

**Title:** Holographic quantum tasks in the static patch

**Speakers:** Victor Franken

**Collection/Series:** Quantum Information

**Subject:** Quantum Information

**Date:** October 16, 2024 - 11:00 AM

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**Abstract:**

Static patch holography is a conjectured duality between the static patch of an observer in de Sitter spacetime and a quantum theory defined on its (stretched) cosmological horizon. We illustrate from entanglement wedge reconstruction how a closed and connected de Sitter spacetime can emerge in this framework from the entanglement between the two holographic screens of two antipodal observers. In holographic spacetimes, a direct scattering in the bulk may not have a local boundary analog, imposing the existence of  $O(1/G)$  mutual information on the boundary. This statement is formalized by the connected wedge theorem, which is expected to hold beyond the AdS/CFT correspondence from which it originates. We consider scatterings in the static patch of an observer. We argue that for static patch holography to be consistent with the connected wedge theorem, causality on the stretched horizon should be induced from null infinity. In particular, signals propagating in the static patch are associated with fictitious local operators at null infinity. We present a sketch of proof of the connected wedge theorem in asymptotically de Sitter spacetime using induced causality.

# Holographic quantum tasks in the static patch

Victor Franken  
CPHT, Ecole Polytechnique

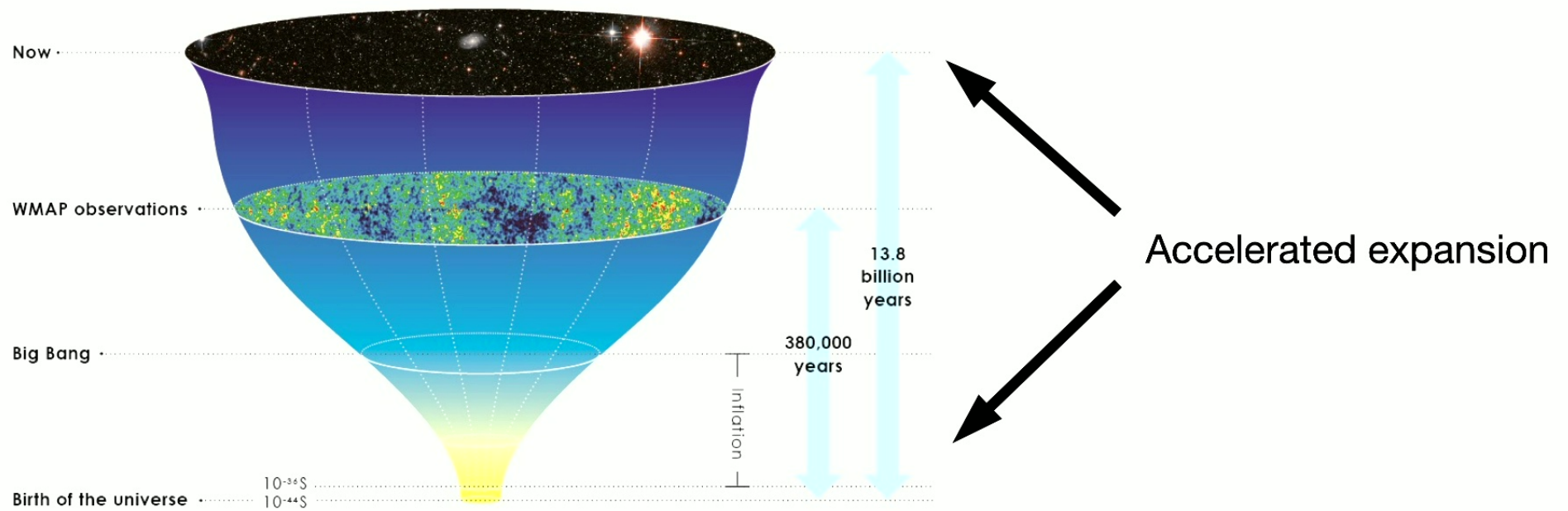
Based on  
ArXiv:2410.09050 with Takato Mori



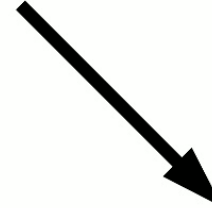
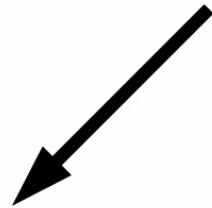
Seminar at Perimeter Institute, October 2024



# Motivations

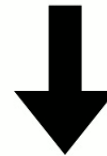


Quantum gravity effects?



Before Planck time  $t_{pl} \sim 10^{-43} s$

Present time:  
Large scales but horizons

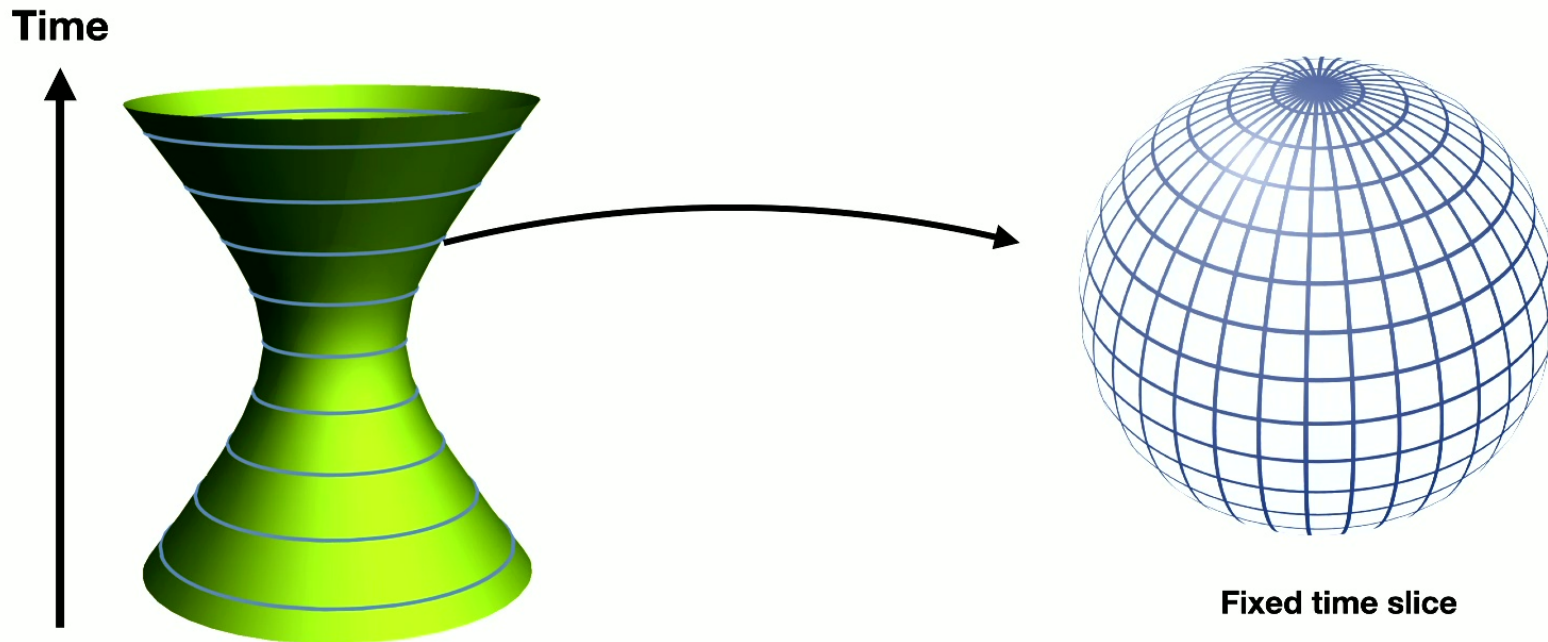


We need a quantum gravitational  
description of our expanding universe

The main idea:

**We can probe holography in cosmology using quantum tasks !**

## Simplest model of our universe: de Sitter spacetime



$$ds^2 = -d\tau^2 + \cosh^2 \tau d\Omega_{d-1}^2$$

The main idea:

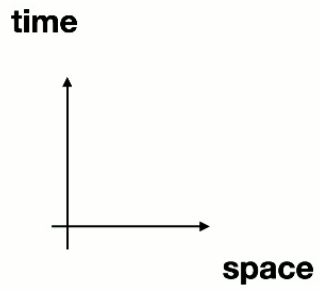
**We can probe holography in cosmology using quantum tasks ! → Part III**



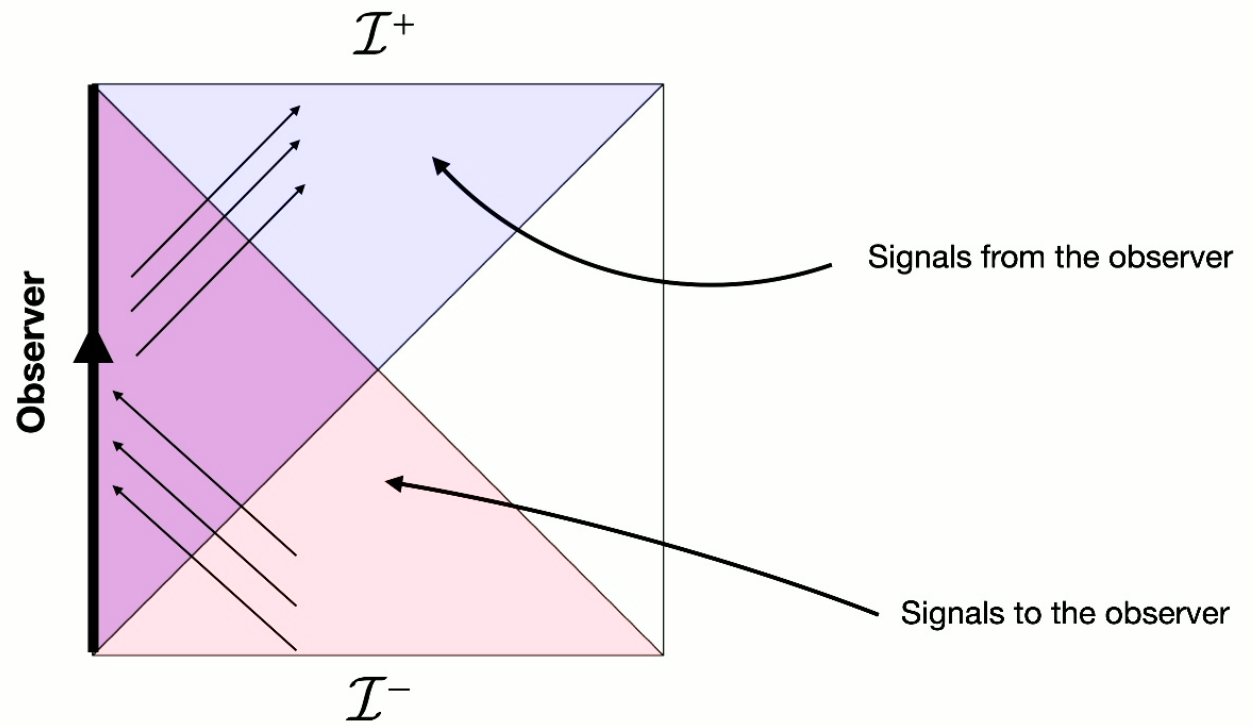
**Part I**



**Part II**



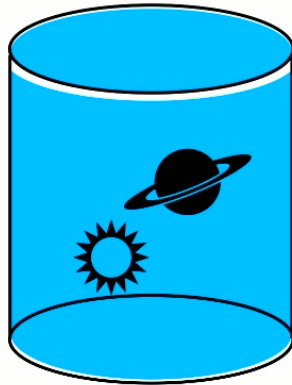
No observer has access  
To the full spacetime!



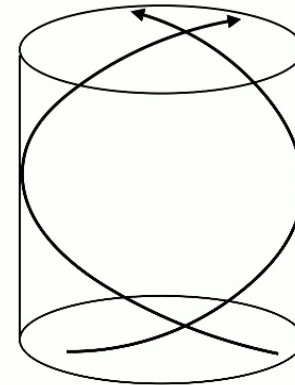


**In anti-de Sitter (AdS) space:**

Quantum Gravity  
in  $D$  dimensions



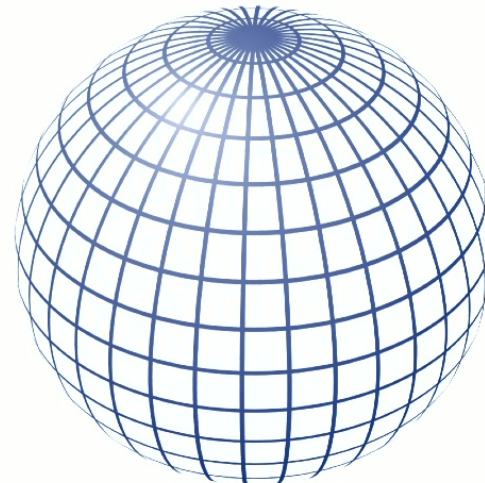
Field Theory  
on spatial boundary  
( $D - 1$  dimensions)





**In de Sitter (dS) space:**

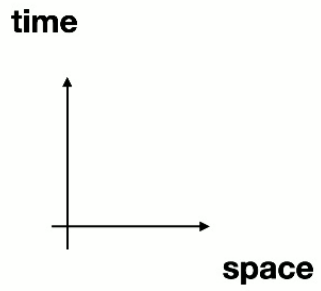
**No boundary...**

**Where is the hologram?**

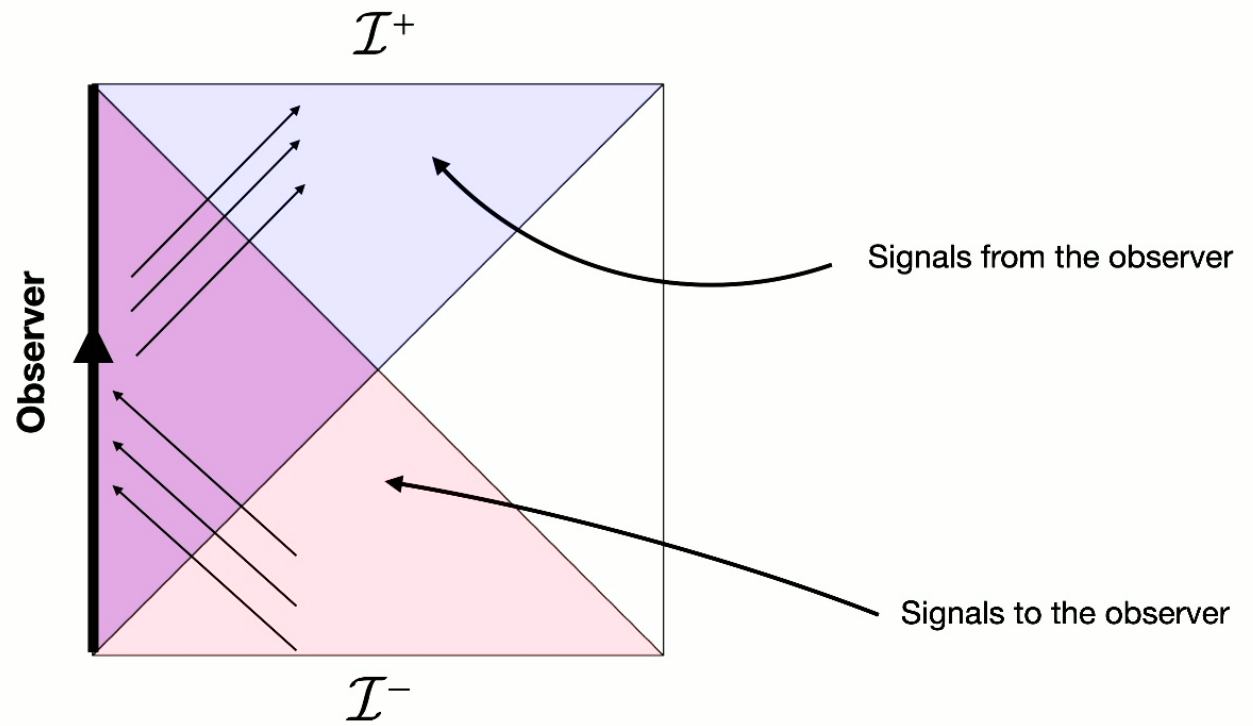


## No spatial boundary: 3 interpretations



- No holography in dS ...
- Holography on time boundaries  dS/CFT [Strominger '21]
- No dynamical degrees of freedom  Hilbert space of dimension 1  
[Almheiri et al.'20] [Shaghoulian '23]



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[Almheiri et al.'20] [Shaghoulian '23]



- Physically meaningful observables should be measured by an **observer**
- Including an observer implicitly selects a **static patch**
- Explicitly including such observer is necessary to get a consistent theory!

[Chandrasekaran, Longo, Penington, Witten '22] [Witten '23]

- Static patch of the observer has an effective boundary

→ Holographic d.o.f. located on the cosmological horizon

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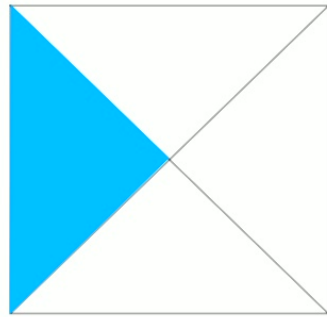
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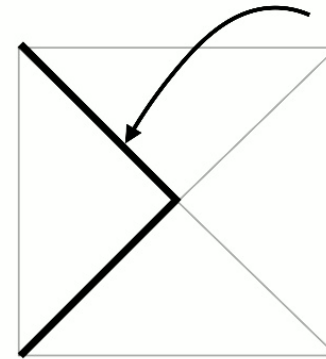


# Static patch holography conjecture

[Susskind '21] :



Gravity in the static patch



'Holographic screen'

Quantum theory on the horizon

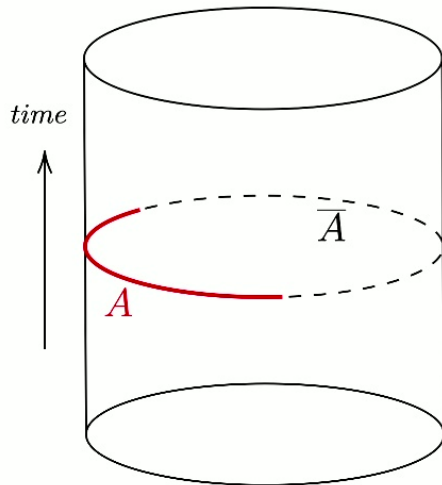
- $S_{GH} = \frac{A_H}{4G}$  is a counting of degrees of freedom
- Conjectured realization in 3D [Susskind '21] [Narovlansky, Verlinde '23]

# Holographic entanglement entropy

Can we probe entanglement on the screen from the bulk?

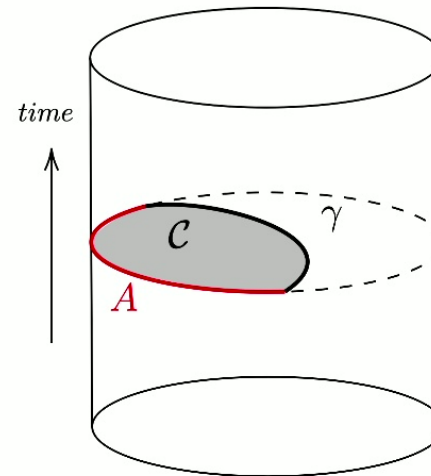
**Yes!** [Ryu, Takayanagi '06] [Hubeny, Rangamani, Takayanagi '07]

$$S(A) = -\text{Tr} \rho_A \log \rho_A$$



Holographic screen

$$S(A) = \min \text{ext} \left[ \frac{\text{area}(\gamma)}{4G} \right]$$



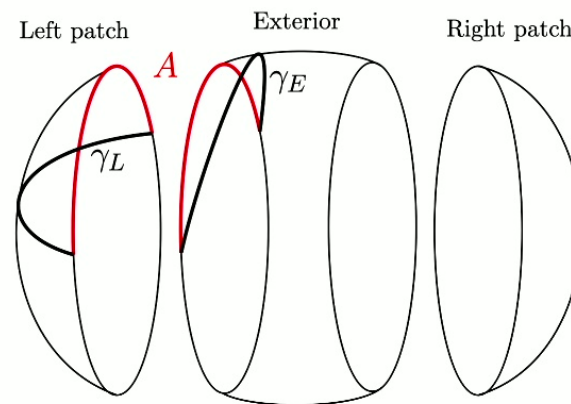
Bulk

$$\partial \mathcal{C} = A \cup \gamma$$

## Comment:

In the de Sitter case, one should also take into account the exterior of the static patch

[Shaghoulian, Susskind '21'22] [VF, Partouche, Rondeau, Toumbas '23]

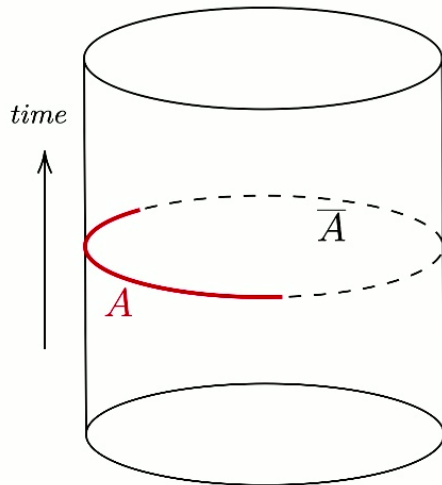


# Holographic entanglement entropy

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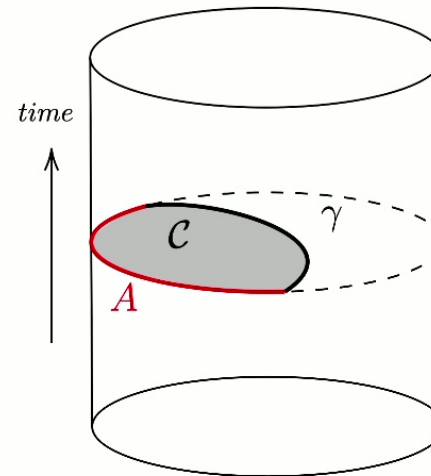
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## What can we say about the dual theory, given:

- Location of the dual theory
- Method to compute entanglement
- Bulk causality

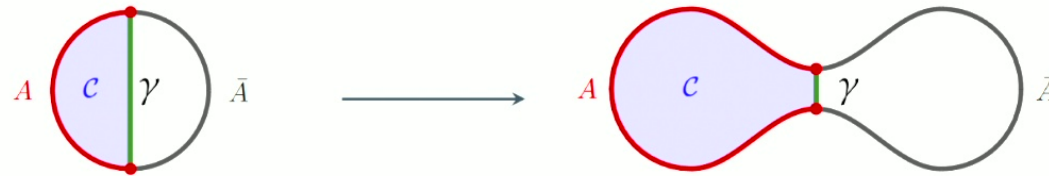
?

[Van Raamsdonk '10]

Entanglement



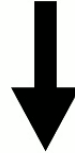
**Spatial connectivity**



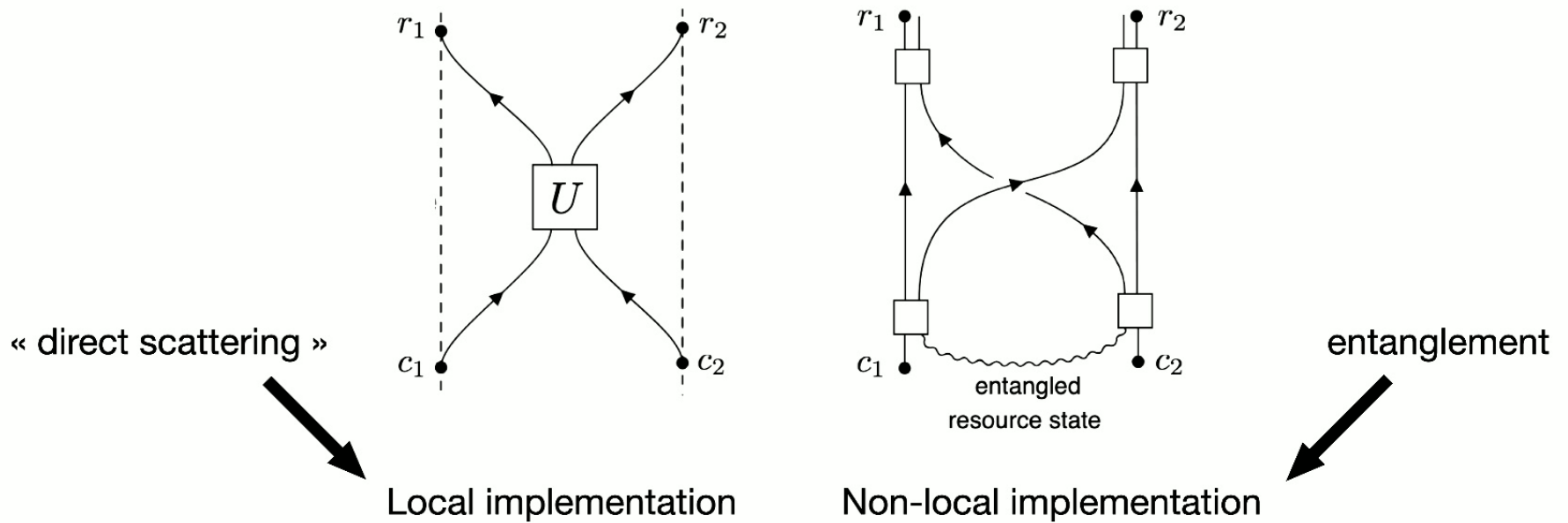
Entanglement tells us about spatial features

What about Lorentzian geometry ?

# What about Lorentzian geometry ?

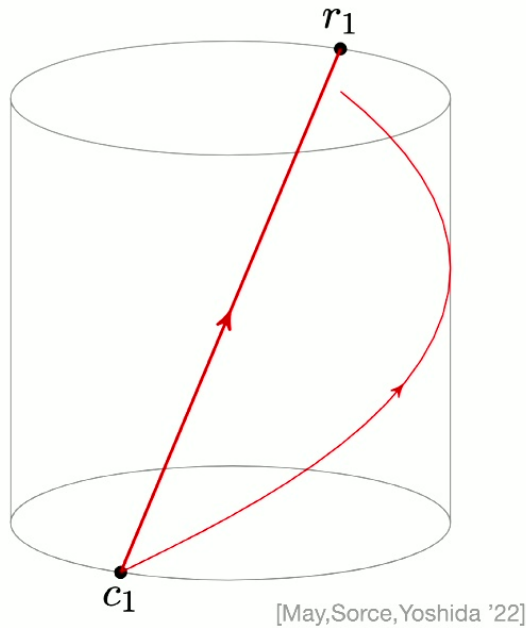


[May]: Consider quantum tasks on the holographic screen



**AdS/CFT: Can the bulk be used as a shortcut for quantum tasks on the boundary?**

1-to-1 task: **NO** [Gao,Wald '00]



$p, q$  on the boundary

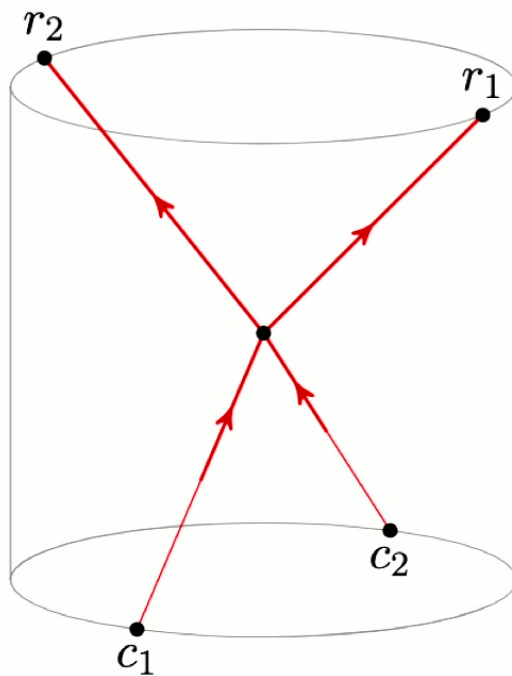
$$q \in J_{bulk}^+(p) \Rightarrow q \in J_{boundary}^+(p)$$

**'1-to-1 bulk causality is weaker than  
1-to-1 boundary causality'**



# AdS/CFT: Can the bulk be used as shortcut for quantum tasks on the boundary?

2-to-2 task: **YES** [Gao,Wald '00]



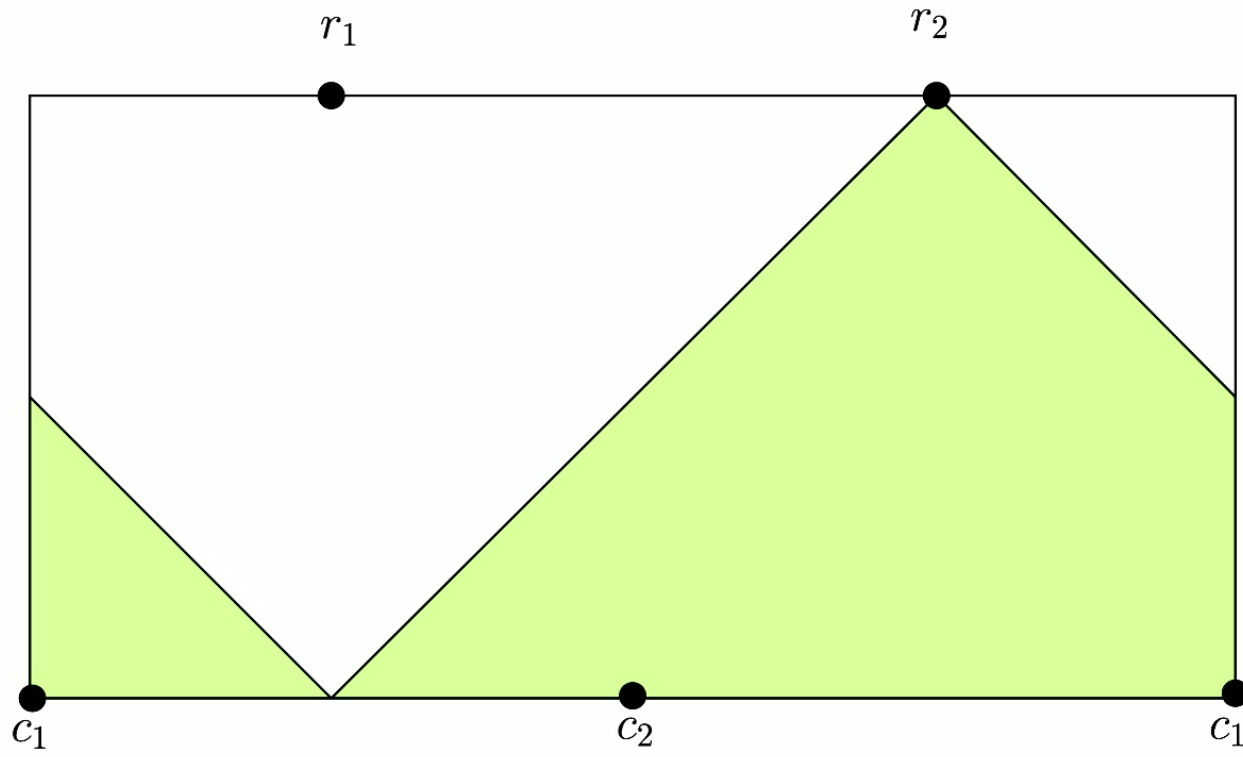
Bulk scattering region

$$J_{12 \rightarrow 12} = J^+(c_1) \cap J^+(c_2) \cap J^-(r_1) \cap J^-(r_2) \neq \emptyset$$

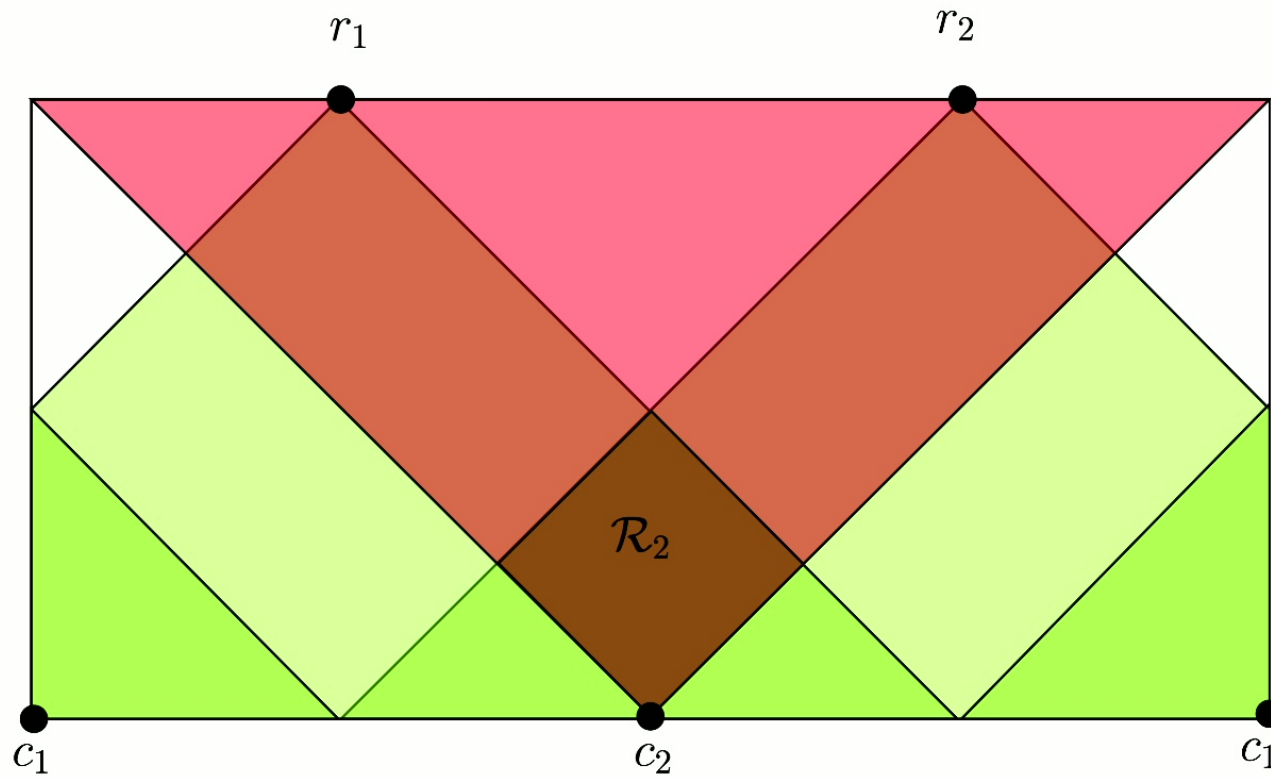
Boundary scattering region

$$\hat{J}_{12 \rightarrow 12} = \hat{J}^+(c_1) \cap \hat{J}^+(c_2) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$$

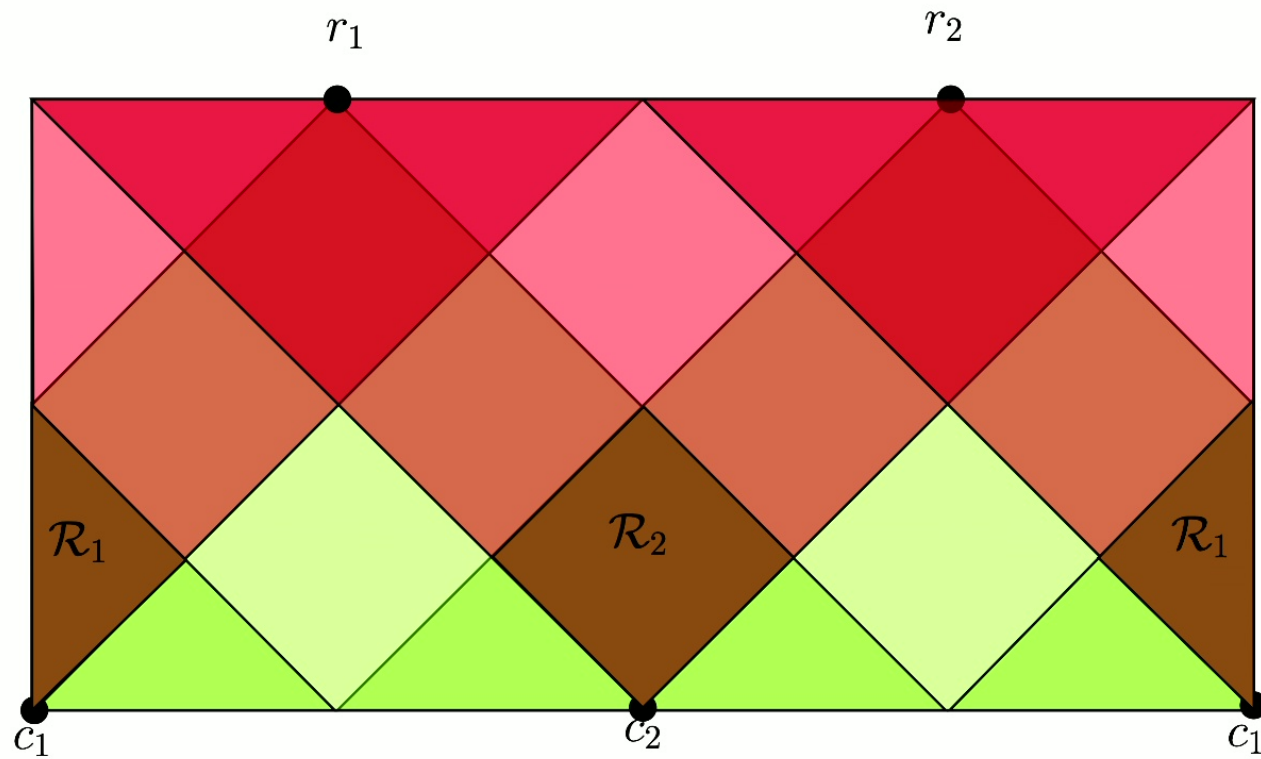
?



Decision region:  $R_i = \hat{J}^-(r_1) \cap \hat{J}^-(r_2) \cap \hat{J}^+(c_i)$



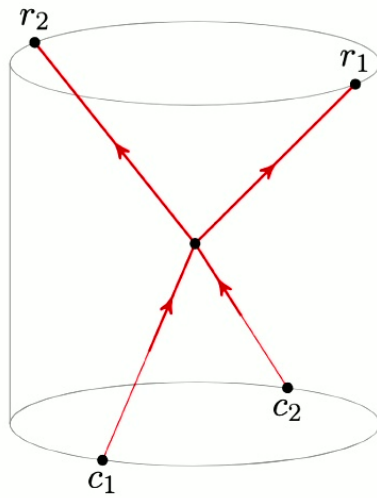
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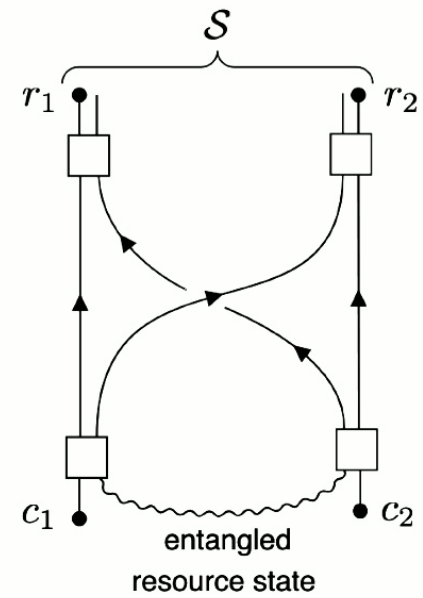
In this example,

$$\hat{J}_{12 \rightarrow 12} = \hat{J}^+(c_1) \cap \hat{J}^+(c_2) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2) = \emptyset$$

Bulk direct scattering

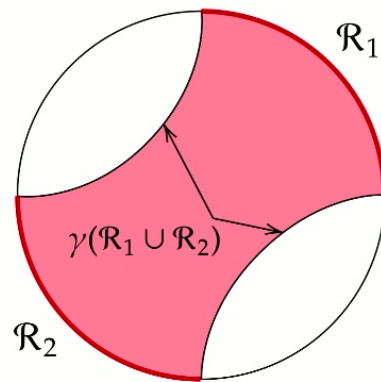


Non-local quantum task

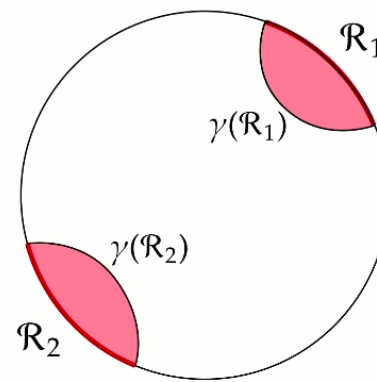


## Connected wedge theorem [May '20][May, Penington, Sorce '20]

Let  $\{c_1, c_2, r_1, r_2\}$  be a bulk-only scattering configuration on the boundary of an asymptotically AdS spacetime with a holographic dual. Then  $R_1$  and  $R_2$  are connected by an entanglement wedge

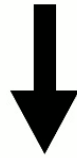


$$I(\mathcal{R}_1 : \mathcal{R}_2) = O(1/G_N)$$



$$I(\mathcal{R}_1 : \mathcal{R}_2) < O(1)$$

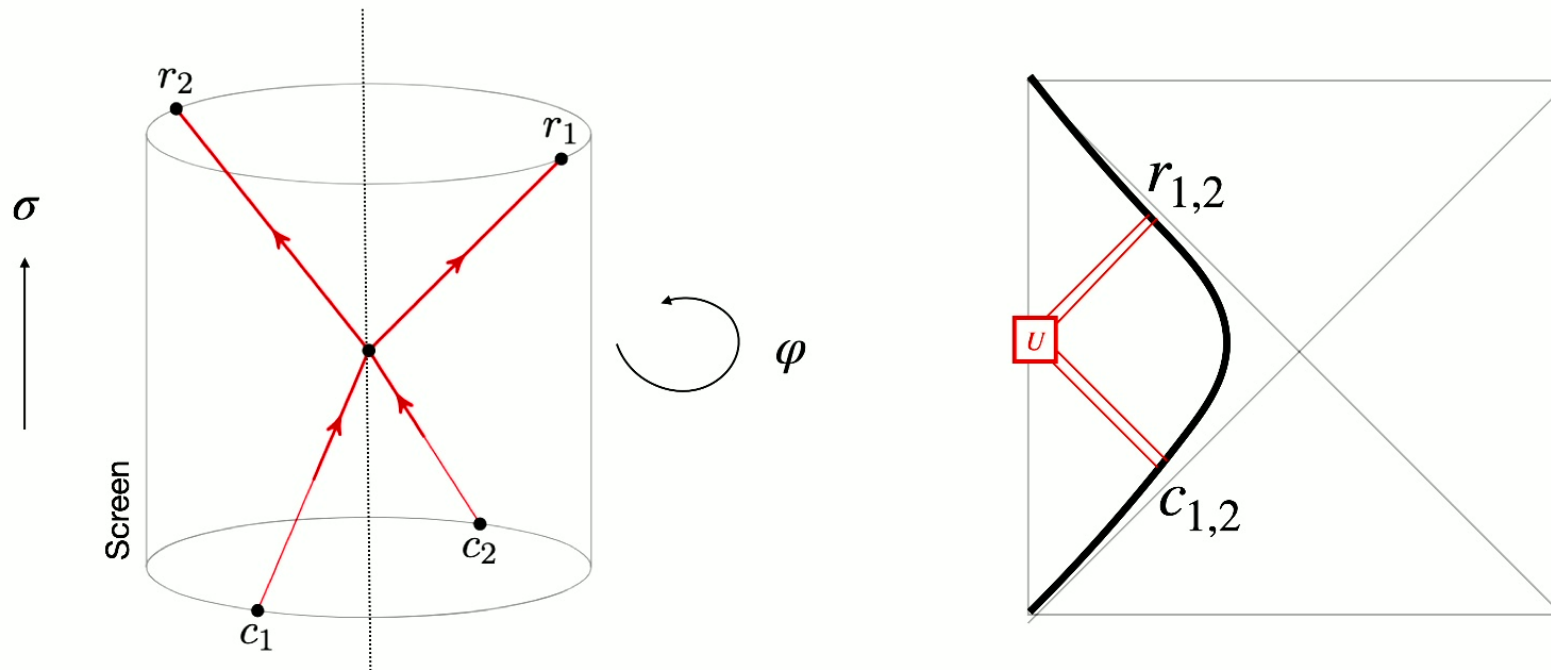
- Holographic version proven from focussing arguments in general relativity
- The statement about mutual information is entirely proven from quantum information theory [May '20][May, Penington, Sorce '20]
- Should be valid in any holographic duality



Great non-trivial check for holographic proposals!

Consider scattering similar to the AdS one

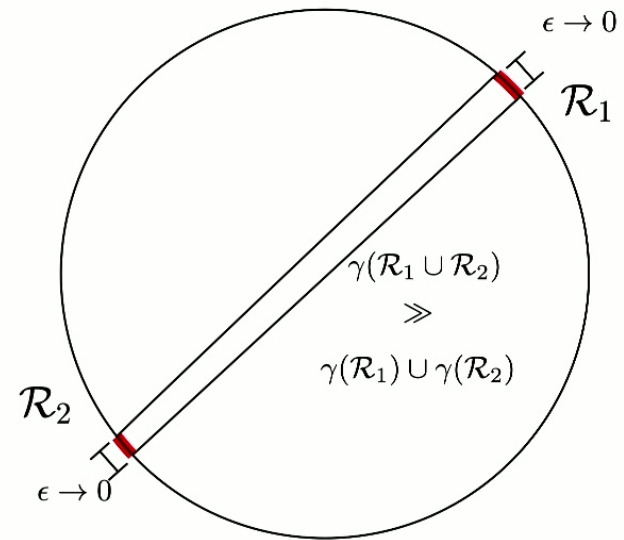
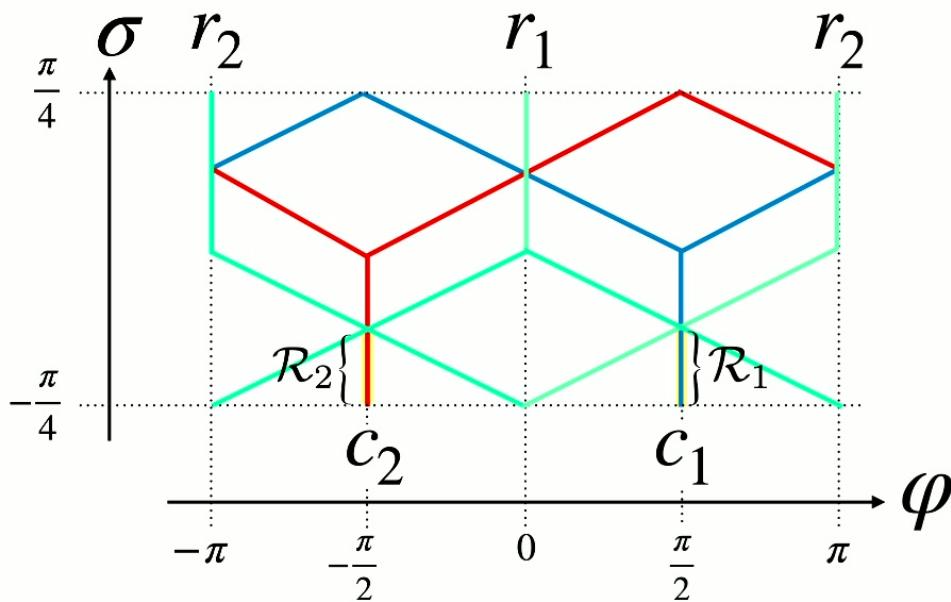
$$J_{12 \rightarrow 12} = J^+(c_1) \cap J^+(c_2) \cap J^-(r_1) \cap J^-(r_2) = \text{one point bulk point}$$





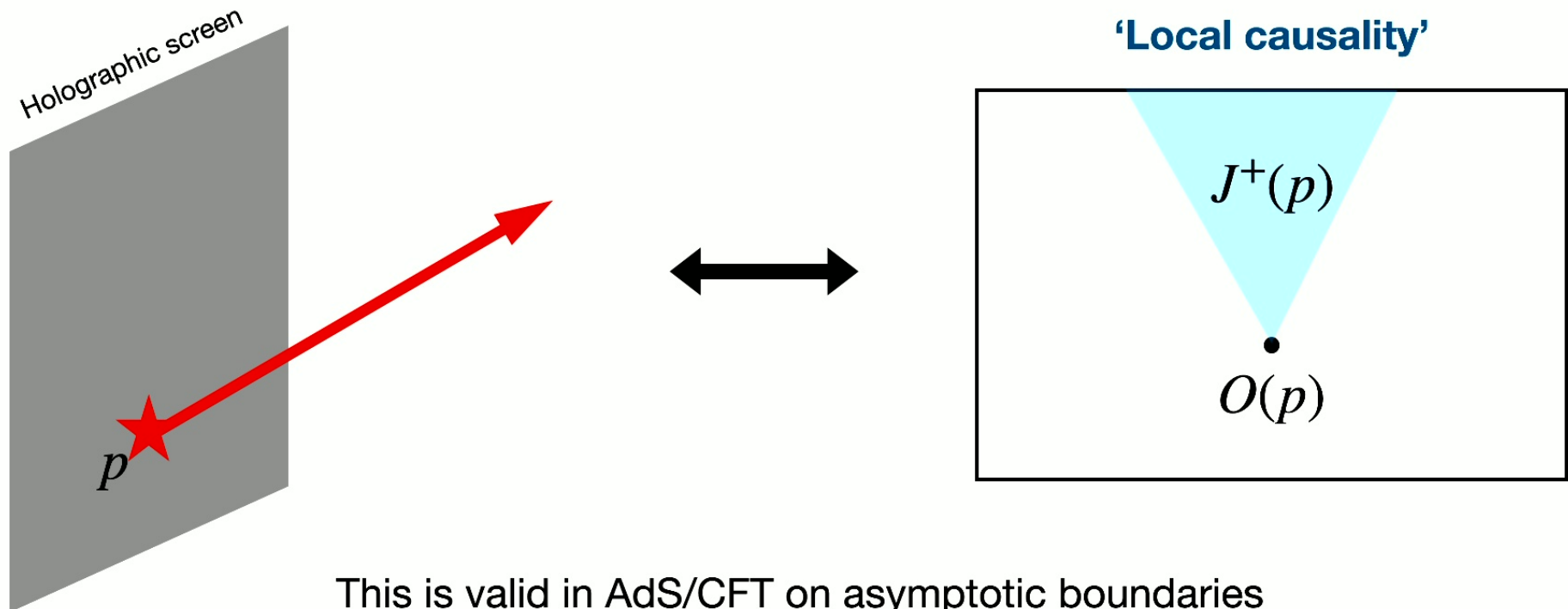
Construct decision region:

$$R_i = \hat{J}^-(r_1) \cap \hat{J}^-(r_2) \cap \hat{J}^+(c_i)$$



**The entanglement wedge is disconnected!**

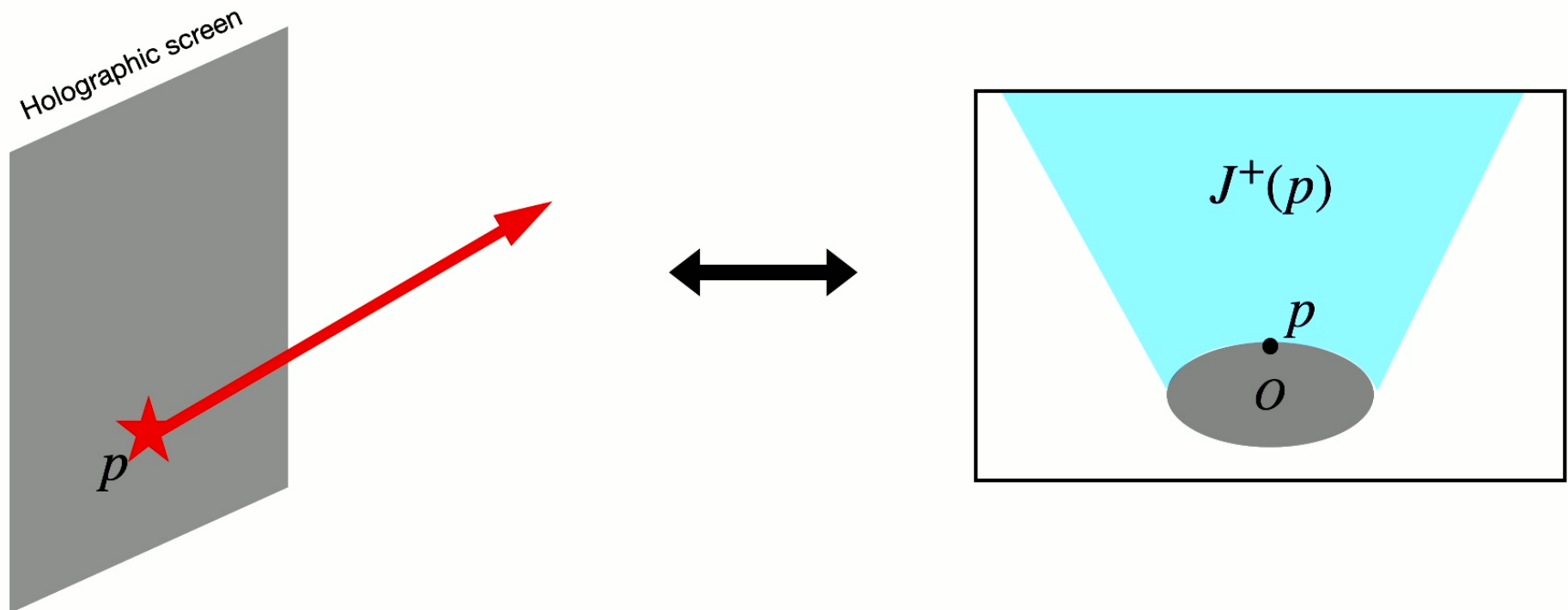
Something that we have implicitly assumed:



This is valid in AdS/CFT on asymptotic boundaries

**NOT** in general [Mori, Yoshida '23]

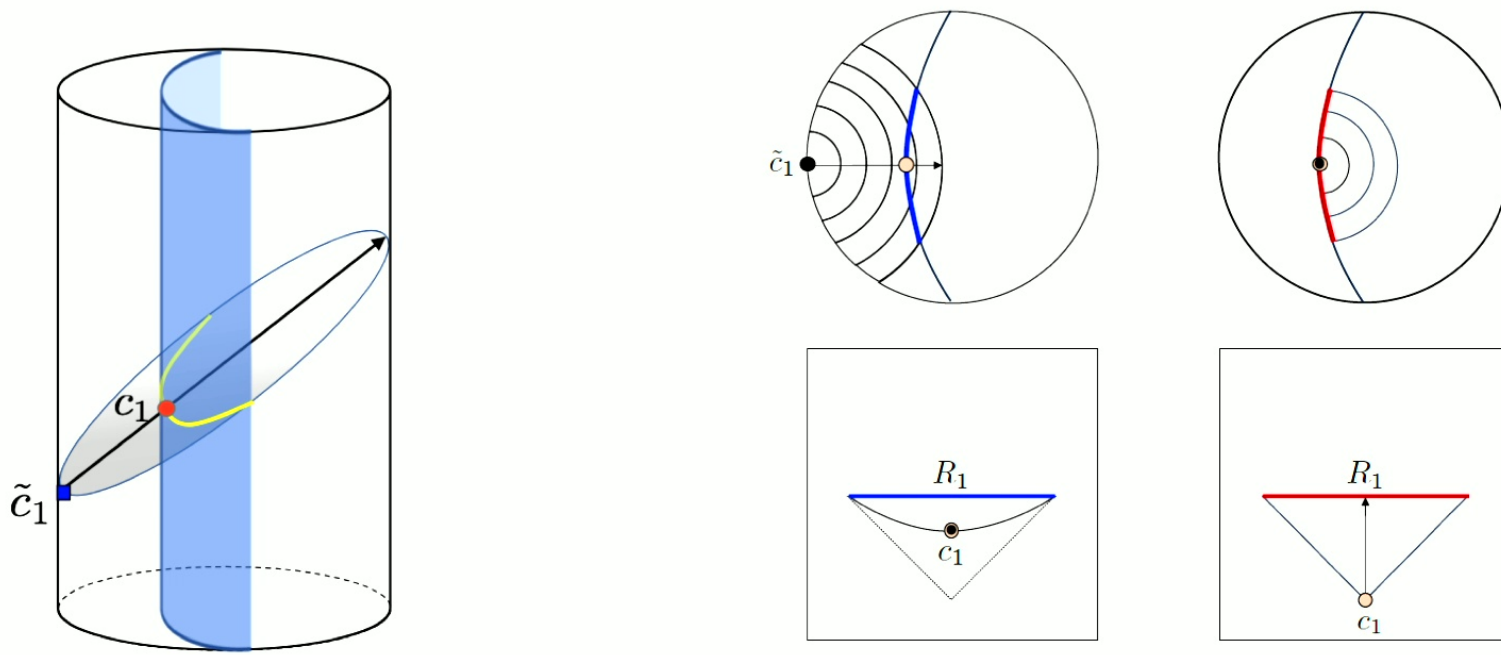
## What if the dual theory is non-local?



# The case of AdS braneworld holography [Mori, Yoshida '23]

Non-local operators need some preparation

➔ Apparently superluminal signals from the local perturbation

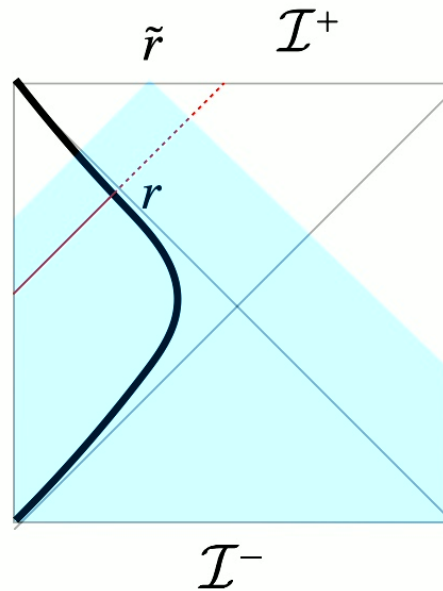


Proposal in dS: relate perturbations on the holographic screens to points on  $\mathcal{I}^\pm$

[VF, Mori '24]

$$r \in J^-(\tilde{r})$$

$$\hat{J}^-(r) = J^-(\tilde{r}) \cap S$$



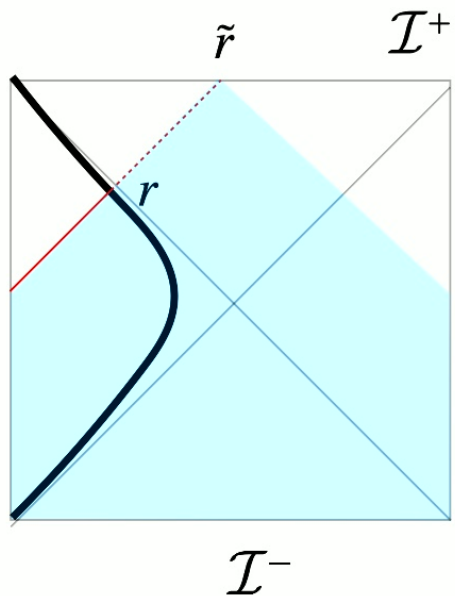
**'Induced causality'**

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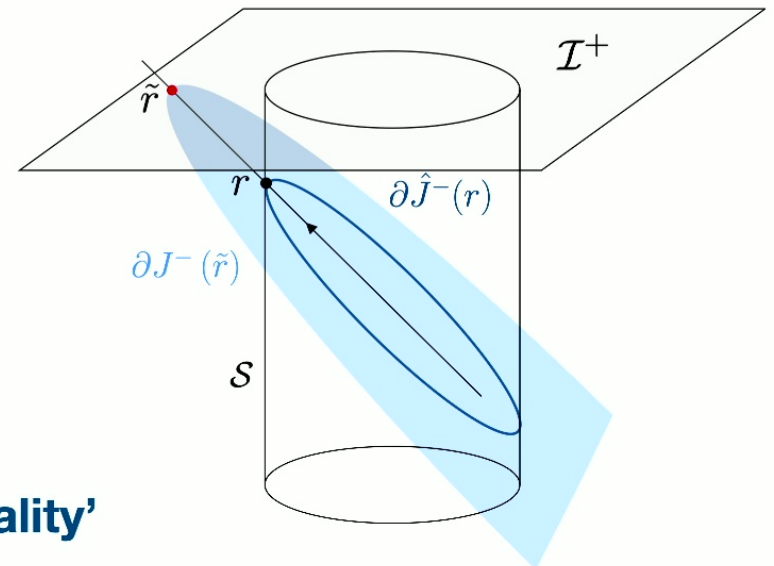
[VF, Mori '24]

$$\hat{J}^-(r) = \min_{\tilde{r}} (J^-(\tilde{r}) \cap S)$$

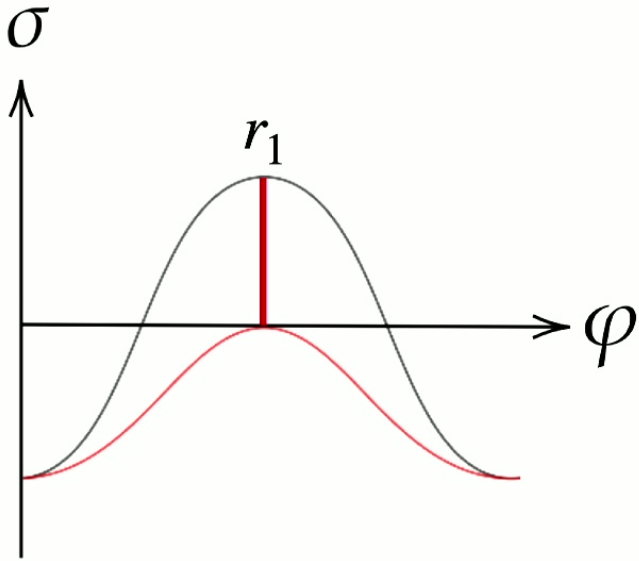
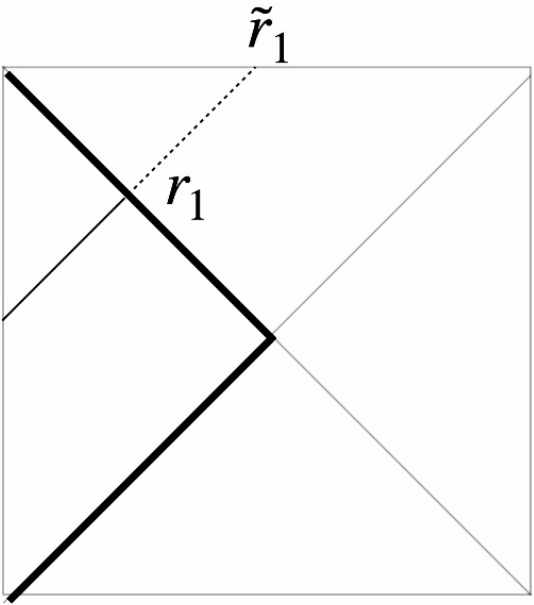
'min':  $\tilde{r}$  such that  $J^-(\tilde{r}) \cap S \in J^-(\tilde{r}') \cap S$  for any  $\tilde{r}'$



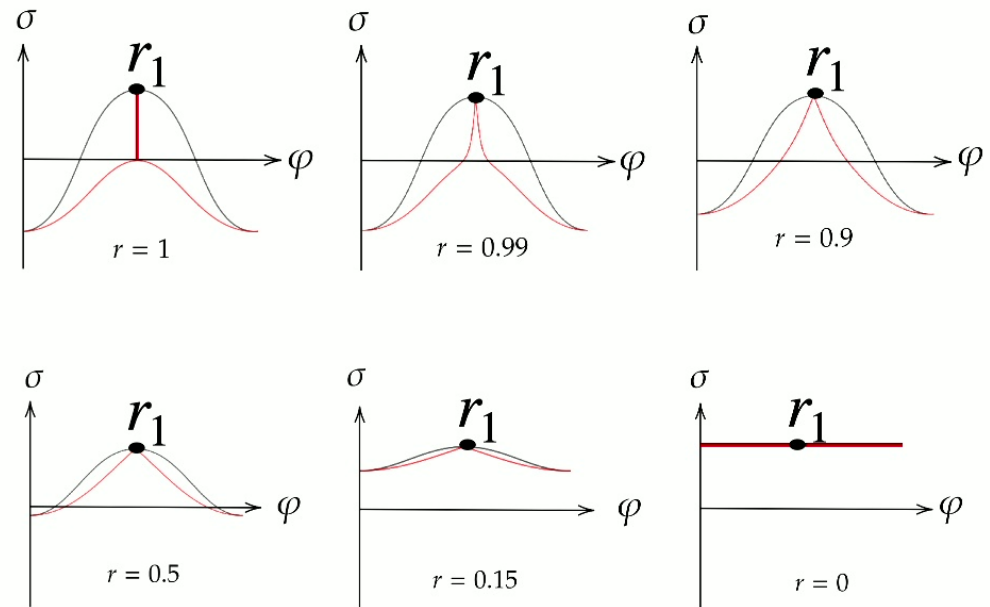
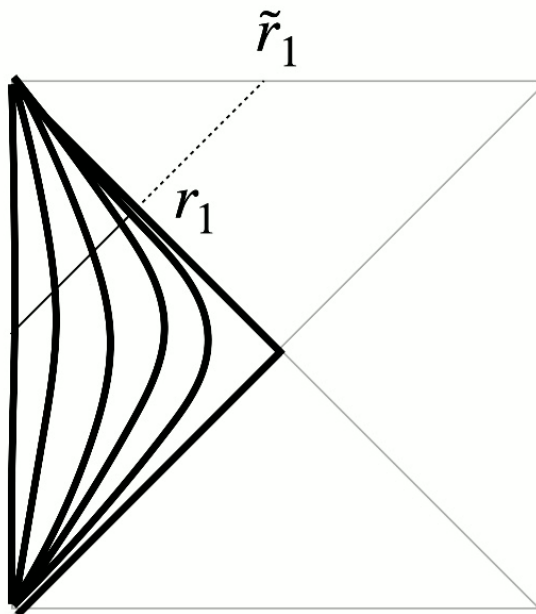
**'Induced causality'**



# Induced causality vs local causality



# Induced causality vs **local causality**

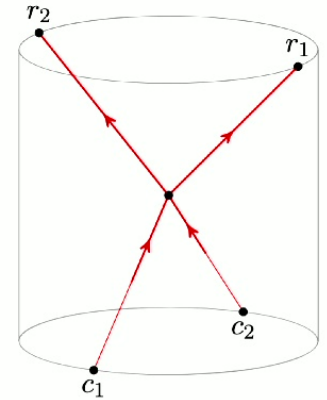


$r =$  distance between the screen and the observer

Pushing the screen inside the patch localizes the theory



## Resolution of the apparent contradiction



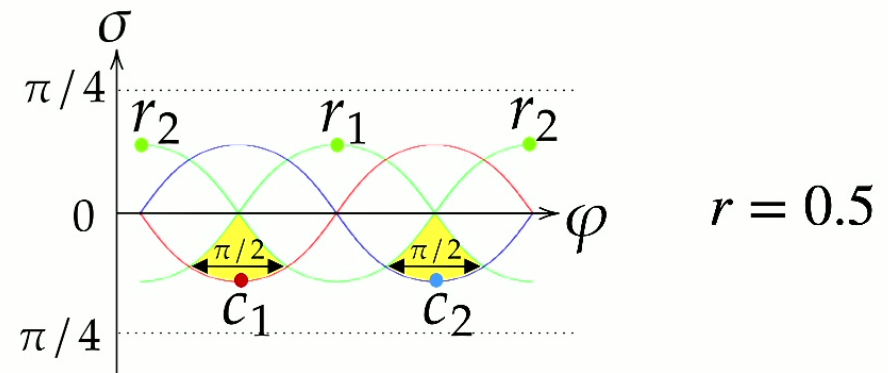
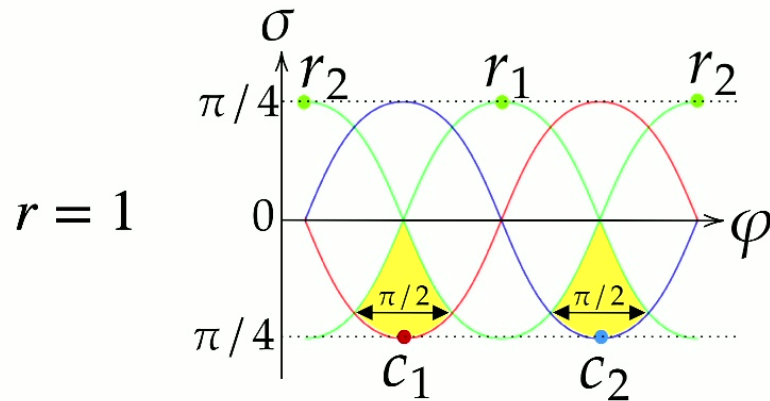
Use induced causality:  $\mathcal{R}_i = \hat{J}^+(c_i) \cap \hat{J}^-(r_1) \cap \hat{J}^-(r_2)$

The connected wedge theorem is satisfied!

$\mathcal{R}_1 \cup \mathcal{R}_2$  covers **half of the screen**



Independant of the screen!



$\mathcal{R}_1 \cup \mathcal{R}_2$  covers half of the screen  Good news!

$J_{12 \rightarrow 12} =$  one point bulk point

$$S(\mathcal{R}_1) + S(\mathcal{R}_2) = S(\mathcal{R}_1 \cup \mathcal{R}_2)$$

## **Connected wedge theorem in the static patch** [VF, Mori '24]

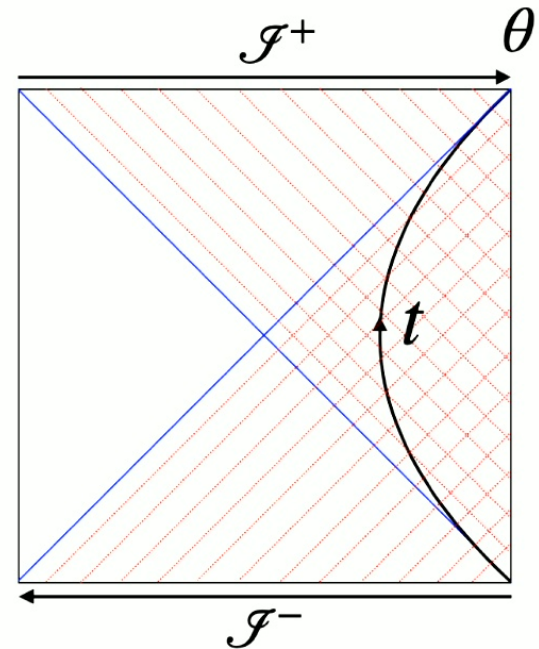
Assuming static patch holography and its associated entropy prescription, if the  $\{c_1, c_2, r_1, r_2\}$  boundary scattering configuration is possible in the static patch, and not on  $S$  from induced causality, then  $R_1$  and  $R_2$  have a mutual information  $O(1/G)$ , and their entanglement wedge is connected in the static patch

# Summary

- de Sitter holography important for both early and present universe
- Quantum description should be observer depend
- The connected wedge theorem is a general feature of holographic dualities
- Static patch holography: To maintain consistency, causality on the holographic screen should be induced from null infinities
- Local perturbation on the screen = local operator at  $\mathcal{I}^\pm$

# Outlook

- Relating static patch holography to dS/CFT
- Precise nature of the holographic screen?
- Scattering outside the static patch
- Generalization to celestial holography?



**Thank you!**

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