Title: Asymptotic Safety and Canonical Quantum Gravity

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Subject: Quantum Gravity

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Abstract:

First, I will argue how background independent QFT-based regularisation methods can alleviate some important problems in quantum gravity.
Secondly, I will bring into closer contact the asymptotic safety (ASQG) and canonical (CQG) approach to quantum gravity. AS is a QFT-based approach to quantum gravity, which we will use to construct the generating functional of the n-point correlation functions. In particular, I will work with the Lorentzian version of the functional renormalisation group which we relate to the reduced phase space formulation of CQG.

Asymptotic safety
&
Canonical quantum gravity

Renata Ferrero

based on work in collaboration
with Thomas Thiemann 2404.18220, 2404.18224 [hep-th]
with Martin Reuter and Roberto Percacci 2203.08003, 2404.12357 [hep-th]

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Motivation

ASQG

- QFT-approach: non-perturbative RG flow with UV fixed point
- Mostly uses Euclidean signature
- Background dependence?
- Truncations of the flow equations

[Percacci's book (2017)
Reuter and Saueressig book (2019)]

CQG

[Rovelli's book (2004),
Thiemann's book (2019)]

- Lorentzian signature
- Manifestly background-independent
- No truncations performed

[Manrique, Rechenberger, Saueressig 1102.5012 (2011),
Fehre, Litim, Pawlowski, Reichert, 2111.13232 (2011),
Banerjee, Niedermaier 2201.02575 (2022),
D'Angelo, Pinamonti, Rejzner 2202.0758 (2022)]

Lorentzian version of the flow equation possible

Background-independent via the background field method

Truncations required in practice in any RG as an approximation scheme

[Thiemann 2003.13622 (2022)]

Relational formulation

Idea

Path integral treated with methods of **ASQG** in the Lorentzian version

a) Reduced phase space formulation of **CQG**

b) Construction of time-ordered correlation functions as a path integral

Machinery

Framework

First application

3.
- Einstein-Hilbert action coupled to 4 massless scalar fields
- Development of Lorentzian heat kernel cutoff functions

Quantum Gravity:
Shared principles

Background Independence

None of the theory's ingredients, predictions and assumptions should depend on any given fixed metric

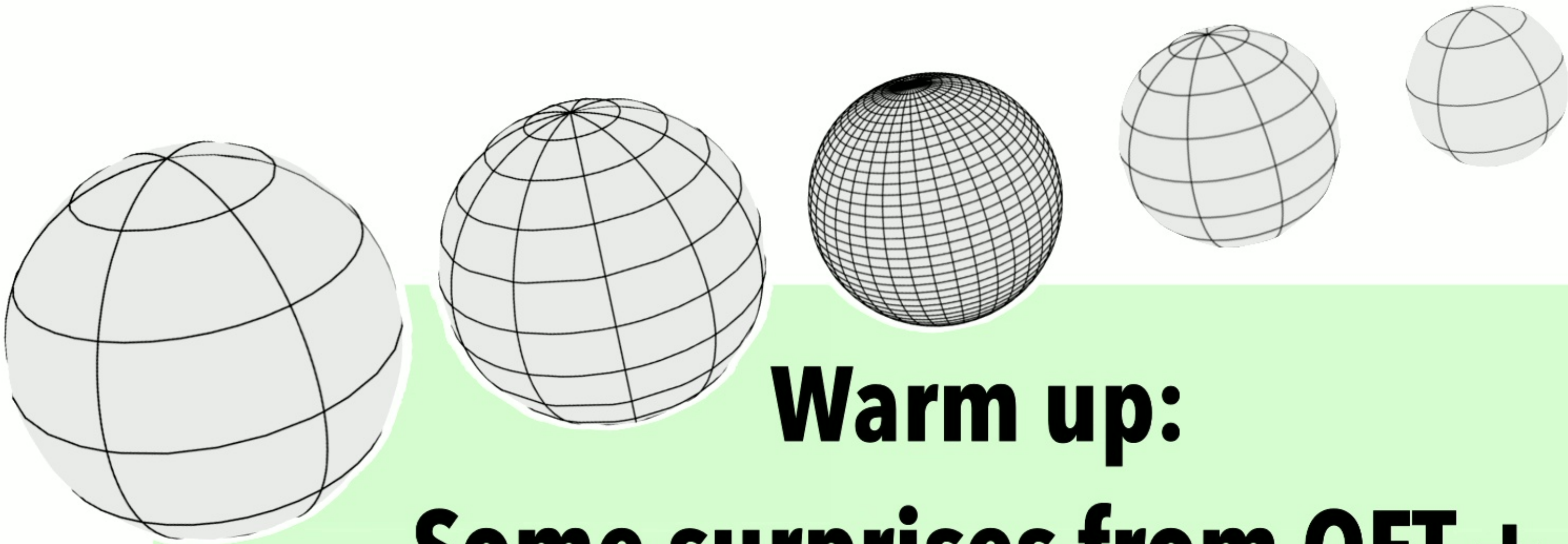
- No background structure at all **Loop Quantum Gravity**
- Background field method $g \to \bar{g} + h$ where $\bar{g}$ is arbitrary [DeWitt, Dynamical Theory of Groups and Fields (1965)]

  Physical metrics are self-consistently resulting from the dynamics of quantum gravity **Asymptotic Safety**

  not realized in perturbative approaches [Goroff and Sagnotti (1985)]

Non-perturbativity

No expansion in powers of the carrier fields (e.g., metric fluctuations) & no expansion in small couplings



Warm up:
Some surprises from QFT + background independence

Warm up: if I fix a background…

Pauli computation:

Sum up modes **around a fixed flat spacetime**: $\rho_{vac}[g_{\mu\nu} = \delta_{\mu\nu}] = \frac{1}{2}\int_{|\vec{p}|\leq \mathcal{P}} \frac{d^3|\vec{p}|}{(2\pi)^3}|\vec{p}| \sim \mathcal{P}^4 = m_{Pl}^4 = 10^{76} GeV^4$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \approx 10^{-47} GeV^4$$

Deviation theory-experiment

of ca. 123 orders!

Cosmological constant problem stemming out from

summing up vacuum energies on a fixed background.

Revisit Pauli computation in a self-consistent backround independent regularization of the modes sum:

the N-cutoffs

Goroff-Sagnotti computation:

Use perturbation theory & renormalization **around a fixed flat spacetime**: gravity non-renormalizable at two-loops

Revisit the renormalization program in a nonperturbative backround independent fashion:

the asymptotic safety scenario

Revisit Pauli computation: The N-cutoffs

[RF, Percacci (2024) 2404.12357 [hep-th]

[Becker, Reuter 2021, Becker, Banjeree, RF 2023]]

Three main ingredients

- Use a dimensionless cutoff
- Go on-shell selfconsistently
- Dynamical gravity

- Consider a free **scalar field coupled to gravity**

$$S_H(g) = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,[2\Lambda - R] \;\; ,$$

$$S_m(\phi; g) = \int d^4x\sqrt{g}\frac{1}{2}(\partial\phi)^2$$

- The backreaction of the scalar field on the metric is **encoded in the effective action**. At one loop

$$\Gamma(g,\phi) = S_H(g) + S_m(g,\phi) + \frac{1}{2}\mathrm{Tr}\log(\Delta/\mu^2)$$

- Variation of the EA with respect to the metric yields the **semiclassical Einstein equations**

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda_B g_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$$

There is no cosmological constant problem

- Let us work on a **dynamical sphere** S^4

the radius is dynamical
it makes the use of a dimensionless cutoff particularly natural!

angular momentum $\lambda_\ell = \frac{R}{12}\ell(\ell+3)\ , \quad m_\ell = \frac{1}{6}(\ell+1)(\ell+2)(2\ell+3)\ , \quad \ell = 1, 2 \ldots$

the operation Tr in the definition of the EA can be written explicitly as a sum over all eigenstates of the Laplacian

Dimensionless cutoff: N-cutoff

We regulate the sum by putting an upper bound N on the quantum number ℓ

$$\frac{1}{2}\mathrm{Tr}_N \log(\Delta/\mu^2) = \frac{1}{2}\sum_{\ell=1}^{N} m_\ell \log\left(\frac{\lambda_\ell + E}{\mu^2}\right) = \frac{1}{2}\sum_{\ell=1}^{N} m_\ell \log\left(\frac{R}{12\mu^2}\ell(\ell+3)\right)$$

There is no cosmological constant problem

Self-consistent on-shellness
the radius (Ricci scalar)
is dynamical

$$\frac{24\pi}{GR^3}(-R+4\Lambda) = \frac{1}{24R}N(N+4)(N^2+4N+7) =: \frac{1}{12R}f(N)$$

$$R = \frac{24\pi}{Gf(N)}\left(-1 \pm \sqrt{1+\frac{G\Lambda f(N)}{3\pi}}\right) \sim \frac{1}{N^2} \quad \to 0 \text{ when } N \to \infty$$

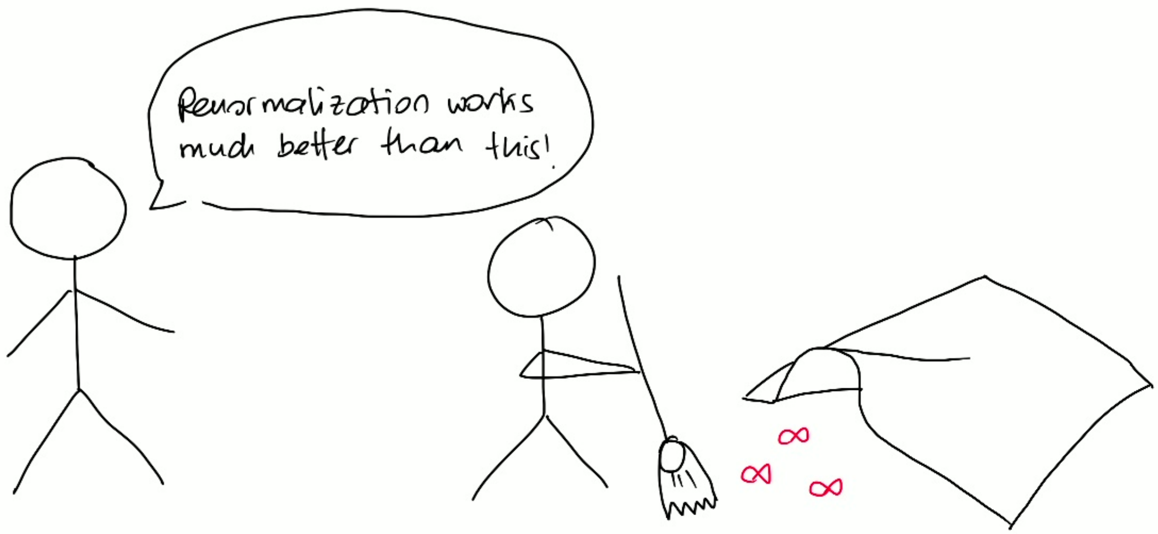
The curvature of spacetime decreases when more quantum modes are included in the calculation!

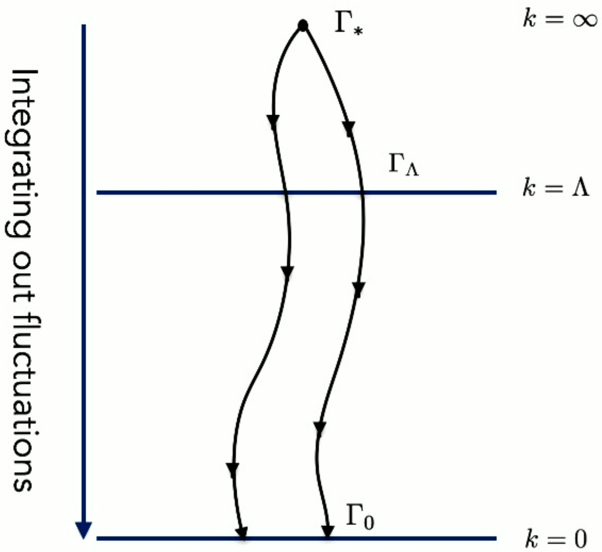
Asymptotic Safety

Weinberg's conjecture: There exists a nonperturbative dynamical mechanism which renders physical scattering amplitudes finite and computable at energy scales exceeding the Planck scale: a nontrivial UV fixed point.

Technically investigated in the continuum via the **Functional Renormalization Group**

in the the discrete in EDT/CDT [Ambjørn and co. 2408.07808 [hep-lat]]





UV fixed point

Microscopic model: Asymptotic Safety, Dynamical Triangulations

Initial condition

Effective action

Callan-Symanzik equation

RG in perturbation theory

only the finitely many beta functions that are related to the relevant couplings are considered

Callan (1970), Symanzik (1970)

The RG

Exact RGE

Wilsonian Exact RG

the quantum fluctuations in the path integral can be integrated out progressively

Wilson (1971), Kadanoff (1966)

Functional RGE

Functional RG

scale-dependent version of the effective action, the Effective Average Action

Wetterich (1991)
Reuter and Wetterich (1994)

Functional Renormalization Group

Alternative manipulation of the path integral

implement the underlying RG idea already at the level
of the scale dependent action

Asymptotic Safety via FRG:
Investigate acceptable UV limit, if there exist a trajectory whose endpoint in the UV
is given by the nontrivial fixed point of the RG flow.

Effective average action - Cutoff function

- Generating functional $W[J] = i^{-1} \ln(Z[J])$
- Effective action: Legendre transform $\Gamma[\hat{h}] := [L \cdot W][\hat{h}] = \text{extr}_J(< J, \hat{h} > - W[J])$

- **Effective average action**

$$Z_k[J, \bar{g}] = \int [dh]\, e^{-iS[\bar{g}+h]}\, e^{i<J,h>} e^{-i\frac{1}{2}<h,R_k(\bar{g})\cdot h>},$$

$$W_k[J, \bar{g}] = i^{-1} \ln(Z[J, \bar{g}]),$$

$$\Gamma_k[\hat{h}, \bar{g}] = \text{extr}_J\, (< J, \hat{h} > - W_k(J, \bar{g})) - \frac{1}{2} < \hat{h}, R_k(\bar{g}) \cdot \hat{h} >$$

cutoff function

[Wetterich (1992), Morris (1994), Reuter (1996)]

where $k \to R_k(\bar{g})$ is a 1-parameter family of integral kernels which only depend on the background d'Alembertian

⟶ **Euclidean: integrate out momenta**

⟶ **Lorentzian: oscillating with $R_k = 0$ for $k = 0$, s.t.** $\Gamma[\hat{h}, \bar{g}] = \Gamma_{k=0}[\hat{h}, \bar{g}]$

Remark: everything is defined in Lorentzian signature - no analytic continuation needed

Flow equation - Lorenzian version

- **Lorentzian version of the Wetterich equation:**

$$k\ \partial_k\ \Gamma_k[\hat{h},\bar{g}] = \frac{1}{2i}\ \mathsf{Tr}([R_k(\bar{g}) + \Gamma_k^{(2)}(\hat{g},\bar{g})]^{-1}\ [k\ \partial_k R_k(\bar{g})]), \qquad \Gamma^{(2)}[\hat{h},\bar{g}] := \frac{\delta^2\Gamma^{(2)}[\hat{h},\bar{g}]}{\delta\hat{h}\otimes\delta\hat{h}}$$

Exact and **non-perturbative** identity and **can be used to construct a well defined Γ rather than using $Z[J]$**

- To solve it : Taylor expand both LHS and RHS in powers of $g_{\mu\nu}$ and compare coefficients (**truncation**)

$$\mathsf{Tr}[R_k(\bar{g}) + \Gamma_k^{(2)}(\hat{h}, \bar{g})]^{-1}$$

In order to evaluate the traces in a **background independent** way, we can use

Background field method

+

Lorentzian heat kernel

Lorentizan heat kernel

- Using the **Schwinger proper time integral**

$$P_k^{-1} = B^{-1}\,[\bar{\Box} + C_k]^{-1} = -\frac{i}{B}\,[\int_0^\infty dt\; e^{it\bar{\Box} - t\epsilon}]_{\epsilon \to -iC_k}$$

- Traces rewritten by the spectral theorem (involving both **minimal and non-minimal** operators)

$$O_k(\bar{\Box}) = \int_{-\infty}^{\infty} dt\; \hat{O}_k(t)\; H_t, \quad H_t := e^{it\bar{\Box}}$$

Heat kernel **on general manifold**: $H_t(x,y) = [4\pi|t|]^{-d/2}\; e^{i\frac{\pi}{4}\mathrm{sgn}(t)[2-d]}\; e^{\frac{i}{2t}\,\sigma(x,y)}\;\Omega_t(x,y)$

Synge's world function

[Benedetti, Groh, Machado, Saueressig 1012.3081 (2010), Groh, Saueressig, Zanusso 1112.4856 (2011)]

[Christensen (1976), Fulling's book (1989), Moretti (1999), Decanini, Folacci (2006), Parker, Toms' book (2009)]

- The trace can be expanded

$$\mathrm{Tr}[H_t(\bar{\Box})] = \frac{e^{i\pi/4\mathrm{sgn}(t)(2-d)}}{(4\pi|t|^{d/2})}\left(1 + \frac{i}{6}\bar{R}t + \cdots\right)$$

Analytic continuation of the proper time variable (not the coordinate time!)

choosing a suitable cutoff function… we get convergent traces.

Lorentizan heat kernel - New cutoff function

Choose a suitable cutoff function to get convergent traces

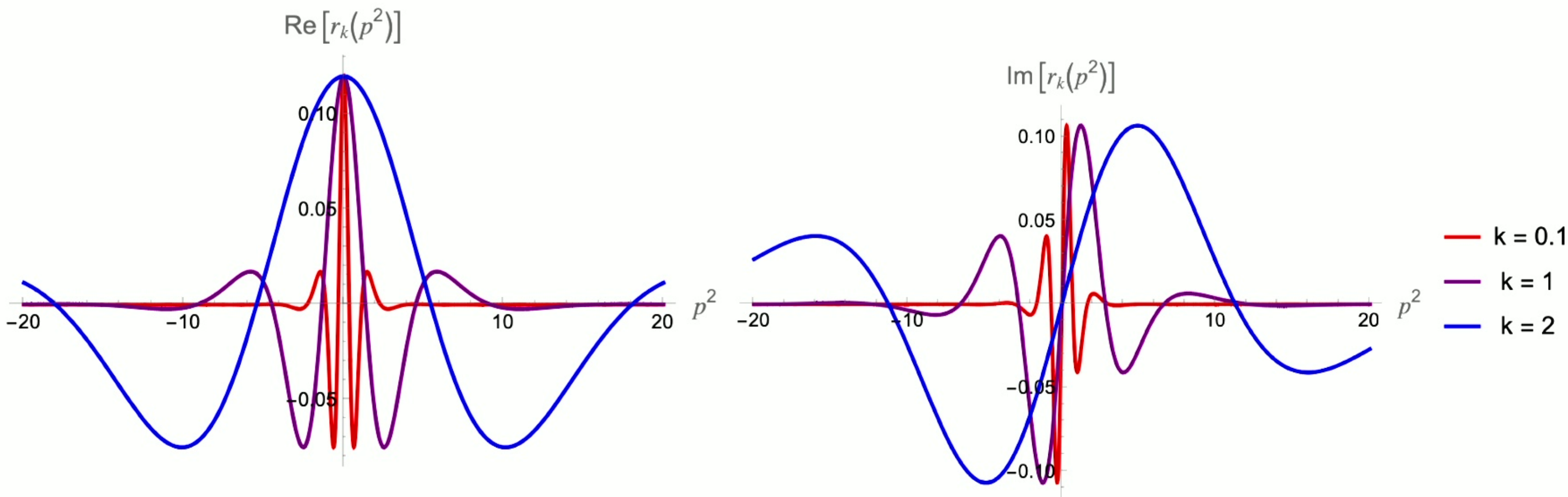
$$R_k(\overline{\Box}) = f_k\, k^2\, r\left(\frac{\overline{\Box}}{k^2}\right), \qquad r\left(\frac{\overline{\Box}}{k^2}\right) = \int_0^\infty dt\, e^{-t^2-t^{-2}}\, e^{it\overline{\Box}/k^2}$$

- Fourier transform $\hat{r}(t)$ has **rapid decrease** at $t = 0, \infty$ smoothly joins the constant function $\hat{r}(t) \equiv 0,\, t \leq 0$
- **Convergence** guaranteed by $k\partial_k R_k$ producing an $e^{-t_0^2 - t_0^{-2}}$ factor
- **Positive support**: heat kernel time integrals involve the heat kernel $s = t_0 + \cdots + t_n$ and the heat kernel itself contains $|s|^{-d/2}$ as a prefactor

Price to pay: complex valued flow.

- **Admissable trajectories**: flow to real valued dimensionful coupling constants when $k \to 0$

Lorentizan cutoff function



What is missing as a candidate theory of quantum gravity?

Extract physical degrees of freedom

Explicit canonical quantization

Relational observables & Reduced phase space

Constrained Hamiltonian system

[Henneaux and Teitelnboim's book (1994)]

Reduced phase space approach

[Dittrich gr-qc/0507106 (2006), Thiemann gr-qc/0411031 (2006)]

- physical Hamiltonian of the **relational Dirac observables**
- **gauge fixing approach:** select the "true dofs" and construct the **reduced Hamiltonian:** function of the true dofs which generate the eoms as the primary Hamiltonian when restricted to the reduced phase space
- The **physical interpretation of the true dofs and the reduced Hamiltonian depends on the choice of gauge fixing**: Q, P are those observables which on the gauge cut $G_j(0) = 0$ coincide with the gauge invariant observables $O_P(0),\ O_Q(0)$ and their evolution is generated by the reduced Hamiltonian induced by $G_j(t) = 0$.

Quantization and Path integral

- **Canonical quantization**: representation of the canonical commutation relations and *−relations
- Weyl elements $W(f,g) := e^{i\,[f_a Q^a + g^a P_a]}$
- GNS data $(\mathcal{H},\ \Omega,\ \rho)$ of a state ω on $\mathfrak{U}$ $\omega(a) =< \Omega, \rho(a)\Omega >_{\mathcal{H}}$

[Haag's book (1984), Glimm and Jaffe's book (1987), Bratteli and Robinson's book (1997)]

[Thiemann 2003.13622 (2022)]
UV-regularization on the modes

- **Path integral formulation**: time ordered correlation functions and its generating functional,

supposing that H is bounded from below and has a unique ground state Ω_0

$$F_N((t_1, f^1), .., (t_N, f^N)) :=< \Omega_0,\ \mathcal{T}[W_{t_N}(f^N,0)..W_{t_1}(f^1,0)]\ \Omega_0 >_{\mathcal{H}}$$

Lorentzian signature

where $U(t) = \exp(-itH)$

$$W_t(f,g) := U(t)W(f,g)U(t)^{-1}$$

Generating functional $\chi(f) :=< \Omega_0,\ \mathcal{T}[e^{i\int_{\mathbb{R}} dt\ f_a(t)\ Q^a(t)}]\ \Omega_0 >_{\mathcal{H}}$

Not so practically useful as we have not explicit access to Ω_0, but rather to Ω_M.

Quantization and Path integral

- Suppose that f has compact support on $[-\tau, \tau]$, then for $T \gg \tau$ and $\Delta_N = \frac{\tau}{N}$

partition function $\quad \chi(f) = \lim_{M\to\infty} \lim_{T\to\infty} \lim_{N\to\infty} \frac{Z_{M,T,N}(f)}{Z_{M,T,N}(0)}$

$$Z_{M,T,N}(f) =< \Omega_M, e^{i\Delta_N H_M} \; e^{i\Delta_N f_a(t_{N-1})Q^a} \; e^{i\Delta_N H_M} \; .. \; e^{i\Delta_N H_M} \; e^{i\Delta_N f_a(t_{-N})Q^a} e^{i\Delta_N H_M} \; \Omega_M >$$

- Using **Feynman-Kac arguments** to send $N \to \infty$, one arrives at

$$Z(f) = \int [dp]\,[dq]\, \overline{\Omega(q(\infty))}\; \Omega(q(-\infty))\; e^{i\int\; dt\; f_a(t)q^a(t)}\; e^{-i\int\; dt\; [p_a(t)\dot{q}^a(t)\; - H(q(t),p(t))]}$$

- Observations:
  - **Integral over phase space** and not over configuration space to begin with. Integrating out the momenta explicitly is only possible when H has a sufficiently simple dependence on p and it may modify the "Lebesgue measure" $[dq]$.
  - **Dependence** of $Z(f)$ on the cyclic vector Ω or equivalently the corresponding **state** ω.

RF, Reuter [2203.08003 [hep-th]]
State-dependence in de Sitter

CQG derivation

The model: $$S = G_N^{-1} \int_M d^d x \sqrt{-\det(g)}\, [R[g] - 2\Lambda - \frac{G_N}{2} S_{IJ}\, g^{\mu\nu}\, \phi^I_{,\mu}\, \phi^J_{,\nu}]$$

[Giesel, Thiemann 1206.3807 (2012), [Giesel, Vetter 1610.07422 (2016)]]

- **Reduced phase space approach**: impose d gauge fixing conditions on the configuration variables and solve the constraint for the d conjugate momenta
- Convenient set of gauge

$$G^I := \phi^I - k^I = 0\,, \qquad \det(\partial k/\partial x) \neq 0\,, \qquad \partial k^I/\partial x^\mu \text{ independent of } x^0$$

- Gauge dofs. $(\phi^I), (\pi_I)$ and **true dofs.** (q_{ab}, p^{ab})
- Impose **gauge stability** condition: solved by $N^\mu := N^\mu_*$

CQG derivation

- The cyclic rep. correspond to states ω wrt. which we can compute **time-ordered correlation functions** [Brattelli, Robison (1997)]
- They are obtained from a **generating functional**

$$Z_s[f] = \int [dq\ dp]\ \overline{\Omega(q(\infty))}\Omega(q(-\infty))\ e^{-\int\ dx^0\ (i <p,\dot{q}> + (-i)^s\ H[p,q])}\ e^{i^s\ \int\ dx^0\ <f,q>}$$

- **Problem: H involves a square root**
- Solution: unfolding the reduced phase space path integral to the unreduced phase space [Henneaux, Teitelboim's book (1992)]
  - Extend the integration to ϕ^I, π^I through $\delta(C)$, $\delta(G)$

$$C_I = \pi_I + h_I = 0\ , \qquad G = \phi^I - k^I = 0 \qquad \longrightarrow \qquad H = \dot{k}^I h_I = -\dot{\phi}^I \pi_I$$

$\longrightarrow$ this forces us to work with **Lorentzian signature** $s = 1$.

$$Z_1[f] = \int [dq]\ \overline{\Omega(q(\infty))}\Omega(q(-\infty))\ e^{-iS_1[g]}\ \ e^{i \int_M\ d^dx\ f^{ab} q_{ab}},\ \ S_1[g] = \frac{1}{G_N}\int_M\ d^dx\sqrt{-det(g)}[R[g] - 2\Lambda - \frac{G_N}{2} g^{\mu\nu}\ S_{IJ}\ \ k^I_{,\mu}\ k^J_{,\nu}]$$

- Ghost matrix K_ω

$$\overline{\Omega(q(\infty))}\Omega(q(-\infty)) = \left| \int [d\rho\ d\eta]\ e^{-i \int\ d^dx\ \eta^\mu\ [K_\omega]^I_\mu(q)\ \rho_I} \right|$$

ASQG model

- For a first investigation we do not consider the state dependence.

Ansatz: $\Gamma_k(\hat{h},\bar{g}) := \frac{1}{G_{N,k}} \int d^dx \left[-\det(g)\right]^{1/2} \left[R[g] - 2\Lambda_k - \frac{\kappa_k G_{N,k}}{2} g^{\mu\nu}\, \delta_{\mu\nu} \right]_{g=\bar{g}+\hat{h}}$

where we specialized to the gauge $\phi^I = k^I$, s.t. $\kappa^I_\mu = k^I_{,\mu} = \text{const.}$ and $\kappa_{\mu\nu} := S_{IJ}\kappa^I_\mu \kappa^J_\nu = \kappa_k \delta_{\mu\nu}$

[Baldazzi, Falls, RF 2112.02118 (2021)]

ASQG model

Results: matter coupling

- At the level of our truncation in $d = 4$, the coupling constant κ_k **is not flowing** and the flow of the gravitational couplings completely disentangles

  The matter contribution to $\mathrm{Tr}[K^{-1}U_k]$ and $\mathrm{Tr}[K^{-1}U_k]^2$ vanish in $d = 4$

  The additional matter term contributes to the phase space reduction and indicates how scalar field degrees of freedom are transformed into metrical ones.

  However, at this level of the truncation, it **neither explicitly contributes to the flow of the couplings related to the physical degrees of freedom, nor is it itself affected by their running**.

ASQG model

Results: gravitational coupling

- Define in general the integrals

$$I_{m,n} = \int_0^\infty dt_1 \cdots dt_m \frac{e^{-t_1^2 - t_1^{-2}} \cdots e^{-t_m^2 - t_m^{-2}}}{(t_1 + \cdots + t_m)^n} \,,$$

$$J_{m,n} = \int_0^\infty dt_1 \cdots dt_m \frac{\frac{d}{dt_1}(e^{-t_1^2 - t_1^{-2}}) \cdots e^{-t_m^2 - t_m^{-2}}}{(t_1 + \cdots + t_m)^n}$$

- **Beta functions:**

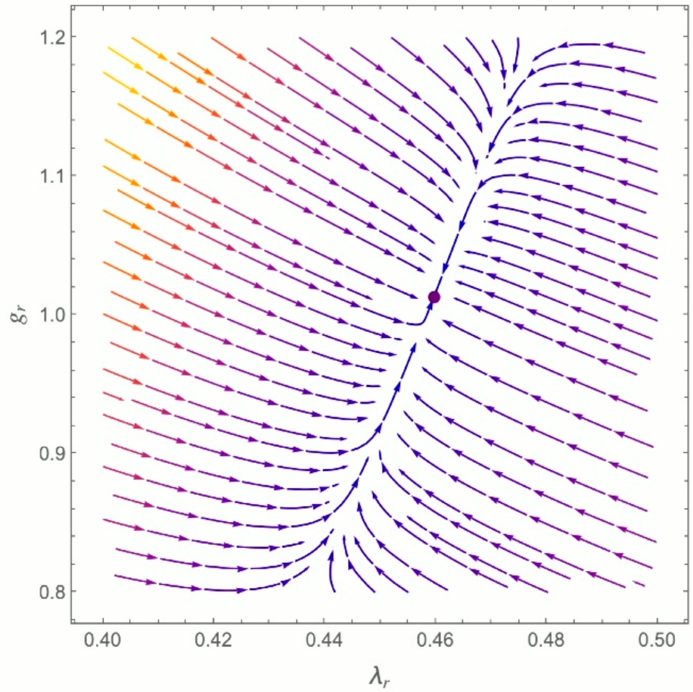
$$\begin{aligned} k\partial_k \lambda_k = \; & -4\lambda_k + \eta_N \lambda_k - \frac{g}{4\pi}\frac{1}{2\lambda_k}\left((2-\eta_N)\left(I_{1,2} + \frac{1}{2\lambda_k}\left(I_{2,2} - 5I_{1,3} + \frac{4}{3}\lambda_k I_{1,2}\right)\right.\right. \\ & + \frac{1}{(2\lambda_k)^2}\left(I_{3,2} + 2\left(I_{2,2} - 5I_{1,3} + \frac{4}{3}\lambda_k I_{1,2}\right) - i\frac{16}{9}\lambda_k^2 I_{1,2} + \frac{2}{3}I_{1,3}\right)\Bigg) \\ & + 2\left(J_{1,2} + \frac{1}{2\lambda_k}\left(J_{2,2} - 5J_{1,3} + \frac{4}{3}\lambda_k J_{1,2}\right)\right. \\ & \left.\left. + \frac{1}{(2\lambda_k)^2}\left(J_{3,2} + 2\left(I_{2,2} - 5J_{1,3} + \frac{4}{3}\lambda_k J_{1,2}\right) - i\frac{16}{9}\lambda_k^2 J_{1,2} + \frac{2}{3}J_{1,3}\right)\right)\right) \end{aligned}$$

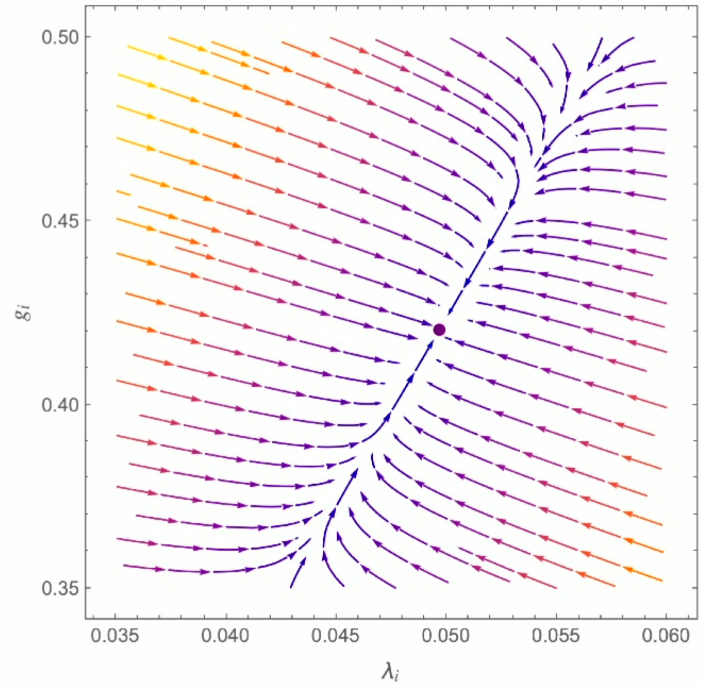
$$\begin{aligned} k\partial_k g_k = \; & 2g_k - \frac{g_k^2}{2\pi}\frac{1}{2\lambda_k}\left((2-\eta_N)\left(\frac{i}{6}I_{1,1} + \frac{1}{2\lambda_k}\left(\frac{i}{6}I_{2,1} + 3I_{1,2} + i\frac{2}{9}\lambda_k I_{1,1} - i\frac{5}{12}I_{1,2}\right)\right.\right. \\ & + \frac{1}{(2\lambda_k)^2}\left(\frac{i}{6}I_{3,1} + 2\left(\frac{i}{6}I_{2,1} + 3I_{2,2} + i\frac{2}{9}\lambda_k I_{2,1} - i\frac{5}{12}I_{2,2}\right) - i\frac{8}{27}I_{1,1} + \frac{3}{2}I_{1,3} + i\frac{1}{18}\lambda_k I_{1,2}\right)\Bigg) \\ & + 2\left(\frac{i}{6}J_{1,1} + \frac{1}{2\lambda_k}\left(\frac{i}{6}J_{2,1} + 3J_{1,2} + i\frac{2}{9}\lambda_k J_{1,1} - i\frac{5}{12}J_{1,2}\right)\right. \\ & \left.\left. + \frac{1}{(2\lambda_k)^2}\left(\frac{i}{6}J_{3,1} + 2\left(\frac{i}{6}J_{2,1} + 3J_{2,2} + i\frac{2}{9}\lambda_k I_{2,1} - i\frac{5}{12}J_{2,2}\right) - i\frac{8}{27}J_{1,1} + \frac{3}{2}J_{1,3} + i\frac{1}{18}\lambda_k J_{1,2}\right)\right)\right) \end{aligned}$$

ASQG model

Results: gravitational coupling

- **UV fixed point** $\lambda_* = 0.460 + 0.050\ i\ ,\quad g_* = 1.013 + 0.420\ i$ very close to the Reuter's fixed point
- Critical exponents $\theta_1 = 12.24 - 0.07\ i\ ,\quad \theta_2 = 0.95 + 0.017\ i$ **two relevant directions?**
- Projection into real and imaginary part:



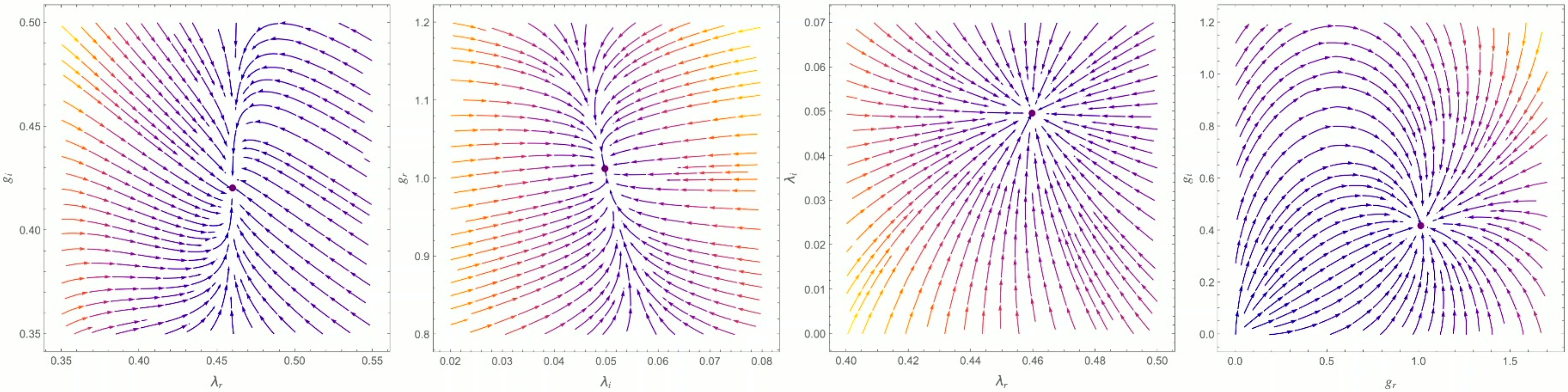


ASQG model

Results: gravitational coupling

- Other projections:

Attractive fixed point



Conclusions

- **ASQG**: systematic procedure
    1. Well defined construction of the effective action
    2. Machinery to compute correlators of the Hamiltonian theory
- **CQG**: input how to define the effective average action
    3. State underlying the Hamiltonian theory
    4. Restriction on correlation functions of the true dofs
- Einstein-Klein-Gordon theory as a showcase model to demonstrate that ASQG and CQG can be fruitfully combined
- Techn. development: regularized **Lorentzian** heat kernel proper time
- **Results**: Complexed value UV fixed point

  Existence of admissible trajectories

Outlook

- Classification of Lorentzian cutoff functions
- Incorporation of ghost matrix term or use of field redefinitions
- Higher order truncations
- More realistic matter coupling where the Euclidean version is possible