Title: The Quantum Mechanics of Spherically Symmetric Causal Diamonds

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Abstract:

We construct the phase space of a spherically symmetric causal diamond in (d+2)-dimensional Minkowski spacetime. Utilizing the covariant phase space formalism, we identify the relevant degrees of freedom that localize to the d-dimensional bifurcate horizon and, upon canonical quantization, determine their commutators. On this phase space, we find two Iyer-Wald charges. The first of these charges, proportional to the area of the causal diamond, is responsible for shifting the null time along the horizon and has been well-documented in the literature. The second charge is much less understood, being integrable for $d \geq 2$ only if we allow for field-dependent diffeomorphisms and is responsible for changing the size of the causal diamond.

The QM of Spherically Symmetric Causal Diamonds

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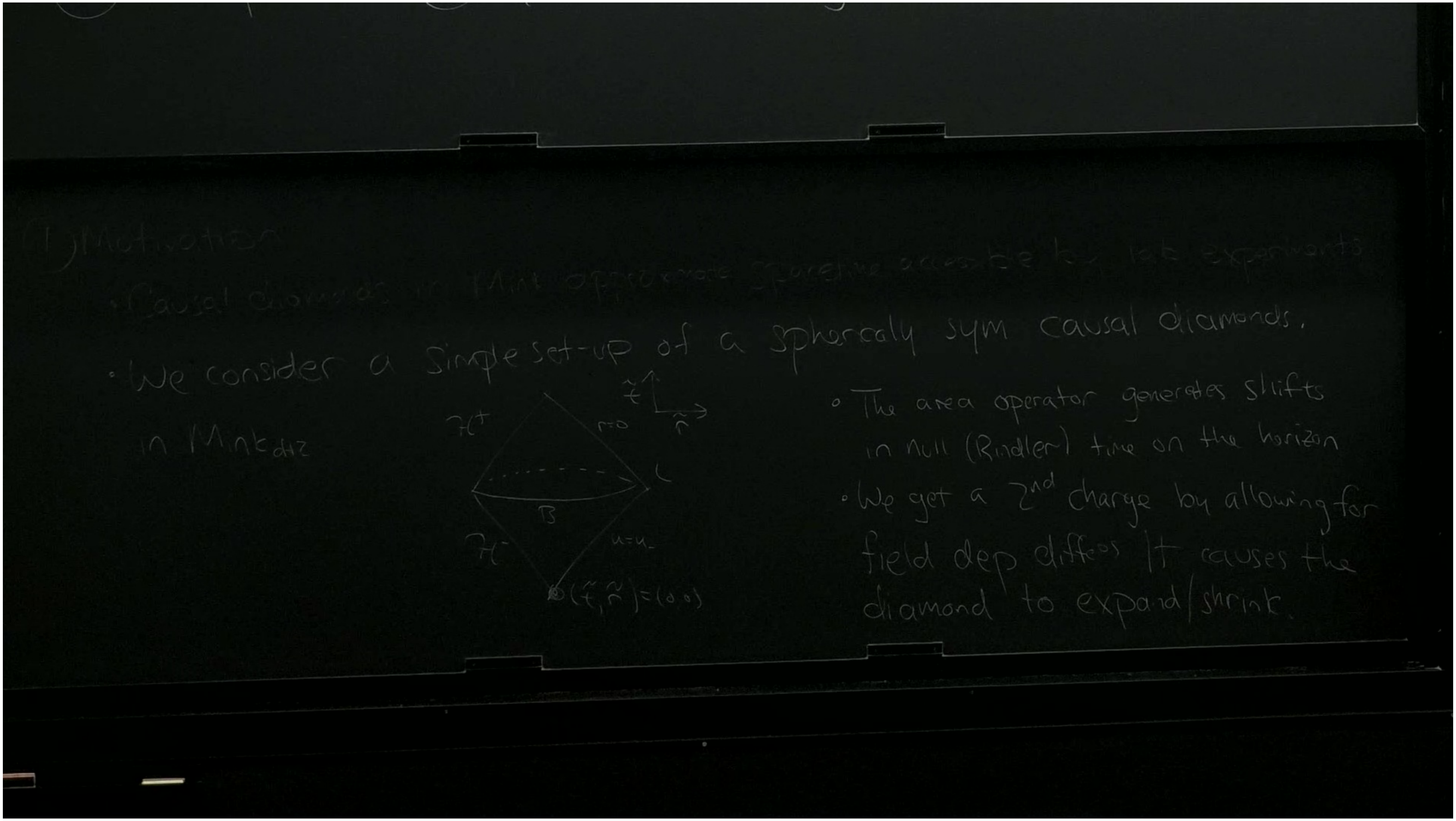
1. Motivation
2. Set up
3. Symplectic form
4. Hamiltonian Charges



(1) Motivation
• Causal diamonds in Mink approximate spacetime accessible to lab experiments
• We consider a simple set-up of a spherically sym causal diamonds in Minkowski.
$\tilde{t}$
$\tilde{r}$
$\mathcal{H}^+$
$r=0$
$\mathcal{B}$
$\mathcal{H}^-$
$u=u_-$
$(\tilde{t},\tilde{r})=(0,0)$
• The area operator generates shifts in null (Rindler) time on the horizon
• We get a 2nd charge by allowing for field dep diffeos. It causes the diamond to expand/shrink.

② Set-up

The metric for Mink is

sphere

$$ds^2 = -d\tilde{u}^2 - 2\,d\tilde{u}\,d\tilde{r} + \tilde{r}^2\,d\Omega_d^2$$

$\tilde{u} = \tilde{t} - \tilde{r}$

We consider diamond of size L

$$|\tilde{t} + L| + \tilde{r} \le L \iff \underbrace{-2L}_{\mathcal{H}^-} \le \tilde{u} \le \underbrace{-2\tilde{r}}_{\mathcal{H}^+}$$

Make coord change

② Set-up

The metric for Mink is

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We consider diamond of size L

$$|\tilde{t} + L| + \tilde{r} \le L \iff \underbrace{-2L \le \tilde{u}}_{\mathcal{H}^-} \underbrace{\le -2\tilde{r}}_{\mathcal{H}^+}$$

Make coord change $\tilde{u} = -2\Phi_0(u)$ $\quad \tilde{r} = \Phi(u,r)$

Require $\mathcal{H}^-$ be at $u = u_-$ & $\mathcal{H}^+$ be at $r = 0$

$\rightarrow \Phi(u) = ($ $\qquad \Phi(u,0) = \Phi_0(u)$

$ds^2 = -2\kappa_0 r\,du^2 + 2\,du\,dr +$

at $u=u_-$ & it be at

$$\Phi(u,0)=\Phi_0(u)$$

$$\boxed{ds^2=-2\kappa_0 r\,du^2+2\,du\,dr+\Phi^2 d\Omega_d^2}$$

$$re^{2\beta}du^2+2e^{2\beta}du\,dr+\Phi^2 d\Omega_d^2$$

$$=2\partial_u\Phi_0\partial_r\Phi \qquad K=\frac{\partial_u(\Phi_0-\Phi)}{r\partial_r\Phi}$$

$$\Rightarrow \boxed{\Phi=L-\frac{1}{2\kappa_0}e^{\kappa_0 u+\alpha}-re^{-\kappa_0 u-\alpha}}$$

(3) Symplectic form

The symplectic potential is

$$\Theta = \frac{1}{16\pi G_N}\int_\Sigma d\Sigma_\mu \left(g^{\nu\rho}\,\delta\Gamma^{\mu}_{\nu\rho} - g^{\mu\nu}\,\delta\Gamma^{\rho}_{\nu\rho} \right)$$

$$= \frac{\Omega_d}{8\pi G_N}\log|\partial_u\varphi|\,\delta\varphi\Big|_{\mathcal{B}} + \delta(\ldots)$$

$\varphi = \Phi^d$

...ntial is

$$\ldots g^{\nu\rho}\,\delta\Gamma^{\mu}_{\nu\rho} - g^{\mu\nu}\,\delta\Gamma^{\rho}_{\nu\rho}\Big)$$

$$\ldots\varphi|\,\delta\varphi\Big|_{\mathcal{B}} + \delta(\ldots)$$

$$\varphi = \Phi^{d}$$

Symp form

$$\Omega = \delta\Theta$$

$$= \frac{\Omega_d}{8\pi G_N}\,\delta\log|\partial_u\varphi| \wedge \delta\varphi\Big|_{\mathcal{B}}$$

$$= \frac{1}{8\pi G_N}\,\delta\mu \wedge \delta A$$

$$A = \Omega_d L^{d}$$

$$\mu = \kappa_0 u_- + \alpha$$

$\Phi(u_-, 0) = L \Rightarrow u_- = -\infty$

- If $\kappa_0 = \kappa_0(L) \Rightarrow \boxed{\Omega = -\frac{1}{8\pi G_N}\, \delta\alpha \wedge \delta A}$

Invert this to get Poisson bracket

$$\{\alpha, A\} = -8\pi G_N \Rightarrow \boxed{[\alpha, A] = -8\pi i G_N}$$

4 Ham charges

The Iyer-Wald Ham charge is obtained from Ω

$$\delta H_\xi = \Omega[\delta\phi ; \delta_\xi \phi]$$

The diffeo should preserve $\beta = 0$, $\kappa = \kappa_0$

$$\mathcal{L}_\xi g_{uu} = \mathcal{O}(r) \qquad \mathcal{L}_\xi g_{ur} = \mathcal{L}_\xi g_{rr} = 0$$

$$\xi^\mu = (f(u),$$

charge is obtained from Ω as

$[\delta\phi; \delta_\xi \phi]$

$$\xi^\mu = \left(f(u), -r\partial_u f(u), \vec{0}\right)$$

Under $x^\mu \mapsto x^\mu + \xi^\mu$

$\beta = 0, \ \kappa = \kappa_0$

$$\delta_\xi \kappa_0 = -\lambda_\kappa \qquad \delta_\xi L = -\lambda_L$$

$$\mathcal{L}_\xi g_{ur} = \mathcal{L}_\xi g_{rr} = 0$$

$$\delta_\xi \alpha = -\kappa_0 \lambda_\alpha \qquad \kappa_0 = \kappa_0(L)$$

$$\Rightarrow \lambda_\kappa = \kappa_0'(L)\lambda_L$$

$k_0 = \frac{c}{l}$

λ_α parametrizes null time shift

λ_L " change in CD size

1) $\lambda_L = 0$ charge

$$\delta H_\alpha = -\frac{1}{8\pi G_N}\delta_\xi \alpha \, \delta A$$

$$= \frac{\lambda_\alpha \Omega_d}{8\pi G_N} k_0(L)\,\delta(L^d)$$

$$= \delta\Bigg(\underbrace{\frac{\lambda_\alpha d\,\Omega_d}{8\pi G_N}\int_0^L dl'\, k_0(l')\, l'^{\,d-1}}_{H_\alpha}\Bigg) =$$

$$\boxed{\Phi = L - \frac{1}{2k_0} e^{k_0 u + \alpha} - r e^{-k_0 u - \alpha}}$$

$k_0 = \frac{c}{\ell}$

shift

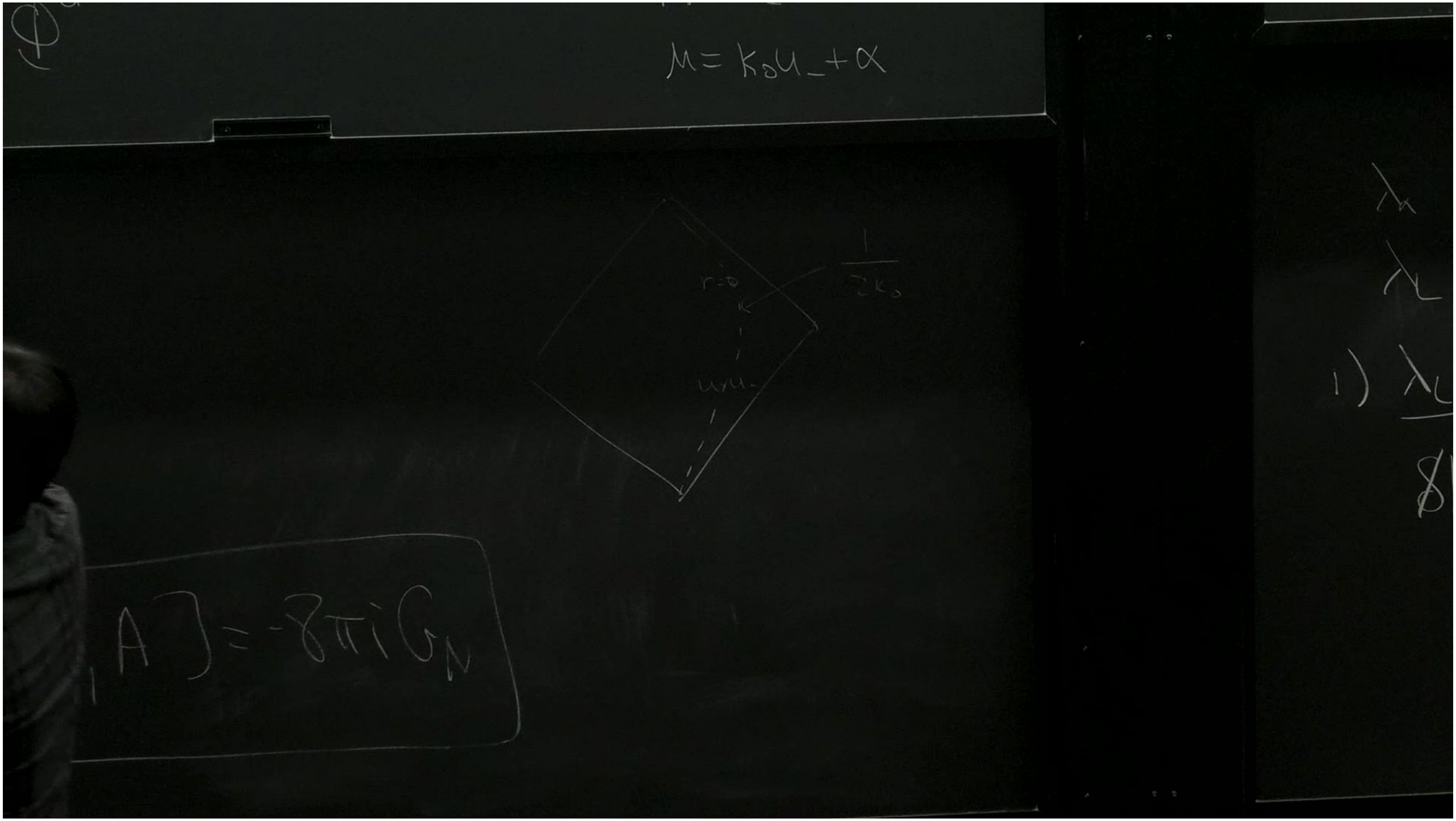
CD size

$$\delta\left(\underbrace{\frac{\lambda_\alpha d\,\Omega_d}{8\pi G_N}\int_0^L dL'\, k_0(L')\, L'^{\,d-1}}_{H_\alpha}\right) = \delta\left(\frac{\lambda_\alpha d}{d-1}\,\frac{k_0(L)}{2\pi}\,\frac{A}{4G_N}\right)$$

H_α generates time translation in u

$$\boxed{= L - \frac{1}{2k_0}e^{k_0 u+\alpha} - r e^{-k_0 u-\alpha}}$$

$\Rightarrow$ $\boxed{H_\alpha \text{ is the boost Ham}}$



A] = -8πiG_N

2) $\lambda_\alpha = 0$ charge

$$\delta H_L = \frac{1}{8\pi G_N}\,\delta\alpha\,\delta_g A$$

$$= -\frac{d\,\lambda_L\,\Omega_d}{8\pi G_N}\,L^{d-1}\,\delta\alpha$$

Consider

$$\lambda_L = \frac{G_N}{L^{d-1}} \Longrightarrow \boxed{H_L = -\frac{d\,\Omega_d}{8\pi}\,\alpha}$$

H_L

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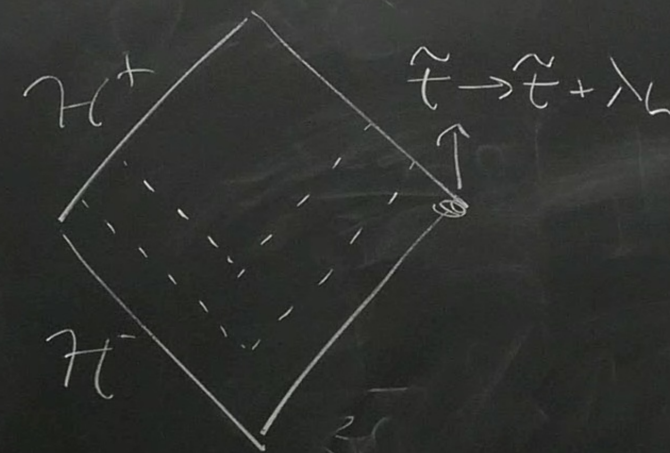
$$\lambda_L = \frac{G_N}{L^{d-1}} \Rightarrow \boxed{H_L = -\frac{d\,\Omega_d}{8\pi}\,\alpha}$$

H_L shif

creates

H_L shifts size of CD and creates nested diamonds



$$-\frac{d\Omega_d}{8\pi}\alpha$$

H_L shifts global Mink $\hat{t}$

At B, $\hat{t} \mapsto \hat{t} + \lambda_L$

Both H_L and H_α correspond to time translations

H_L is not present in standard BMS analysis, if we

shifting L by constant doesn't do anything

2) $\lambda_\alpha = 0$ charge

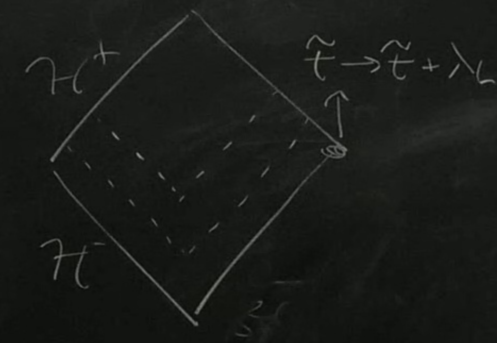
$$\delta H_L = \frac{1}{8\pi G_N} \delta\alpha\, \delta_g A$$

$$= -\frac{d\,\lambda_L\,\Omega_d}{8\pi G_N} L^{d-1}\,\delta\alpha$$

Consider $\boxed{\lambda_L = \frac{G_N}{L^{d-1}}} \Longrightarrow \boxed{H_L = -\frac{d\,\Omega_d}{8\pi}\alpha}$

H_L shifts size of CD a...

creates nested diamon...



H_L is not present in standard BMS analysis. If we take L → ...

shifting L by constant doesn't do anything