Title: Quantum metrology with correlated noise

Speakers: Stanisław Kurdziałek

Collection/Series: Quantum Information

Subject: Quantum Information

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Abstract:

I will present a universal numerical tool for identifying optimal adaptive metrological protocols in the presence of both uncorrelated and correlated noise [arXiv:2403.04854]. Leveraging a novel tensor network decomposition of quantum combs, the algorithm demonstrates efficiency even with a large number of channel uses (N=50). In the second part of the talk, I will explore the generalization of existing metrological upper bounds [Nat. Com. 3, 1063 (2012), PRL 131(9), 090801 (2023)] for correlated noise scenarios [arXiv:2410.01881].

QUANTUM METROLOGY WITH CORRELATED NOISE

Stanisław Kurdziałek

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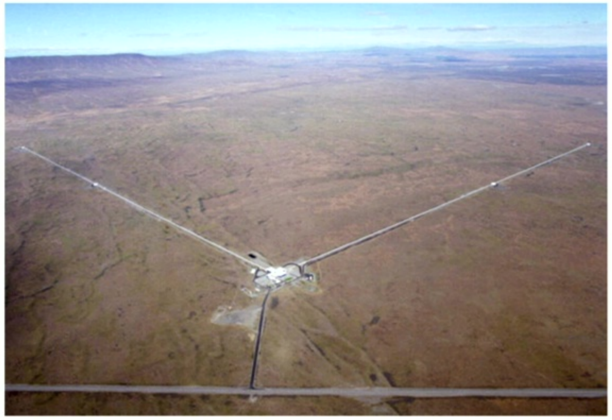
FACULTY OF PHYSICS

Quantum metrology

Gravitational wave detector (LIGO)



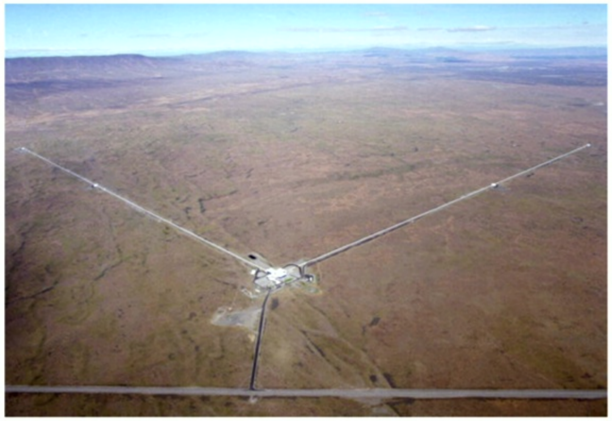
$\Delta L/L \approx 10^{-23}$

Cesium Fountain atomic clock

$\Delta t/t \approx 10^{-16}$

What are the precision limits?

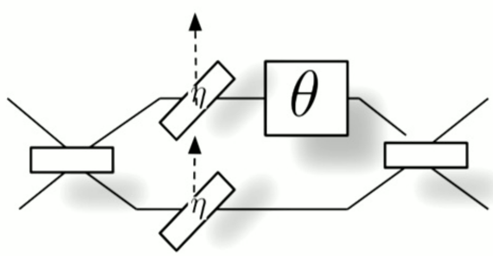
Quantum metrology (but for slightly less naive theorists)



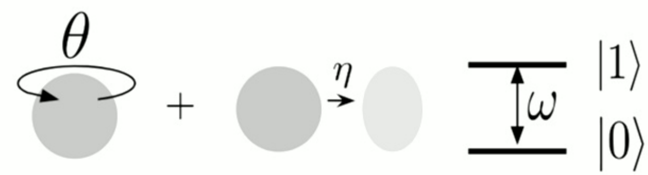
Michelson interferometer

Ramsey interferometry

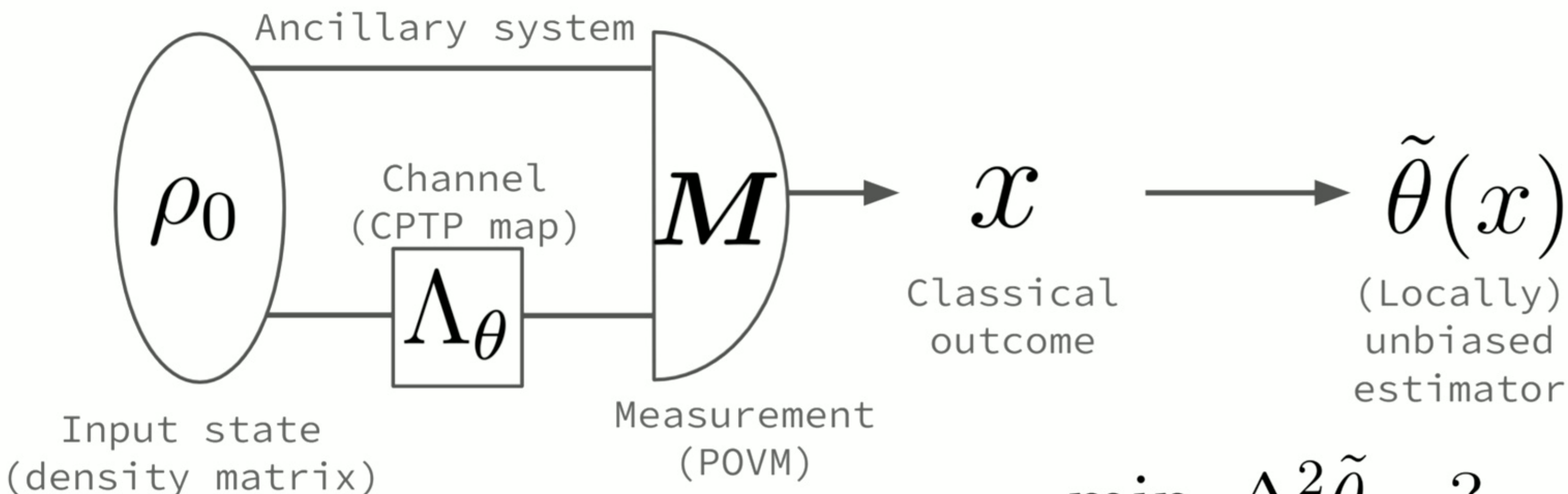
Quantum channel



$$= \Lambda_\theta =$$

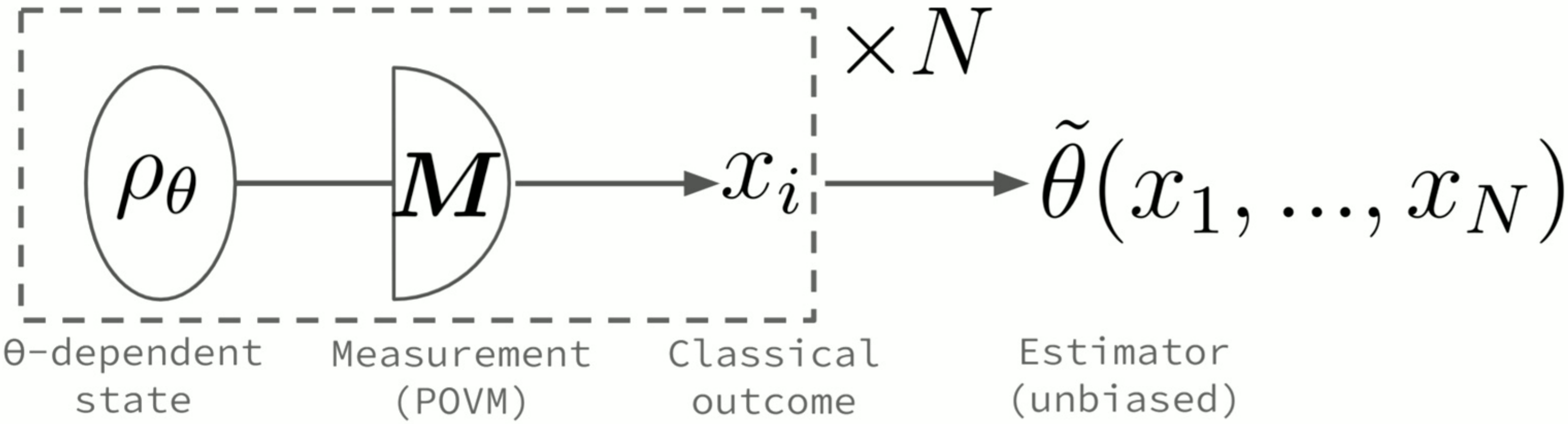


Quantum metrology = quantum channel estimation



$$\min_{\rho_0, \mathbf{M}, \tilde{\theta}} \Delta^2 \tilde{\theta} = ?$$

Easier task: Quantum state estimation



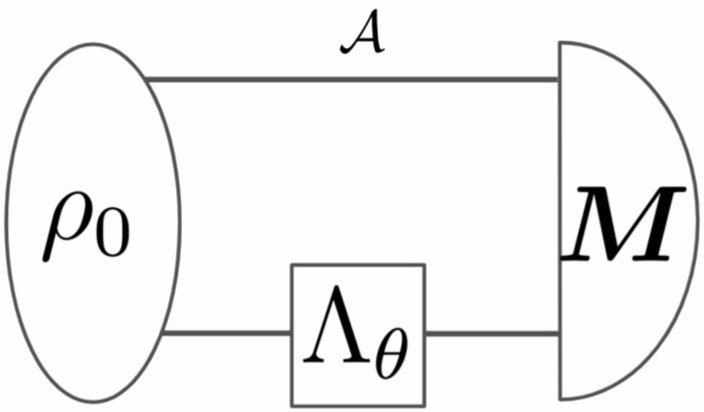
$$\min_{\mathbf{M},\tilde{\theta}} \Delta^2 \tilde{\theta} = \frac{1}{N F(\rho_\theta)}$$

Quantum Fisher Information

$$F(\rho_\theta) = \mathrm{Tr}(\rho_\theta L^2)$$

$$\dot{\rho}_\theta = \frac{1}{2}(\rho_\theta L + L \rho_\theta)$$

Channel estimation: channel QFI



$$\rho_\theta = \Lambda_\theta \otimes \mathcal{I}(\rho_0)$$

Channel QFI

$$\mathcal{F}(\Lambda_\theta) = \max_{\rho_0} F(\Lambda_\theta \otimes \mathcal{I}(\rho_0))$$

$$\min_{\rho_0, \mathbf{M}, \tilde{\theta}} \Delta^2 \tilde{\theta} = \frac{1}{\mathcal{F}(\Lambda_\theta)}$$

It is hard to maximize QFI
over input directly

QUANTUM FISHER INFORMATION: DIFFERENT FORMULAS

$$F(\rho_\theta) = \mathrm{Tr}(\rho_\theta L^2)$$

$$\dot{\rho}_\theta = \frac{1}{2}(\rho_\theta L + L\rho_\theta)$$

Variational formulation

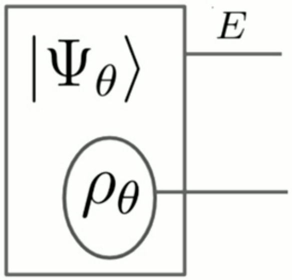
L : free variable

$$\bar{F}(\rho_\theta, L) = 2\mathrm{Tr}(\dot{\rho}_\theta L) - \mathrm{Tr}(\rho_\theta L^2)$$

$$F(\rho_\theta) = \max_{L=L^\dagger} \bar{F}(\rho_\theta, L)$$

Minimization over purifications

$$\rho_\theta = \mathrm{Tr}_E\left[|\Psi_\theta\rangle\langle\Psi_\theta|\right]$$



$$F(|\Psi_\theta\rangle) = 4\left(\langle\dot{\Psi}_\theta|\dot{\Psi}_\theta\rangle - |\langle\dot{\Psi}_\theta|\Psi_\theta\rangle|^2\right)$$

$$F(\rho_\theta) \leq F(|\Psi_\theta\rangle) \leq 4\langle\dot{\Psi}_\theta|\dot{\Psi}_\theta\rangle$$

$$F(\rho_\theta) = \min_{|\Psi_\theta\rangle} 4\langle\dot{\Psi}_\theta|\dot{\Psi}_\theta\rangle$$

Channel QFI: different formulas

$$\mathcal{F}(\Lambda_\theta) = \max_{\rho_0} F(\Lambda_\theta \otimes \mathcal{I}(\rho_0))$$

Iterative see-saw (ISS)

$$\mathcal{F}(\Lambda_\theta) = \max_{\rho_0, L} \bar{F}(\Lambda_\theta \otimes \mathcal{I}(\rho_0))$$

SDP SDP

1) Initialize random ρ_0 and L
2) Optimize over L with fixed ρ_0 (SDP)
3) Optimize over ρ_0 with fixed L (SDP)
4) Repeat until convergence

K. Macieszczak, arXiv:1312.1356

Minimization over purifications (MOP)

$$\Lambda_\theta(\bullet) = \sum_k K_{k,\theta} \bullet K_{k,\theta}^\dagger$$

QFI of a purified channel:

$$\mathcal{F}_{\text{pur}} = 4\|\alpha\|, \quad \alpha = \sum_k \dot{K}_{k,\theta}^\dagger \dot{K}_{k,\theta}$$

Min. over purifications – min. over Kraus representations: $\mathcal{F}(\Lambda_\theta) = 4 \min_{\{K_k\}} \|\alpha\|$

A. Fujiwara, H. Imai, J. Phys. A: Math. Theor. 41, 255304 (2008)

Channel QFI: different formulas

$$\mathcal{F}(\Lambda_\theta) = \max_{\rho_0} F(\Lambda_\theta \otimes \mathcal{I}(\rho_0))$$

Iterative see-saw (ISS)

$$\mathcal{F}(\Lambda_\theta) = \max_{\rho_0, L} \bar{F}(\Lambda_\theta \otimes \mathcal{I}(\rho_0))$$

SDP (for ρ_0) SDP (for L)

1) Initialize random ρ_0 and L
2) Optimize over L with fixed ρ_0 (SDP)
3) Optimize over ρ_0 with fixed L (SDP)
4) Repeat until convergence

K. Macieszczak, arXiv:1312.1356

?

Minimization over purifications (MOP)

$$\Lambda_\theta(\bullet) = \sum_k K_{k,\theta} \bullet K_{k,\theta}^\dagger$$

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$$\mathcal{F}_{\text{pur}} = 4\|\alpha\|, \quad \alpha = \sum_k \dot{K}_{k,\theta}^\dagger \dot{K}_{k,\theta}$$

Min. over purifications – min. over Kraus representations: $\mathcal{F}(\Lambda_\theta) = 4 \min_{\{K_k\}} \|\alpha\|$

A. Fujiwara, H. Imai, J. Phys. A: Math. Theor. 41, 255304 (2008)

MOP: SDP formulation

We don't have to consider all Kraus representations!

It's enough to start from arbitrary and take transformation:

$$K_i \to K_i$$

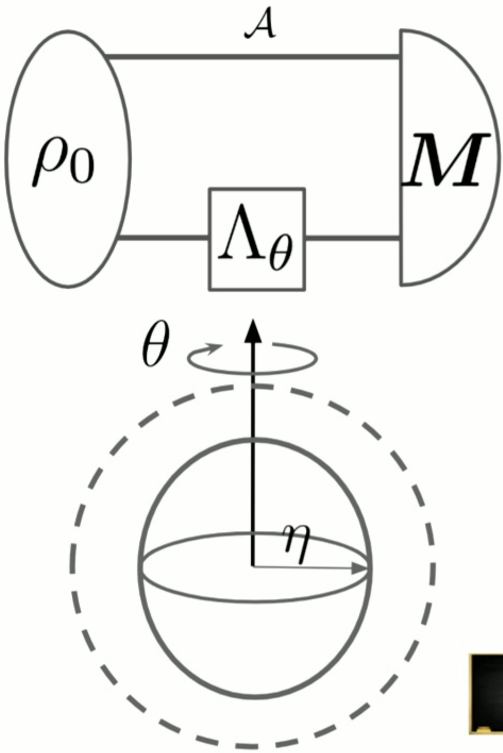
$$\dot{K}_i \to \dot{K}_i - i \sum_j h_{ij} K_j$$

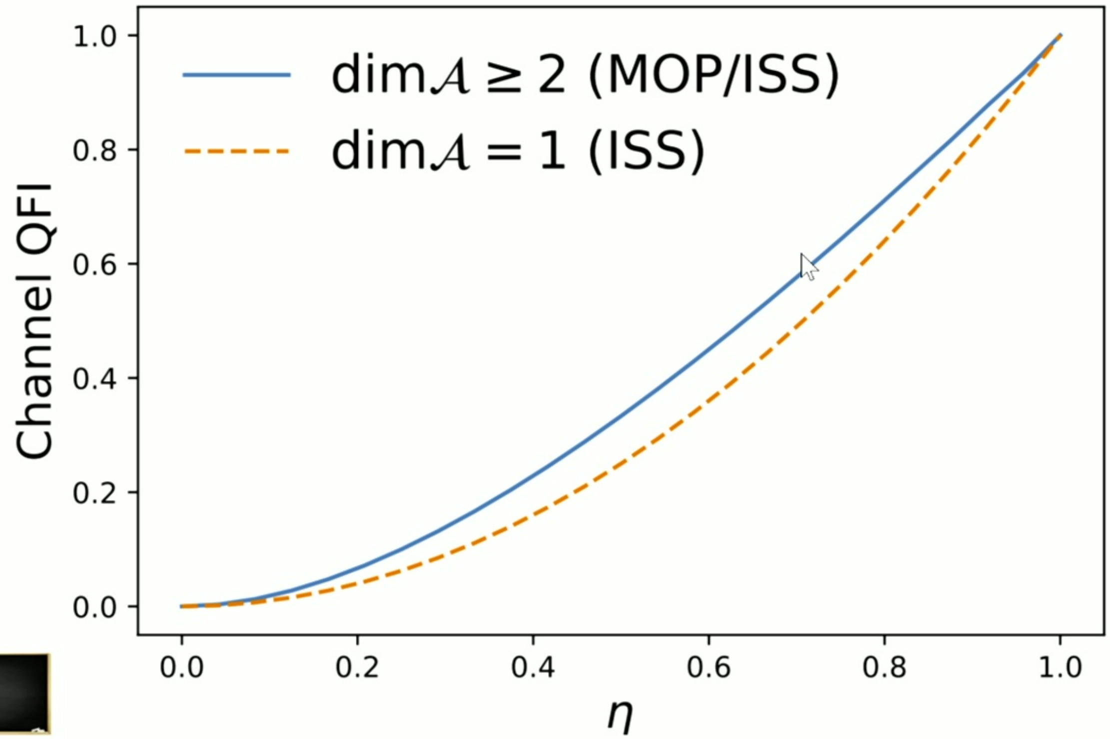
$$h : \text{hermitian matrix}$$

$$\min_{\{K_i\}} \|\alpha\| = \min_h \|\alpha\|$$

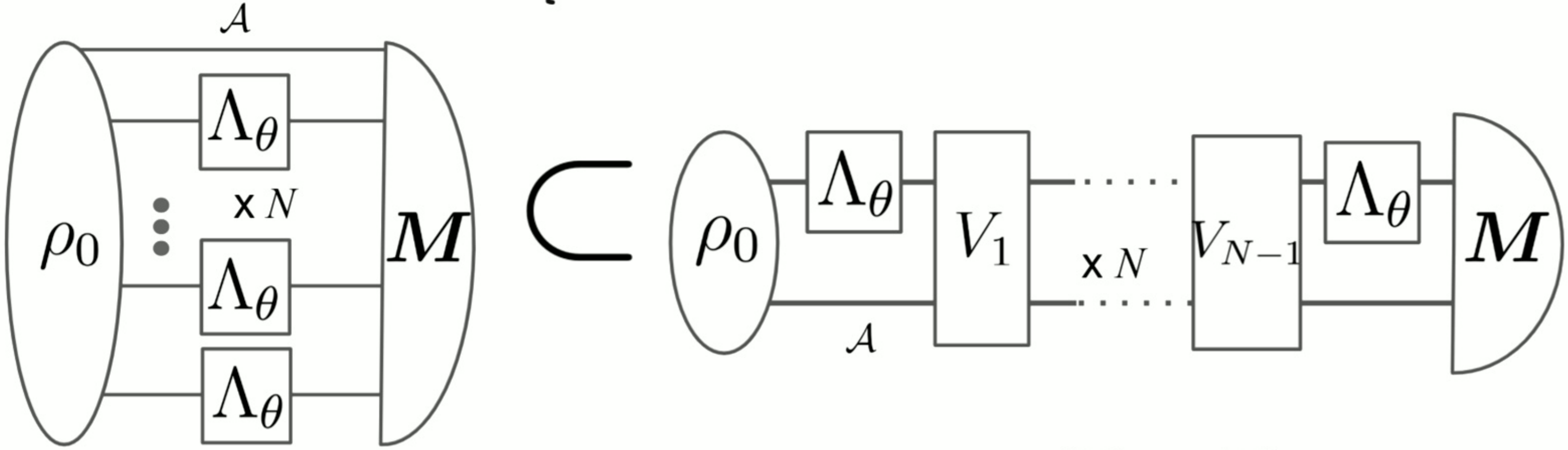
Can be formulated as SDP!

Example: phase estimation with depolarization





Quantum metrology: quantum channels estimation



$$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \equiv \mathcal{F}(\Lambda_\theta^{\otimes N}) \geq N\mathcal{F}(\Lambda_\theta)$$

(usually >)

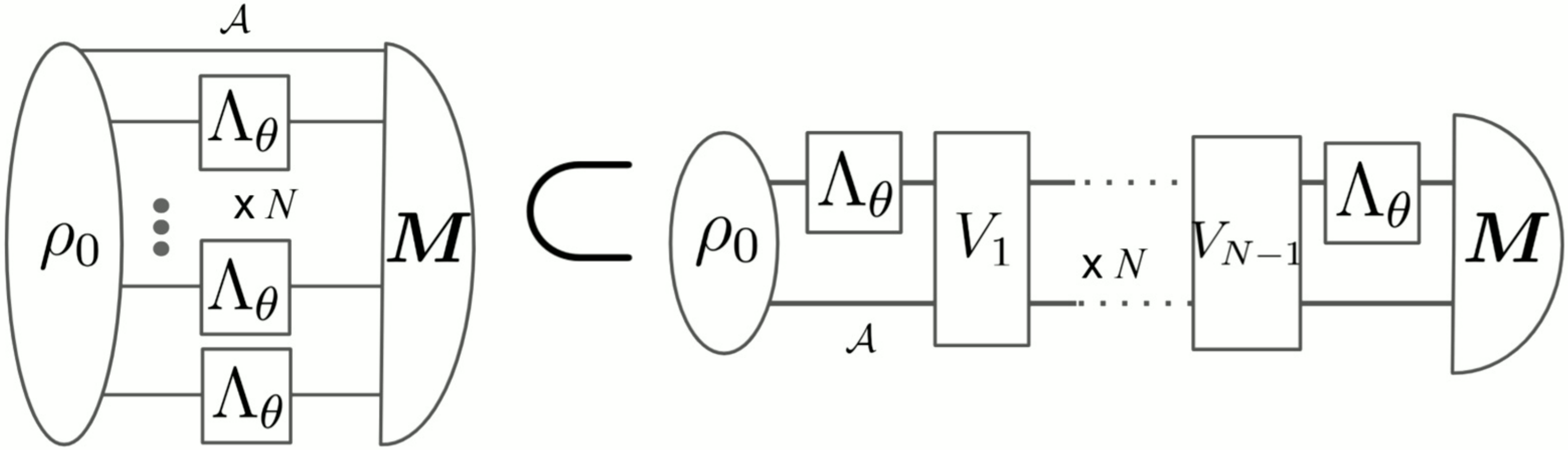
$$\lim_{N\to\infty} \mathcal{F}_{\mathrm{AD}}^{(N)}/\mathcal{F}_{\mathrm{E}}^{(N)} = 1$$

$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \sim N$: standard scaling

$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \sim N^2$: Heisenberg scaling

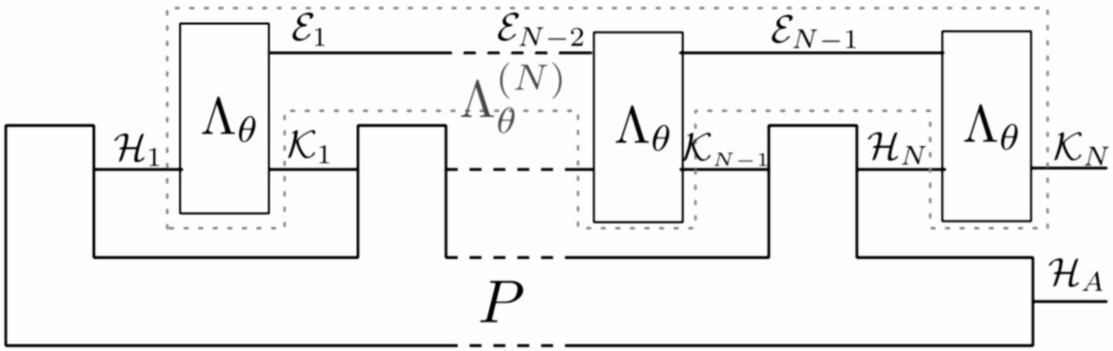
R. Demkowicz-Dobrzański, J. Kołodyński, M. Guta, Nat. Comm. 2012
S. Zhou, L. Jiang, PRX Quantum 2021
SK, W. Górecki, F. Albarelli, R. Demkowicz-Dobrzański, PRL 2023

Adaptive and parallel schemes



Adaptive schemes may give advantage for finite N.
In some cases, they are also easier to implement.
Asymptotic advantage for correlated noise:???

Adaptive scheme as a quantum comb



For correlated noise: $\Lambda_\theta^{(N)} \neq \Lambda_\theta^{\otimes N}$

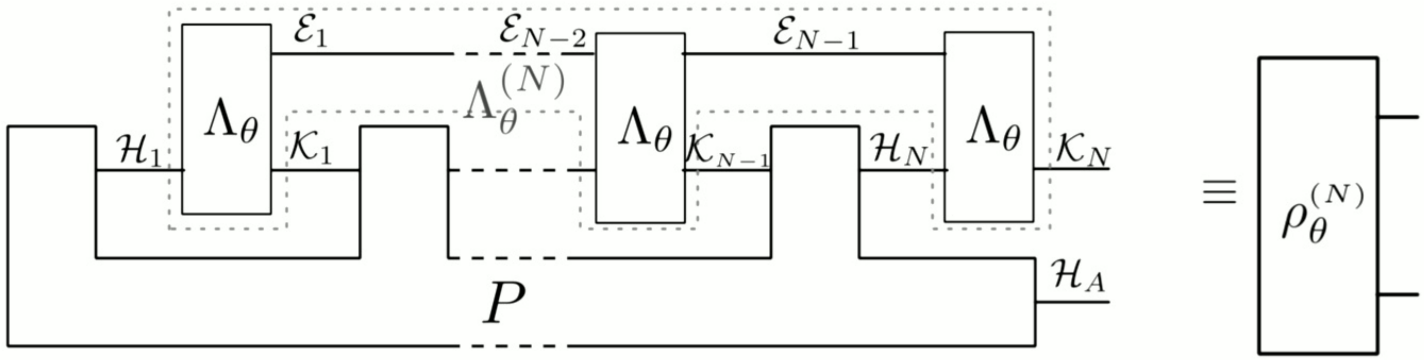
Choi operator of a strategy comb satisfies:

$$P \in \mathrm{Lin}\left(\mathcal{H}_1 \otimes \ldots \otimes \mathcal{H}_N \otimes \mathcal{H}_\mathcal{A} \otimes \mathcal{K}_1 \otimes \ldots \otimes \mathcal{K}_{N-1}\right)$$

$$P \geq 0,\ \mathrm{Tr}_{\mathcal{H}_A \otimes \mathcal{H}_N} P = P^{(N-1)} \otimes \mathbb{1}_{\mathcal{K}_{N-1}},$$

$$\underset{1<k<N}{\forall} \mathrm{Tr}_{\mathcal{H}_k} P^{(k)} = P^{(k-1)} \otimes \mathbb{1}_{\mathcal{K}_{k-1}},\ \mathrm{Tr}_{\mathcal{H}_1} P^{(1)} = 1$$

Optimization of QFI over combs



$$\rho_\theta^{(N)} = \Lambda_\theta^{(N)} * P$$

Link product, linear in both arguments

$$\mathcal{F}_{\mathrm{AD}}^{(N)} = \max_P F(\rho_\theta^{(N)})$$

Can be formulated as SDP using MOP!

Practical limitations: ***N<5***

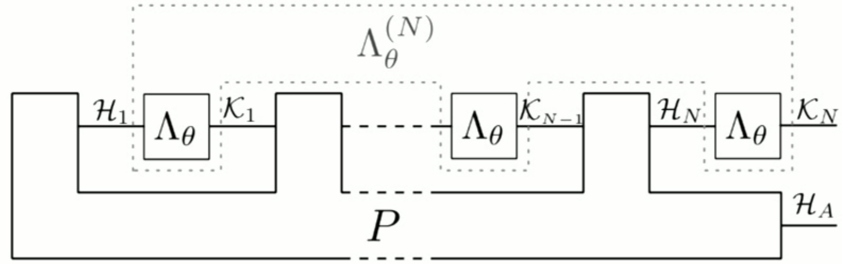
A. Altherr, Y. Yang, PRL 127, 060501 (2021)

Q. Liu, Z. Hu, H.Yuan, Y. Yang, PRL 130, 070803 (2023)

N >> 1

Go and no-go theorems

ISS optimization for combs



$$\bar{F}(\rho_\theta, L) = 2\mathrm{Tr}(\dot{\rho}_\theta L) - \mathrm{Tr}(\rho_\theta L^2)$$

$$\mathcal{F}_{\mathrm{AD}}(\Lambda_\theta^{(N)}) = \max_{P,L} \bar{F}(\Lambda_\theta^{(N)} \star P, L)$$

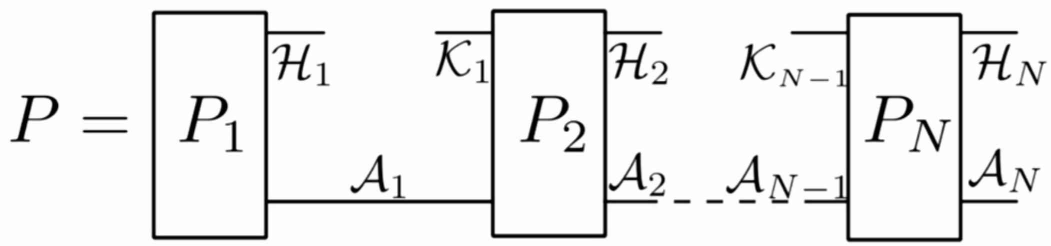
SDP: linear function, linear and positivity constraints for P

SDP: quadratic function, L hermitian

ISS method works again! but…

- We control output ancilla size, but not ancilla size during the protocol
- Complexity still exponential with N

Break comb into teeth



$$P = P_1 * P_2 * ... * P_N$$

$$d_{\mathcal{A}_i} = d_{\mathcal{A}}$$

$$d_{\mathcal{H}_i} = d_{\mathcal{K}_i} = d_{\mathcal{H}}$$

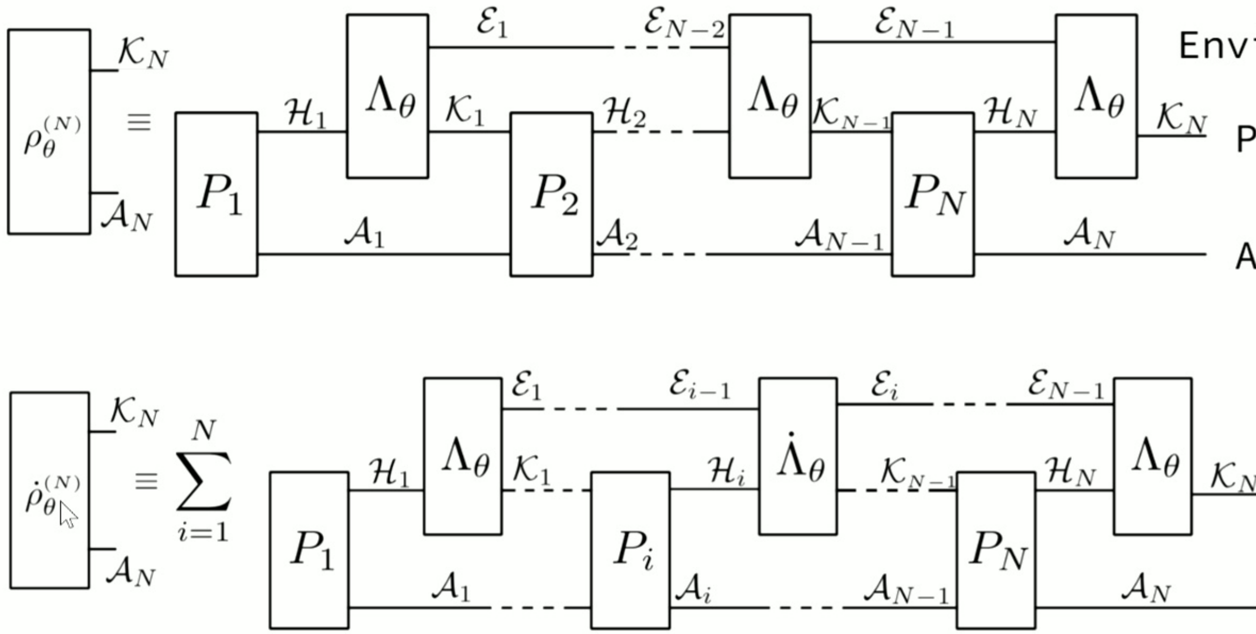
Storing P requires $d_{\mathcal{H}}^{4N-2} d_{\mathcal{A}}^2$ variables

Storing $P_1 * P_2 * ... * P_N$ requires $d_{\mathcal{H}}^2 d_{\mathcal{A}}^2 + (N-1) d_{\mathcal{H}}^4 d_{\mathcal{A}}^4$ variables

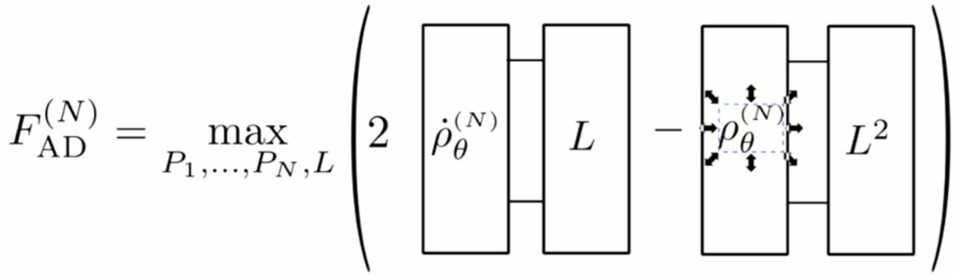
Exponential gain for limited ancilla size!

ISS optimization with tensor networks

We write the figure of merit as a tensor network.
Nodes: Choi operators, links: link products



ISS optimization with tensor networks

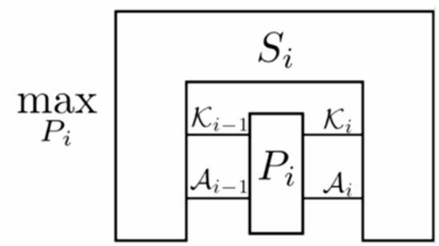


Optimization over L

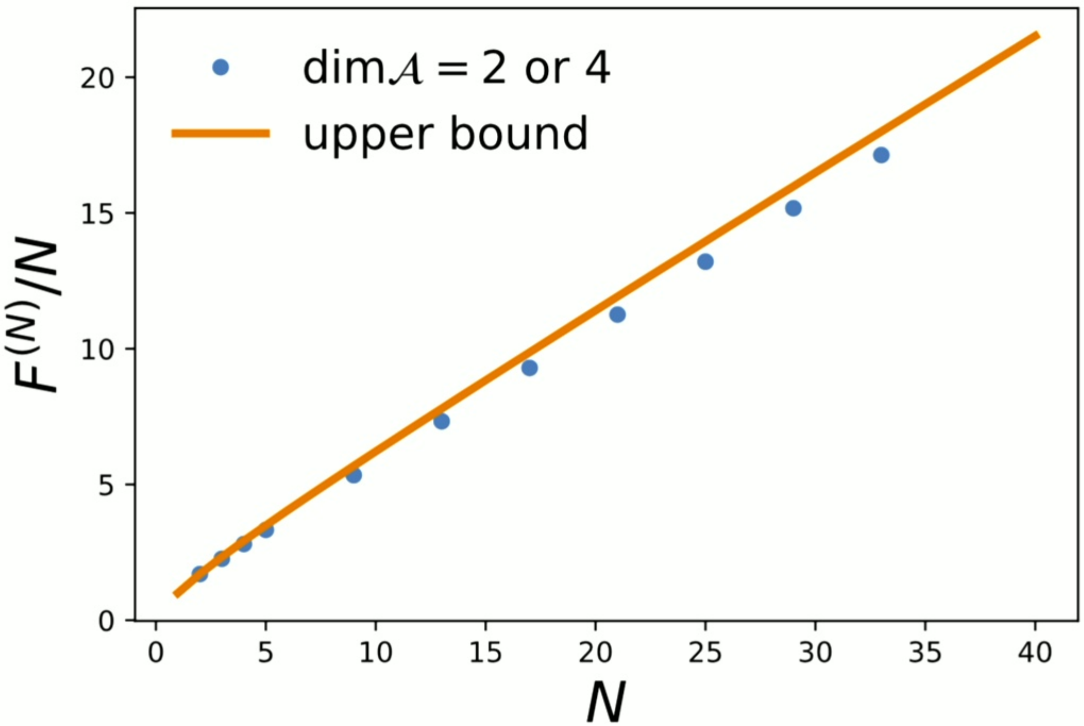
Contract all the indices in $\rho_\theta^{(N)}, \dot{\rho}_\theta^{(N)}$ (order matters!) then solve standard SDP for L

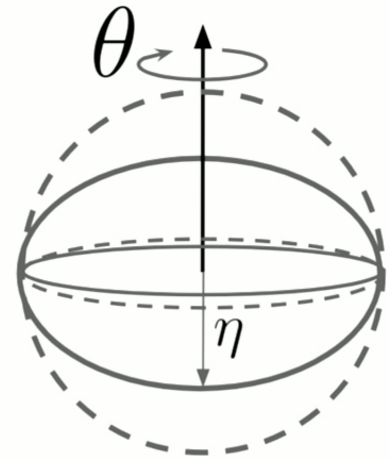
Optimization over P_i

Contract all the indices in the figure of merit apart from P_i indices. Then solve



Example: dephasing perpendicular to rotation

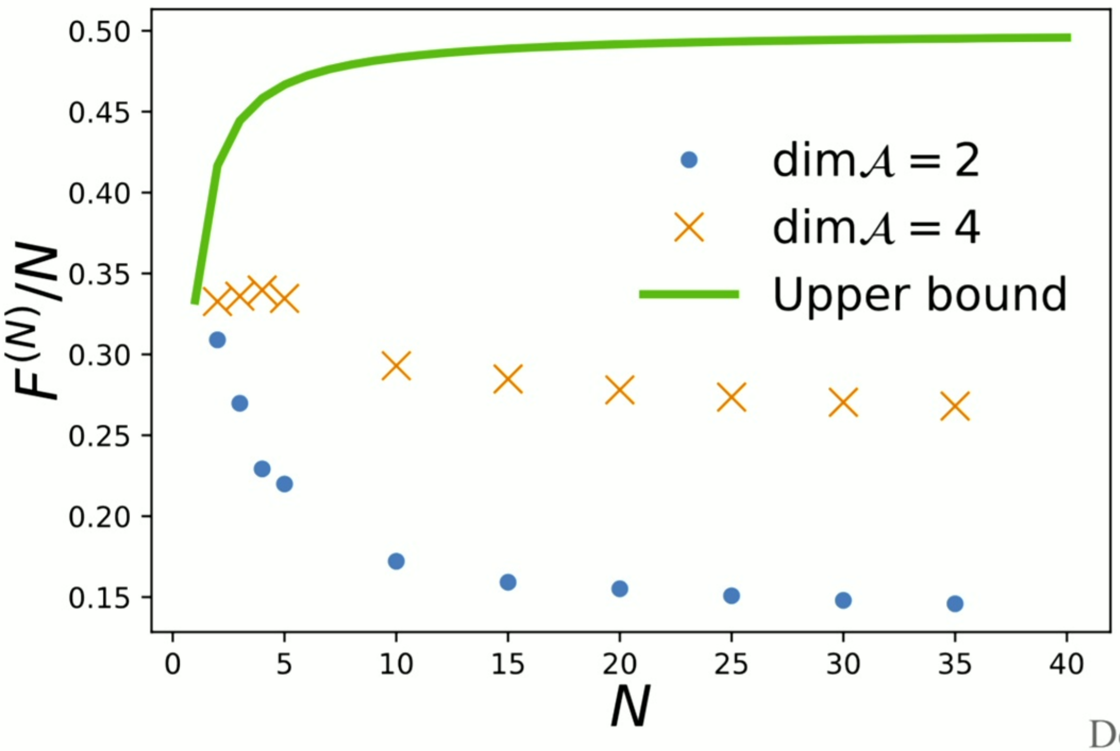


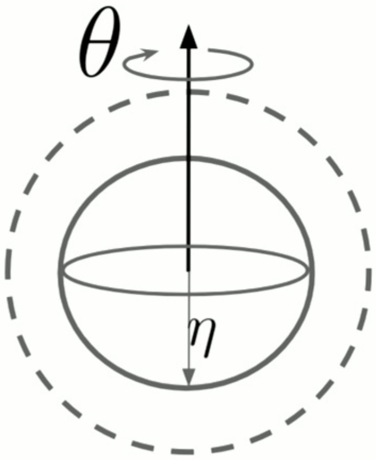


$$\eta = 0.85$$

Upper bounds:
S. Kurdziałek, W. Górecki, F. Albarelli, R. Demkowicz-Dobrzański, PRL 2023

Example: Depolarization

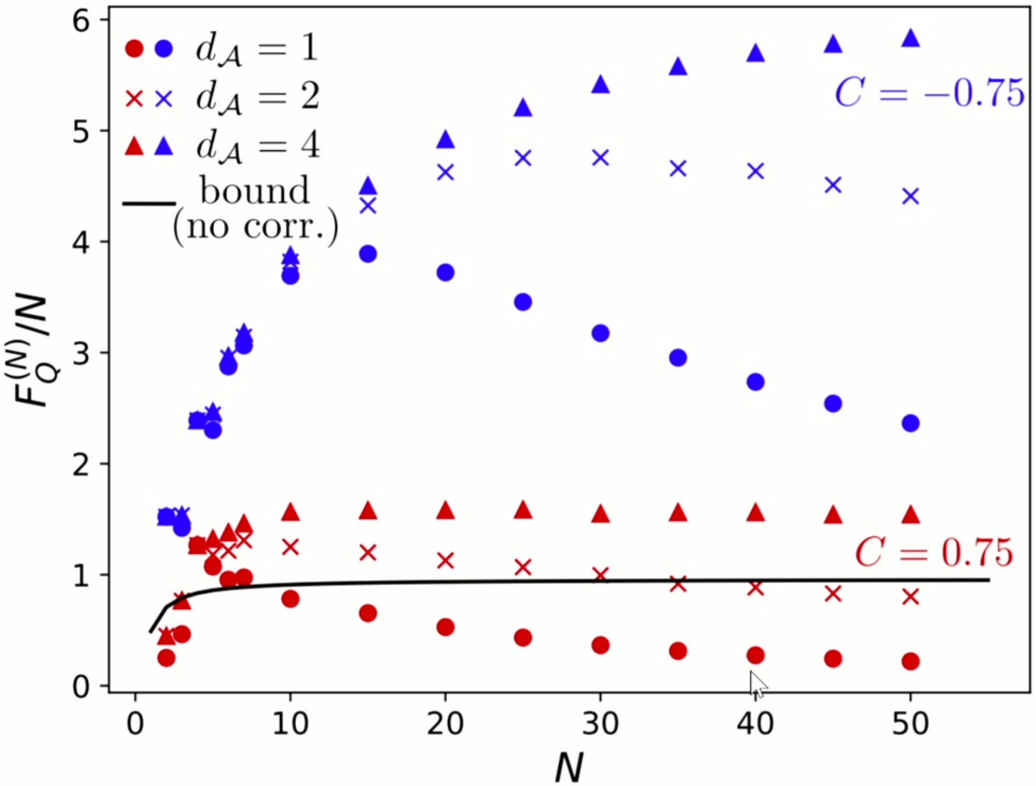


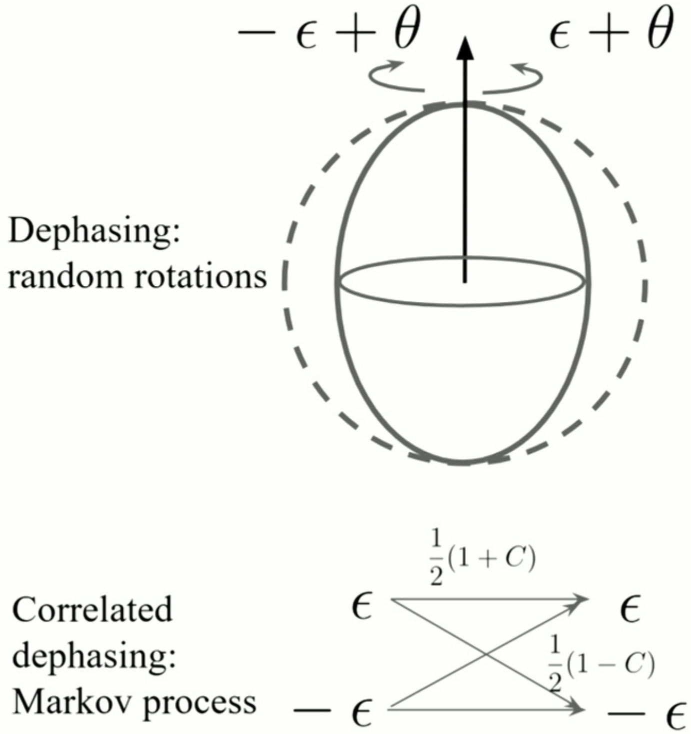


$\eta = 0.5$

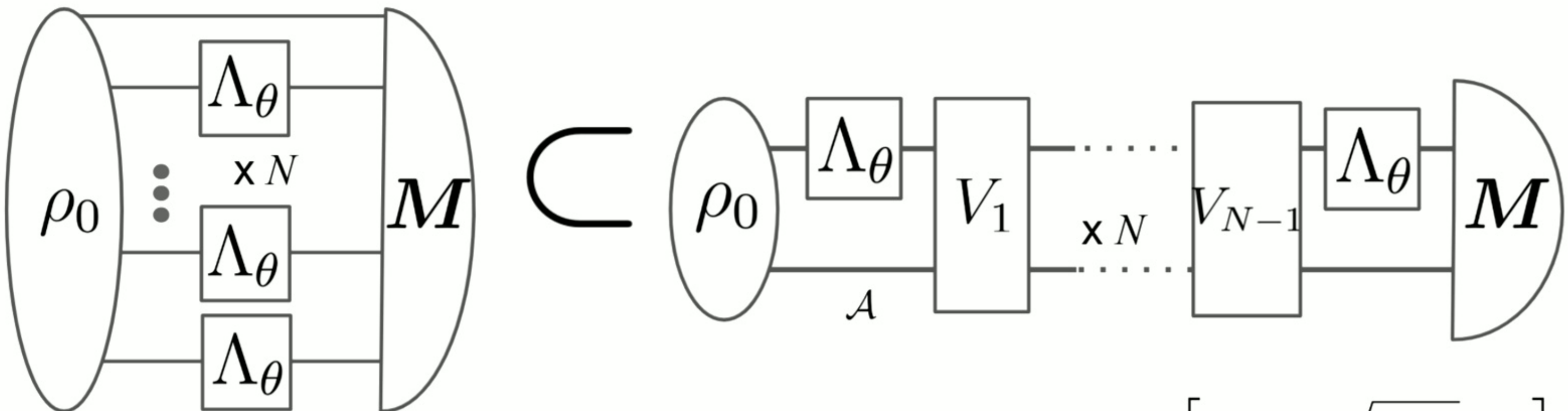
Upper bounds:
S. Kurdziałek, W. Górecki, F. Albarelli, R. Demkowicz-Dobrzański, PRL 2023

Example: Parallel dephasing with correlations





NO-GO THEOREMS FOR UNCORRELATED NOISE



$$\mathcal{F}_{\mathrm{E}}^{(N)} \leq 4 \min_h N\|\alpha\| + N(N-1)\|\beta\|^2$$

$$\alpha = \sum_k \dot{\tilde{K}}_k^\dagger \dot{\tilde{K}}_k, \beta = \sum_k \dot{\tilde{K}}_k^\dagger K_k$$

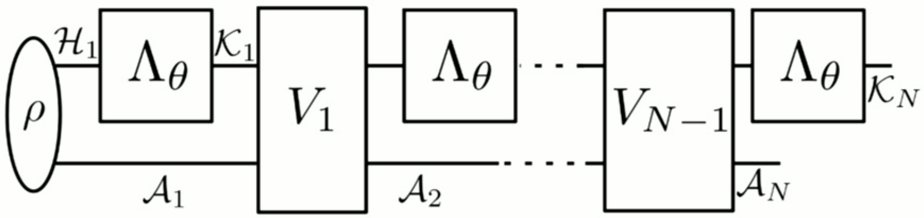
$$\mathcal{F}_{\mathrm{AD}}^{(i+1)} \leq \mathcal{F}_{\mathrm{AD}}^{(i)} + 4 \min_h \left[\|\alpha\| + \sqrt{\mathcal{F}_{\mathrm{AD}}^{(i)}} \|\beta\| \right]$$

$$\mathcal{F}_{\mathrm{AD}}^{(N)} \leq 4 \min_h N\|\alpha\| + N(N-1)\|\beta\|^2 \left(1 + \frac{c \log n}{n-1}\right)$$

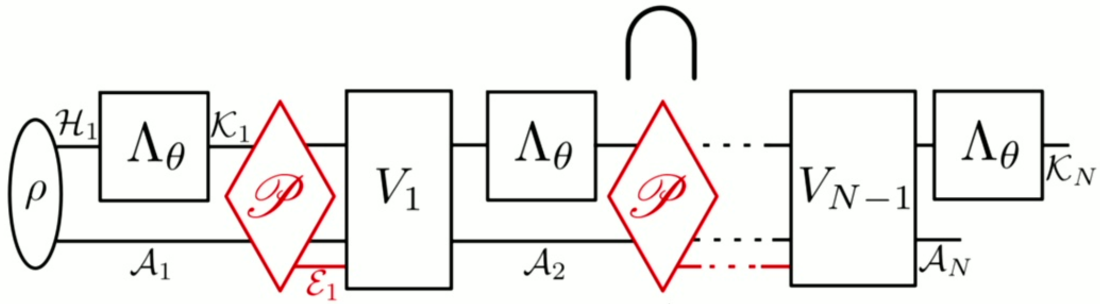
R. Demkowicz-Dobrzański, J. Kołodyński, M. Guta, Nat. Comm. 2012
S. Zhou, L. Jiang, PRX Quantum 2021
SK, W. Górecki, F. Albarelli, R. Demkowicz-Dobrzański, PRL 2023

What makes adaptive bound not tight?

$$\mathcal{F}_{\rm AD}^{(i+1)} \leq \mathcal{F}_{\rm AD}^{(i)} + 4\min_h \left[\|\alpha\| + \sqrt{\mathcal{F}_{\rm AD}^{(i)}}\|\beta\|\right]$$



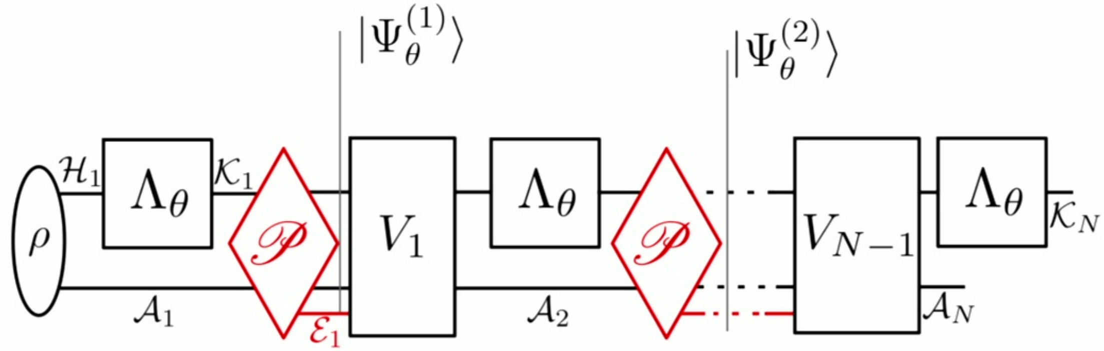
Due to its iterative nature, the bound is also valid for this scheme:

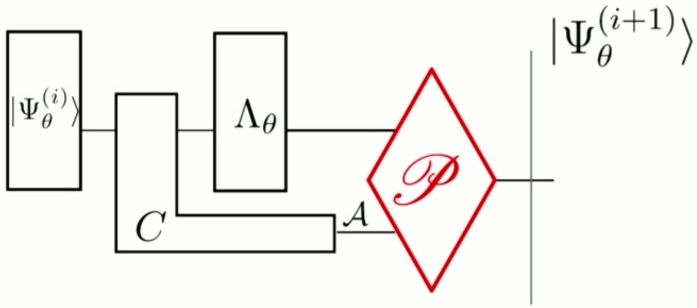


1) QFI non-increasing purification can be made each step.

2) Some non-tight algebraic inequalities were used.

TIGHTER ADAPTIVE BOUND





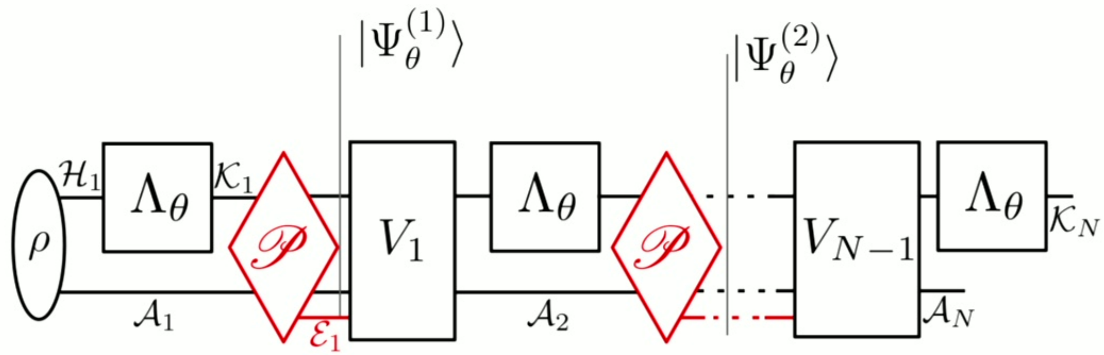
$$\mathbb{\Lambda}_\theta = |\Psi_\theta^{(i)}\rangle \langle \Psi_\theta^{(i)}| \otimes \Lambda_\theta$$

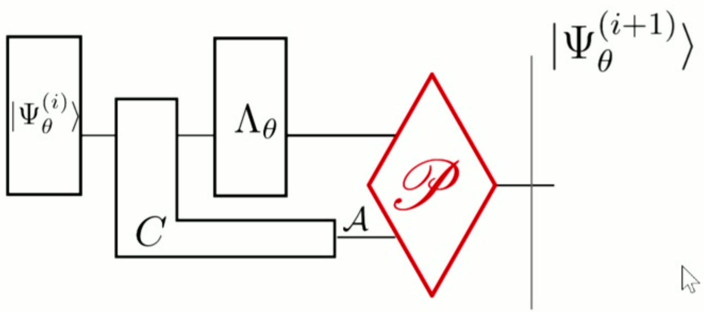
$$\mathcal{F}_{\mathrm{AD}}^{(i+1)} \leq \max_C F(\mathbb{\Lambda}_\theta \star C)$$

We can solve this using MOP formulation of comb QFI

?

TIGHTER ADAPTIVE BOUND





$$\mathbb{\Lambda}_\theta = |\Psi_\theta^{(i)}\rangle \langle \Psi_\theta^{(i)}| \otimes \Lambda_\theta$$

$$\mathcal{F}_{\mathrm{AD}}^{(i+1)} \leq \max_C F(\mathbb{\Lambda}_\theta \star C)$$

We can solve this using MOP formulation of comb QFI

$$|\Psi_\theta^{(i)}\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

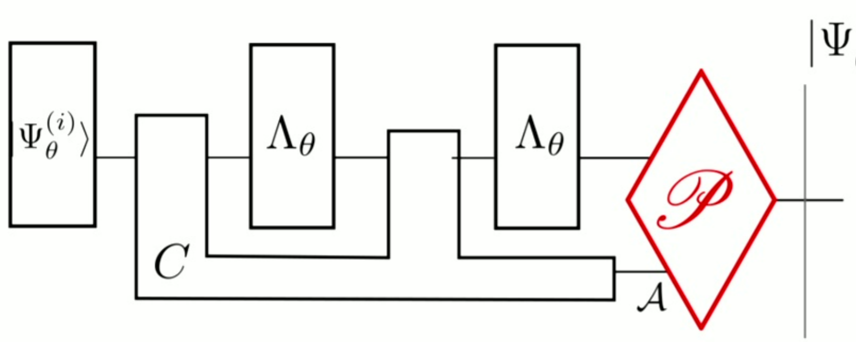
$$|\dot{\Psi}_\theta^{(i)}\rangle = \begin{bmatrix} 0 \\ \sqrt{\mathcal{F}_{\mathrm{AD}}^{(i)}/2} \end{bmatrix}$$

1) QFI non-increasing purification can be made each step.

2) ~~Some non-tight algebraic inequalities were used.~~

All other cases are isomorphic!

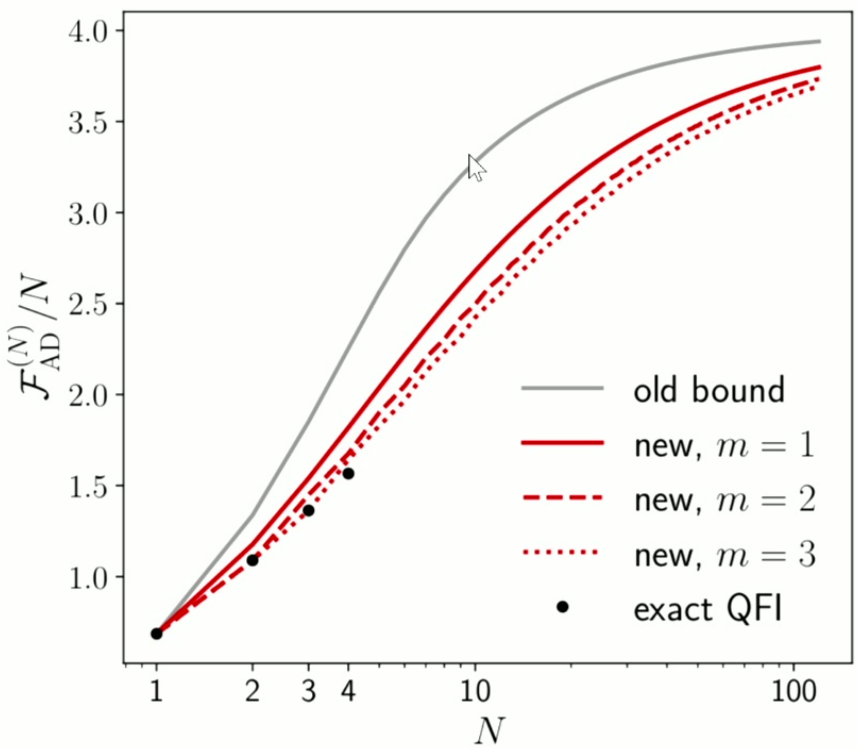
Even tighter adaptive bound



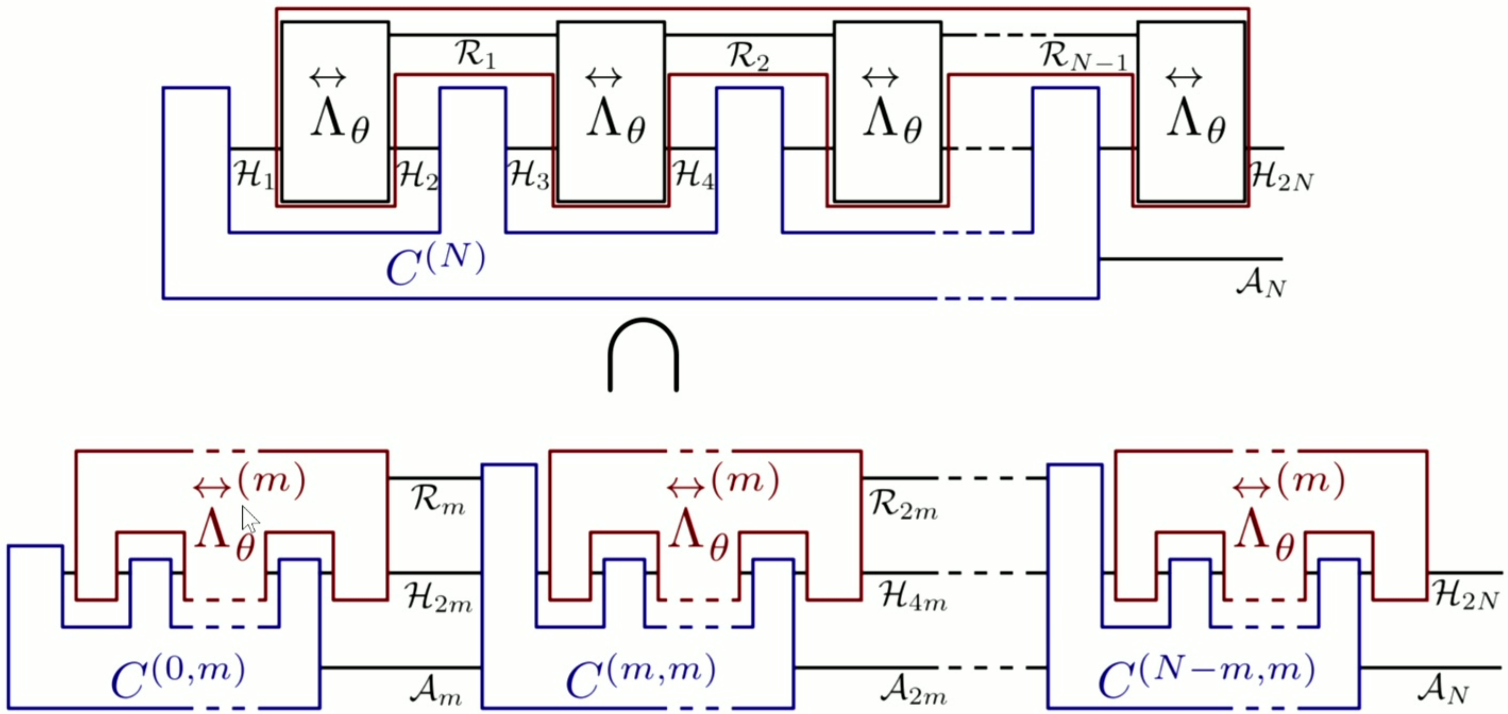
$$\mathbb{\Lambda}_\theta = |\Psi_\theta^{(i)}\rangle \langle \Psi_\theta^{(i)}| \otimes \Lambda_\theta^{\otimes m}$$

$$\mathcal{F}_{\mathrm{AD}}^{(i+m)} \leq \max_C F(\mathbb{\Lambda}_\theta \star C)$$

Example (phase estimation + amplitude damping noise)



Generalization to correlated noise



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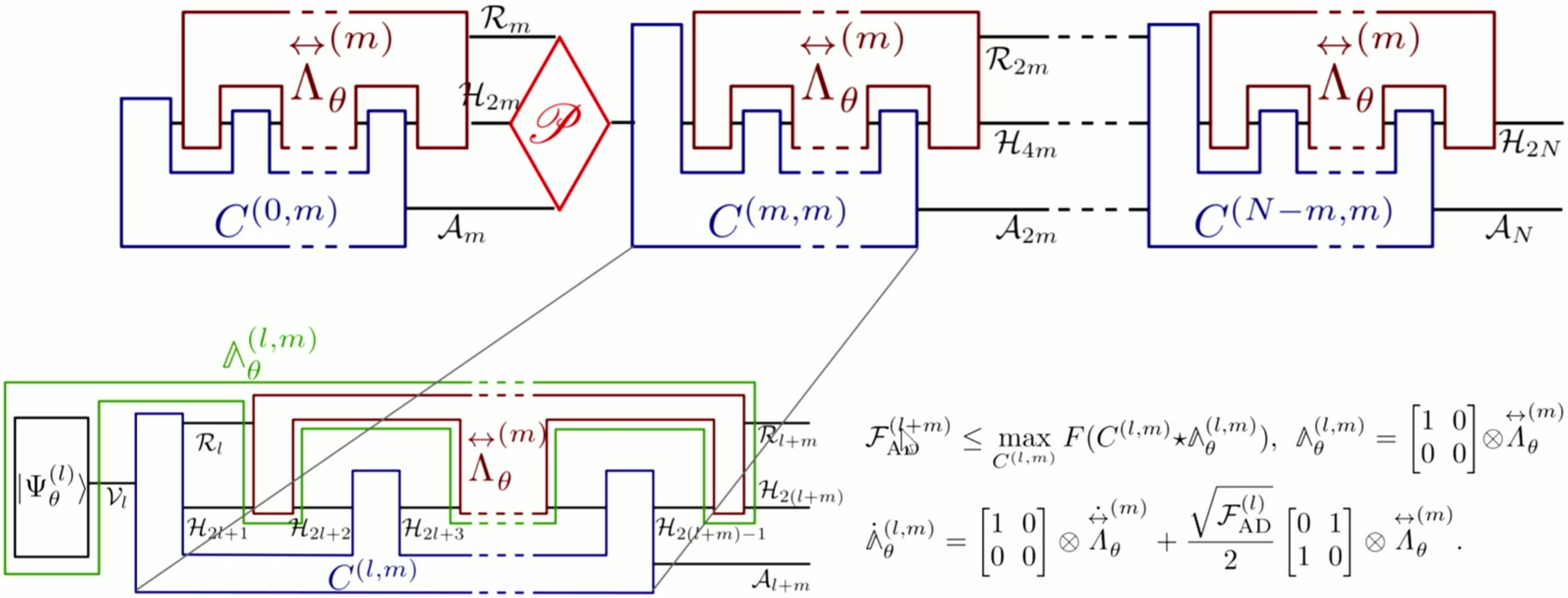
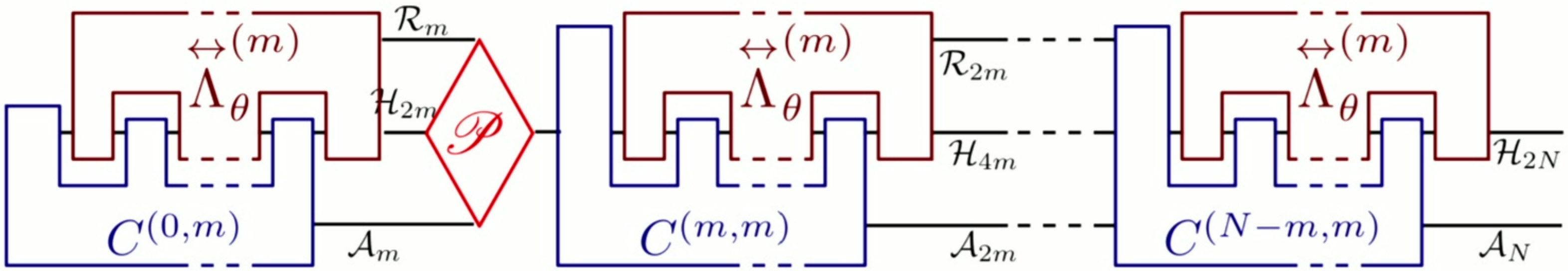
Generalization to correlated noise



$$\mathcal{F}_{\mathrm{AD}}^{(l+m)} \leq \max_{C^{(l,m)}} F(C^{(l,m)} \star \mathbb{\Lambda}_\theta^{(l,m)}), \quad \mathbb{\Lambda}_\theta^{(l,m)} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \overset{\leftrightarrow}{\Lambda}_\theta^{(m)}$$

$$\dot{\mathbb{\Lambda}}_\theta^{(l,m)} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \dot{\overset{\leftrightarrow}{\Lambda}}_\theta^{(m)} + \frac{\sqrt{\mathcal{F}_{\mathrm{AD}}^{(l)}}}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \overset{\leftrightarrow}{\Lambda}_\theta^{(m)} .$$

Generalization to correlated noise



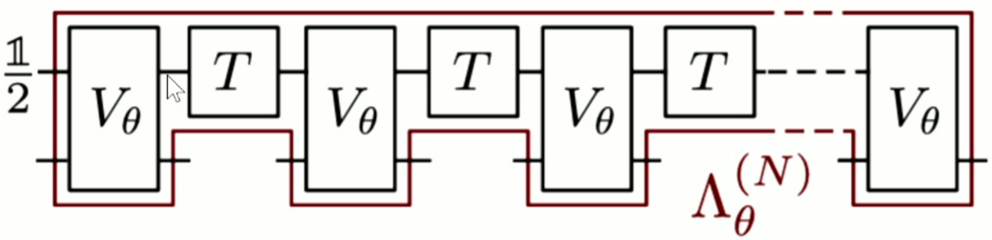
$$\mathcal{F}_{\text{AD}}^{(l+m)} \leq \max_{C^{(l,m)}} F(C^{(l,m)} \star \mathbb{\Lambda}_\theta^{(l,m)}), \;\; \mathbb{\Lambda}_\theta^{(l,m)} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \overset{\leftrightarrow}{\Lambda}_\theta^{(m)}$$

$$\dot{\mathbb{\Lambda}}_\theta^{(l,m)} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \dot{\overset{\leftrightarrow}{\Lambda}}_\theta^{(m)} + \frac{\sqrt{\mathcal{F}_{\text{AD}}^{(l)}}}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \overset{\leftrightarrow}{\Lambda}_\theta^{(m)}.$$

1) QFI non-increasing purification can be made each *m* steps.

2) Information leaks from environment every *m* steps.

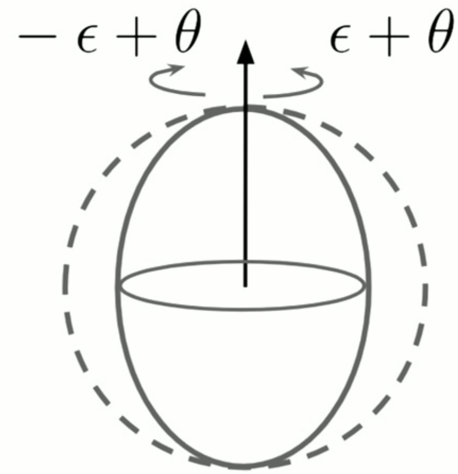
Example: dephasing with correlations

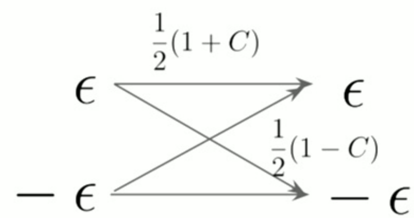


$$V_\theta = U_{+\epsilon+\theta} \otimes |+\rangle\langle+| + U_{-\epsilon+\theta} \otimes |-\rangle\langle-|$$

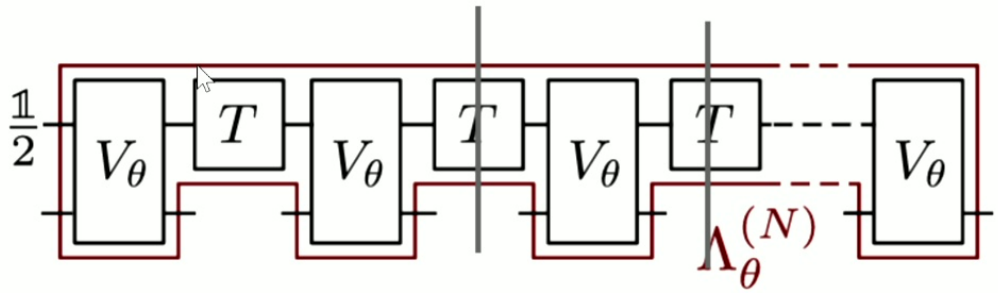
$$T(+|+) = T(-|-) = \frac{1}{2}(1+C)$$

$$T(+|-) = T(-|+) = \frac{1}{2}(1-C)$$





Example: dephasing with correlations

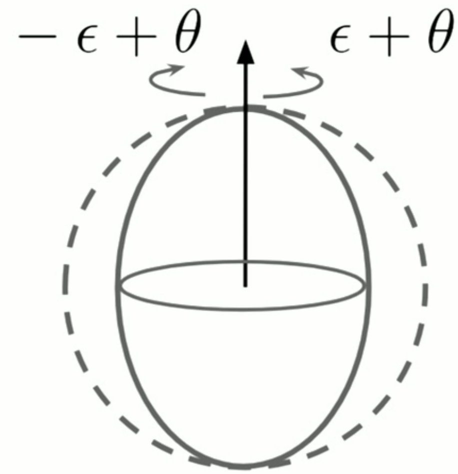


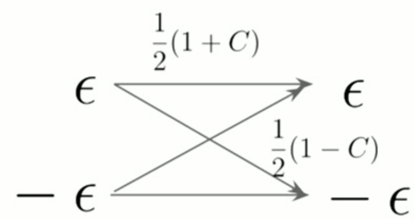
$$V_\theta = U_{+\epsilon+\theta} \otimes |+\rangle\langle+| + U_{-\epsilon+\theta} \otimes |-\rangle\langle-|$$

$$T(+|+) = T(-|-) = \frac{1}{2}(1 + C)$$

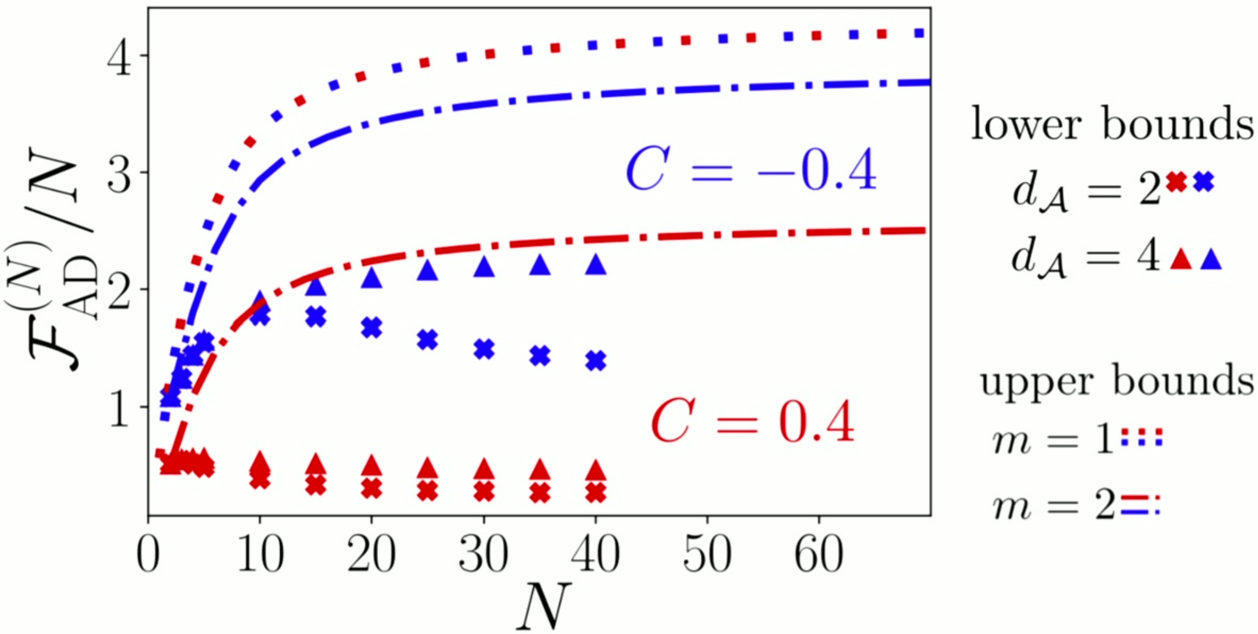
$$T(+|-) = T(-|+) = \frac{1}{2}(1 - C)$$

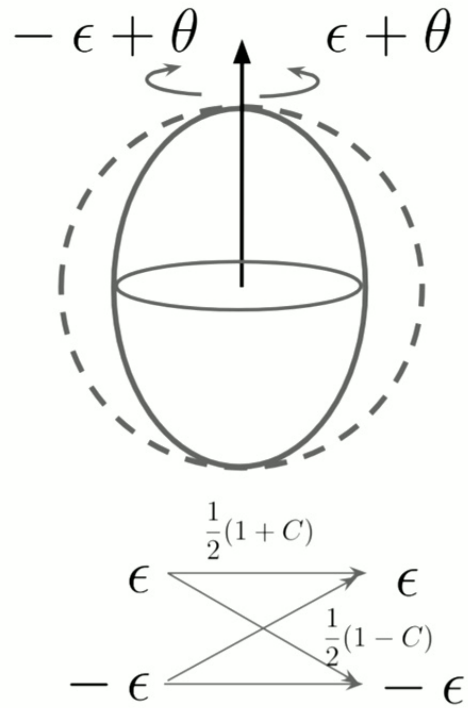
How to cut the chain into pieces?



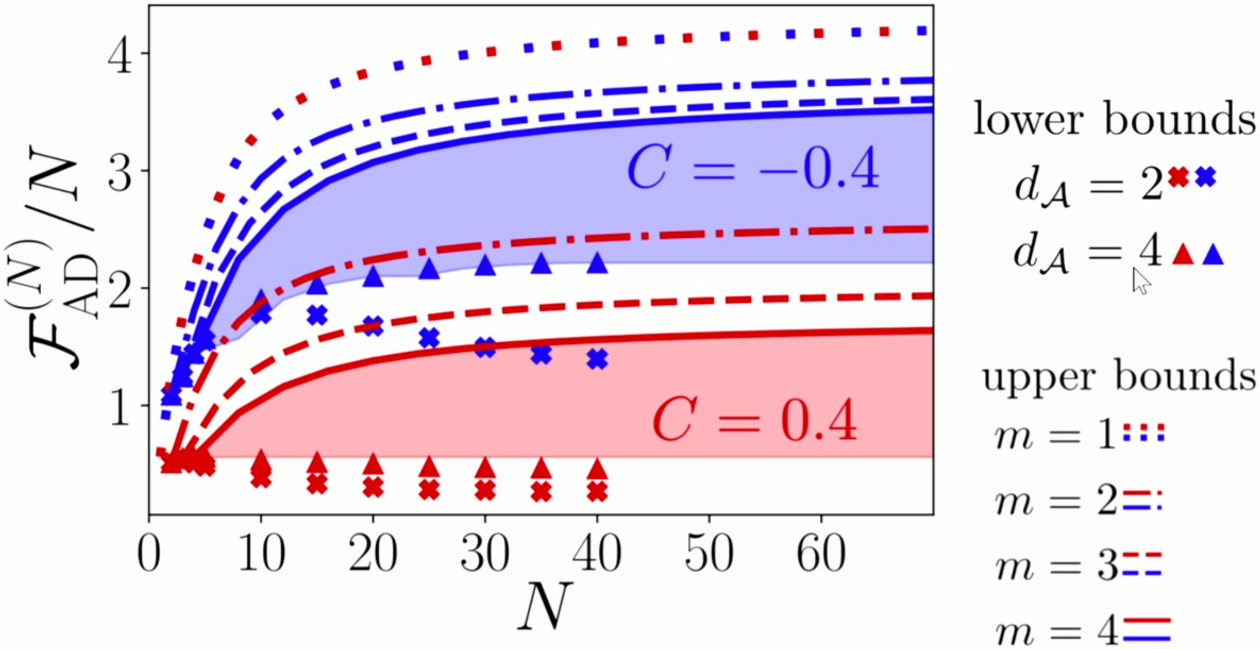


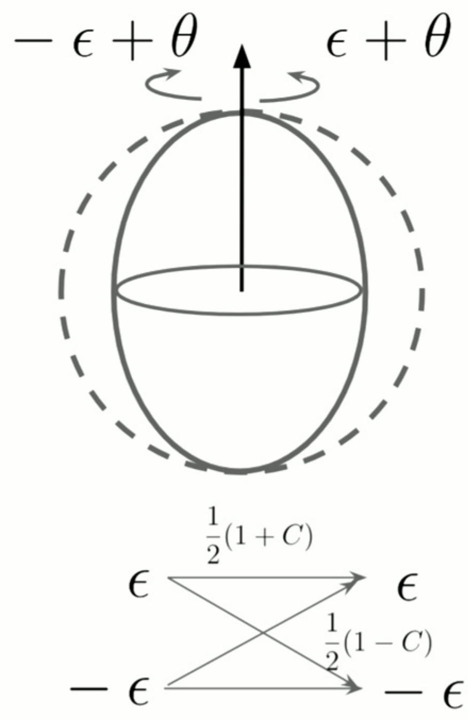
Example: dephasing with correlations



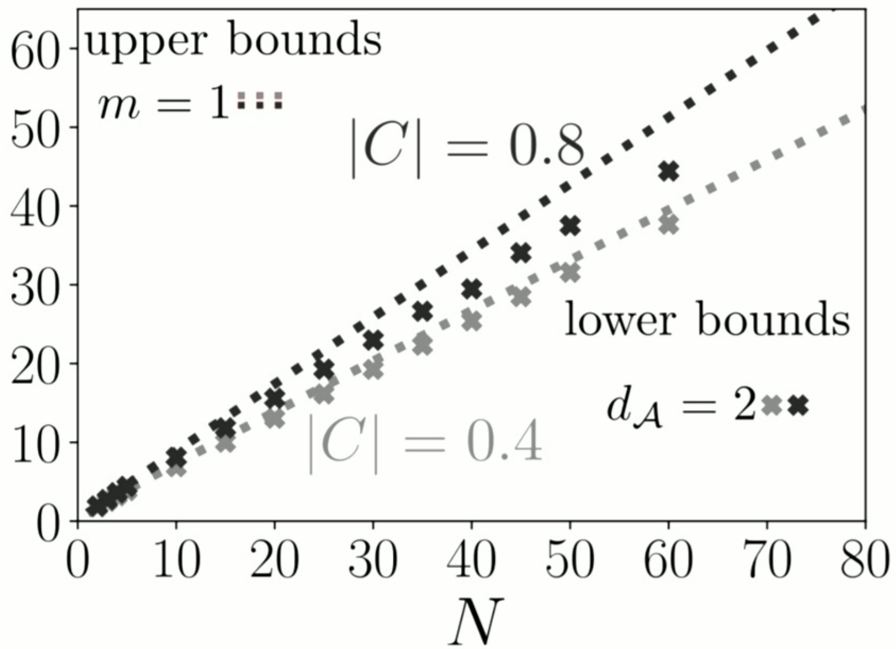


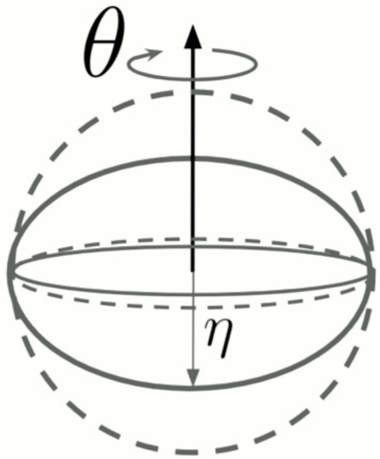
Example: dephasing with correlations

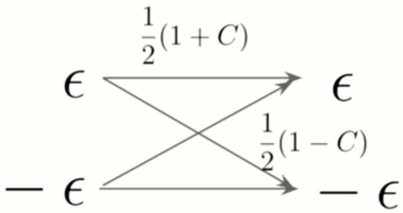




Example: perpendicular dephasing with correlations







Bounds equally tight for all *m*!

Thank you for your attention

arXiv:2403.04854

Quantum metrology using quantum combs and tensor network formalism

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arXiv:2410.01881

Universal bounds in quantum metrology in presence of correlated noise

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