Title: TBA - Quantum Fields & Strings Seminar

Speakers: Daine Danielson

Collection/Series: Quantum Fields and Strings

Subject: Quantum Fields and Strings

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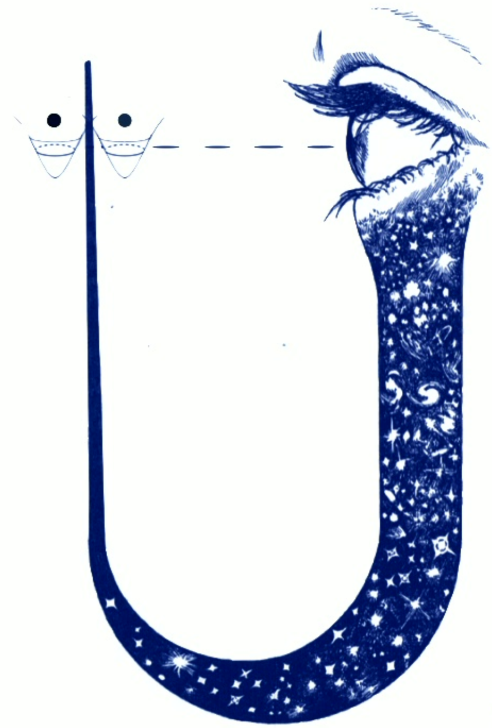
Horizons as Eavesdroppers: Horizon Algebras & Soft Quantum Information



G.S.

Daine L. Danielson[1], G. Satishchandran[2], R. M. Wald[1]
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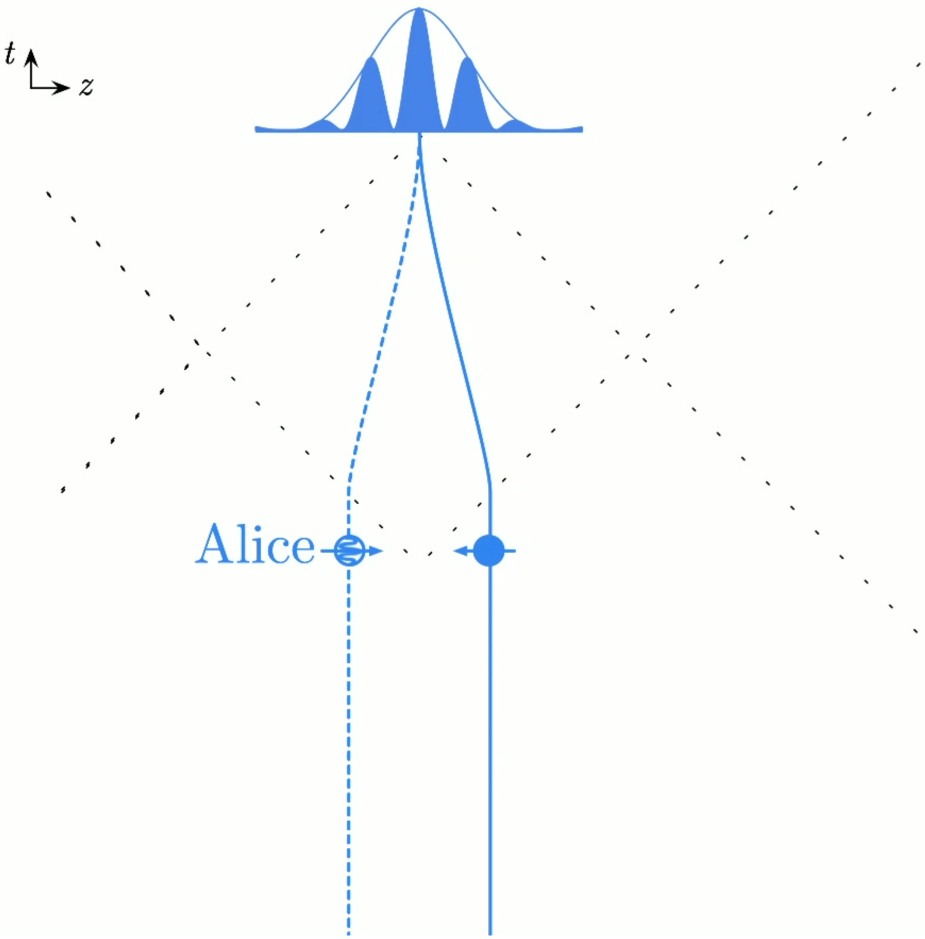
Perimeter Institute for Theoretical Physics, October 29, 2024

D.L.D., G. Satishchandran, R.M. Wald, *Int. J. Mod. Phys. D* 2241003 (2022), arXiv:2205.06279
Gravity Research Foundation Essay Competition, Third Award

D.L.D., G. Satishchandran R.M. Wald (2024), *Phys. Rev. D* 108, 025007 (2024), arXiv:2401.00026

D.L.D., G. Satishchandran, R.M. Wald, arXiv:2407.02567

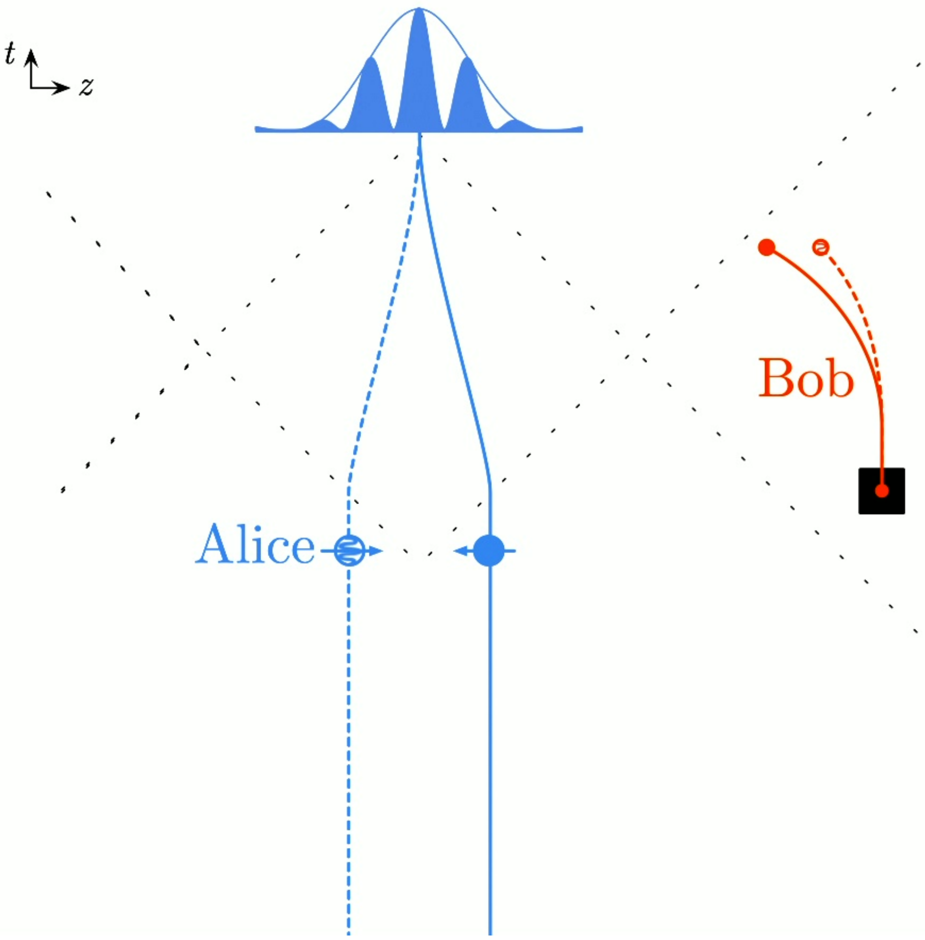
D.L.D., Jonah Kudler-Flam, G. Satishchandran (to appear)



In the past, Alice used a Stern-Gerlach apparatus to produce a **spatial superposition of a massive (or charged) body,** $\frac{1}{\sqrt{2}}\left(|\uparrow_z, A_1\rangle + |\downarrow_z, A_2\rangle\right)$

Later, she attempts an interference experiment and looks for signs of decoherence.

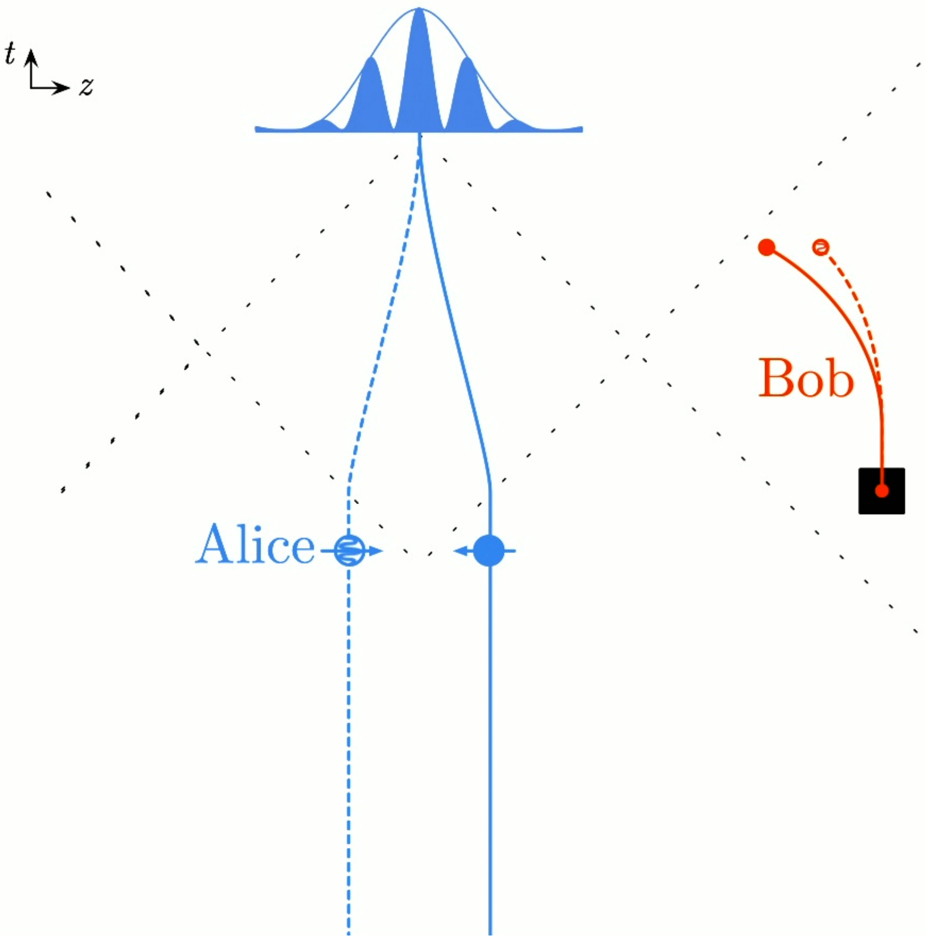
She can do this by, e.g., looking for coherent interference of the spin. She can measure spin along the *x*-axis. If she sees spin down even once, she knows her superposition has decohered.



Alice's particle is entangled with its own "Newtonian" field. Formally,

$$\frac{1}{\sqrt{2}}\left(|\uparrow, A_1\rangle \otimes |\psi_1\rangle + |\downarrow, A_2\rangle \otimes |\psi_2\rangle\right).$$

In a *spacelike-separated region,* Bob may attempt to measure Alice's superposed gravitational field by releasing a particle from a trap.



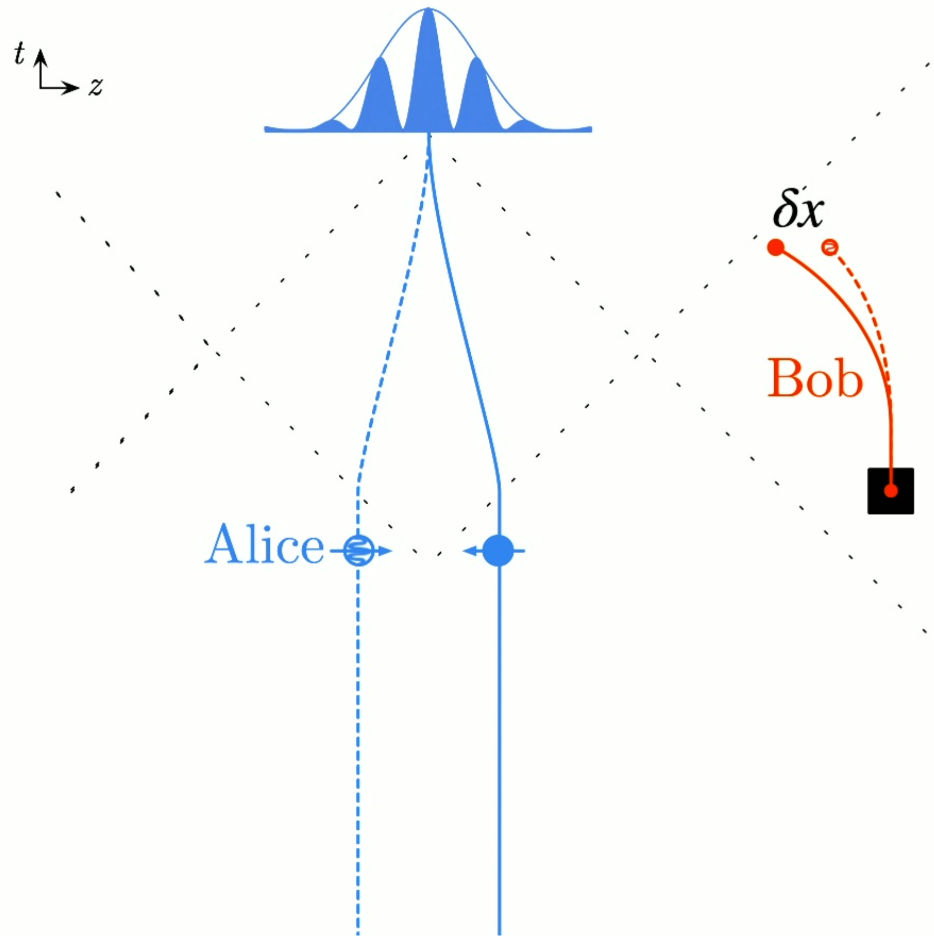
Bob can *choose* to measure, or not to measure, the field.

If Bob successfully measures the field, Alice's particle is decohered.

But Alice can tell whether her particle is decohered!

This seems paradoxical.

Paradox resolved...



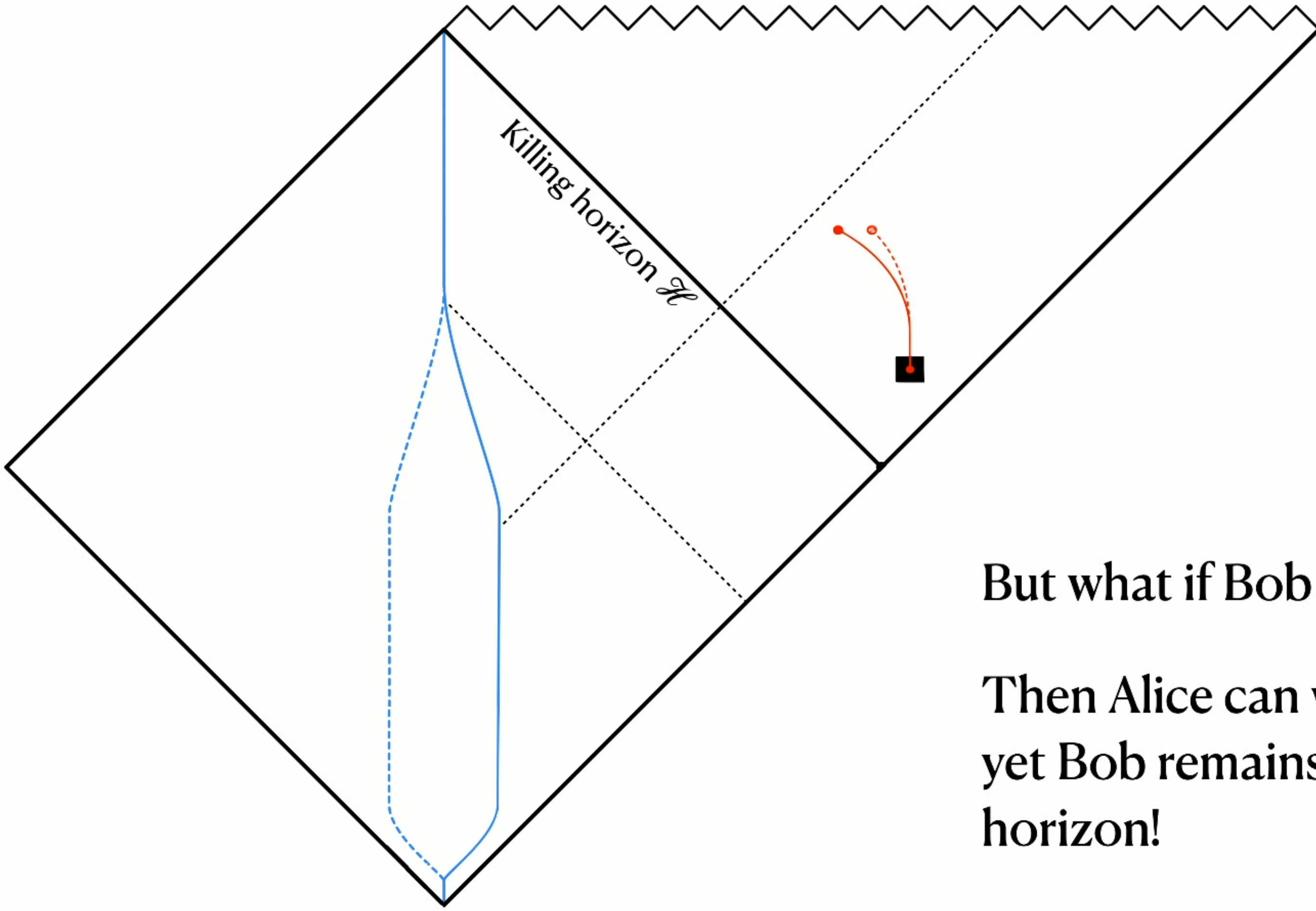
Bob's precision is limited by **vacuum fluctuations** of the metric, requiring $\delta x > \Delta x$. In QED, $\Delta x \sim q/m$. In gravity, $\Delta x \sim l_{\mathrm{P}}$.

Alice needs to recombine slowly to avoid producing **entangling radiation:** $\langle N \rangle \ll 1$. $2\,|\rho_{\mathrm{L,R}}| = |\langle h_{\mathrm{L}} | h_{\mathrm{R}} \rangle| = e^{-\frac{1}{2}\langle N \rangle}$.

If Alice goes slower, Bob must measure from farther away to remain spacelike. Thus he measures a weaker field, requiring more time.

[DLD, Satishchandran, Wald (2022). Belenchia *et al.* (2019).]

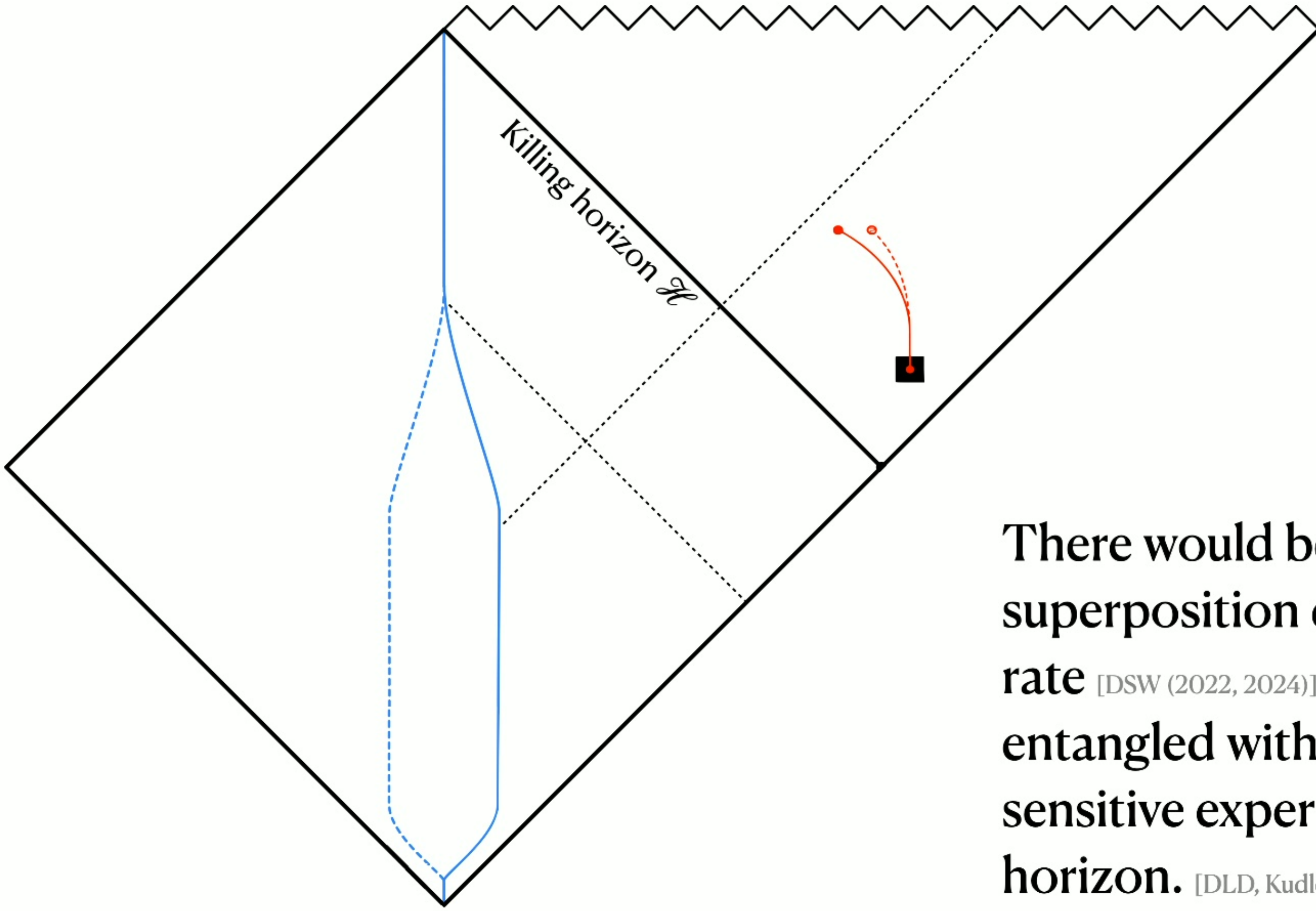
Paradox restored?



But what if Bob is behind a horizon?

Then Alice can work as slowly as she likes, yet Bob remains spacelike, just beyond the horizon!

Universal decoherence?



There would be no paradox if Alice's superposition decoheres at a constant rate [DSW (2022, 2024)] *as if* she were becoming entangled with any number of optimally-sensitive experiments hidden behind the horizon. [DLD, Kudler-Flam, Satishchandran, (to appear)].

Black hole decoherence effect

for a charged particle

- Plugging in numbers, Alice's superposition decoheres after
$$T_D \sim \frac{\hbar c^6 D^6}{G^3 M^3 q^2 d^2} \sim 10^{43} \text{ years } \left(\frac{D}{\text{a.u.}}\right)^6 \cdot \left(\frac{M_\odot}{M}\right)^3 \cdot \left(\frac{e}{q}\right)^2 \cdot \left(\frac{\text{m}}{d}\right)^2.$$

- If our Sun were a black hole, an electron on Earth superposed by a meter would decohere in 10^{32} times the age of the universe. But, if this experiment were done at the innermost stable orbit, then $T_D \sim 5$ minutes!

Black hole decoherence effect

for a massive particle

- The analysis proceeds exactly as before in (linearized) quantum gravity.
- *All objects* source gravity!
- Any superposed body will therefore be decohered by soft horizon gravitons after a time,

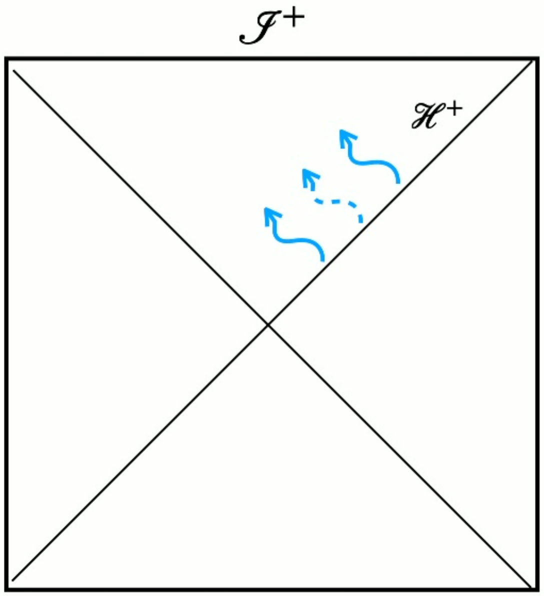
$$T_D^{\mathrm{GR}} \sim \frac{\hbar c^{10} D^{10}}{G^6 M^5 m^2 d^4} \sim 10\ \mu\mathrm{s} \left(\frac{\mathrm{D}}{\mathrm{a.u.}}\right)^{10} \cdot \left(\frac{\mathrm{M}_\odot}{\mathrm{M}}\right)^{5} \cdot \left(\frac{\mathrm{M}_{\mathrm{Earth}}}{\mathrm{m}}\right)^{2} \cdot \left(\frac{\mathrm{R}_{\mathrm{Earth}}}{\mathrm{d}}\right)^{4}.$$

- The effect is weak, but universal.

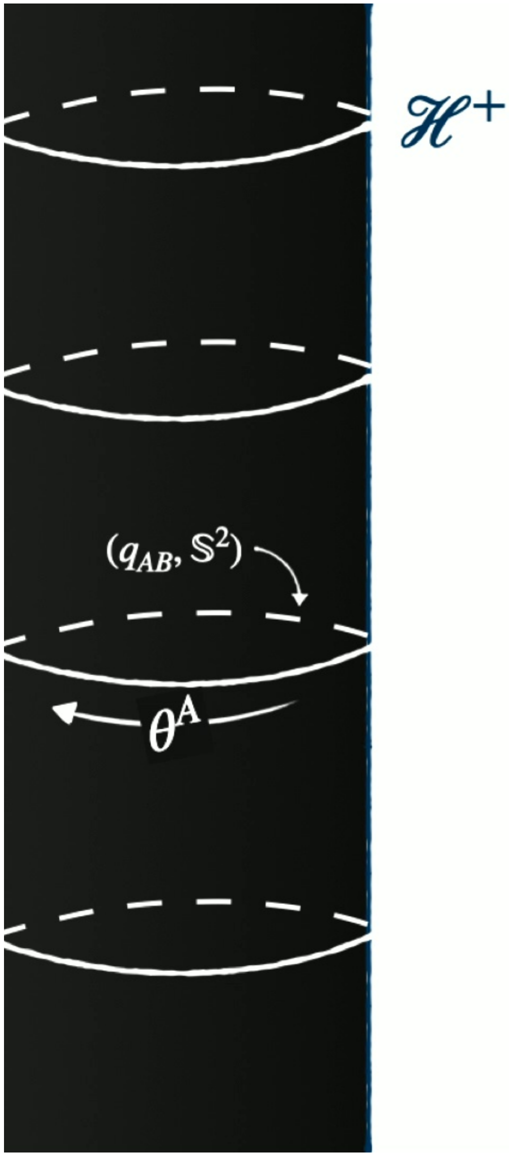
Generalizations

- Wei and Gralla (arXiv:2311.11461): the effect also arises in the presence of a rotating black hole, now also depending on the angular momentum.
    - Also see for discussion of extremal black holes, involving *black hole Meisner effect*.
- Cosmological horizons…

Cosmological Horizons



- In de Sitter spacetime with a horizon radius R_H, the electromagnetic decoherence time is $T_{\rm D}^{\rm EM} \sim \dfrac{\hbar \epsilon_0 R_H^3}{q^2 d^2}$.
- The quantum gravitational decoherence time is $T_{\rm D}^{\rm GR} \sim \dfrac{\hbar R_H^5}{G m^2 d^4}$.
- Since $d \ll R_H$, the decoherence time will be much larger than the Hubble time R_H/c unless q is extremely large relative to the Planck charge $q_P \equiv \sqrt{\epsilon_0 \hbar c} \sim 11e$, or m much larger than the Planck mass $m \gg m_P \sim 10\mu\text{g}$.
- Nevertheless, we see that decoherence does occur despite the fact that Alice's lab is *inertial* in this case.



What is the mechanism?

Suppose Alice performs her experiment in a stationary laboratory outside a black hole.

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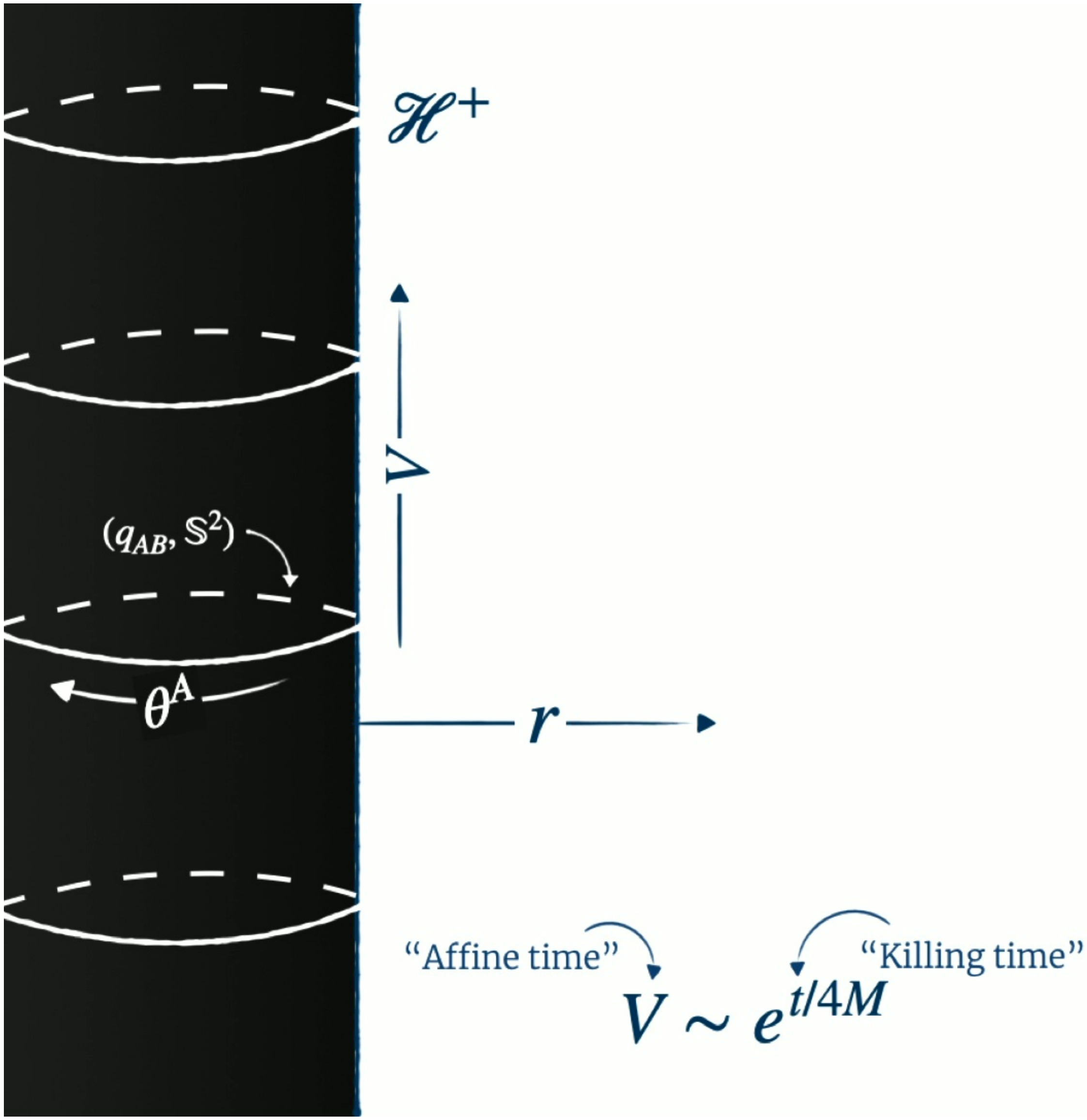
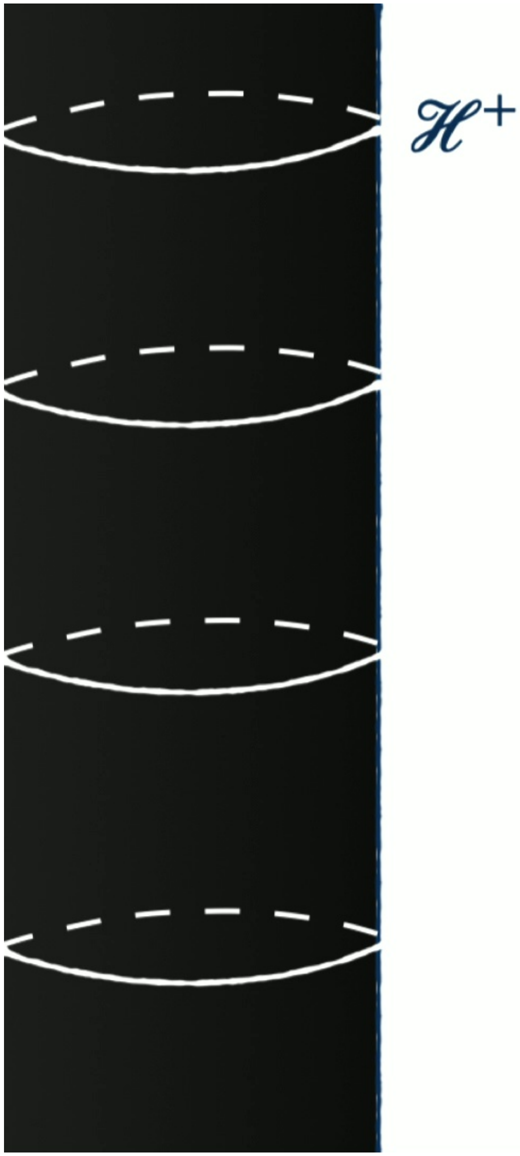
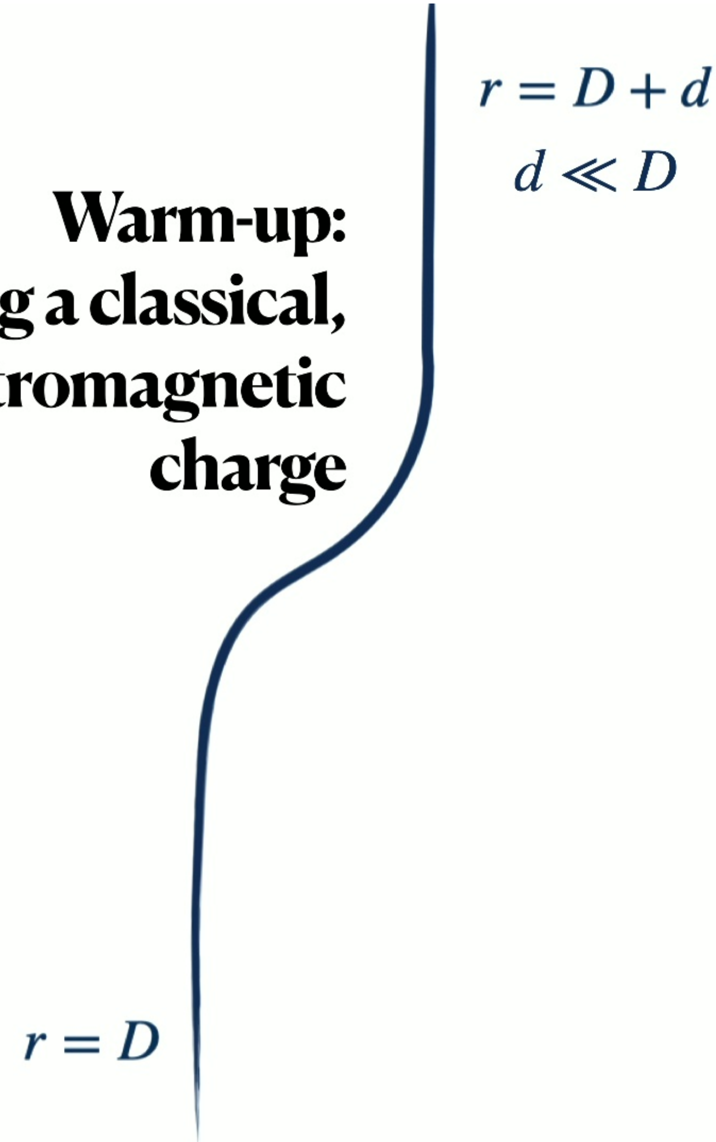


Daine Danielson, Perimeter, 2024



Warm-up: displacing a classical, electromagnetic charge



Unavoidable horizon radiation

V: affine time along the horizon

(angular indices on the horizon cross section)

$$\partial_V E_r = -\,\mathscr{D}^A E_A \qquad\qquad E_A = -\,\partial_V A_A$$

Evolving Coulomb field on the horizon

Radiation into the black hole

$$\Delta E_r \neq 0 \implies \int_{-\infty}^{V} E_A \neq 0 \implies \Delta A_A \neq 0$$

Net change in the potential!

Daine Danielson, Perimeter, 2024

A "memory effect" on the black hole horizon

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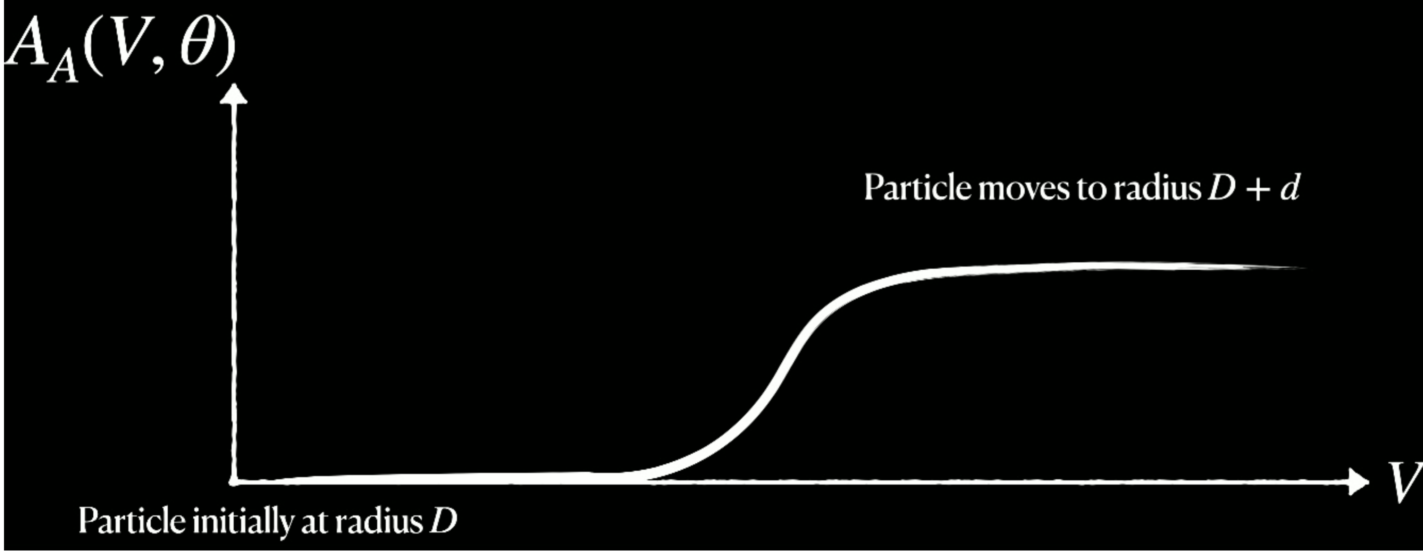


- A direct mathematical analog of the "memory effect," but on a black hole horizon.
- Unlike the memory effect at null infinity, this occurs on horizons in all dimensions in which radiation exists.

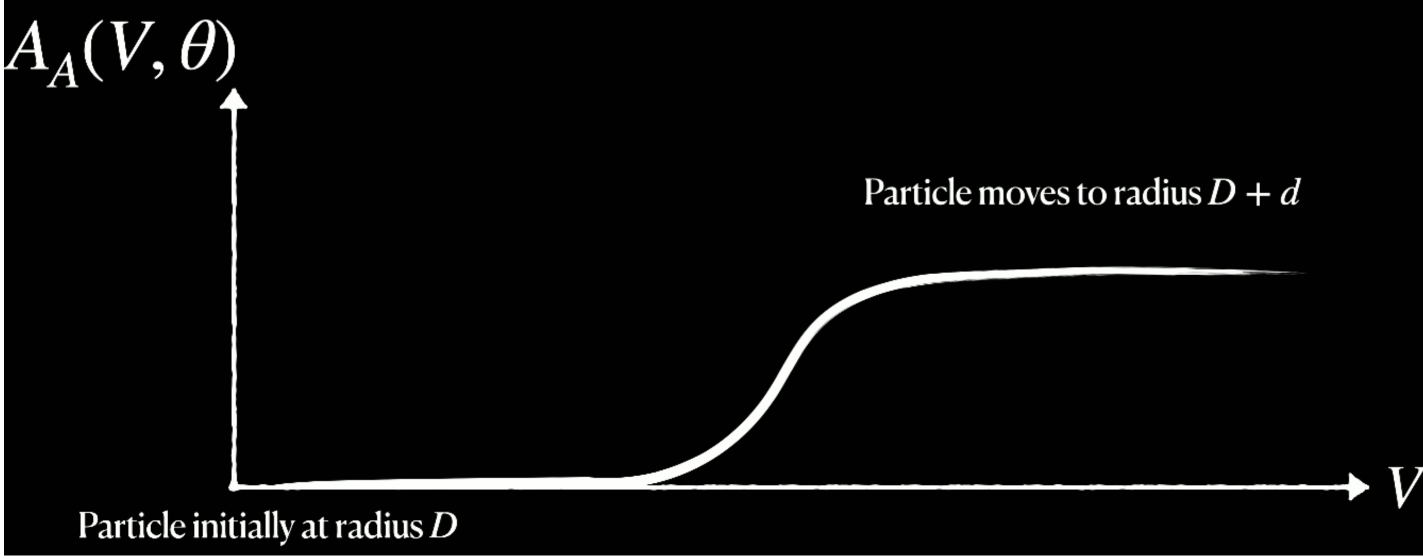
Soft photons on the black hole horizon

$$\hat{A}_A(\omega) \sim \frac{1}{\omega}$$

so large ΔV means arbitrarily low energy...

$$\langle N \rangle \propto \int_{S^2} d\Omega \int_0^\infty d\omega \; \omega \hat{A}^B \hat{A}_B \to \infty$$

but *many* (soft) photons!
For a permanently displaced particle, the permanently shifted Coulomb field produces an "infrared divergence."



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Horizon Algebras

DLD, G. Satishchandran, J. Judler-Flam (to appear)

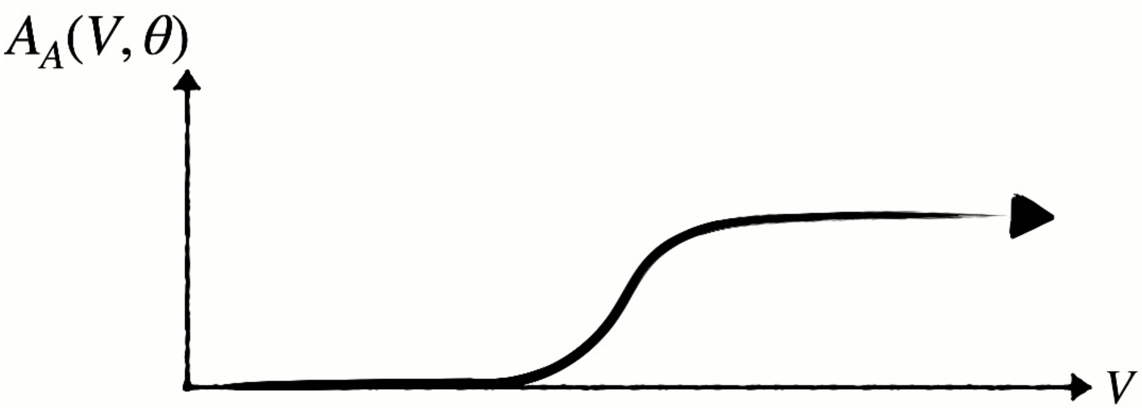
Notation: $\hat{E}(A) \equiv \int_{\mathscr{H}} dV dx_A \hat{E}_B A^B$

Coherent state operator: $\exp\left(i \int_{\mathscr{H}} dV dx_A \hat{E}_B A^B\right) = e^{i\hat{E}(A)}$

What about coherent states with memory?

Let A_Δ^B be a four-potential with nonzero memory.

This would require A_Δ^B to have non-compact support. But as we have seen,



$\implies \langle N \rangle = \langle \Omega \hat{E}(A) \,|\, \hat{E}(A)\Omega \rangle \to \infty$

Therefore $\hat{E}(A)$ cannot be represented as an operator on the standard Fock space!

Horizon Algebras

DLD, G. Satishchandran, J. Judler-Flam (to appear)

We can extend the algebra to include the nonperturbative operators $\hat{U}(A_\Delta)$ defined by the Weyl relations $\hat{U}(A_1)\hat{U}(A_2) = \hat{U}(A_1 + A_2)e^{-i\Omega(A_1,A_2)/2}$ where Ω is the symplectic form.

The Poincare-invariant vacuum $\omega_\Omega : \mathscr{A} \to \mathbb{C}$ admits a *unique extension* onto this enlarged algebra $\mathscr{A}$!

$$\omega_\Omega(\boldsymbol{U}^*(f_1)\boldsymbol{U}(f_2)) = \begin{cases} e^{i\Omega(f_1,f_2)/2} & \partial_V f_1 = \partial_V f_2 \\ e^{-\frac{1}{2}\|f_1-f_2\|^2} e^{i\Omega(f_1,f_2)/2} & \Delta_{f_1} = \Delta_{f_2} \\ 0 & \Delta_{f_1} \neq \Delta_{f_2}. \end{cases}$$

The resulting IR-finite states on the algebra, $\hat{U}(A)\,|\,\Omega\rangle$, include coherent states with memory. For the special case where $\partial_V A = 0$, $\hat{U}(A) = e^{i\hat{\Delta}_B(A)}$ is an element of the horizon symmetry group (e.g., an asymptotic symmetry transformation at null infinity), and measures the *memory* of coherent state operators:

$$\hat{U}(\lambda)^*\hat{U}(A_\Delta)\hat{U}(\lambda) = \hat{U}(A_\Delta)e^{i\Delta^B(\mathscr{D}_B\lambda)}$$

Soft Quantum Information Theory

DLD, G. Satishchandran, J. Judler-Flam (to appear)

The resulting Hilbert space admits a Tomita-Takesaki theory, which allows us to quantify the information content of soft modes.

Consider the coherent states (possibly with memory)

$$|\psi\rangle = \hat{U}(f^{\Psi})|\Omega\rangle \text{ and } |\phi\rangle = \hat{U}(f^{\Phi})|\Omega\rangle.$$

Suppose I am interested in the mixed states that results from restricting these sates to the past of a cut V_0 of the horizon.

Then the relative Tomita operator is,

$$\hat{S}_{\Psi\to\Phi} = e^{i\Omega(f^{\Phi}(V_0),f^{\Phi})/2}\hat{U}(F_-^{\Psi})\hat{U}(F_+^{\Phi})\hat{S}_{\Omega}\hat{U}(F_-^{\Phi})^*\hat{U}(F_+^{\Psi})^*e^{-i\Omega(f^{\Psi}(V_0),f^{\Psi})/2}$$

where

$$f_{\pm}(V) := \Theta(\pm(V-V_0))f(V_0) + \Theta(\pm(V_0-V))f(V)$$

$$F_{\pm}(V) := f_{\pm} - f(V_0)$$

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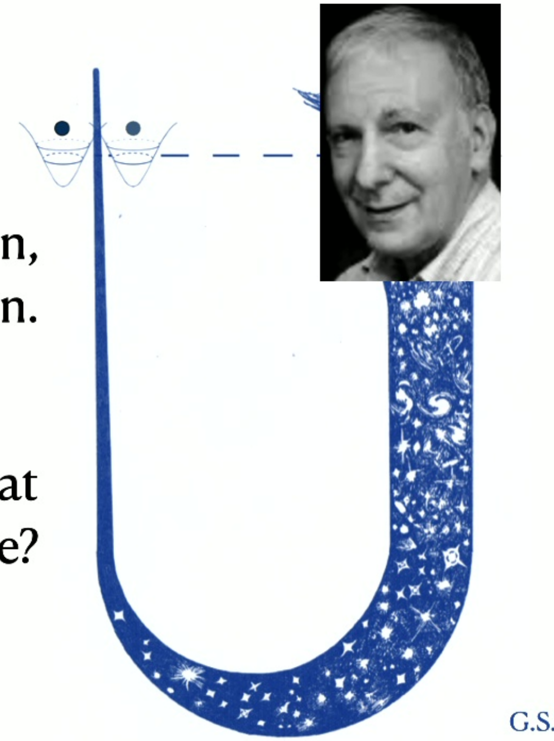
Incidentally, this solves the so-called "Hard Case" of Casini *et al.* (2019).

Evidently, quantum information theory is easier in gapless gauge theories with soft modes.

(The same construction goes through in the null quantization of General Relativity.)

A black hole, or cosmological horizon, will eventually decohere any quantum superposition.

But what about the “Bobs” in the interior that motivated this whole adventure?



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Soft Quantum Information Theory

DLD, G. Satishchandran, J. Judler-Flam (to appear)

- We can think of the black hole as a quantum channel $\mathcal{N}$, acting on an *arbitrary* quantum experiment carried out by Alice.

Decoherence of Alice's system due to the black hole is given by

$$D(\mathcal{N}) := 1 - \frac{1}{1 - 1/\mathrm{Tr}\mathbf{1}} \left(\sup_{\mathcal{D}} F(\mathcal{D} \circ \mathcal{N}, \mathrm{Id})) - 1/\mathrm{Tr}\mathbf{1} \right)$$

- F is the "channel fidelity," a measure on distinguishability of channels. The above can be explicitly calculated in Tomita-Takesaki theory, when suitably generalized to include soft modes (to appear).
- $\mathcal{D}$ is an optimal recovery channel which Alice applies after her experiment, attempting to recover some coherence.

Plain English: $D(\mathcal{N})$ is the decoherence of Alice's degrees of freedom due to the black hole.
In the previous examples, this reduces to the familiar $D(\mathcal{N}) = 1 - |\langle \Psi_1 | \Psi_2 \rangle|$.

24

[DLD, Kudler-Flam, Satishchandran (to appear)]

How well can Bob really do?

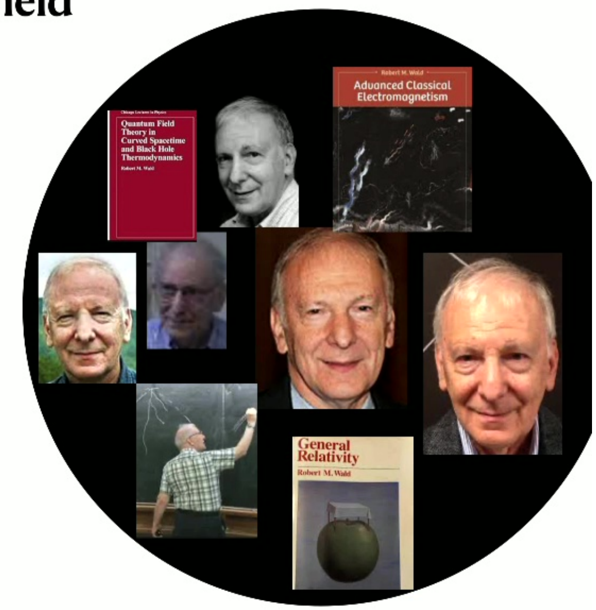
- The “which path information available to Bob” can be generalized.

  Suppose we fill the black hole interior with any number of degrees of freedom, arranged so as to perform an optimal experiment to distinguish Alice’s field states.

How well can Bob really do?

- **The "which path information available to Bob" can be generalized. Suppose we fill the black hole interior with any number of degrees of freedom, arranged so as to perform an optimal experiment to distinguish Alice's field states.**

- The effect of Alice's experiment on the interior is captured by the "complementary channel" $\mathscr{N}^c$.



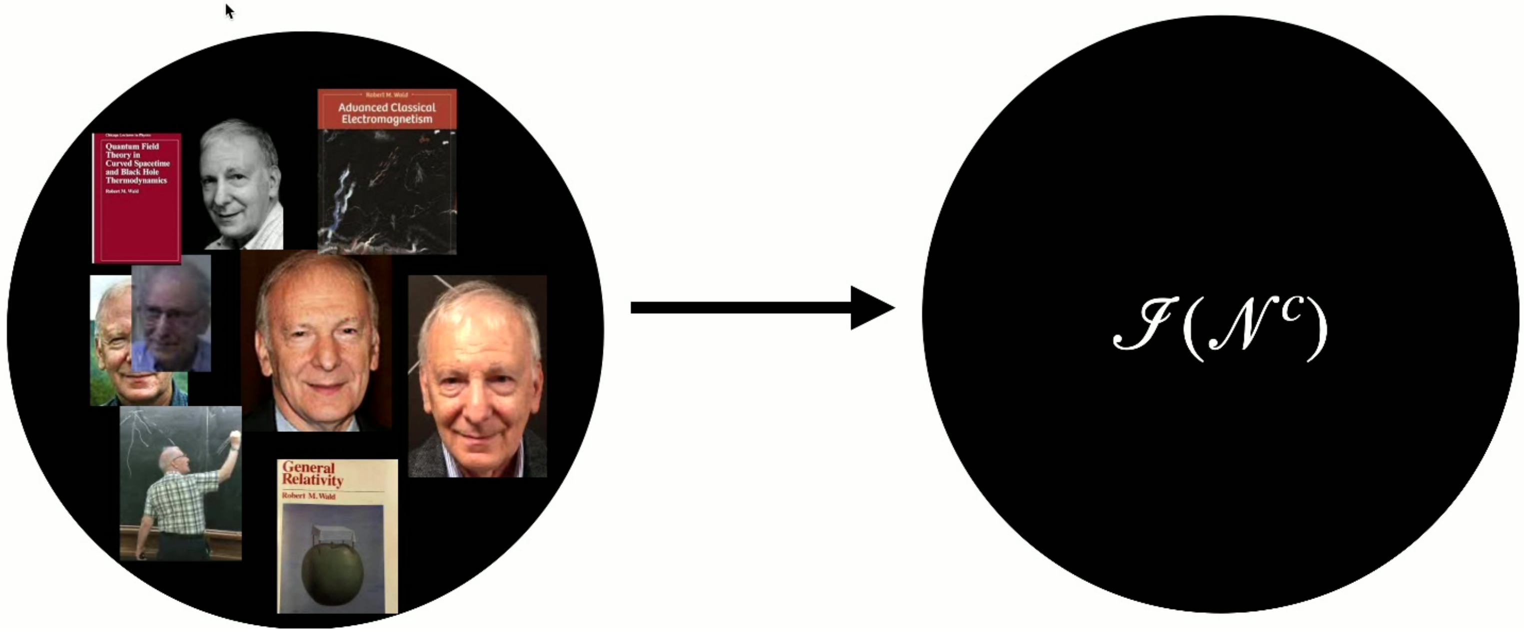
> The distinguishability of the resulting interior states is determined by the **information content** of the complementary channel:
> $$\mathscr{I}(\mathscr{N}^c) := 1 - \frac{1}{1 - 1/\mathrm{Tr}\mathbf{1}}\left(\mathscr{F}(\mathscr{N}^c) - 1/\mathrm{Tr}\mathbf{1}\right)$$

  - $\mathscr{F}(\mathscr{N}^c) = F(\mathscr{N}^c, R)$ where R is a channel that throws away all information about what Alice did.

- **Plain English: $\mathscr{I}(\mathscr{N}^c) \iff$ what is the probability of correctly determining the correct internal state given an optimal measurement?**

- The distinguishability of the resulting interior states is determined by the "channel information"
$$\mathscr{I}(\mathscr{N}^c) := 1 - \frac{1}{1 - 1/\mathrm{Tr}\mathbf{1}} \left(\mathscr{F}(\mathscr{N}^c) - 1/\mathrm{Tr}\mathbf{1}\right)$$

- This is equivalent to the distinguishability of the black hole interior states themselves. I.e., from an exterior perspective, the "degrees of freedom" may as well be the black hole itself, without mention of Bob...

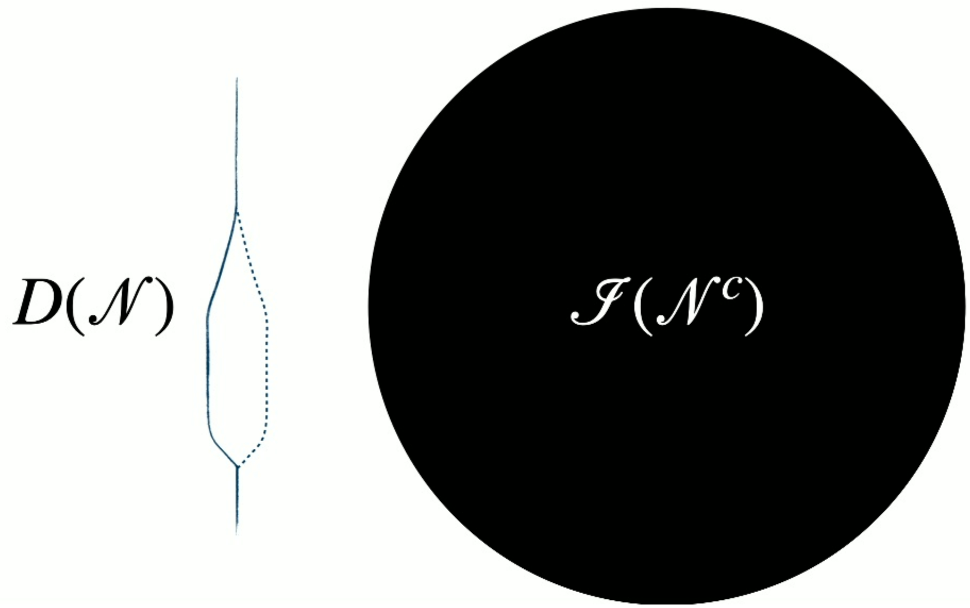


Theorem: $D(\mathcal{N}) = \mathcal{I}(\mathcal{N}^c)$

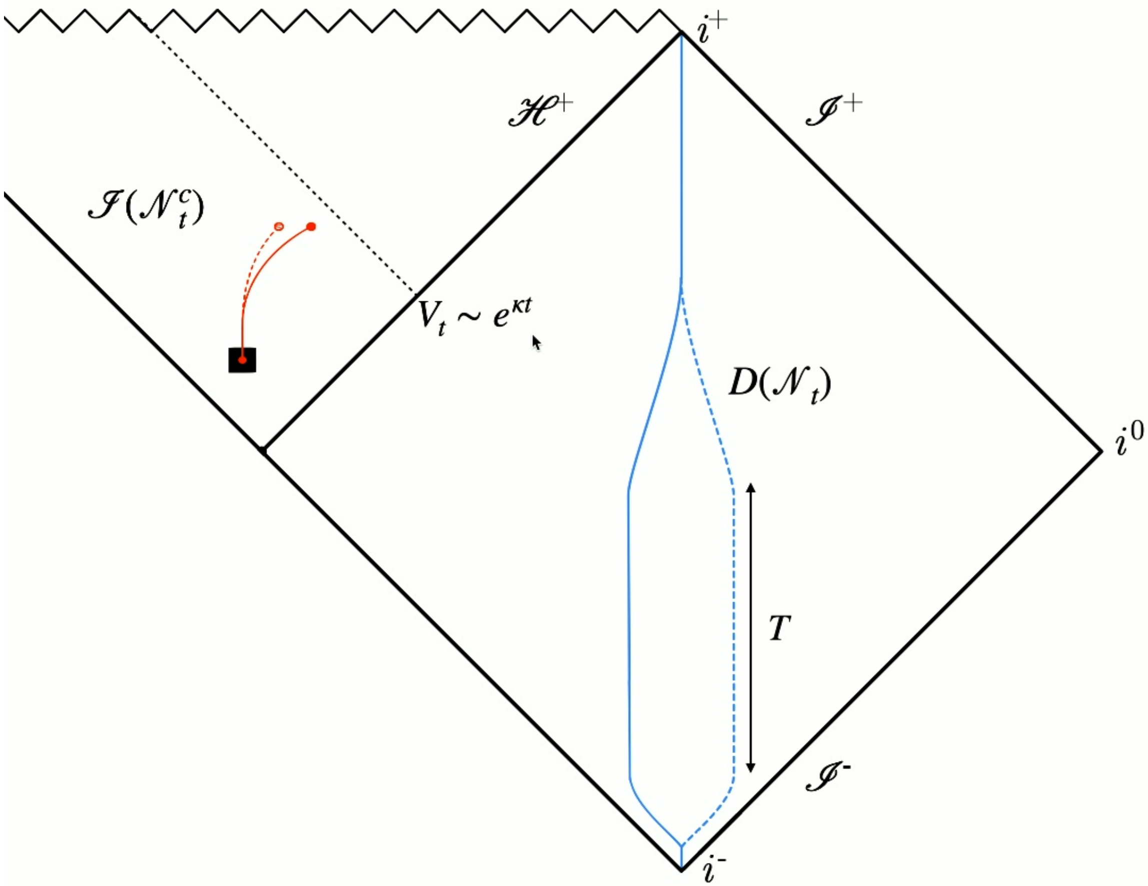
[DLD, Kudler-Flam, Satishchandran (to appear)]

- **The decoherence $D(\mathcal{N})$ of Alice experiment is equal to the information about Alice's experiment $\mathcal{I}(\mathcal{N}^c)$ available to Bob and his assistants in the interior, performing optimal experiments.**

- Equivalently, if we pretend the black hole is just some "quantum degrees of freedom," then the decoherence of Alice is precisely equal to the distinguishability of the resulting states on hypothetical "quantum degrees of freedom" of the black hole itself.



Fidelity Calculation



- Theorem: $D(\mathcal{N}) = \mathscr{I}(\mathcal{N}^c)$
- Calculation of $D(\mathcal{N}_t)$ requires calculating the quantum fidelity f between the two coherent states Ψ, Ω on the horizon, for an arbitrary cut V_t. This is Casini's "hard case."
- Using the horizon algebra this simplifies to:
  $f(\Psi, \Omega) = \langle \Psi \,|\, \hat{\Delta}^{1/2}_{\Psi \to \Omega} \,|\, \Psi \rangle = e^{-\frac{1}{2}\|F_- + P_{V_t} F_-\|^2}$

where $\hat{\Delta}_{\Psi \to \Omega} = \hat{U}(F_+)\hat{\Delta}_\Omega \hat{U}(F_+)^*$
and P_{V_t} is reflection in affine time about the cut V_t.

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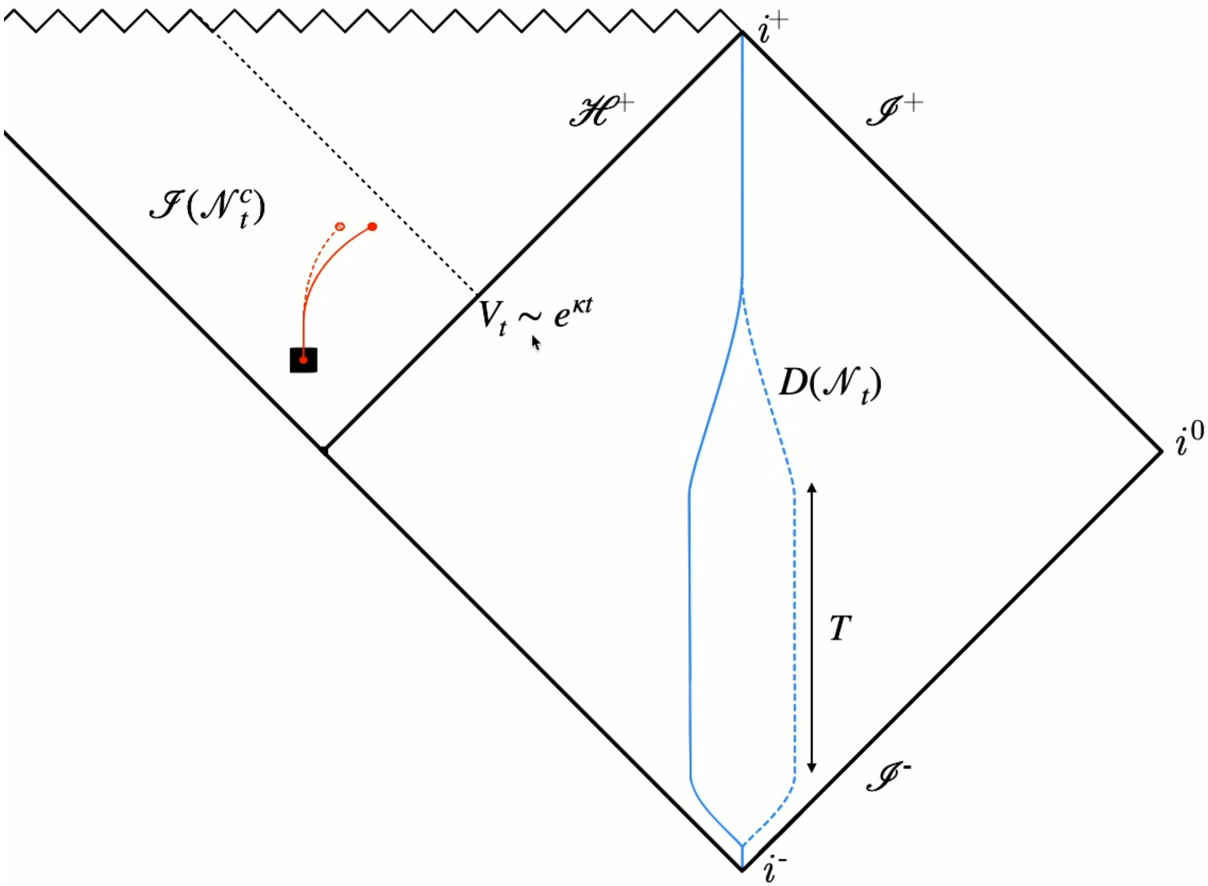
Complementarity between Interior and Exterior



- Indeed, we prove that for the states of the field interior to the black hole,
  - Decoherence $\mathcal{I}(\mathcal{N}_t^c)$ is a strictly increasing function of time
  - $\mathcal{I}(\mathcal{N}_t^c) < \exp\left(-\frac{1}{2}\langle N \rangle_t\right)$ where $\langle N \rangle_t \sim (q^2 d^2 M^3 / b^6) \cdot t$
  - $\lim_{t\to\infty} \mathcal{I}(\mathcal{N}_t^c) = \exp\left(-\frac{1}{2}\langle N \rangle_T\right)$
- Put another way: The decoherence due to the black hole (or any horizon) is equivalent to the maximal amount of which-path information that can be encoded on any **interior degrees of freedom** coupling to the quantum field.

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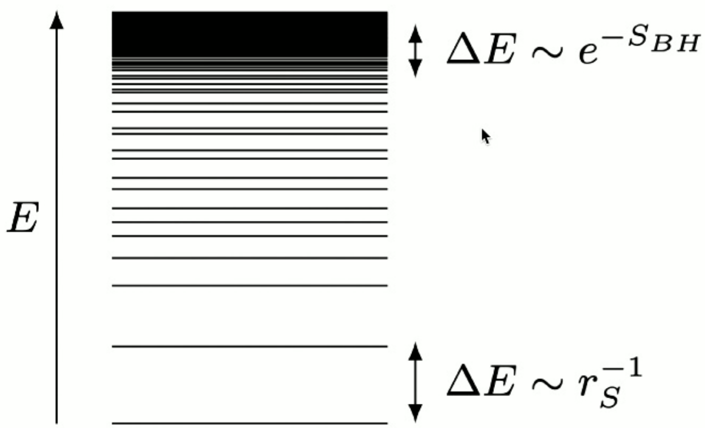
Central Dogma of Black Hole Physics



The so-called "central dogma of black hole physics": as seen from the outside, a black hole can be described as a unitary quantum system with $e^{S_{\rm BH}}$ degrees of freedom. [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini (2021); Shaghoulian (2022)]

We've shown the black hole is an optimally efficient absorber of low frequency radiation. For an ordinary body, level spacings are of order the inverse size of the object—far too large to absorb "soft gravitons."

In quantum gravity, it is often argued that black holes have a dense energy spectrum with level spacings of order $e^{-S_{BH}}$, such as for a highly-excited state of a quantum mechanical system, or a fast scrambler.

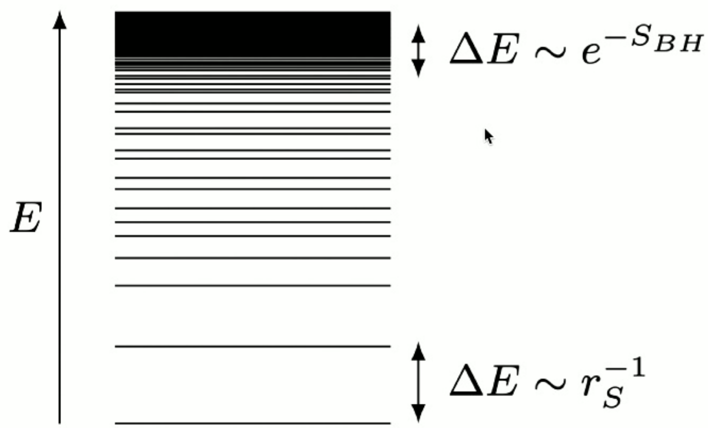
As we have seen, the mere existence of a causally-consistent interior description *requires* such a level spacing, if the central dogma holds.

Daine Danielson, Perimeter

Local Description: Stimulated Emission

- Wilson-Gerow, Dugad, and Chen ('24, arXiv:2405.00804), DSW ('24, arXiv:2407.02567):
  - At low frequencies, the vacuum in Rindler spacetime or outside a black hole is populated by a low-energy population of modes down to zero frequency. This gives rise to *stimulated emission* of soft radiation from Alice's superposition into the horizon!
- Outside a black hole, the vacuum exhibits multipole fluctuations that whose spectrum approaches a *constant* at low frequencies. I.e.,
  - $$\Delta |\vec{P}_{\rm EM}|(\omega) \sim \frac{\sqrt{\epsilon_0 \hbar} G^{3/2} M^{3/2}}{c^3} \sim 10 \ \frac{\mathrm{e \cdot m}}{\sqrt{\mathrm{Hz}}} \left(\frac{M}{M_\odot}\right)^{3/2},$$
  - $$\Delta |Q_{\rm GR}|(\omega) \sim \frac{\sqrt{\hbar} G^2 M^{5/2}}{c^5} \sim 10^{-1} \ \frac{\mathrm{g \cdot m^2}}{\sqrt{\mathrm{Hz}}} \left(\frac{M}{M_\odot}\right)^{5/2}$$

Central Dogma of Black Hole Physics



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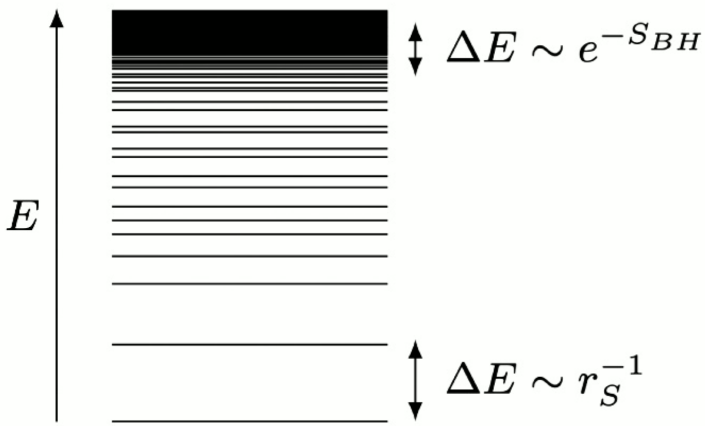
Central Dogma: Multiple Alices?



The so-called "central dogma of black hole physics": as seen from the outside, a black hole can be described as a unitary quantum system with $e^{S_{\mathrm{BH}}}$ degrees of freedom. [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini (2021); Shaghoulian (2022)]

Furthermore, suppose there are N "Alices" performing experiments in the exterior. Alternatively, suppose Bob is trying to measure all multiples of Alice's experiment up through N.

If Alice completes her experiment over a timescale N/ϵ, then the change in the black hole area due to Alice can be kept fixed.

Thus a semiclassical black hole can simultaneously decohere a number of Alices that can be made larger than the hypothetical Hilbert space dimension A of the black hole. This can be done in a time much shorter than the Page time for a large black hole.

It would appear that, if the central dogma holds, then semiclassical physics should break down after a time N/ϵ (although verifying the breakdown will indeed be exponentially complex, e^N).

Daine Danielson, Perimeter

$$T^{\text{BH}}_{\text{Deco.}} \sim \frac{\hbar c^{10} D^{10}}{G^6 M^5 m^2 d^4}$$

$$T^{\text{dS}}_{\text{Deco.}} \sim \frac{\hbar R_H^5}{G m^2 d^4}$$

Questions?

Daine Danielson, Perimeter

"exterior decoherence is interior distinguishability"

$$D(\mathcal{N}_t) = \mathcal{I}(\mathcal{N}_t^c)$$

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