Title: Where are the supermassive black holes measured by PTAs?

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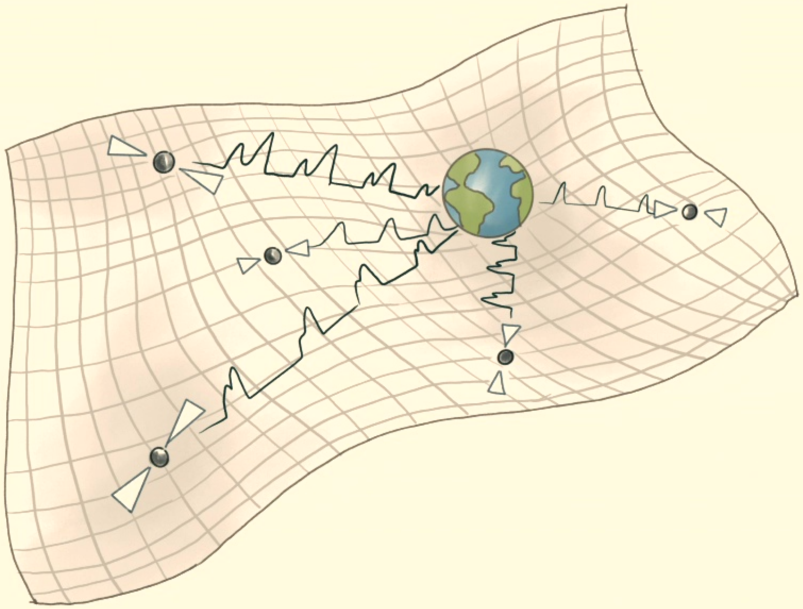
Abstract:

Pulsar timing arrays (PTAs) consist of a set of regularly monitored millisecond pulsars with extremely stable rotational periods. The arrival time of pulses can be altered by the passage of gravitational waves (GWs) between them and the Earth, thus serving as a galaxy-wide GW detector. Evidence for the first detection of low-frequency (~nHz) gravitational waves has recently been reported across multiple PTA collaborations, opening a new observational window into the Universe. Although the origin of the GW signal is yet to be determined, the dominant sources are expected to be inpiralling supermassive black holes (SMBHs). I will discuss a recent work in which we compare the GW detections by PTAs with the expected signal implied by existing electromagnetic observations in a simple but robust manner. This study suggests that the currently measured GW amplitude is larger than expected by a significant amount. I will then show that additional information regarding the typical number of sources contributing to the background can already be inferred from current PTA data.

Where are the supermassive black holes measured by PTAs?



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Summary

- EM and GW-based measurements of supermassive black holes are already precise enough to be interesting
- The **amplitude** of the background can be robustly estimated from measurements of the present-day SMBH mass function and is significantly lower than the value measured by PTAs
- Current PTA measurements can already constrain the **typical number of sources** contributing to the background

GSP, Zaldarriaga, Quataert
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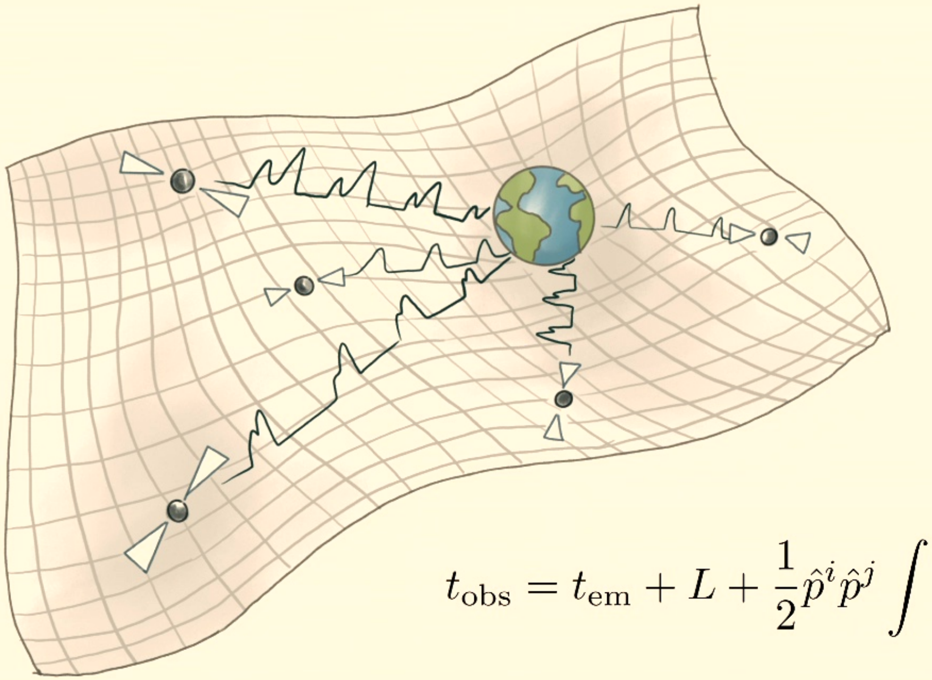
GSP, Zaldarriaga
2406.17010

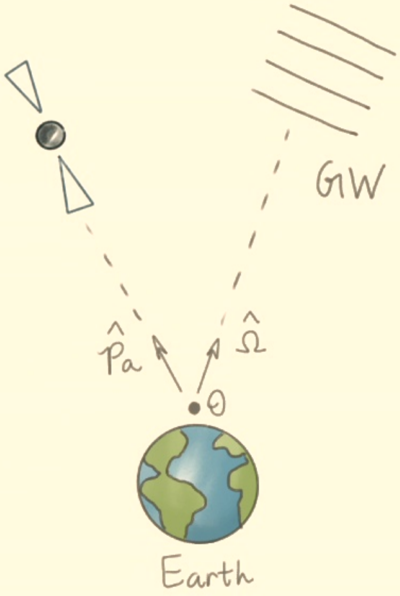
Galaxy-wide detector

Gravitational waves shift the pulse arrival time



$$t_{\mathrm{obs}} = t_{\mathrm{em}} + L + \frac{1}{2}\hat{p}^i\hat{p}^j \int dt'\, h_{ij}^{TT}\left[t', \boldsymbol{x}(t')\right]$$

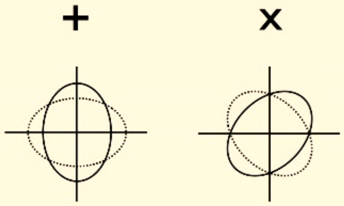
Pulsar timing



Gravitational waves shift the pulse arrival time by

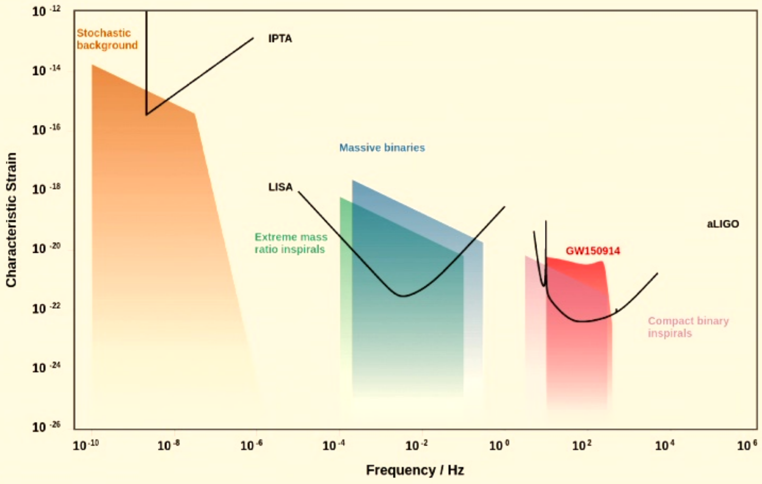
$$z_a(t) = \frac{\Delta T_a}{T_a}(t) = \frac{\hat{p}_a^i \hat{p}_a^j}{2(1+\hat{\Omega}\cdot\hat{p}_a)}\left[h_{ij}(t,\boldsymbol{x}=0) - h_{ij}(t-\tau_a,\boldsymbol{x}_a)\right]$$

e.g. for a monochromatic GW traveling in the $\hat{z}$ direction:



$$h_{ij}^{\mathrm{TT}} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos[\omega(t-z/c)]$$

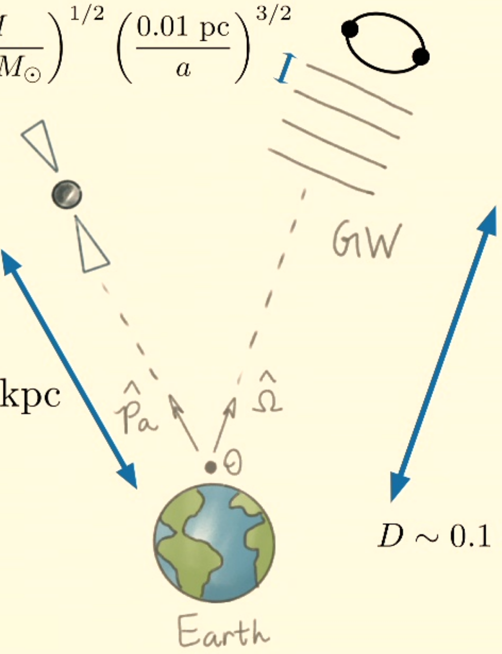
GW detector and sources



$$f_{\rm min} = \frac{1}{T} \sim 3 \times 10^{-9}{\rm Hz}$$

$$f_{\rm max} = \frac{1}{2\Delta t} \sim 10^{-6}{\rm Hz}$$

$$f_{\rm gw} \simeq 10\ {\rm nHz} \left(\frac{M}{10^9\ M_\odot}\right)^{1/2} \left(\frac{0.01\ {\rm pc}}{a}\right)^{3/2}$$



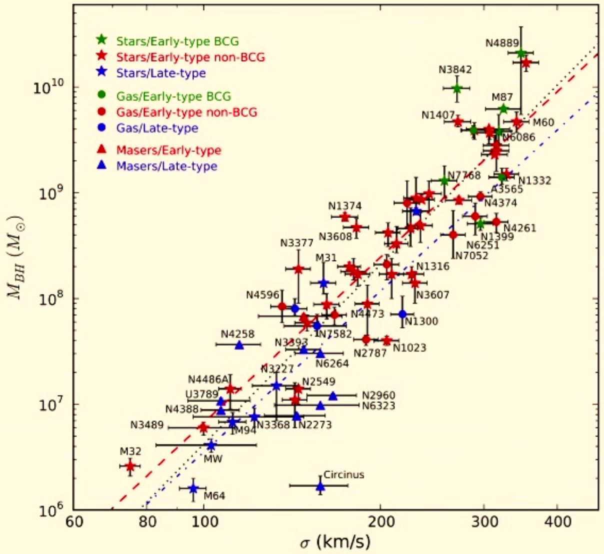
$L_a \sim {\rm kpc}$

$D \sim 0.1 - 1{\rm Gpc}$

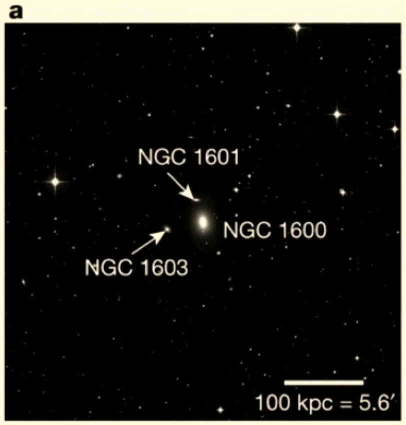
Signal is expected to be sourced by inspiralling **supermassive black holes**

Supermassive black holes

that live nearby



McConnell & Ma 2013



Thomas et al. 2016

Tight relations between SMBH mass and properties of host galaxies directly observed in the local Universe

Supermassive black holes

that accrete

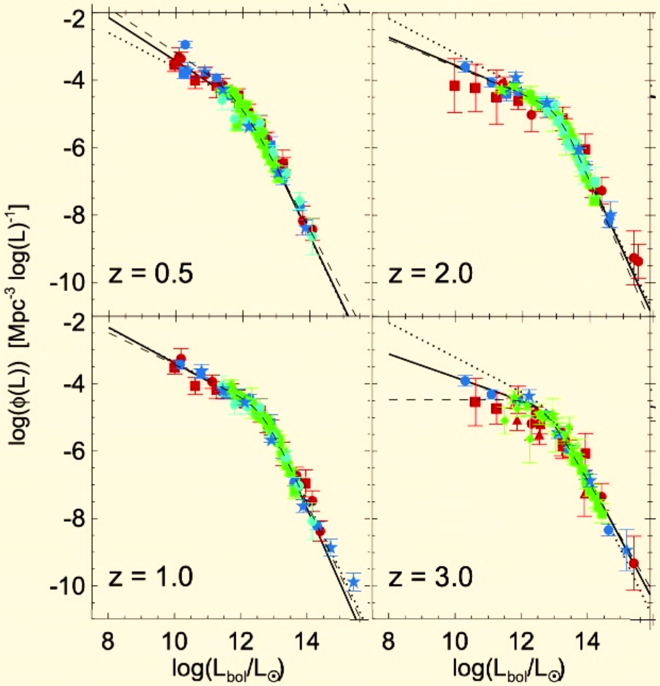
Mass accumulated in black holes through accretion is proportional to the integrated luminosity of quasars

$$L = \epsilon_r \dot{M} c^2$$

$$\rho_{\rm BH} = \frac{1-\epsilon_r}{\epsilon_r c^2} \int dz \frac{dt}{dz} \int d\log L \phi(L,z) L$$

Evolution of the SMBH mass function can be modeled through the continuity equation

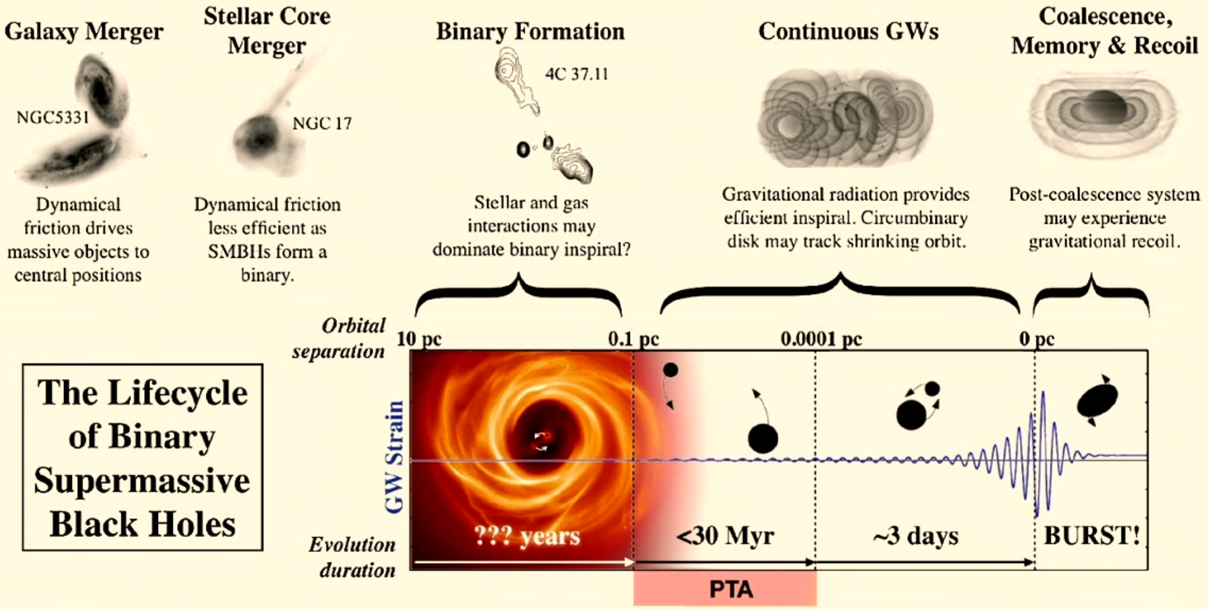
(e.g. Small & Blandford 1992, Yu & Tremaine 2002)



Hopkins et al. 2007

Supermassive black holes

that merge



Dynamical friction drives massive objects to central positions

Dynamical friction less efficient as SMBHs form a binary.

Stellar and gas interactions may dominate binary inspiral?

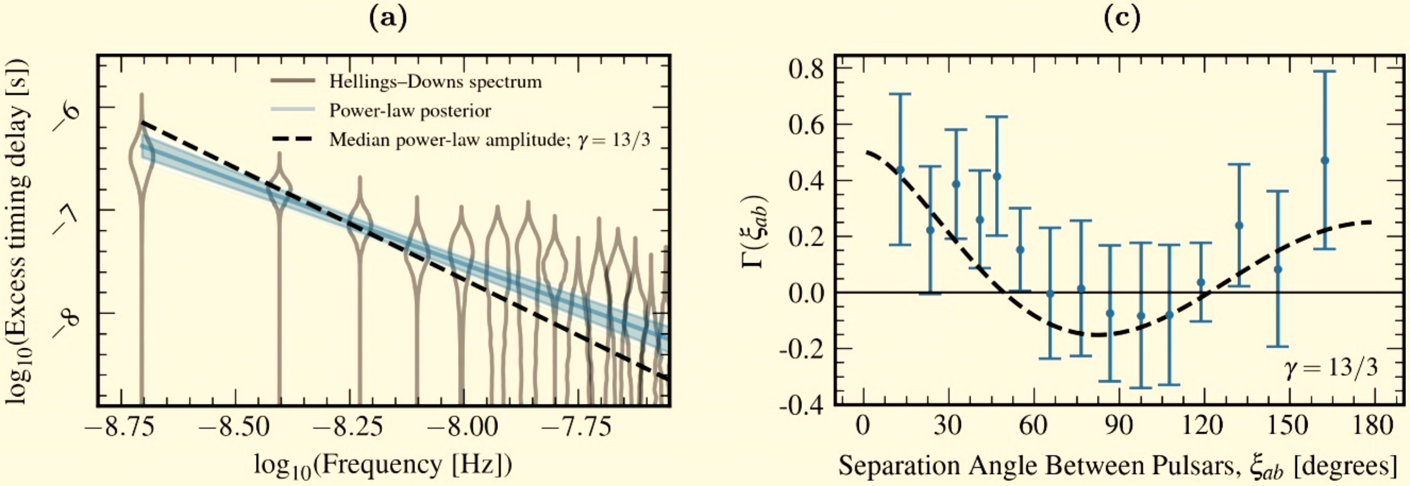
Gravitational radiation provides efficient inspiral. Circumbinary disk may track shrinking orbit.

Post-coalescence system may experience gravitational recoil.

Burke-Spolaor et al. 2019

First detection

By multiple PTA collaborations (EPTA+InPTA, Antoniadis et al. 2023; NANOGrav, Agazie et al. 2023; PPTA, Reardon et al. 2023; CPTA, Xu et al. 2023)



Agazie et al. 2023

Correlation of timing residuals $\langle z_a(t) z_b(t')\rangle = \underline{C(\theta_{ab})} \int df\ \underline{S_h(f)} e^{2\pi i f(t-t')}$

Angular correlation

Energy spectrum ($\propto h_c^2 \propto \rho_{\rm GW}$)

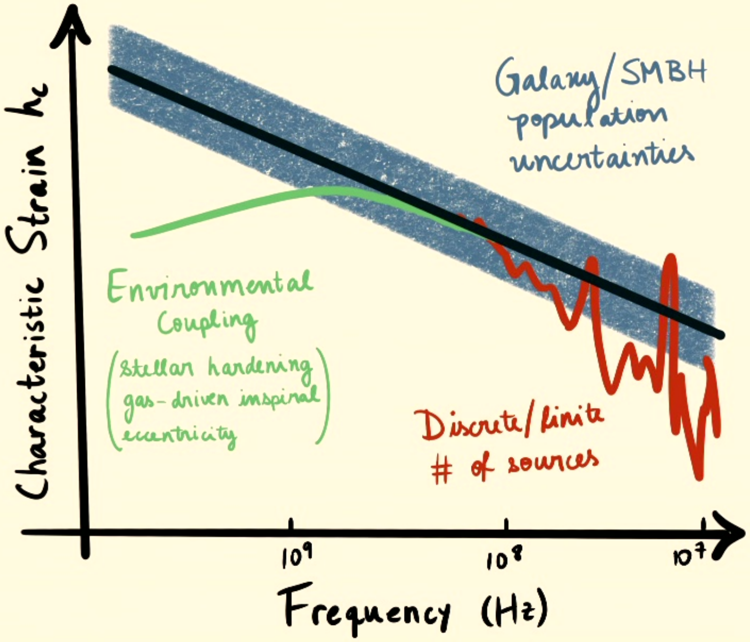
GW spectrum

Depends on the mass function of binary SMBHs

$$\frac{d\rho_{\rm gw}}{d\log f}(f) = \int d\mathcal{M} \int dz \underbrace{\frac{dn}{dz d\mathcal{M}}}_{\text{Binary merger rate density}} \frac{1}{1+z} \underbrace{\left(\frac{dE_{\rm gw}}{d\log f_r}\right)^{*}}_{\text{Energy spectrum}}\Bigg|_{f_r=(1+z)f}$$

$$\frac{d\rho_{\rm gw}}{d\log f}(f) = \frac{\pi c^2}{4G} f^2 h_c^2(f)$$

*Deviations at low frequencies may indicate additional processes shrinking the orbit



Adapted from Burke-Spolaor et al. 2015

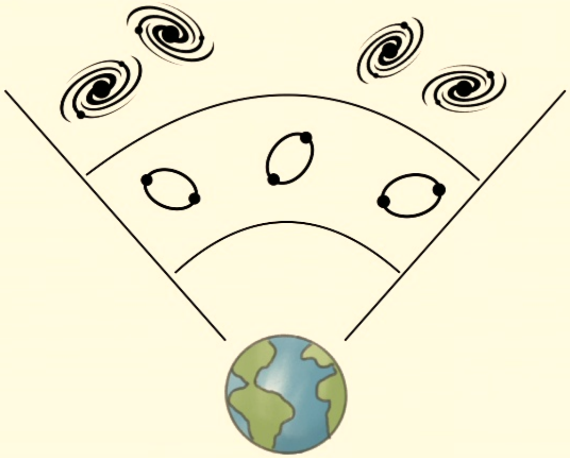
GW spectrum

$$h_c^2(f) = \frac{4G}{\pi c^2 f^2}\int d\mathcal{M}\int \frac{dz}{1+z}\,\underbrace{\frac{dn}{dz\,d\mathcal{M}}}_{\text{Binary merger rate density}}\left(\frac{dE_{\rm gw}}{d\log f_r}\right)\Bigg|_{f_r=(1+z)f}$$

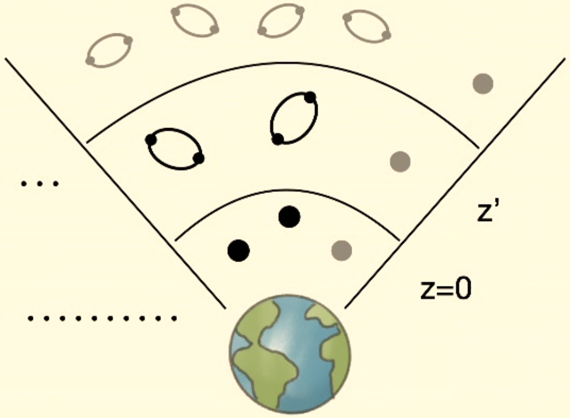
SMBH-galaxy/DM halo connection

Pair fraction

Time delay



$$\frac{dn_{\rm binary}}{dM_{\rm BH}} \cdots = \frac{dn_{\rm single}}{dM_{\rm BH}} \cdots\cdots\cdots$$



Phinney 2001

Connection between GWs and EM observations

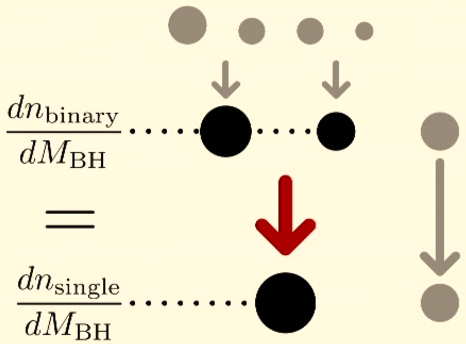
$$h_c^2(f) = \frac{4G}{\pi c^2 f^2} \int d\mathcal{M} \int \frac{dz}{1+z} \underbrace{\frac{dn}{dz\,d\mathcal{M}}}_{\text{Binary merger rate density}} \underbrace{\left(\frac{dE_{\rm gw}}{d\log f_r}\right)}_{\text{Energy spectrum}} \Bigg|_{f_r=(1+z)f}$$

Phinney 2001

Assuming all binaries merge once and changing variables from chirp to total mass

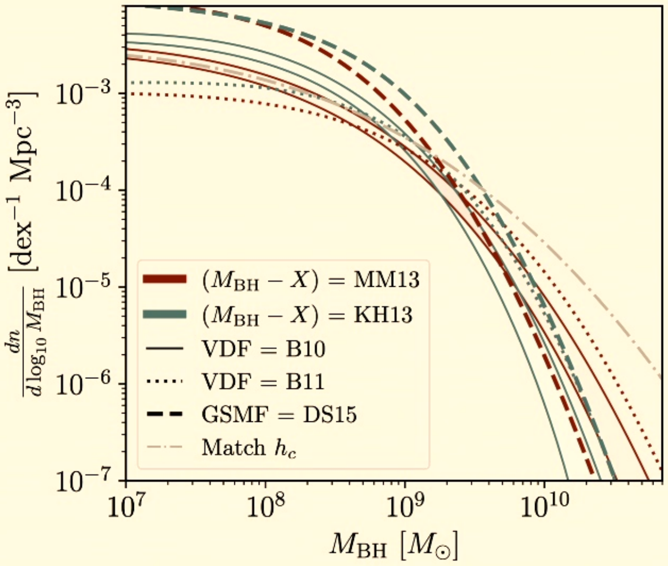
$$\propto f^{-4/3} \langle (1+z)^{-1/3} \rangle \langle \eta \rangle \int dM_{\rm BH} \underbrace{\frac{dn}{dM_{\rm BH}}}_{\text{Single BH mass function}} M_{\rm BH}^{5/3}$$

where $\mathcal{M} = \eta^{3/5} M, \quad \eta = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{q}{(1+q)^2}$

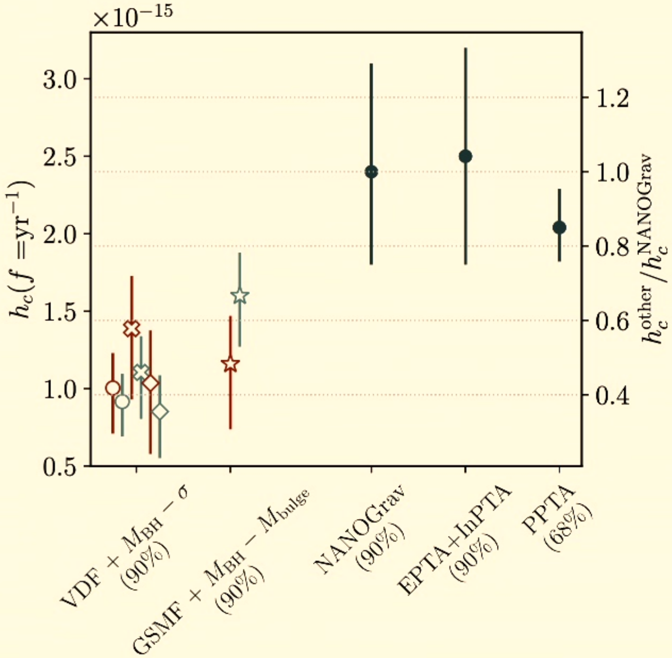


$$\frac{dn_{\rm binary}}{dM_{\rm BH}} = \frac{dn_{\rm single}}{dM_{\rm BH}}$$

GWs are a bit large…



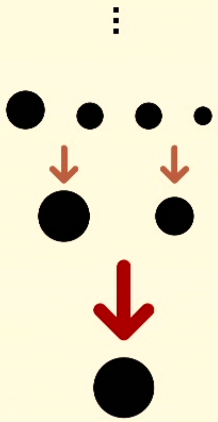
SMBH mass functions computed from local scaling relations and galaxy catalogs



Comparison with GW background measured by PTAs

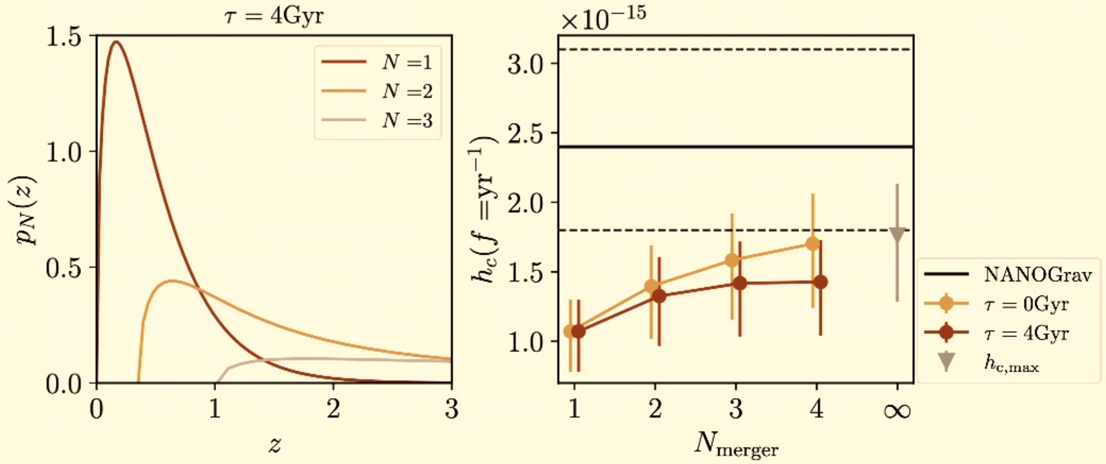
GSP, Zaldarriaga, Quataert 2023

Could it be the merger history?



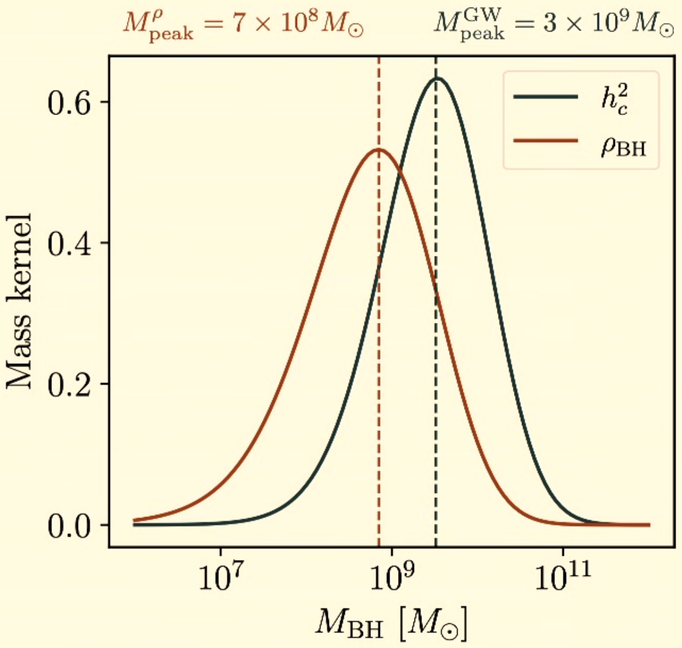
$$h_{c,N}^2 = h_{c,0}^2 \left(1 + \sum_{n=1}^{N} 2^n \times 2^{-5n/3}\right)_{N \to \infty} = 2.7 h_{c,0}^2$$

h_c can only be a factor of 1.64 larger, even in the limit of **infinite** and **instantaneous** mergers, with **no accretion**



Could it be the mass function?

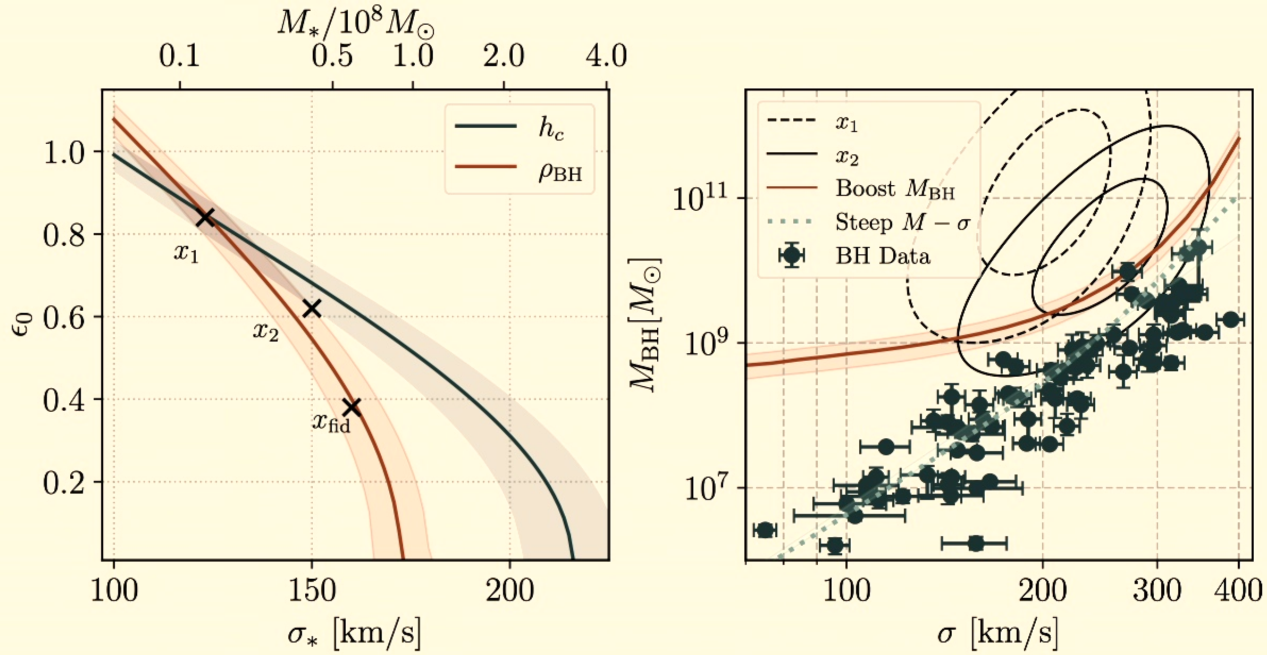
- BH mass density is a useful point of comparison between local observations and quasars
- GW background is sensitive to higher masses



$\rho_{\rm BH} \propto \langle M_{\rm BH} \rangle$

$h_c^2 \propto \langle M_{\rm BH}^{5/3} \rangle$

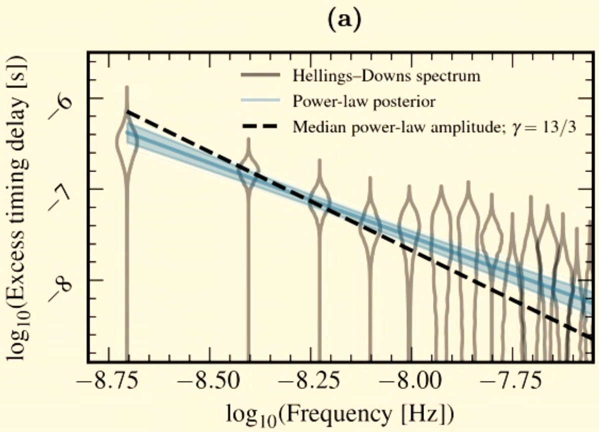
Could it be the mass function?



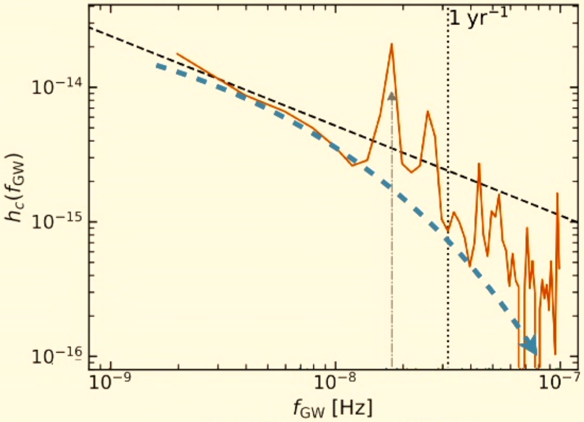
Changes to the mass function that maintain consistency with $\rho_{\rm BH}$ require higher abundance/masses of the most massive SMBHs

~4-10 larger BH abundance or $M_{\rm peak} = 3 \times 10^9 M_\odot \rightarrow 3 \times 10^{10} M_\odot$

Can we learn more from the **shape** of the spectrum?



NANOGrav, Agazie et al. 2023

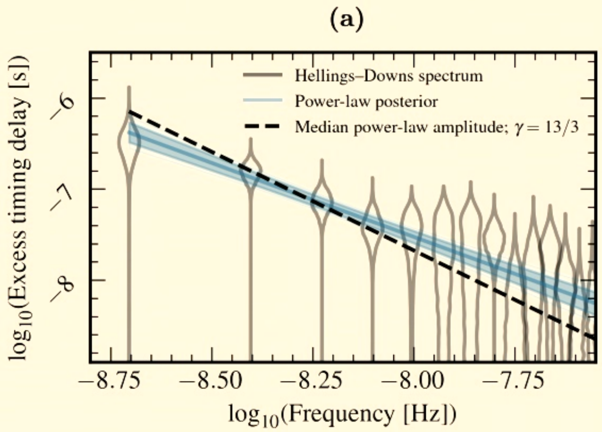


NANOGrav, Agazie et al. 2024

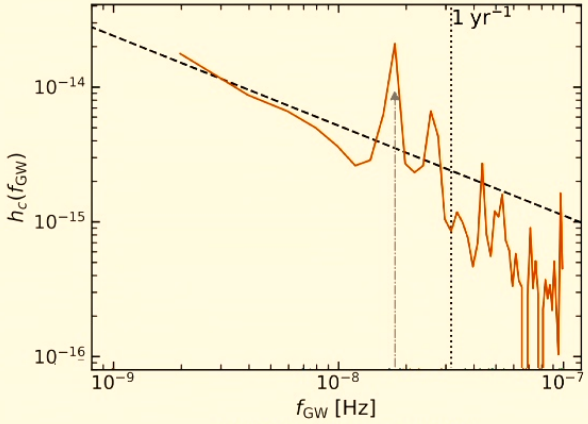
A finite source population leads to deviations in the spectrum (Sesana et al. 2008)

1. Frequency-to-frequency fluctuations
2. Drop-off of the median

Can we learn more from the **shape** of the spectrum?



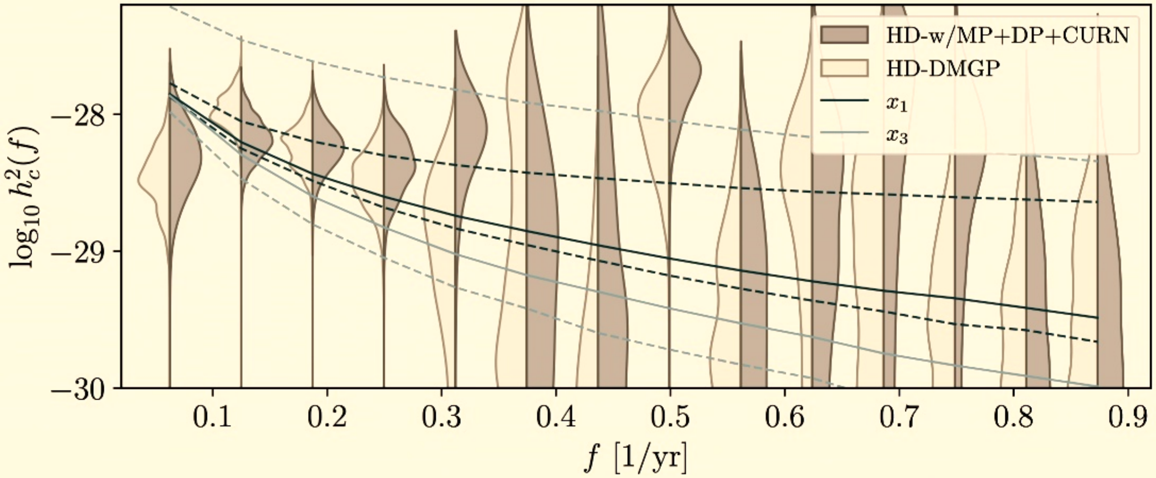
NANOGrav, Agazie et al. 2023



NANOGrav, Agazie et al. 2024

Is likely to be important at some frequency, since $N \propto M_{\rm peak}^{-5/3} f^{-11/3}$

To use the full information in the spectrum, we want a model for the distribution $p(h_c^2(f)\,|\,\vec{\theta}_{\rm SMBH})$



GSP, Zaldarriaga, 2406.17010

Finite sources

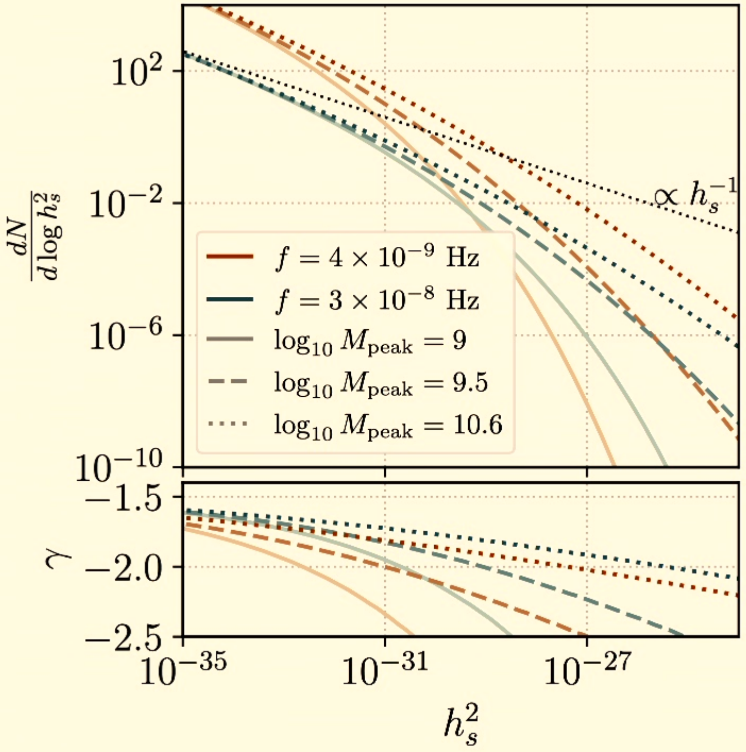
At each frequency $h_t^2 = \sum_s h_s^2(f)$

Define the “luminosity” function of GW sources:

$$\frac{d^3N}{dM\,dz\,dq} \rightarrow \frac{dN}{dh_s^2}$$

amplitude of a single source: $h_s^2 \propto \frac{\eta^2 M^{10/3} f^{4/3}}{d_L^2} \frac{f}{\Delta f}$

e.g. $\bar{N} = \int dh_s^2 \frac{dN}{dh_s^2}$ $\bar{h_t^2} = \int dh_s^2 \frac{dN}{dh_s^2} h_s^2$



Finite sources

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Define the "luminosity" function of GW sources:

$$\frac{d^3N}{dM\,dz\,dq} \to \frac{dN}{dh_s^2}$$

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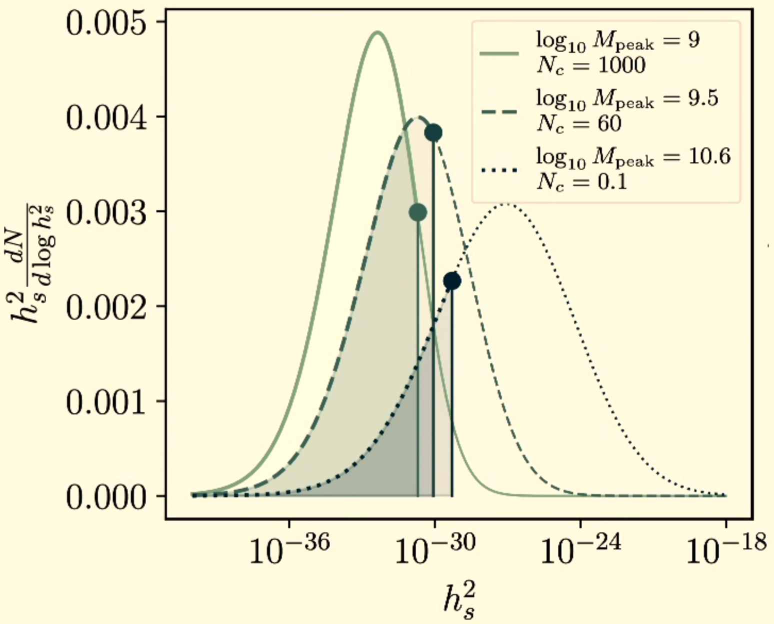


Useful to define the characteristic number of sources

$$N_c(f) \equiv \frac{h_c^2(f)}{h_s^2|_{\rm peak}(f)} \propto M_{\rm peak}^{-5/3} f^{-11/3}$$

Distribution of the total background

Single source probability: $P(h_t^2|N_s=1) = \frac{1}{\bar{N}}\frac{dN}{dh_s^2}$

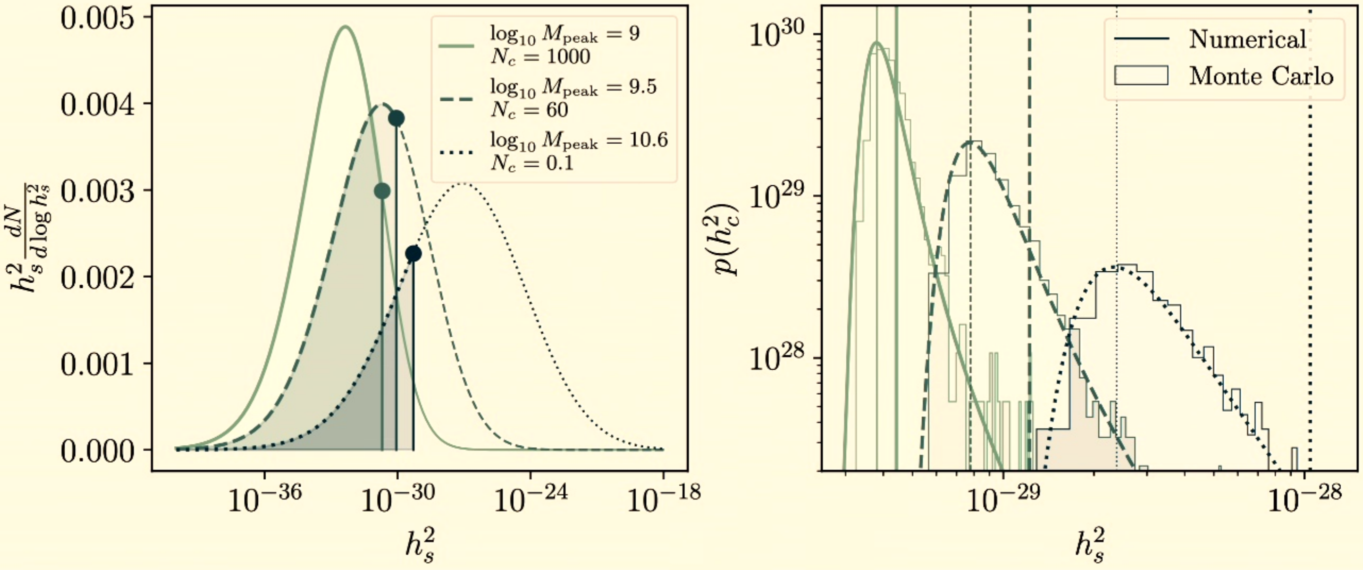
Distribution of the sum of sources: $P(h_t^2) = \sum_{N_s=0}^{\infty} P_{\text{Poiss}}(N_s|\bar{N}) \underbrace{P(h_t^2|N_s)}_{\propto P(h_t^2|N_s=1) * \dots * P(h_t^2|N_s=1)}$

Scheuer 1957
Condon 1974

(Simple in Fourier space $\hat{P}(\omega) = \exp\left\{\bar{N}\hat{P}(\omega|N_s=1) - \bar{N}\right\}$)

Distribution of the total background

Distribution of the sum of sources $P(h_t^2) = \sum_{N_s=0}^{\infty} P_{\rm Poiss}(N_s|\bar{N})P(h_t^2|N_s)$



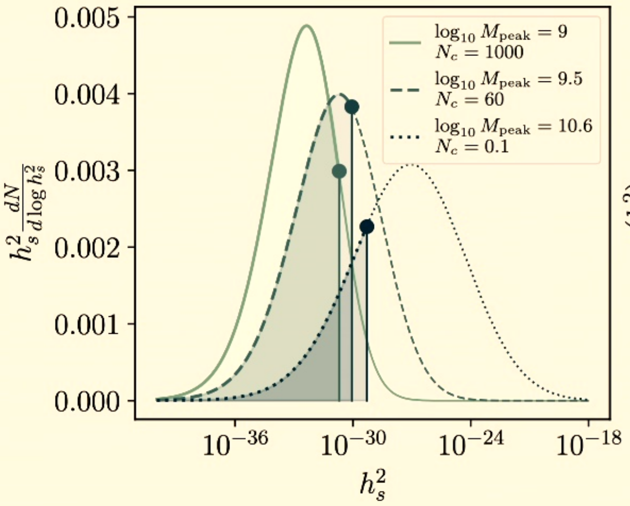
Dot: point after which the total number of sources is =1 $\int_{h_{s,1}^2}^{\infty} dh_s^2 \frac{dN}{dh_s^2} = 1$

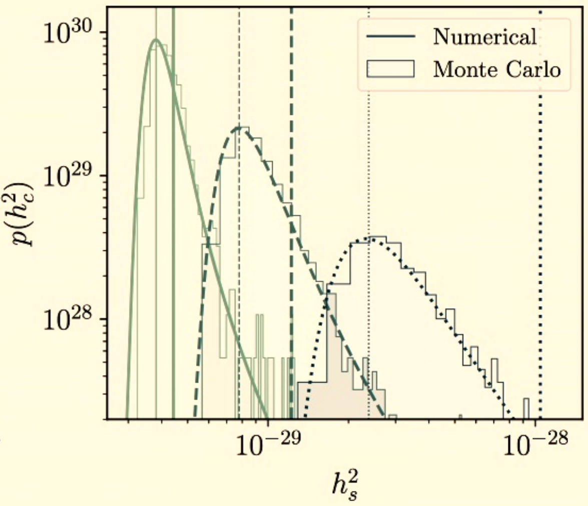
Shade: integral equal to the peak of $p(h_c^2)$ $h_{c,\rm peak}^2 = \int_0^{h_{s,1}^2} dh_s^2 \frac{dN}{dh_s^2} h_s^2$

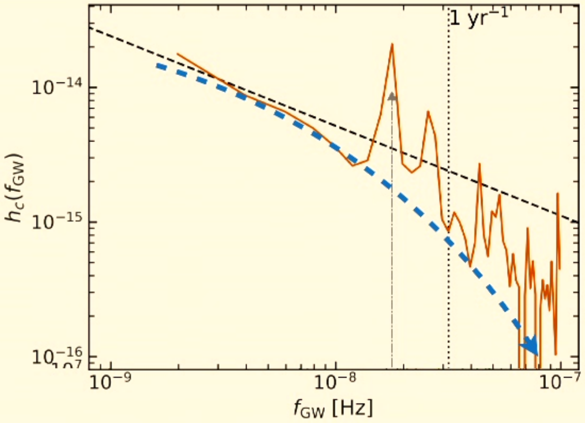
Thin line: $h_{c,peak}^2$

Thick line: h_c^2

Imprints of finite sources on the GW spectrum



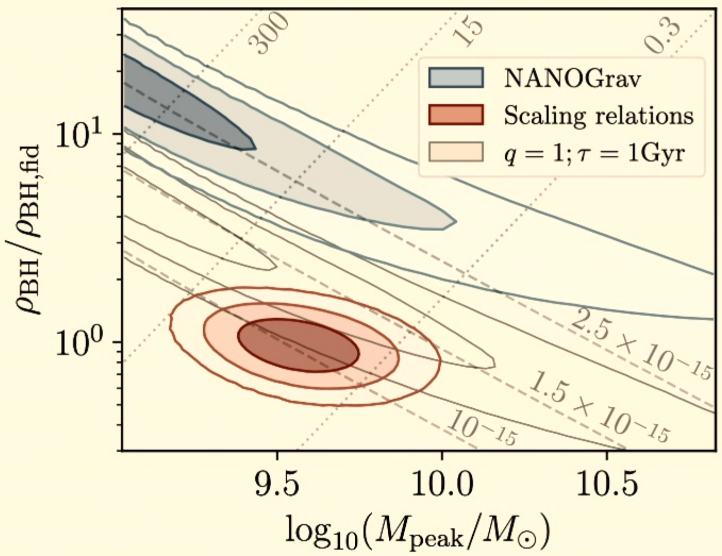




NANOGrav, Agazie et al. 2024

Reanalysis of NANOGrav 15-yr spectrum

Agazie et al. 2023
Lamb et al. 2023

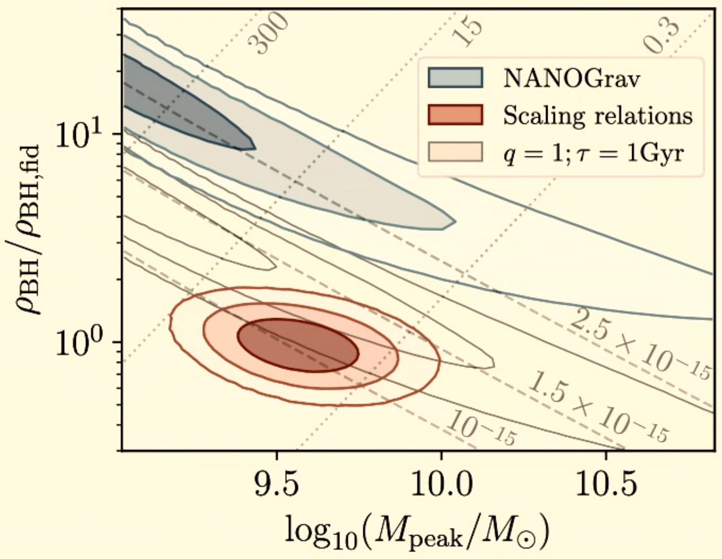


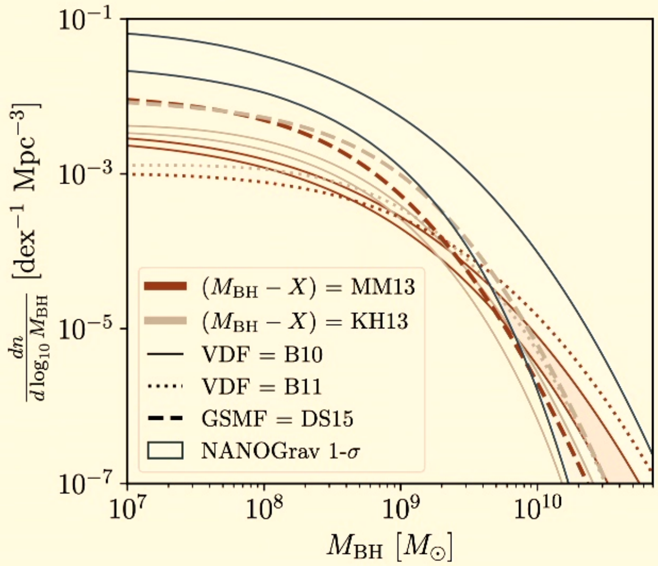
- First fit to take full distribution into account
- Larger BH abundance than implied by EM observations required to match PTAs
- Preference for models with large number of fainter sources

There already is information about the typical number of sources

Reanalysis of NANOGrav 15-yr spectrum

Agazie et al. 2023
Lamb et al. 2023





Big Galaxies and Big Black Holes: The Massive Ends of the Local Stellar and Black Hole Mass Functions and the Implications for Nanohertz Gravitational Waves

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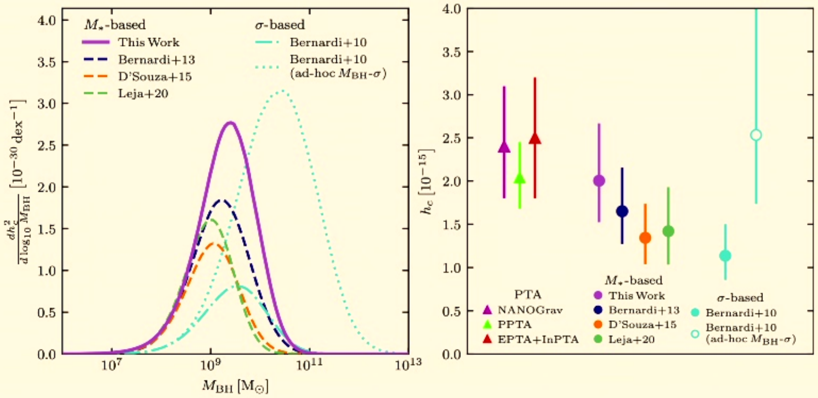

Fewer supermassive binary black holes in pulsar timing array observations

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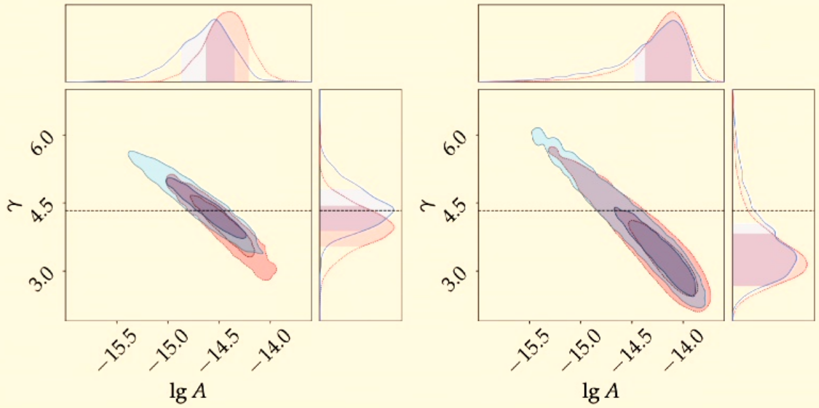
D. Champion,[8] S. Chen,[10] E. F. Keane,[11] G. Shaifullah,[4,5,12] and L. Speri[13]



(a) 25-year EPTA data, Hellings-Downs correlations

(b) 10-year EPTA data, Hellings-Downs correlations

Conclusions

- PTAs opened a new observational window into the Universe
- Simple but robust way to interpret the amplitude of the SGWB
    - Comparing the currently measured GW amplitude with electromagnetic observations suggests higher abundance of the most massive SMBHs
- Full GW spectrum can already tell us more about SMBHs
    - Preference for models with a large number of fainter sources (over few very massive ones)