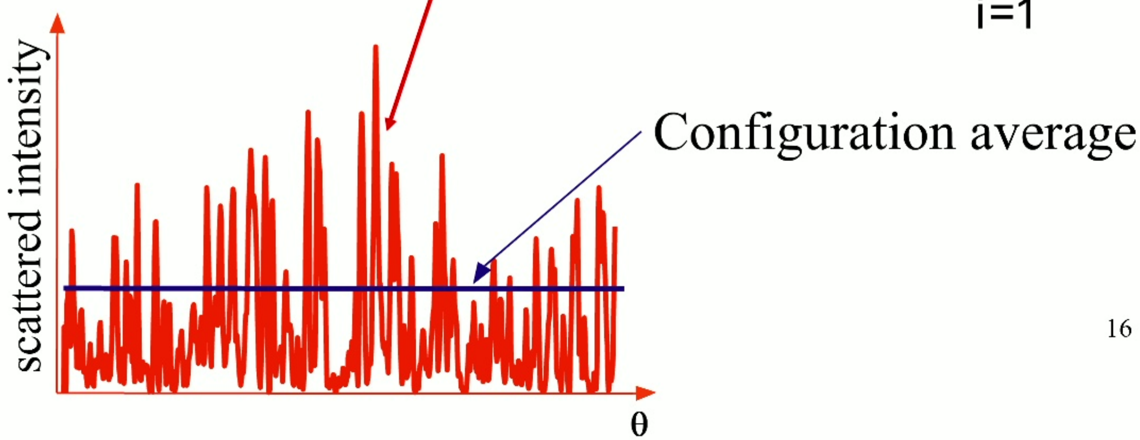
15

Wave effects : Interferences and speckle





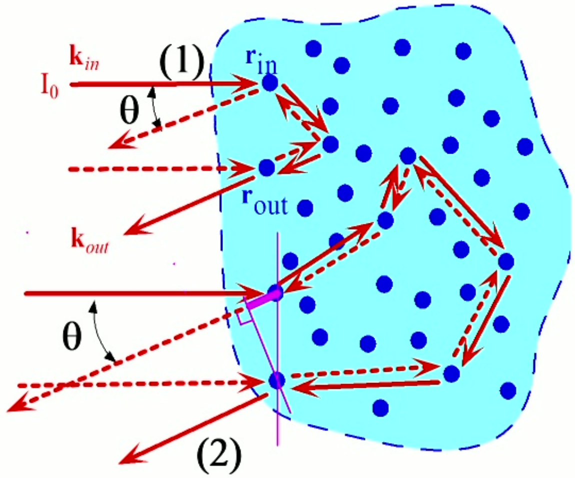
fixed scatterers : speckle pattern $\vec{E} = \sum_{i=1}^{N} \vec{E}_i$



Configuration Averaged Intensity

· **uncorrelated** paths add incoherently

· **correlated** (i.e. reciprocal) paths add coherently



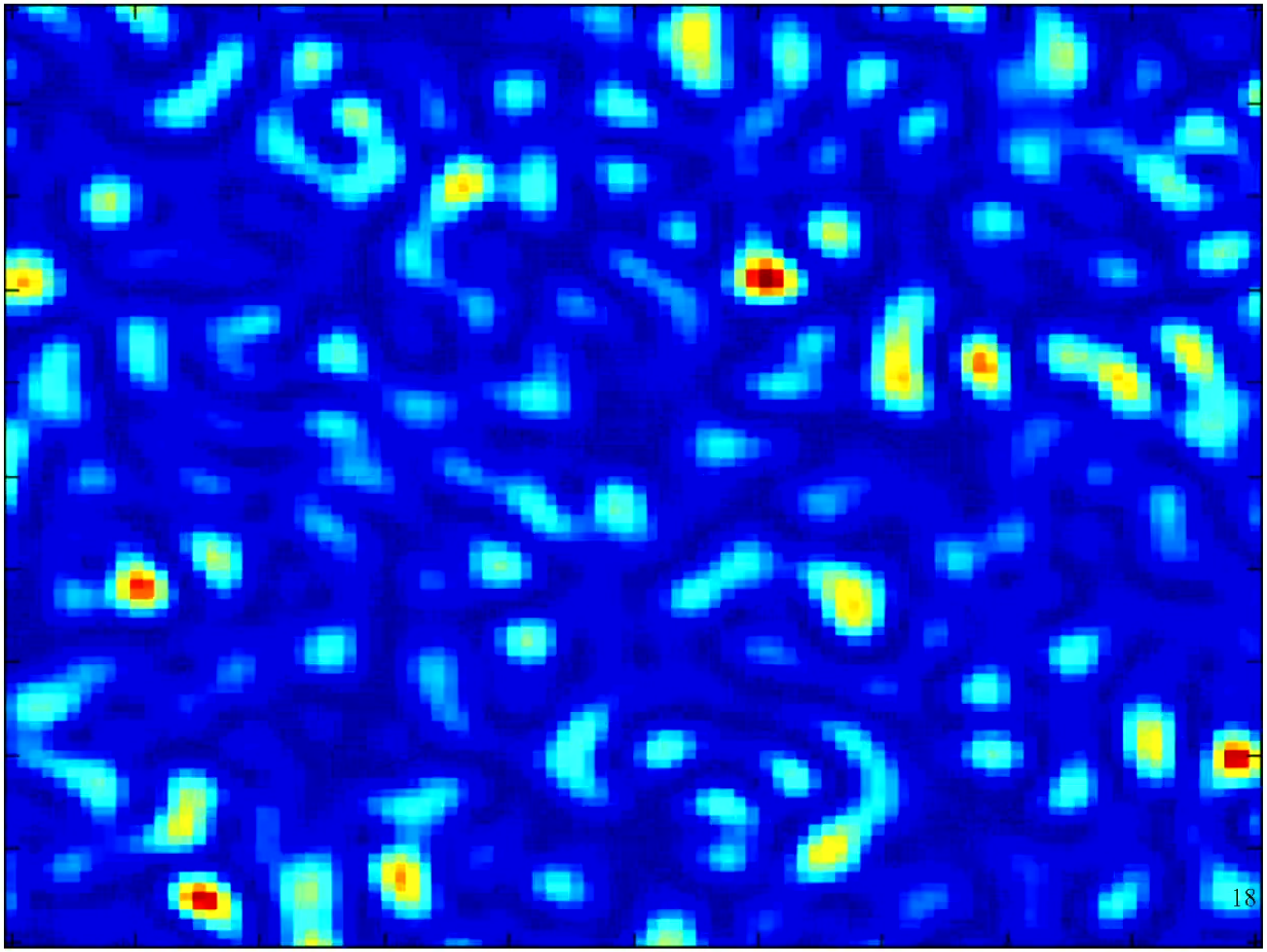
$\Delta\varphi=(k_{in}+k_{out}).(r_{in}-r_{out})$ $\theta=0 \Rightarrow \Delta\varphi = 0$ for any path

Coherent Backscattering $\dfrac{\langle I(0)\rangle}{\langle I(\theta)\rangle} = 2$

17

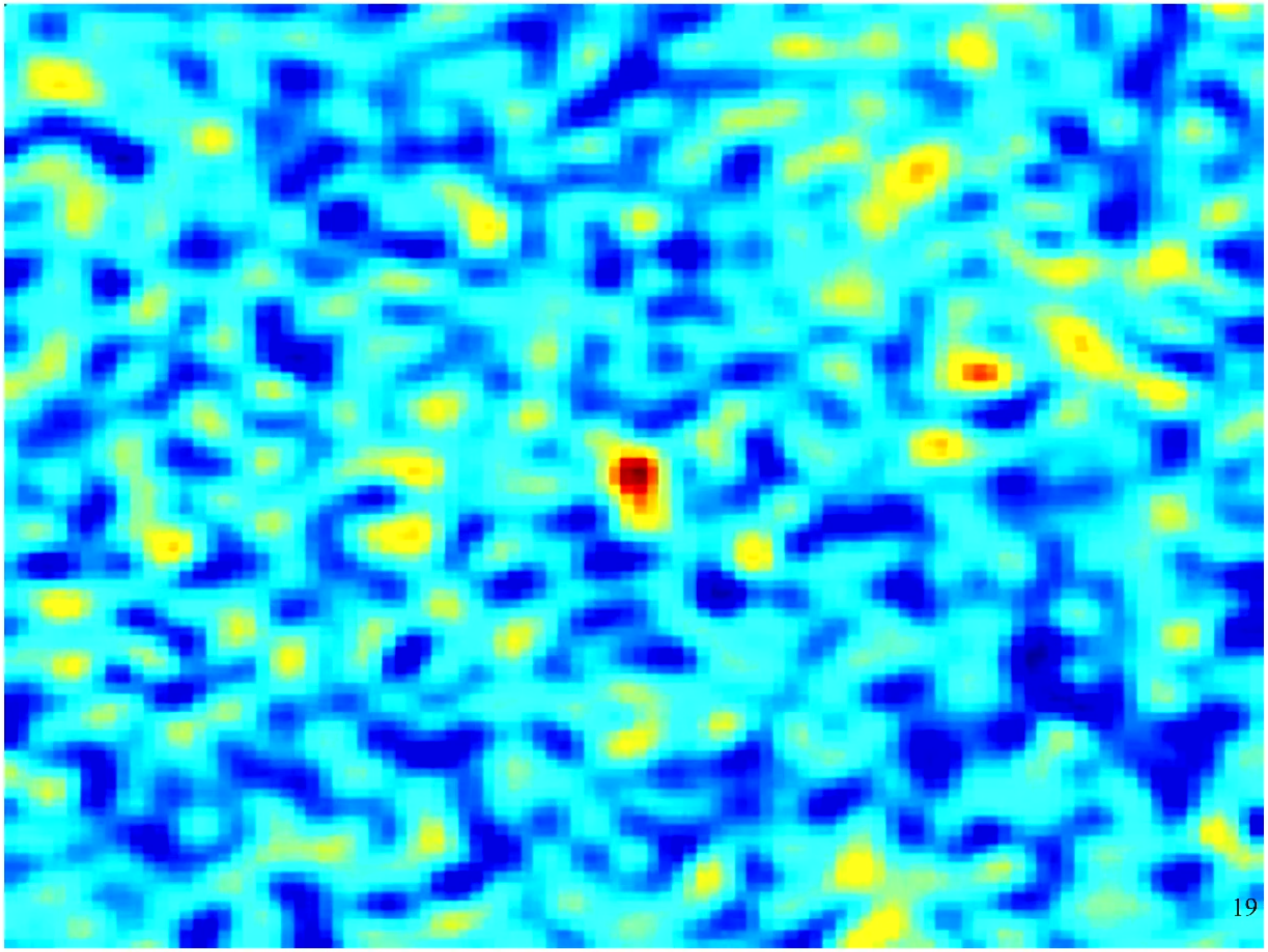
multiple self-aligned Sagnac interferometer

Fluctuating Speckle Pattern



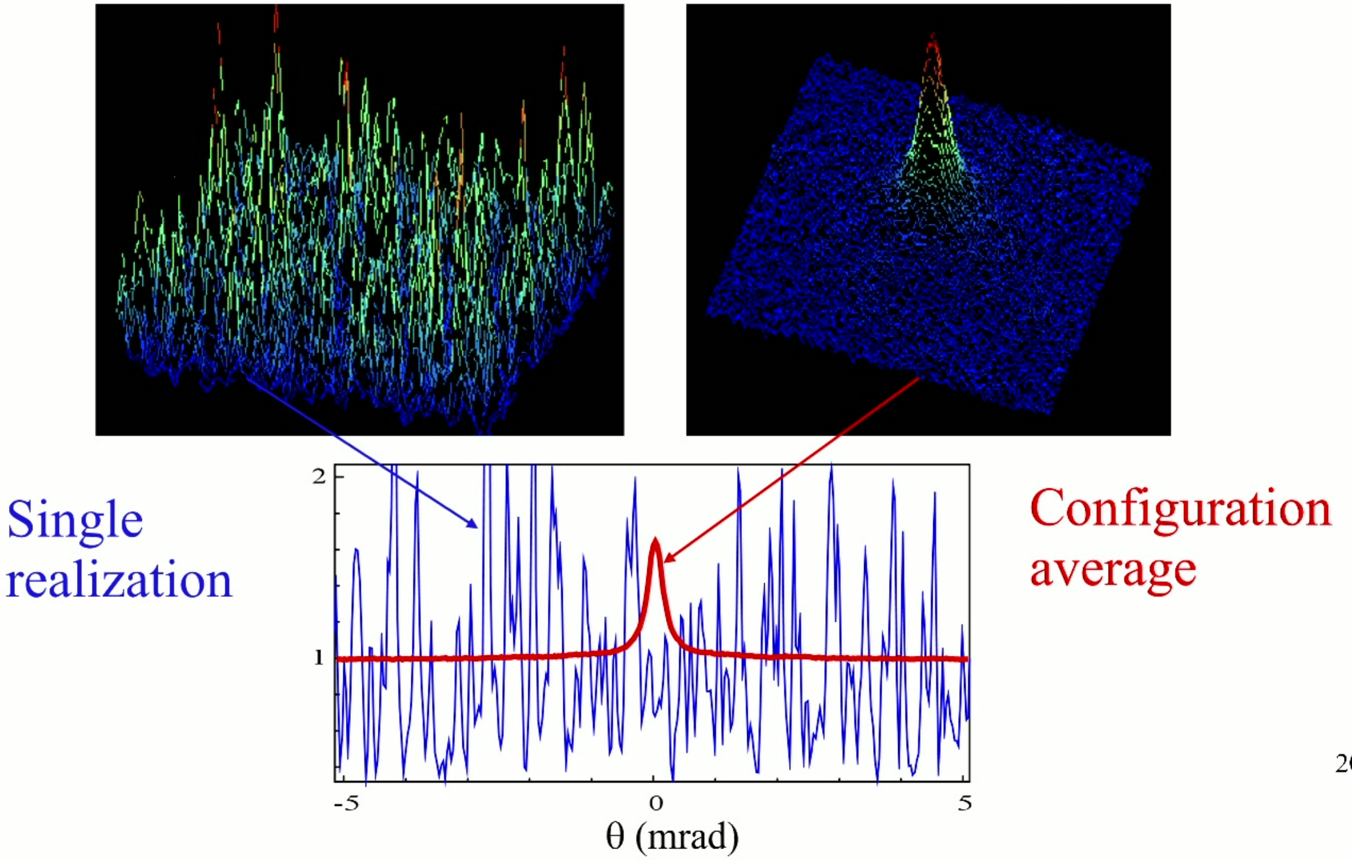
speckle

Integrated signal (configuration average)



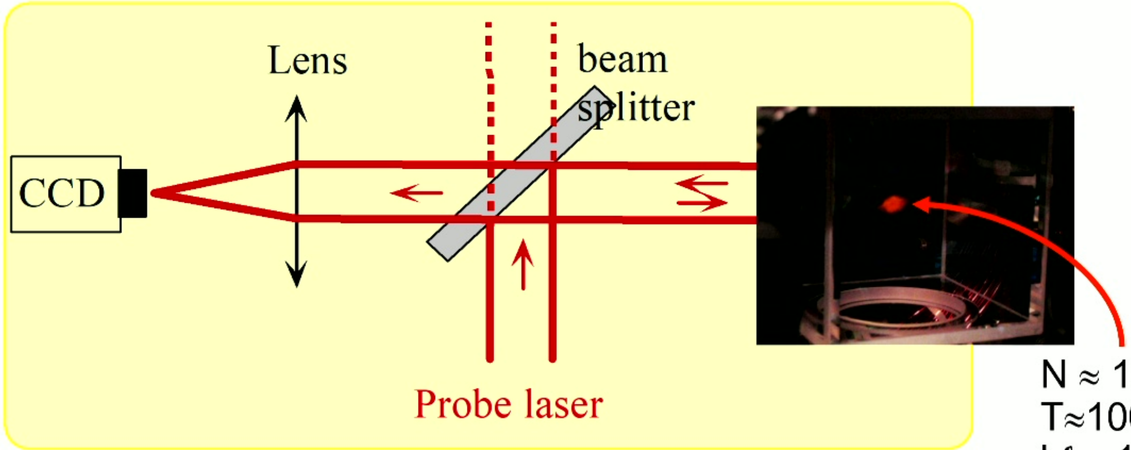
cone

Configuration Average

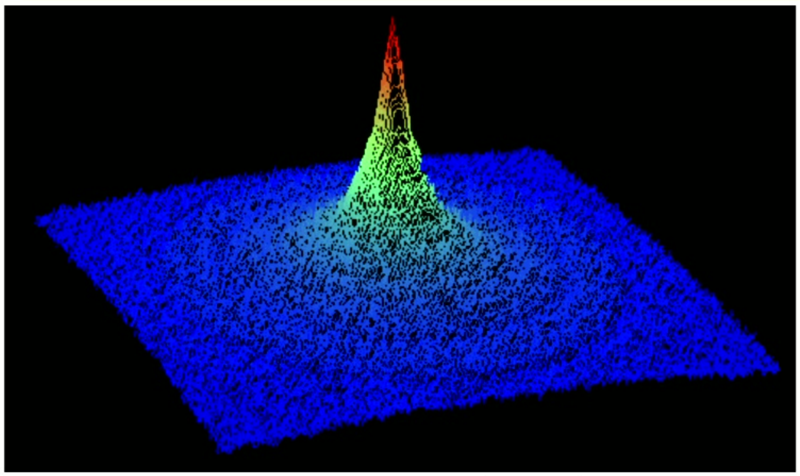


20

Weak Localisation = precursor of strong Localisation?



$N \approx 10^{10}$
$T \approx 100\mu K$
$k\ell \approx 1000$



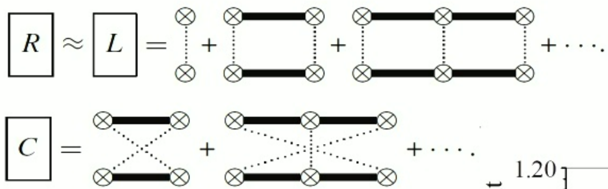
Phys. Rev. Lett., **83**, 5266 (1999)

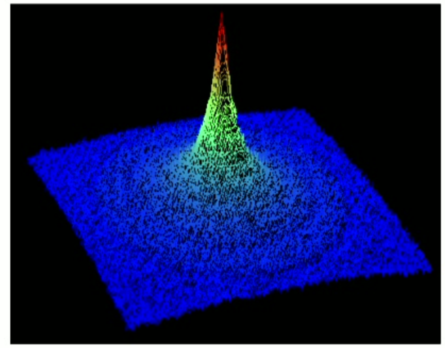
Coherence after resonant scattering with atoms !

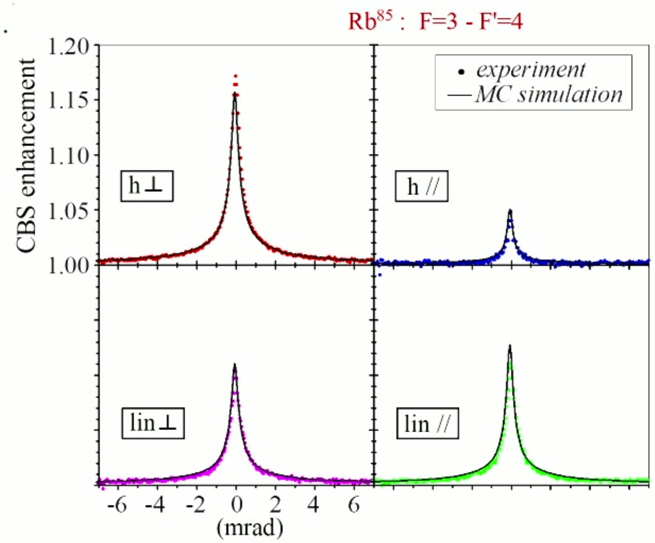
also : M. Havey et al.

Theory :

- no "exact" solution
- diagrammatic approach







Excellent agreement
(no free parameter)

22

T. Jonckheere et al., Phys. Rev. Lett., **85**, 4269 (2000)

Coherent Backscattering vs Weak Localization:

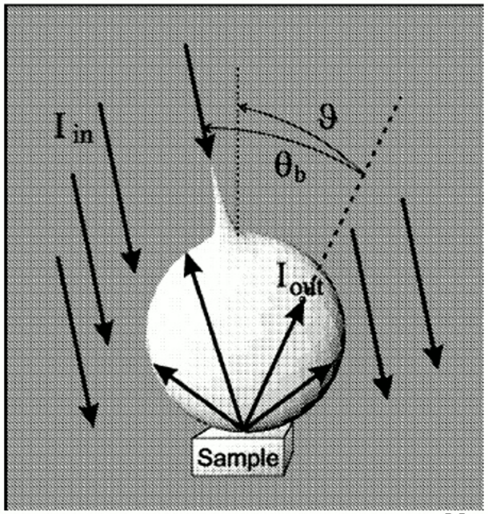
Interference correction to Diffusion coefficient

$$D \approx D_0\,[1 - 1/(k\ell)^2]$$

$$\langle r^2 \rangle \approx l^2\,(1 - PCBS/Ptot)$$

$$\langle r^2 \rangle \approx l^2\left(1 - \Delta\theta_{CBS}^2/4\pi\right)$$

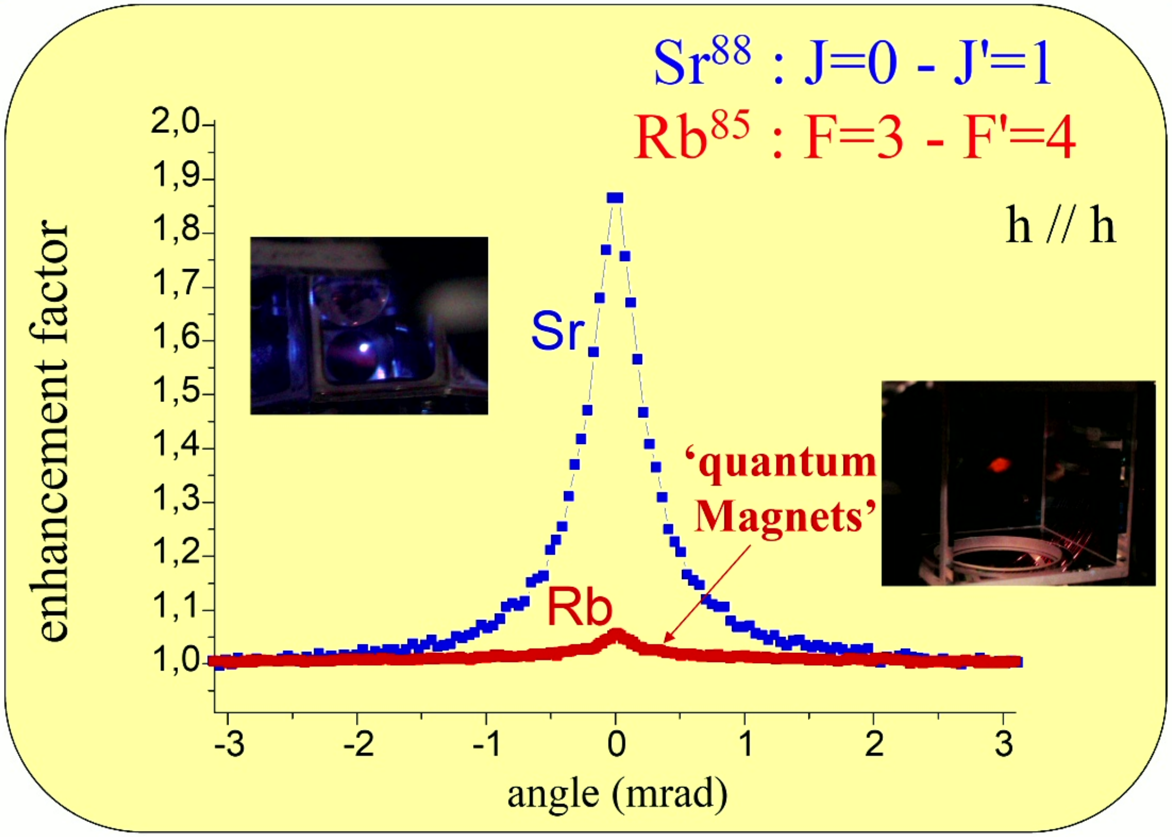
$$\langle r^2 \rangle \approx l^2\left(1 - 1/kl^2\right)$$



23

R. Lenke, G. Maret, in Scattering in Polymeric and Colloidal Systems (2000)

Influence of internal structure



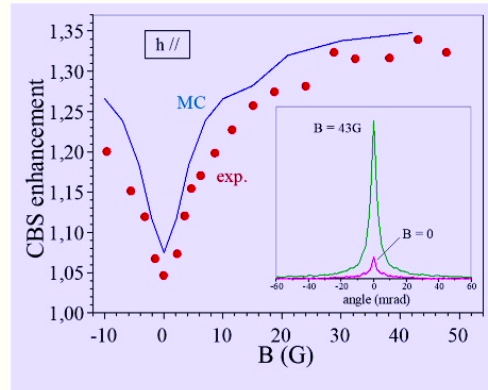
G. Labeyrie et al., Phys. Rev. Lett. **83**, 5266 (1999)   Y. Bidel et al., Phys. Rev. Lett. **88**, 203902 (2002)

24

Atoms with *J=0* to *J=1* behaves like classical Rayleigh scatterer

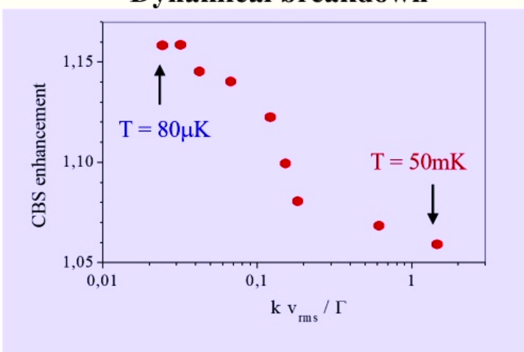
In Summary : on the road towards localization
many problems ☹ … mostly/partially solved ☺

Restoring Coherent Backscattering with Magnetic Fields



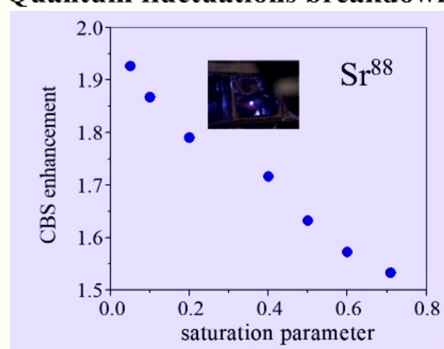
O.Sigwarth et al., Phys. Rev. Lett. 93, 143906 (2004).

Dynamical breakdown



Labeyrie et al., PRL97, 013004 (2006)

Quantum fluctuations breakdown



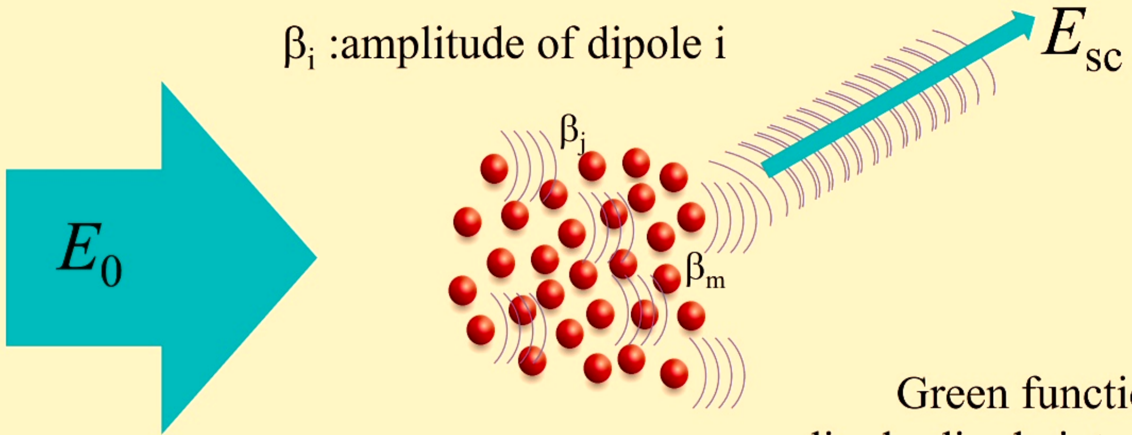
T. Chanclierc et al., Phys. Rev. E 70, 036602 (2004).

Cooperative Scattering and Dicke states

- **Ab initio model for light scattering**
- **Experiments**
- **Effective Hamiltonian approach**

Building up a refractive index « ab inito »
(from individual atoms to macroscopic index)

β_i :amplitude of dipole i



Green function
dipole-dipole interactions

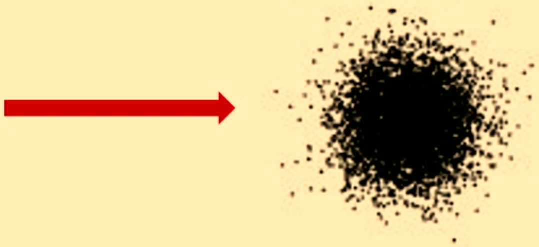
$$\dot{\beta}_j(t) = \boxed{-\frac{i}{2}\Omega e^{i\mathbf{k}_0\cdot\mathbf{r}_j} + \left(i\Delta - \frac{\Gamma}{2}\right)\beta_j(t)} - \boxed{\frac{\Gamma}{2}\sum_{m\neq j}^{N}\beta_m \frac{\exp(ik_0 \mid \mathbf{r}_j - \mathbf{r}_m \mid)}{ik_0 \mid \mathbf{r}_j - \mathbf{r}_m \mid}}$$

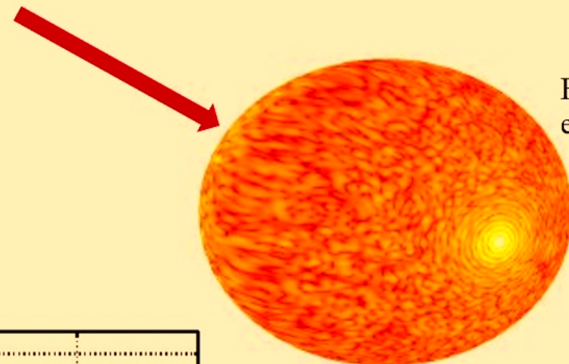
$$E_{\rm sc}(\mathbf{r}) = -\frac{\hbar\Gamma}{2d}\sum_{j=1}^{N}\beta_j \frac{e^{ik_0|\mathbf{r}-\mathbf{r}_j|}}{k_0|\mathbf{r}-\mathbf{r}_j|}$$

27

Spherical gaussian cloud: steady state emission diagram

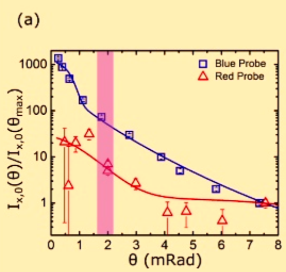
Cloud of atoms

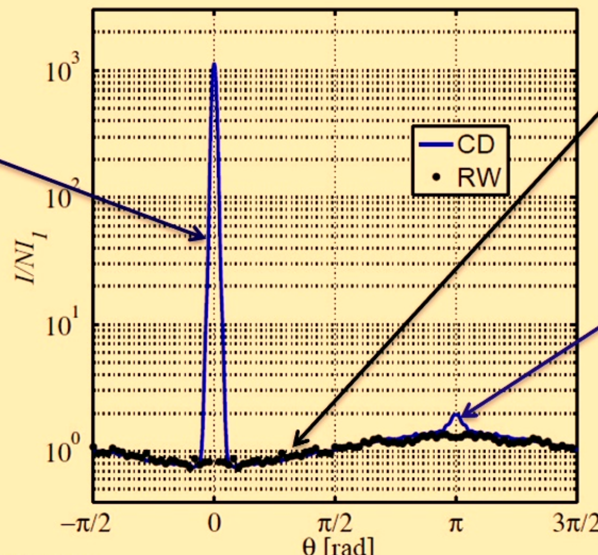




Far field
emission diagram

refractive index
(mean field)





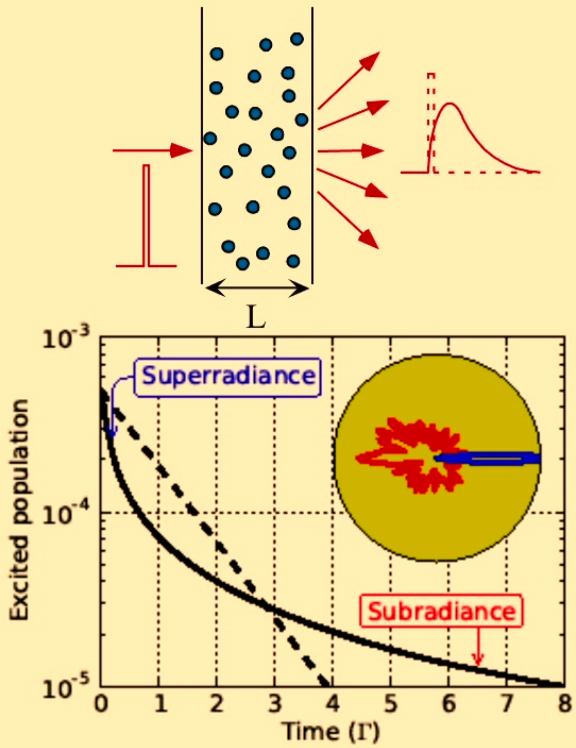
Incoherent model
(particles trajectories,
scattering in 'empty modes')

Mesoscopic physics:
Weak localization
(waves beyond mean field)

S. Bromley et al.,
Nat. Comm. 7, 11039 (2016)

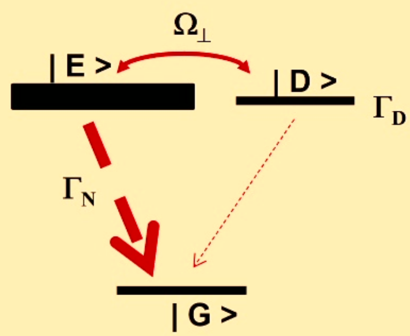
28

Time dependent experiments : coherent scattering

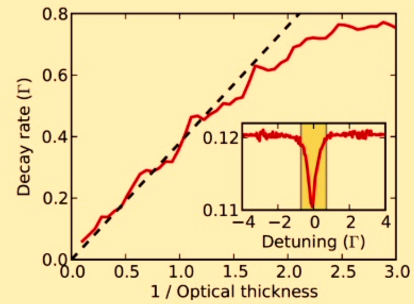


Numerical Simulation of N driven coupled dipoles

Superradiance = bright state
Subradiance = metastable 'dark' states



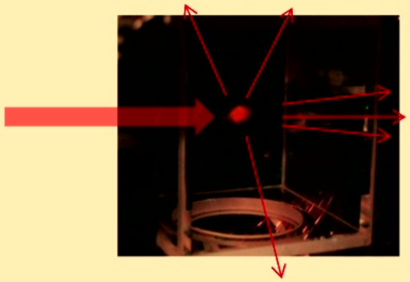
Late decay time $\propto$ b_0



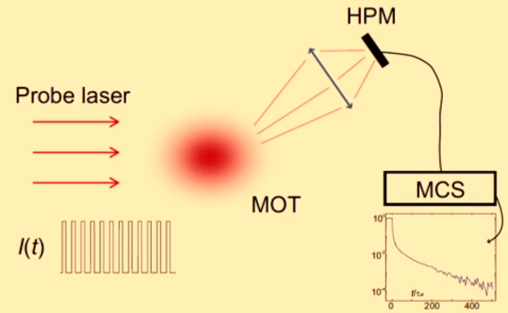
29

T. Bienaimé, N. Piovella, R.K., PRL **108** 123602 (2012)

Experimental results

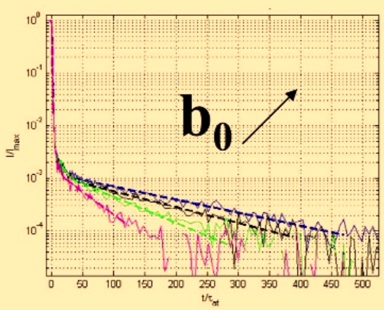


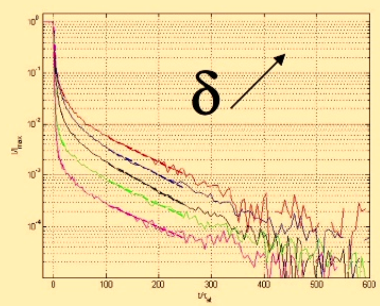
N=10^9 ^{87}Rb
T=50 μK
R=1 mm
ρ=10^{11}/cc
b_0 = 20…100

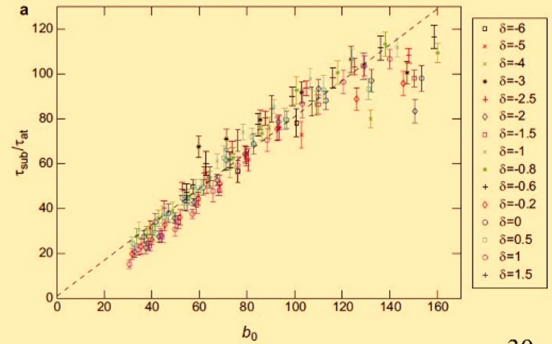


Long decay at b(δ)<1 ☺

Increases as b_0 ☺







30

W. Guerin, M. Araujo, R. K., PRL 116, 083601 (2016)

Theory : Effective Hamiltonian in the single excitation manyfold

$$\boldsymbol{H_{eff}} = (\hbar\omega_0 - \mathrm{i}\frac{\hbar\Gamma_0}{2}) \sum_i S_i^z + \frac{\hbar\Gamma_0}{2} \sum_{i \neq j} V_{ij} S_i^{\ +} S_j^{\ -}$$

Diagonal :
On site energy

Off diagonal :
transport

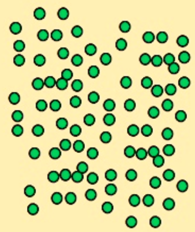
$$V_{ij} = \beta_{ij} - i\gamma_{ij}$$

$$\beta_{ij} = \frac{3}{2}\left[-p\frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q\left(\frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

$$\gamma_{ij} = \frac{3}{2}\left[p\frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q\left(\frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

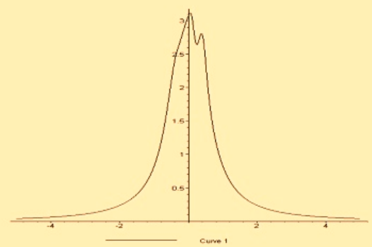
- Open System
- Reminiscent of Anderson Hamiltonian
- Heisenberg model with global coupling
- Long range hopping
- No decoherence (coupling to phonons, …)

Eigenvalues for N coupled dipoles

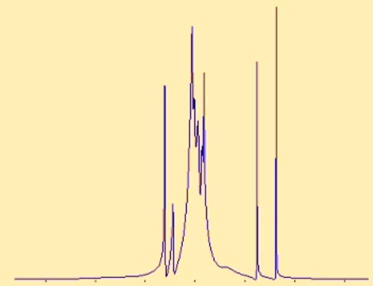


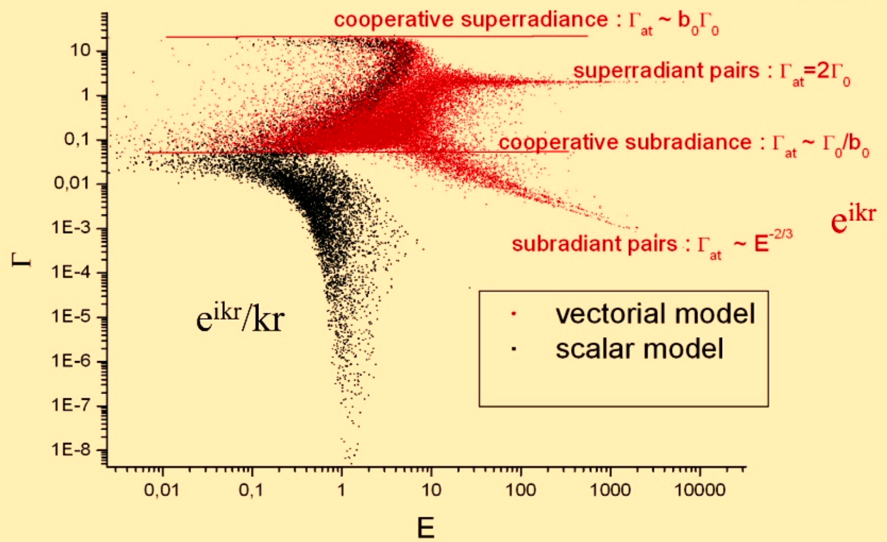
molecular spectrum ?

'dilute' molecule

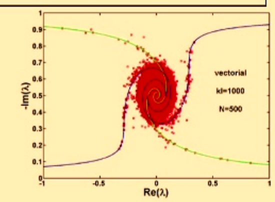


'dense' molecule



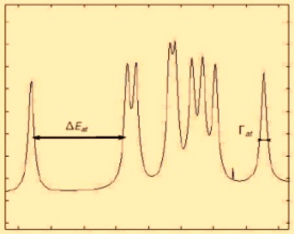


$e^{ikr}\ (1/kr + 1/kr^2 + 1/kr^3)$

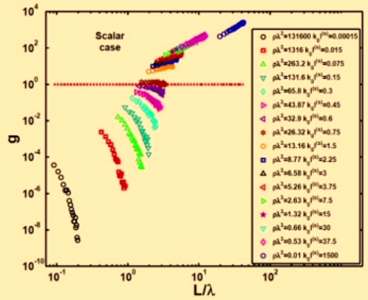


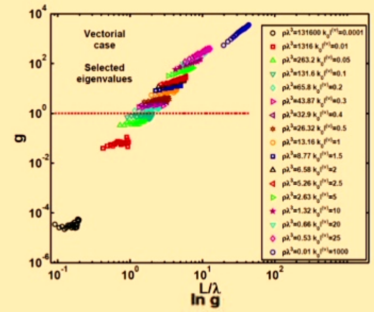
32

Resonance Overlap (« Thouless »)

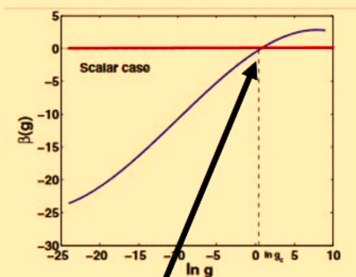


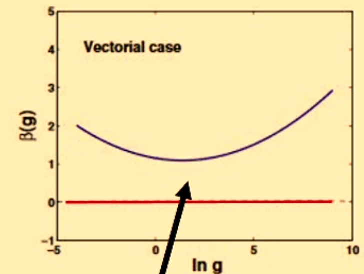
$$g = \left\langle \frac{1}{\langle 1/\Gamma \rangle \langle \Delta E \rangle} \right\rangle$$





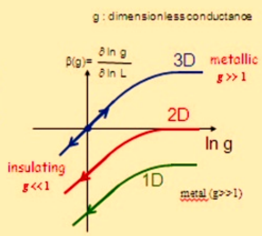
Scaling function β(g)





" $k\ell \approx 1$ "

10^{14} at/cc



Phase transition

No phase transition

No Anderson Localization for Vectorial Light in 3D !

33

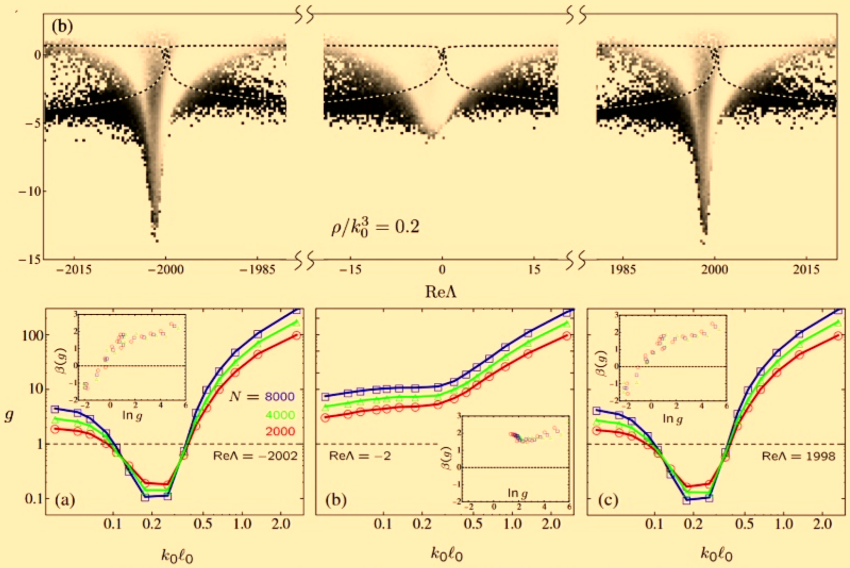
S. Skipetrov, I. Sokolov, PRL 112, 023905 (2014)

L. Bellando et al,. Phys. Rev. A 90, 063822 (2014)

2 solutions proposed since 2014 :

1) Magnetic field assisted Anderson localization

Dense sample + magnetic field



partial **suppression of near field dipole dipole interactions**

S. Skipetrov, I. Sokolov, PRL **114**, 053902 (2015)
S. Skipetrov, PRL **121**, 093601 (2018)

☺

Still requires
large spatial densities

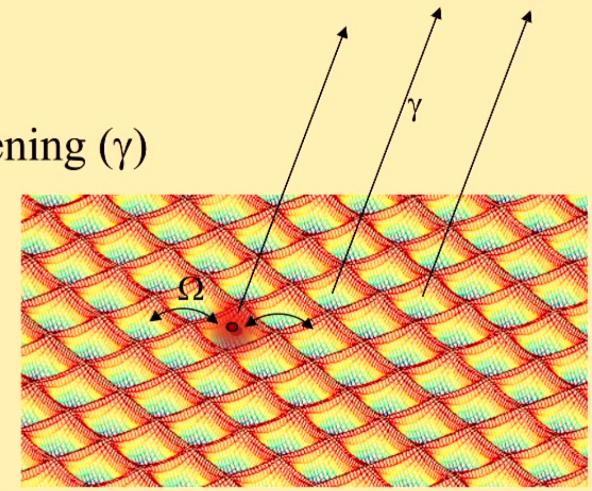
~10^{14} at/cc

☹

2) Diagonal disorder in dilute samples

Combining Anderson and Dicke
Toy Model : Open Disordered System:

3D Anderson model on 10 x10 x10 lattice
hopping (Ω) + on site disorder (W) + opening (γ)



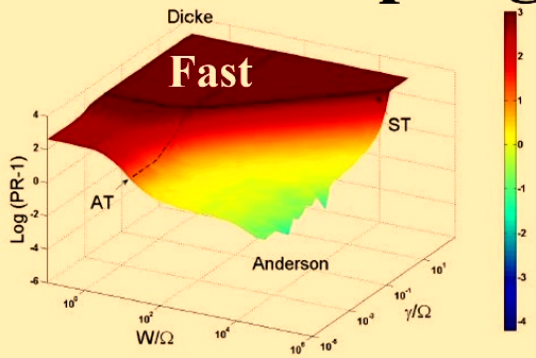
$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|)$$

$$(H_{\text{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2}\sum_c A_i^c (A_j^c)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j}$$

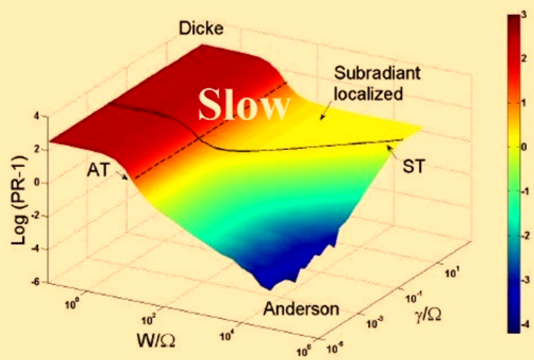
All sites coupled to one single decay channel : $Q_{ij}=1$

35

Toy Model : Anderson lattice model + coupling to one open mode

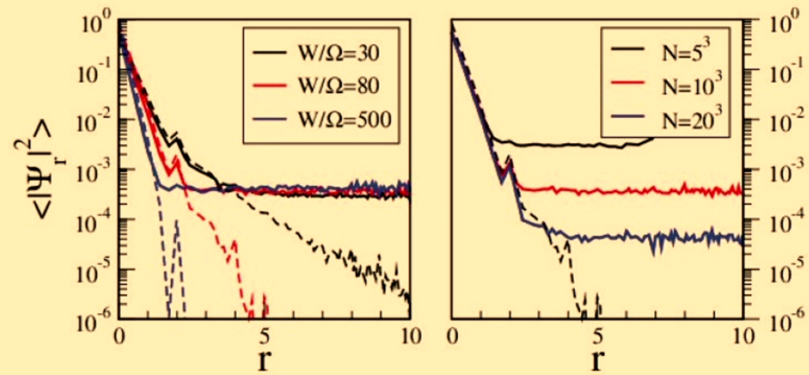


(a)



(b)

Hybrid Subradiant States « decoupled » from outside world



36

A.Biella, et al., EPL, 103, 57009 (2013)

Apply this idea to the coupled dipole model

$$\mathcal{H} = \sum_{i=1}^{N}(E_i - i\frac{\Gamma_0}{2})\,|i\rangle\langle i| + \frac{\Gamma_0}{2}\sum_{i\neq j}^{N} V_{i,j}\,|i\rangle\langle j|$$

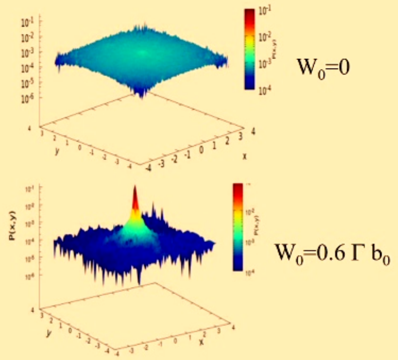
$$V_{i,j} \;=\; \frac{exp(ik_0 \cdot r_{ij})}{k_0 \cdot r_{ij}}$$

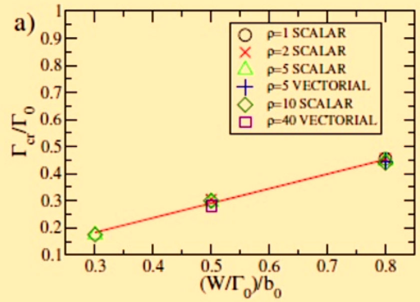
Diagonal disorder : random light shifts : $E_j \in [-W/2, W/2]$

+ remain in the dilute limit : $\rho\lambda^3 < \rho_{cr}\lambda^3 = 24$

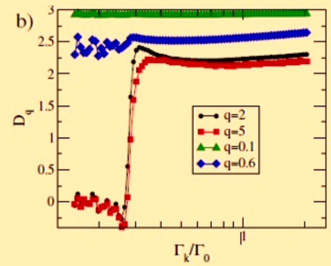
$$PR = \left\langle 1/\sum_i |\langle i|\psi\rangle|^4 \right\rangle$$

Critical disorder :
Mobility edge along the imaginary axis



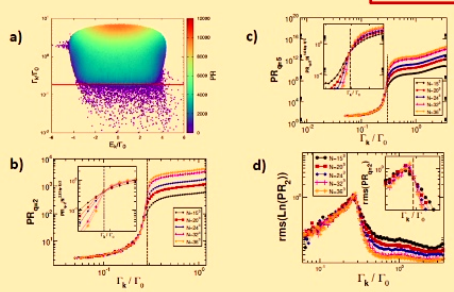


Fractal dimension

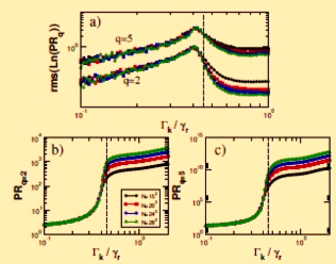


$$PR_q \propto N^{D_q(q-1)/3}$$

Scaling law : $W_{cr} / b_0 \propto \Gamma_{mode}$



also works for
- **vectorial model without magnetic field (even for $\rho\lambda^3 \sim 40$)**
- **on a lattice + diagonal disorder**

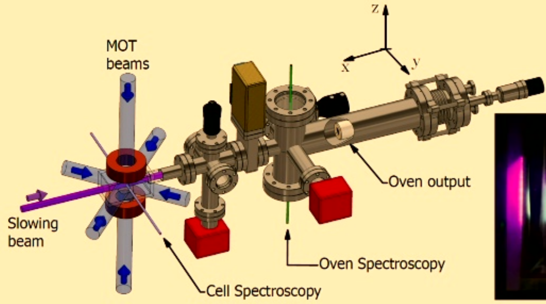


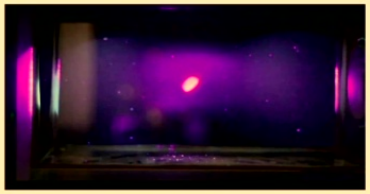
No high densities required ☺

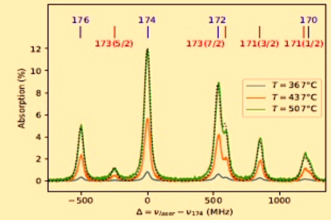
38

L. Celardo, M. Angeli, F. Mattiotti, R. K., EPL 145, 42001(2024)

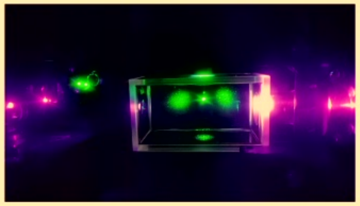
Status of the Yb experiment in Nice

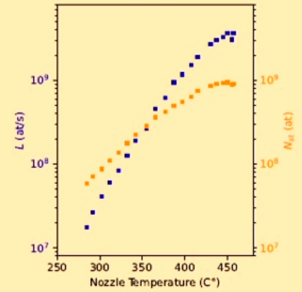






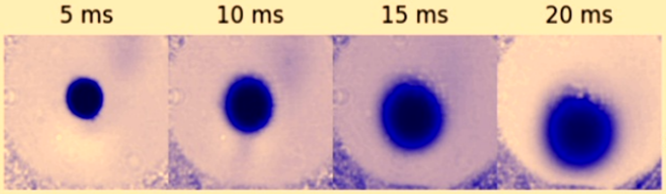
Atom number limited
by light assisted collisions





...

...



OD ~ 50
N = 2 10^8
σ_{rms} ~ 260μm / 400μm

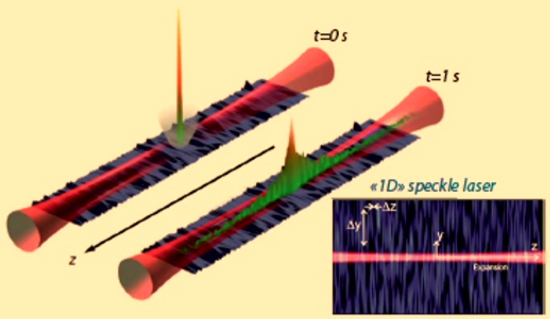
$\rho\lambda^3$ ~ 0.1

39

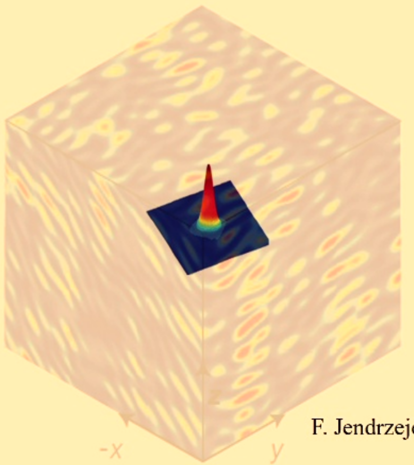
How to prepare an initial excition in the center of a disordered system ?

Matter waves : in 1 & 3D

Time control of the presence of the scattering medium



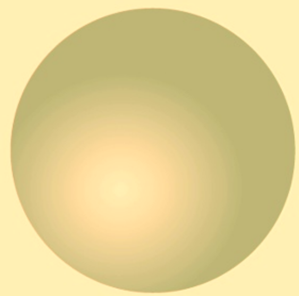
J Billy et al., Nature 453, 891 (2008)

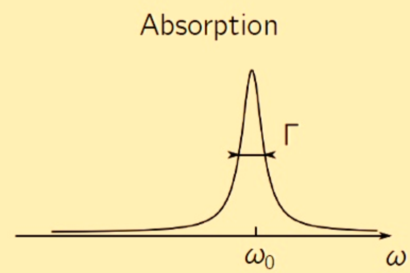


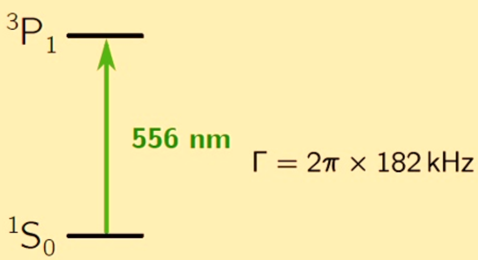
F. Jendrzejewski et al., Nat. Phys. 8, 398 (2012)

^{174}Yb experiment in Nice

T ~ 15 μK
N = 3 10^8
OD : 0 … 100



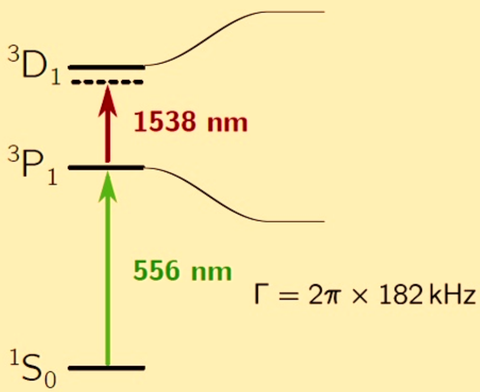


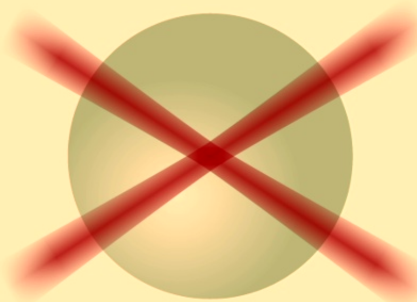


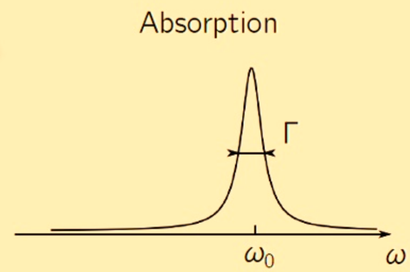
41

^{174}Yb experiment in Nice

$T \sim 15\ \mu K$
$N = 3\ 10^8$
OD : 0 … 100



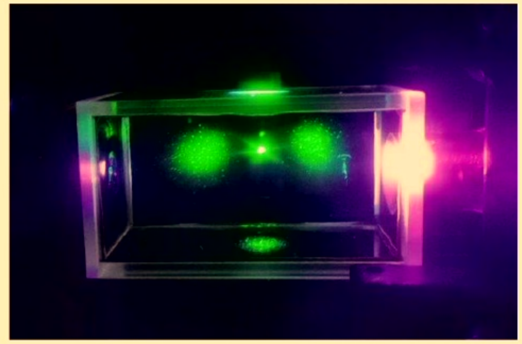


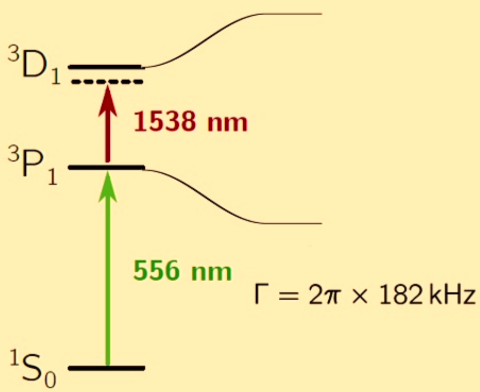


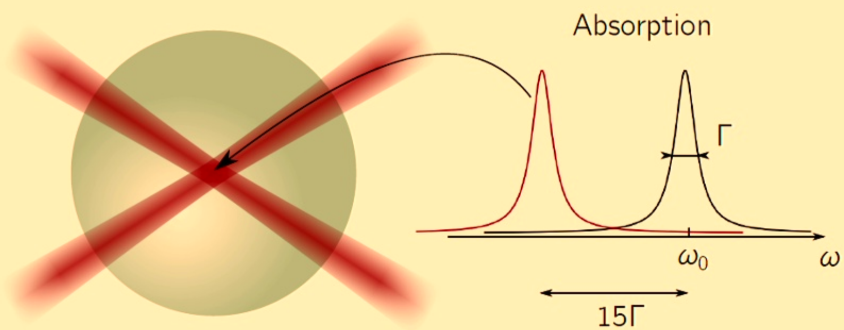
42

^{174}Yb experiment in Nice

T ~ 15 μK
N = 3 10^8
OD : 0 … 100



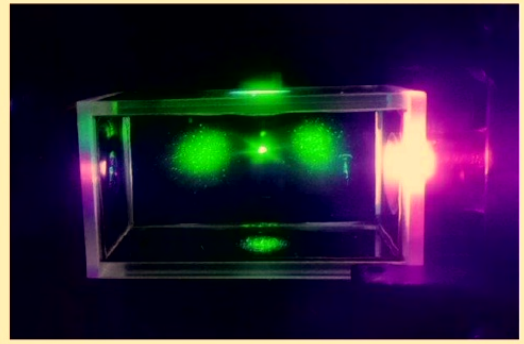


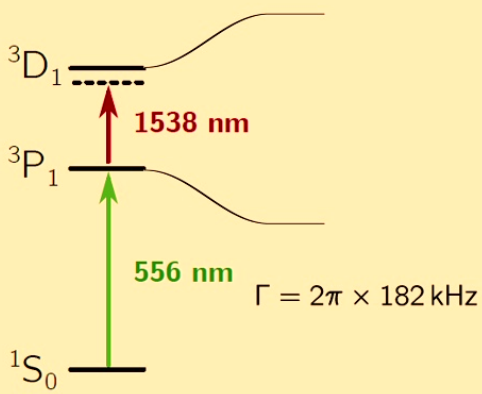


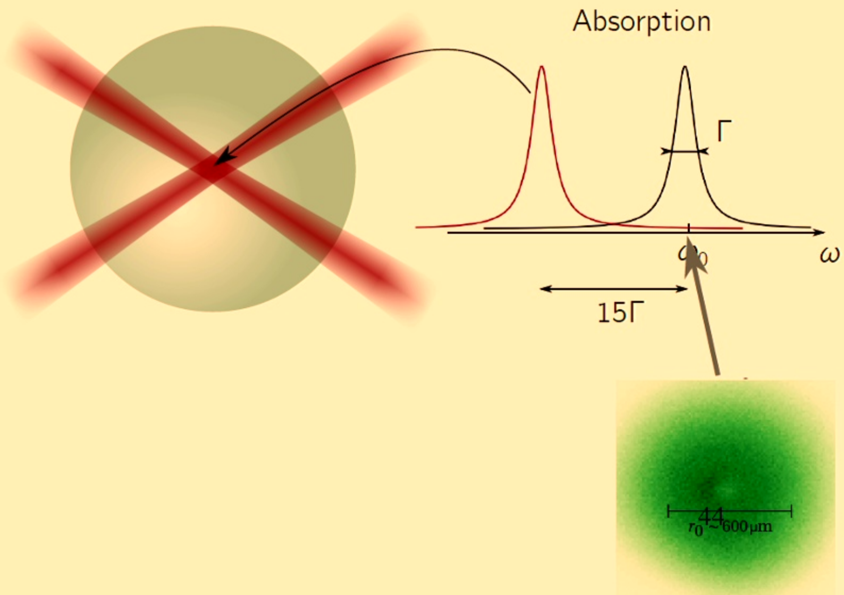
43

^{174}Yb experiment in Nice

$T \sim 15\ \mu K$
$N = 3\ 10^8$
OD : 0 … 100

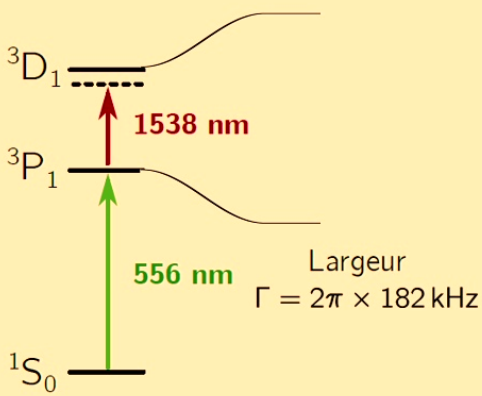


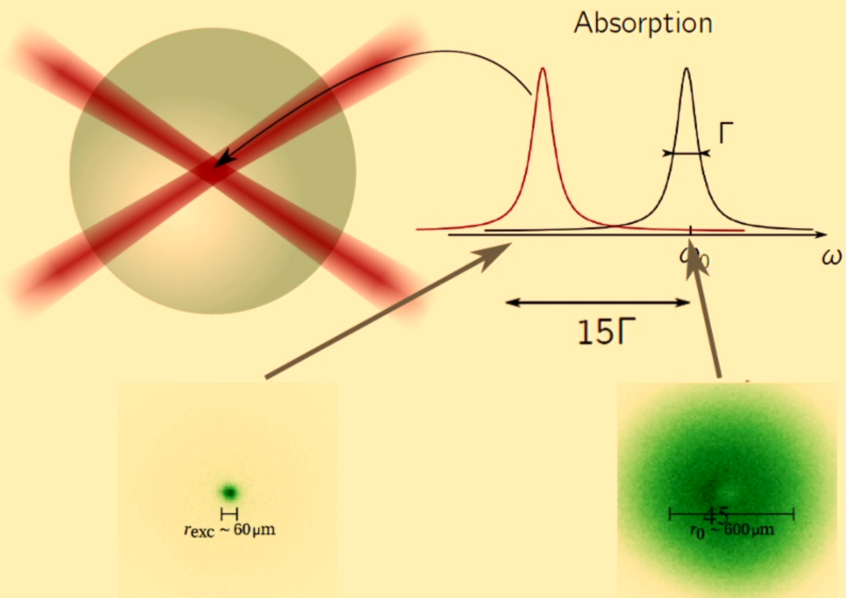




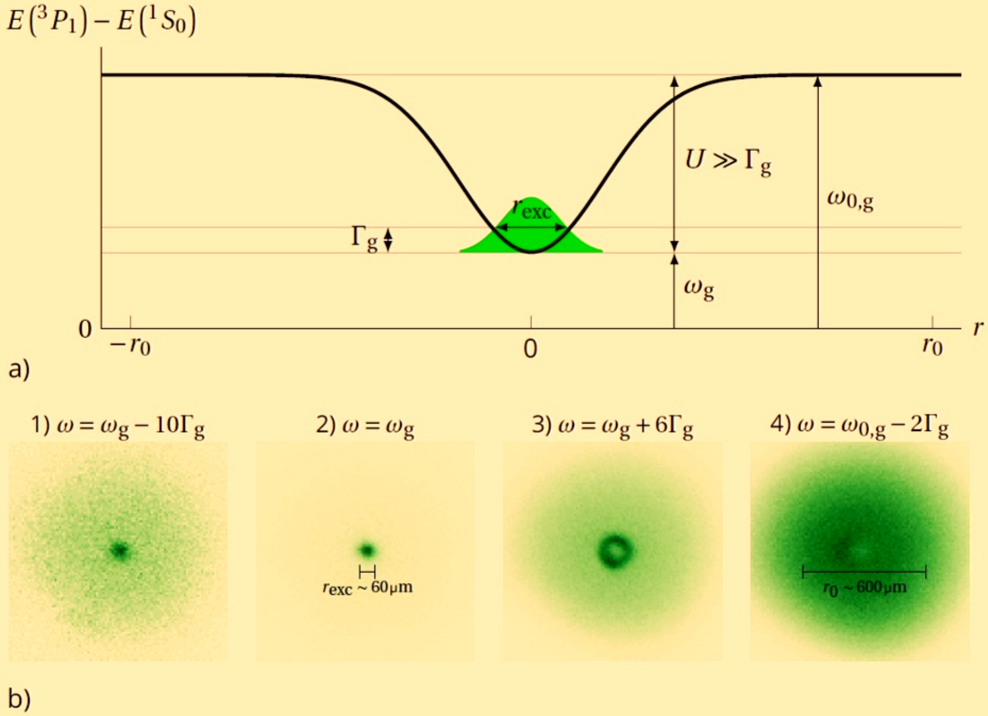
^{174}Yb experiment in Nice

$T \sim 15\ \mu K$
$N = 3\ 10^8$
OD : 0 … 100



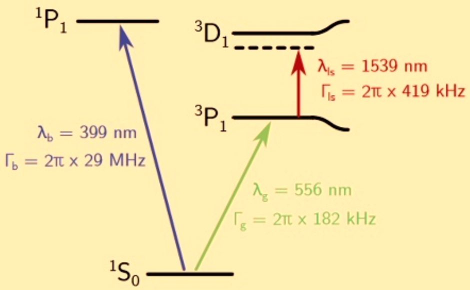


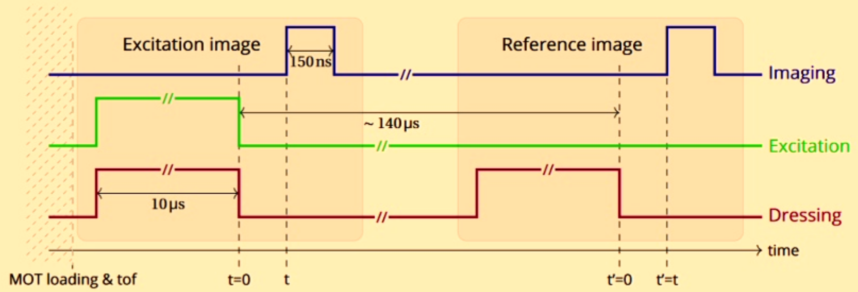
- **Control on in situ spatial excitation**

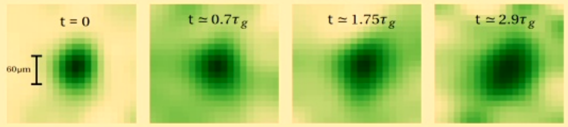


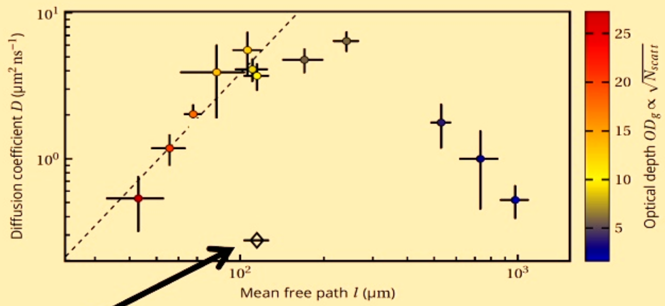
46

'Fast' time resolved in situ spatial detection







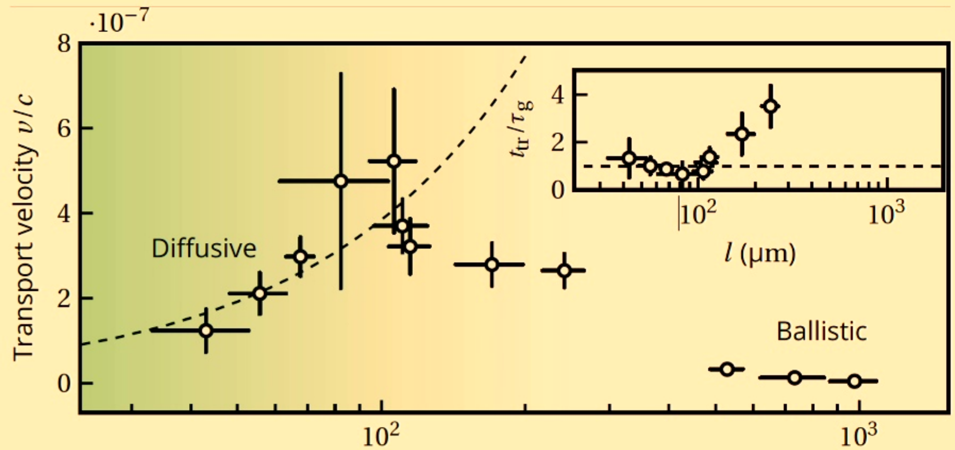


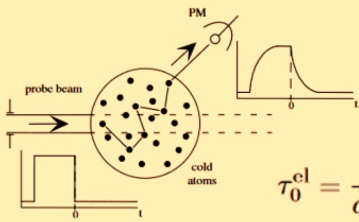
+ control of frequency of launched photons

47

A. Glicenstein et al., Phys. Rev. Lett. in press

In situ

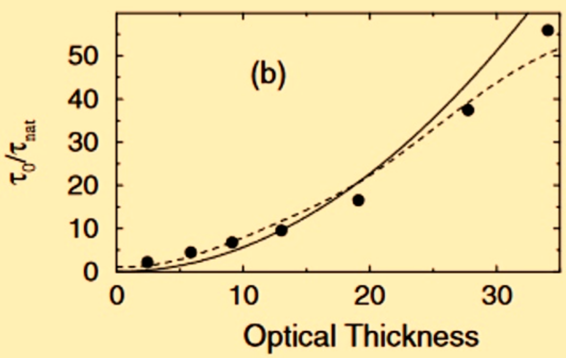


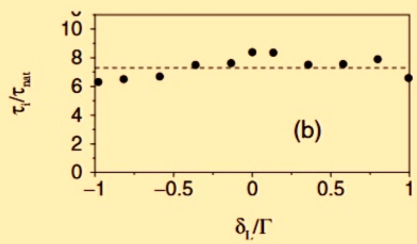


$$\tau_0^{\mathrm{el}} = \frac{3}{\alpha\pi^2} b^2 \tau_{\mathrm{nat}}$$

from outside

$$\tau_{\mathrm{tr}} = \tau_{\mathrm{W}} + \frac{\ell}{v_{\mathrm{g}}} \approx 1/\Gamma = \tau_{\mathrm{nat}}$$





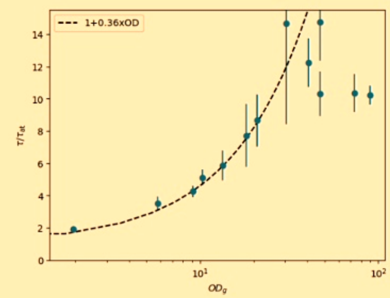
G. Labeyrie, et al.,
Phys. Rev. Lett. **91**, 223904 (2003);

G. Labeyrie, R.K., D. Delande
Applied Physics B **81**, 1001 (2005).

48

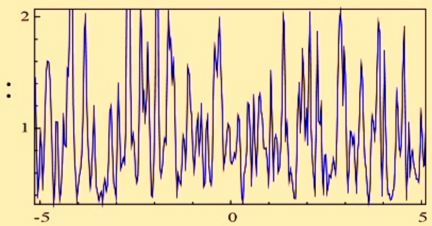
Perspectives

- **Benckmark subradiance with Yb** ☺

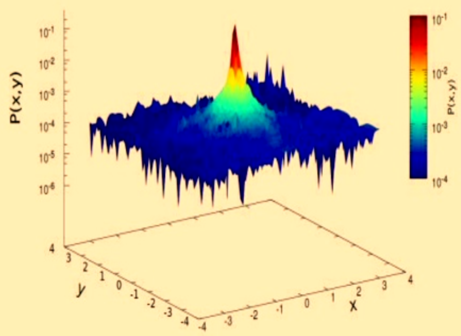


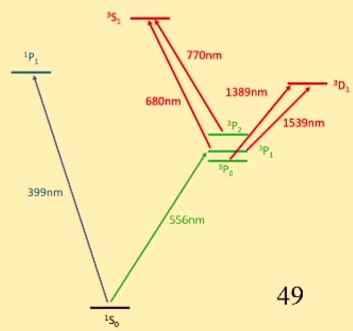
- **Control of phase of launched photons**
- **Add diagonal disorder**

Random Light shift (@1539nm / 680nm) :
diagonal disorder



- **Look for**






Thank you
For your attention