Title: Lecture - Beautiful Papers

Speakers: Pedro Vieira

Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

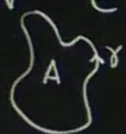
Subject: Other

Date: October 28, 2024 - 9:15 AM

URL: https://pirsa.org/24100070

1) Review   Field Path Integrals $\longleftrightarrow$ Integrals $\underbrace{\text{over loops}}_{\text{WILSON}}$

2) $W \sim e^{-\# A}$ $\leftarrow$ confinement



3) Lattice (where) happens.

Important but we will not discuss much: Continuum Limit.

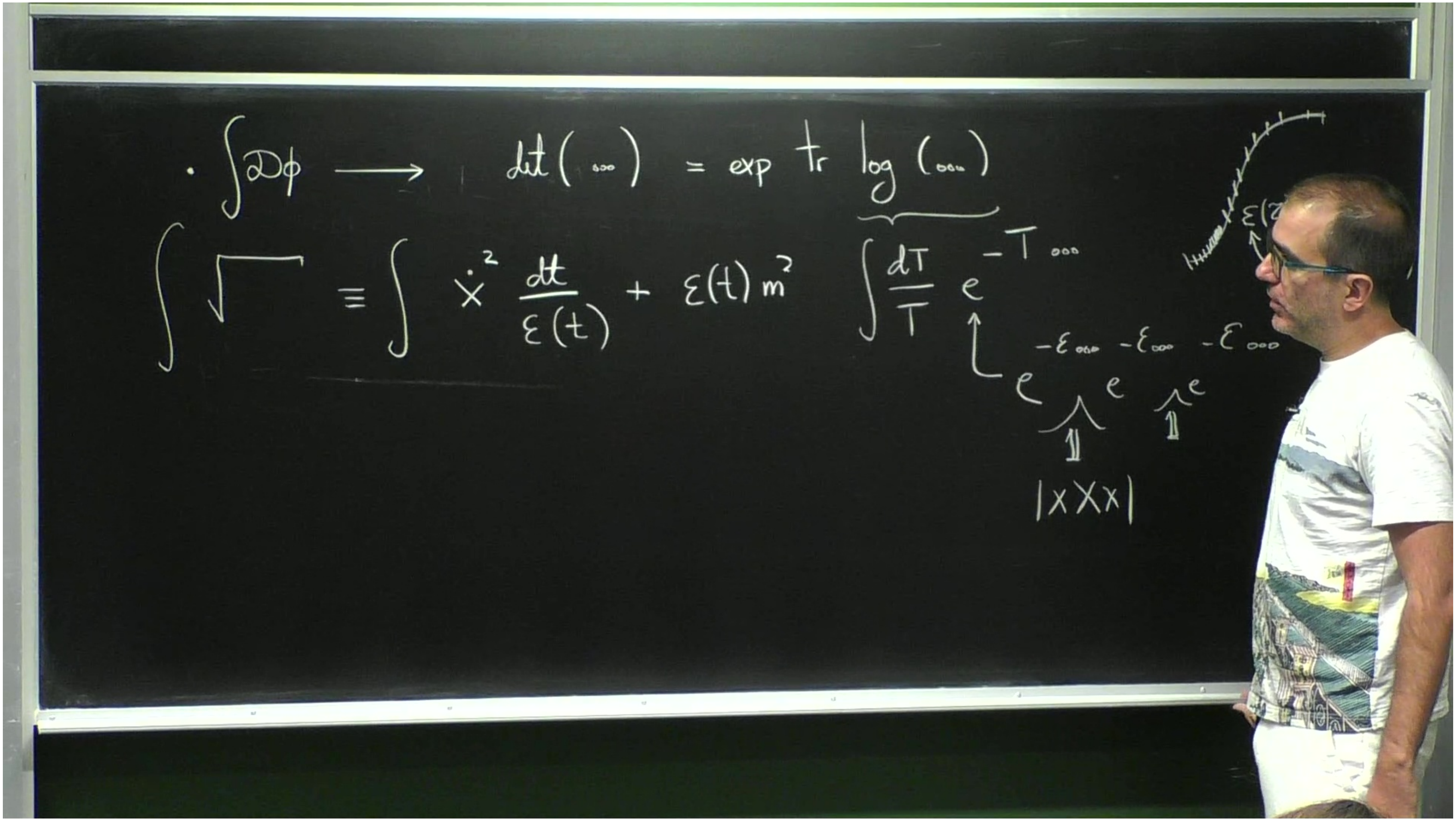
$$\int \mathcal{D}\phi \int \mathcal{D}A_\mu \; e^{-\int \phi\left((\partial + iA)^2 + m^2\right)\phi \; + \; F^2}$$

$$\shortparallel$$

$$\int_{\text{closed Paths } \gamma} \mathcal{D}x(t) \; e^{-m L(\gamma)} \; \underbrace{\int \mathcal{D}A_\mu \, e^{\int_\gamma A_\mu(x(t)) \, \dot{x}^\mu(t) \, dt}}_{\left\langle e^{\int_\gamma A} \right\rangle_A \equiv W_\gamma}$$

$L(\gamma)$: $\int \sqrt{\dot{x}^2}\, dt$, Perimeter

closed Paths γ: vacuum bubbles



$\cdot \int \mathcal{D}\phi \longrightarrow \det(\ldots) = \exp \operatorname{tr} \underbrace{\log(\ldots)}$
$\int \sqrt{} \equiv \int \dot{x}^2 \frac{dt}{\varepsilon(t)} + \varepsilon(t) m^2$
$\int \frac{dT}{T} e^{-T\ldots}$
$e^{-\varepsilon\ldots} e^{-\varepsilon\ldots} e^{-\varepsilon\ldots}$
$\mathbb{1}$ $\mathbb{1}$
$|xXx|$

$$\int \sqrt{} \equiv \int \dot{x}^2 \frac{dt}{\varepsilon(t)} + \varepsilon(t)\, m^2 \qquad \int \frac{dT}{T}\, e^{-T\,\cdots}$$

$\int$ out $\varepsilon(\tau)$

$$e^{-\varepsilon\,\cdots}\ \mathbb{1}\ e^{-\varepsilon\,\cdots}\ \mathbb{1}\ e^{-\varepsilon\,\cdots}$$

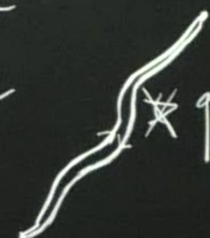
$$|xXx|$$

$$\left\langle \bar{\phi}\phi(0)\ \bar{\phi}\phi(x) \right\rangle = \int \mathcal{D}x(t)\ W\ e^{-mL(\gamma)}$$

if $W \sim e^{-\#\,\text{Area}}$

$\Longleftrightarrow$ quark Confinement.

see a q? vs $\ll$



GOAL

$$W \sim e^{-\text{div}\, L - \mu A}$$

$m^{\text{phys}} = m + \text{div}$ ✓

if $\mu > 0$, $\Rightarrow$ CONF!

NON-ABELIAN

$$\int \mathcal{D}\phi \int \mathcal{D}A_\mu \; e^{-\int \phi\left((\partial+iA)^2+m^2\right)\phi \;+\; \mathrm{tr}\,F^2}$$

$$\equiv \lim_{\Delta x \to 0} \prod \left(\mathbb{1} + A_\mu \, \Delta x^\mu\right)$$

$\int \sqrt{\dot{x}^2}\, dt$, Perimeter

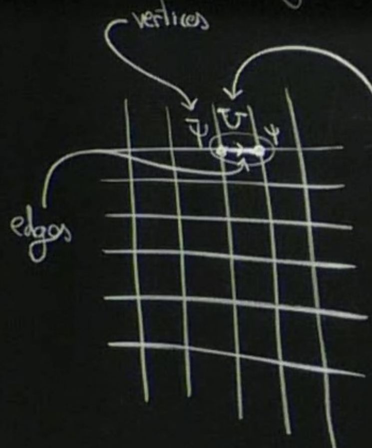
$$=$$

$$\int \mathcal{D}x(t) \; e^{-m L(\gamma)} \; \mathrm{tr}\left[\underbrace{\int \mathcal{D}A_\mu \, P e^{\int_\gamma A_\mu(x(t))\, \dot{x}^\mu(t)\, dt}}_{\left\langle \mathrm{tr}\, P e^{\int_\gamma A} \right\rangle_A \;\equiv\; W_\gamma}\right]$$

closed Paths γ

vacuum bubbles

$$\omega(x,y) \equiv P\exp\int_{\{\gamma\}}^{x} A \;;\quad \omega(x,y) \xrightarrow{GT} \Lambda(x)\,\omega(x,y)\,\Lambda^{-1}(y)$$



$\vec{n}'$ $\vec{n}$

$$\bar\psi_{\vec{x}}\, U_{\vec{x},\hat\mu}\, \psi_{\vec{x}+\hat\mu} \leftarrow GI$$

"e^{A}"

- $\mathrm{tr}\,\omega(x,x) \equiv W$
- $\bar\psi(x)\,\omega(x,y)\,\psi(y)$

$\bar\psi$ ψ

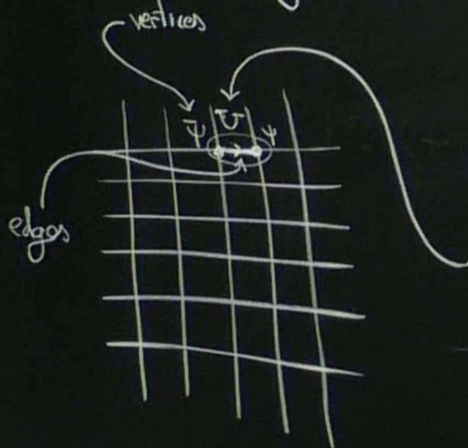
$$U_{x,\mu} = U^{-1}_{x+\mu,-\mu} \neq U_{x+\mu,-\mu} = U^{-1}_{x,\mu}$$

x $x+\mu$

$$W \equiv \mathrm{tr}\, P\exp \oint A$$

$$w(x,y) \equiv P\exp\int_x^y A \; ; \; w(x,y) \xrightarrow{GT} \Lambda(x)\, w(x,y)\, \Lambda^{-1}(y)$$

$\{\gamma\}$



$$\bar\psi_{\vec x}\, U_{\vec x,\hat\mu}\, \psi_{\vec x+\hat\mu} \leftarrow GI$$

e^{A}

- $\mathrm{tr}\, w(x,x) \equiv W$
- $\bar\psi(x)\, w(x,y)\, \psi(y)$

$$U_{x,\mu} = U^{-1}_{x+\mu,-\mu} \neq U_{x+\mu,-\mu} = U^{-1}_{x,\mu}$$

x $\quad$ $x+\mu$

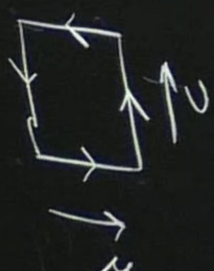
vertices

edges

$$S = \sum_{x} c\,\bar{\psi}_x \psi_x + \sum_{x} \sum_{\mu} \kappa \left(\bar{\psi}_x U_{x,\mu} \psi_{x+\mu} + \bar{\psi}_{x+\mu} U^{-1}_{x,\mu} \psi_x \right)$$

$$+ \frac{1}{2g^2} \sum_{x} \mathrm{tr} \left[U_{x,\mu} U_{x+\mu,\nu} U_{x+\mu+\nu,-\mu} U_{x+\nu,-\nu} \right] \quad (3.29)$$

Plaquettes ≡ faces



vertices

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$$S \equiv \sum_x c\,\bar{\psi}_x \psi_x + \sum_x \sum_\mu \kappa \left(\bar{\psi}_x U_{x,\mu} \psi_{x+\mu} + \bar{\psi}_{x+\mu} U^{-1}_{x,\mu} \psi_x \right)$$

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Lattice QCD

Plaquettes

≡ faces

$$\int \mathcal{D}A \int \mathcal{D}\psi \rightarrow \int \prod_x dU \prod_x d\psi \qquad (3.29)$$

$$\underbrace{\mathrm{tr}\left(U_E^{-1}\, dU_E\right)}_{U_E = e^{i\phi_E} \text{ if Abelian}} = ds_E^2$$

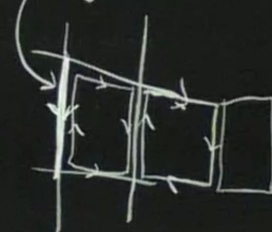
$$dU \to d\phi_E \qquad \int U = 0$$

$$\langle e^{i\phi} \rangle = 0$$

$\langle W \rangle$

$$= \mathrm{tr}\left(U\, U \cdots\cdots U\right)$$

$$\int dU_E\, U_E = 0 \,, \qquad \int dU_E\, U_E \bar{U}_E = 1$$

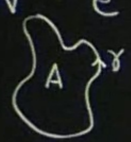


$$\left(\frac{1}{2g^2}\right)^A \times 1 \leftarrow g \text{ large}$$

Area of a surface ending on γ

1) Review Field Path Integrals $\longleftrightarrow$ Integrals $\underbrace{\text{over loops}}_{\text{WILSON}}$

2) $W \sim e^{-\# A} \leftarrow$ confinement



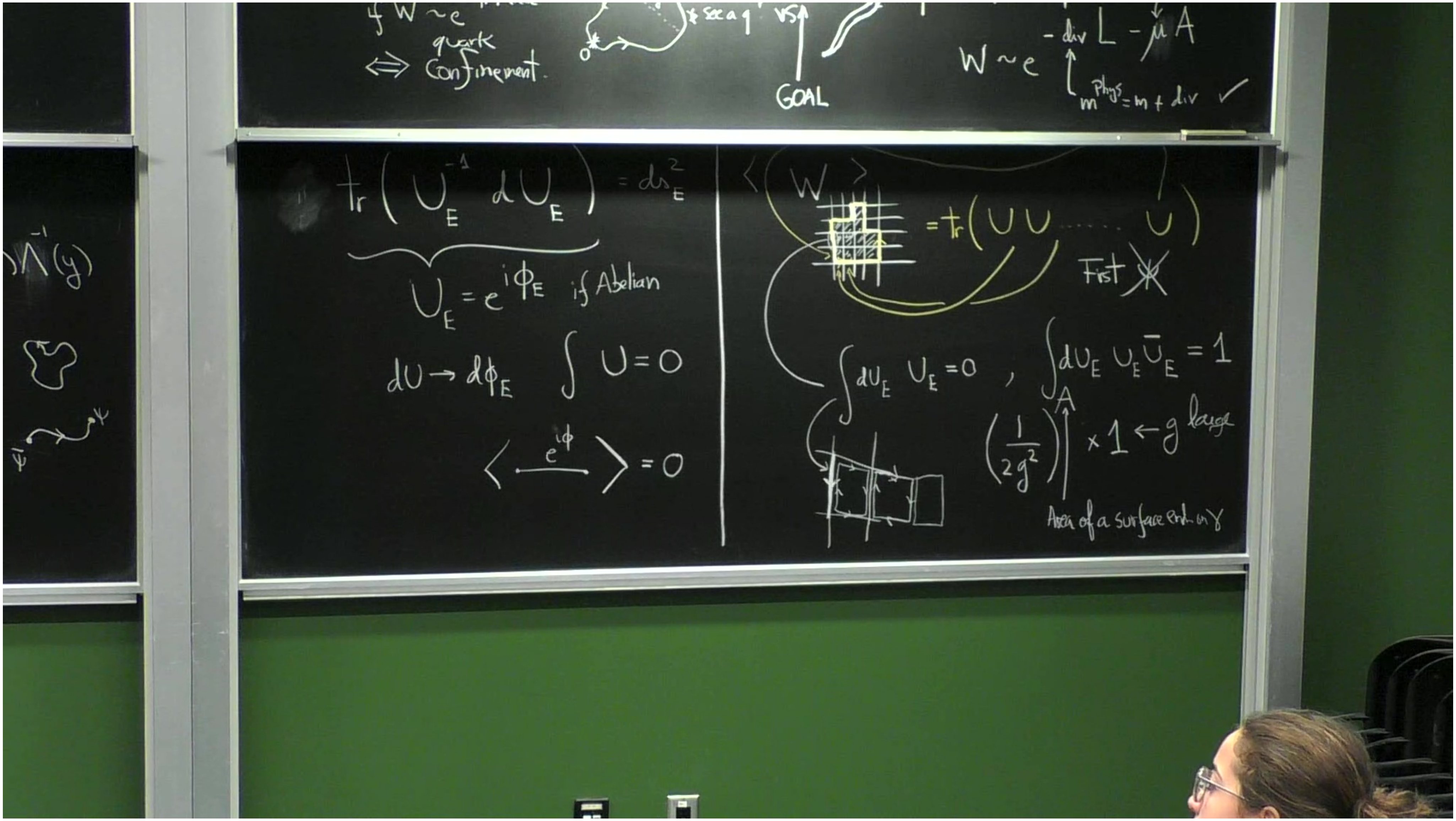
@ $g \to \infty$ (strong coupling)

$$W_\gamma = \sum_{\substack{\text{surfaces that}\\ \text{end in } \gamma}} e^{-\text{Area}\left(\log(2g^2)\right)} \simeq e^{-A^{\text{min surface}}[\gamma]\log 2g^2}$$

Confinement !

3) Lattice (where) happens.

Important but we will not discuss much: Continuum Limit.



quark
⇔ Confinement.
GOAL
W ~ e
m^phys = m + div
tr(U_E^-1 dU_E) = ds_E^2
U_E = e^iφ_E if Abelian
dU → dφ_E
∫ U = 0
⟨e^iφ⟩ = 0
⟨W⟩
= tr(U U ...... U)
∫ dU_E U_E = 0 , ∫ dU_E U_E Ū_E = 1
(1/2g^2)^A × 1 ← g large
Area of a surface ending on γ

1) Review   Field Path Integrals $\longleftrightarrow$ Integrals $\underbrace{\text{over loops}}_{\text{WILSON}}$

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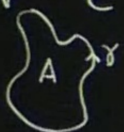

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Confinement ! :-)

even for QED ! :-(

3) Lattice (where) happens.

Important but we will not discuss much: Continuum Limit.

vertices $\boxed{\int \bar{\psi} \not{D} \psi}$ edges $\qquad U_\mu = 1 + \varepsilon a A_\mu$

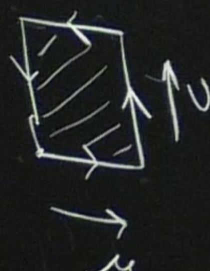
$\psi_x + \varepsilon a \partial_\mu \psi$

$$S \equiv \sum_x c\,\bar{\psi}_x \psi_x + \sum_x \sum_\mu \kappa \left(\bar{\psi}_x U_{x,\mu} \psi_{x+\mu} + \bar{\psi}_{x+\mu} U^{-1}_{x,\mu} \psi_x \right)$$

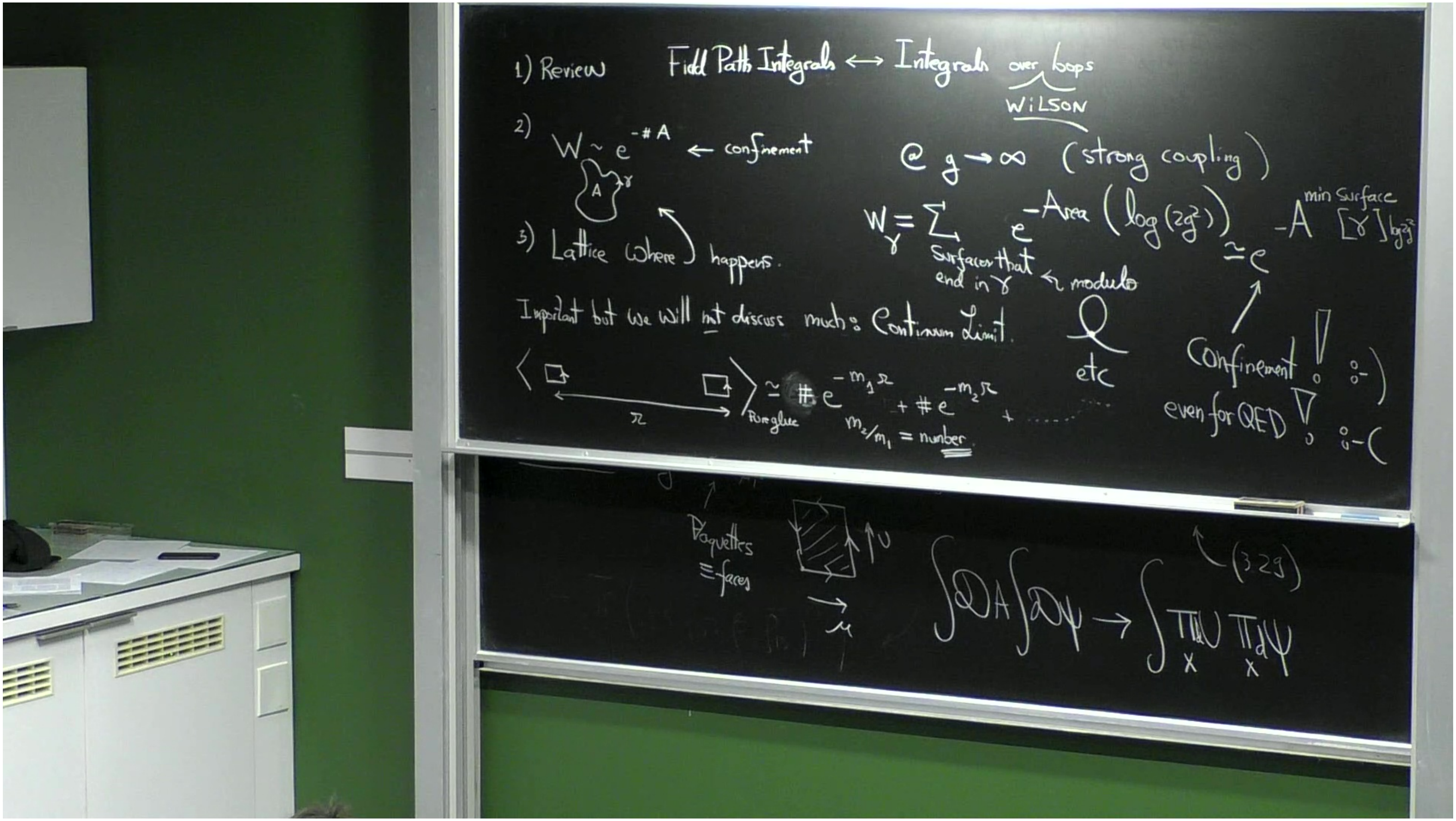
$$+ \frac{1}{2g^2} \sum_x \sum_{\mu,\nu} \mathrm{tr} \left[\underbrace{U_{x,\mu} U_{x+\mu,\nu} U_{x+\mu+\nu,-\mu} U_{x+\nu,-\nu}}_{\boxed{-\frac{1}{4g^2}\int \mathrm{tr} F^2}} \right]$$

Lattice QCD

Plaquettes $\equiv$ faces



$$\int \mathcal{D}A \int \mathcal{D}\psi \to \int \prod_x dU \prod_x d\psi \qquad (3.29)$$


1) Review
Field Path Integrals ⟷ Integrals over loops
WILSON
2) W ~ e^{-# A} ← confinement
@ g → ∞ (strong coupling)
3) Lattice where happens.
W_γ = Σ e^{-Area (log(2g²))} ≃ e^{-A^{min Surface}[γ] log g²}
Surfaces that end in γ
← modulo
etc
Important but we will not discuss much: Continuum Limit.
Confinement !
even for QED
r
Pure glue
m_2/m_1 = number
Plaquettes ≡ faces
(3.29)
∫DA ∫Dψ → ∫ Π_x dU Π_x dψ