Title: Lecture - Beautiful Papers

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Subject: Other

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lattice

$$Z = D \sum_{\substack{\text{loops in the} \\ \underline{\text{original}} \text{ lattice}}} \left(\tanh \frac{J}{T} \right)^{\text{Length Paths}}$$

$$\boxed{\begin{array}{ll} 0 & @\, T \to \infty \\ 1 & @\, T = 0 \end{array}}$$

$$\prod_{i,j} e^{\frac{J}{T} \sigma_i \sigma_j} = \underbrace{\prod_{i,j} \left(\cosh \frac{J}{T} + \sigma_i \sigma_j \sinh \frac{J}{T} \right)}_{\text{many terms } \text{"}1\text{"} + \sigma_1 \sigma_2 + \cdots}$$

$$\sum \sigma_i^{\text{even}} = 2$$

$$\sum \sigma_i^{\text{odd}} = 0$$

$$|\vec{x}-\vec{x}'|^2$$ ← disorder @ high T

$$\exp\left(2\frac{J}{T}\right) = \text{th}\left(J^*/T\right)$$

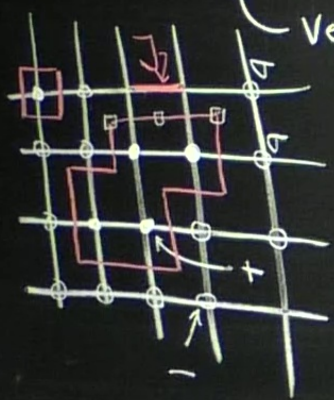
"spins $\overset{+}{\vee}$ everywhere"

J/T

OR C.P.

Single PT $\Rightarrow$ $J^* = J$ @ $T = T_c = 2J/\log(1+\sqrt{2})$

vertex



KW duality — high T / low T duality

$$\boxed{\begin{array}{ll} 1 & @\ T=\infty \\ 0 & @\ T=0 \end{array}}$$

$$Z = c \sum_{\text{loops}} \left(e^{-\frac{2J}{T}}\right)^{\text{Length Path}}$$

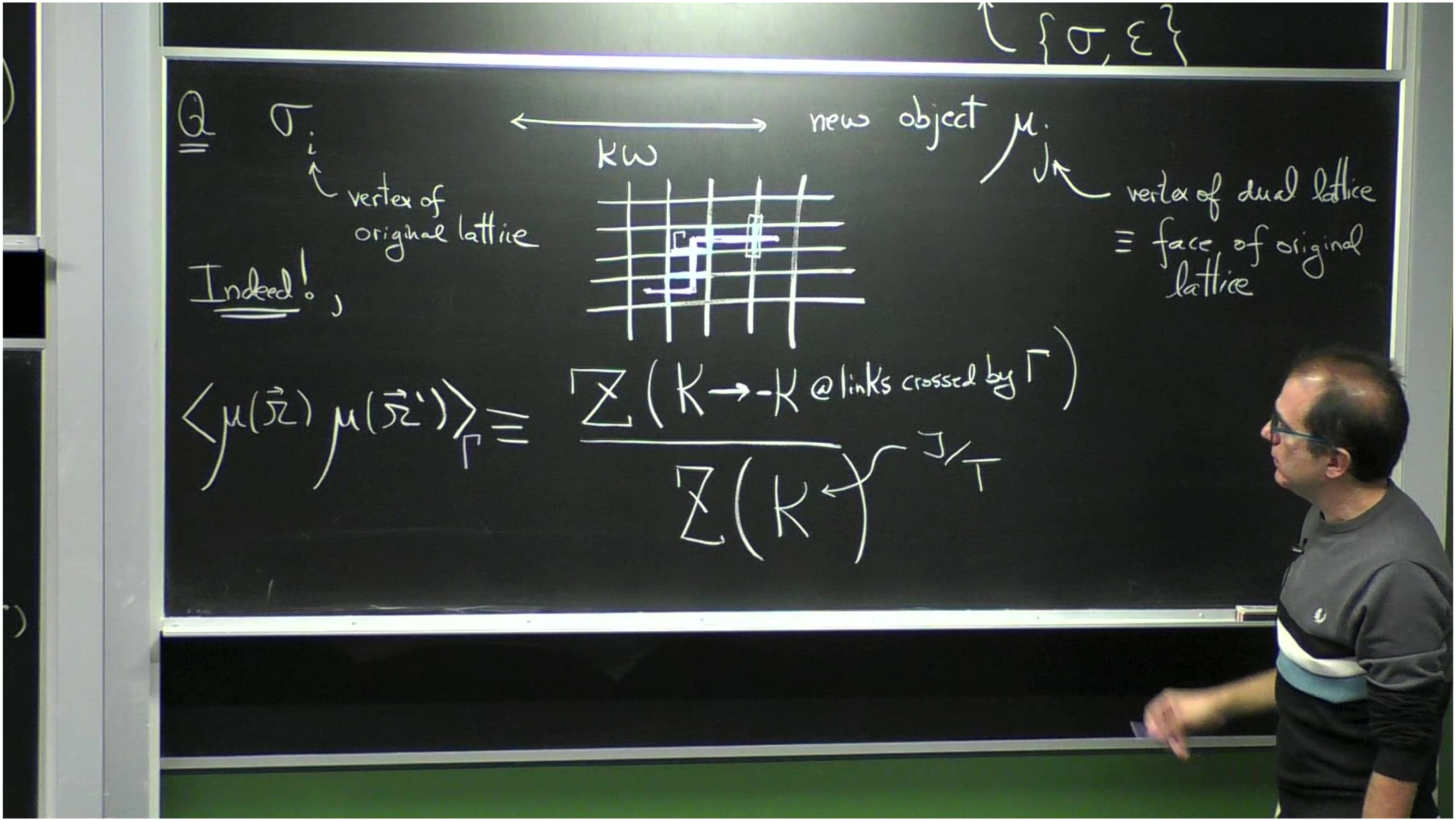
loops ← closed paths in the dual lattice

faces ⟷ vertices

$$Z = D \sum_{\substack{\text{loops in the} \\ \text{original lattice}}} \left(\tanh \frac{J}{T}\right)^{\text{Length Paths}}$$

$$\boxed{\begin{array}{ll} 0 & @\ T\to\infty \\ 1 & @\ T=0 \end{array}}$$

$$\prod e^{\frac{J}{T}\sigma_i\sigma_j} = \prod\left(\cosh \frac{J}{T} \ldots\right)$$


{σ, ε}
Q σ_i ⟷ KW new object μ_j
vertex of original lattice
vertex of dual lattice ≡ face of original lattice
Indeed!,
⟨μ(x⃗) μ(x⃗')⟩_Γ ≡ Z(K → −K @ links crossed by Γ) / Z(K)
J/T

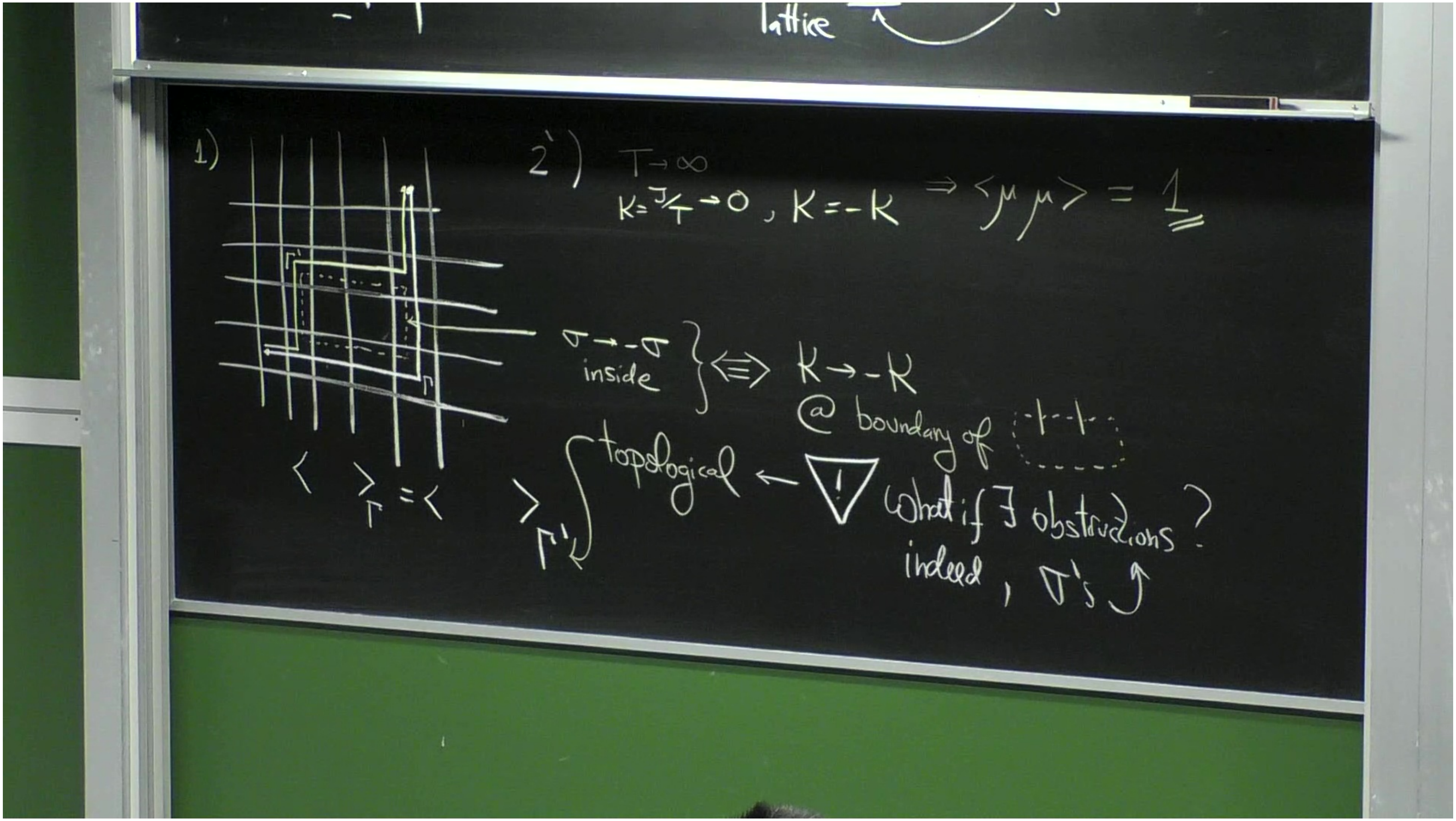
CLAIMS:

1) Γ independent!

2) $\langle \mu(\vec{r})\,\mu(\vec{r}\,')\rangle$ = Same as $\langle \sigma\sigma\rangle$ with $T \gtrless T_c$ swapped with $T \lessgtr T_c$

$\langle \mu \rangle \neq 0$ @ $T > T_c$

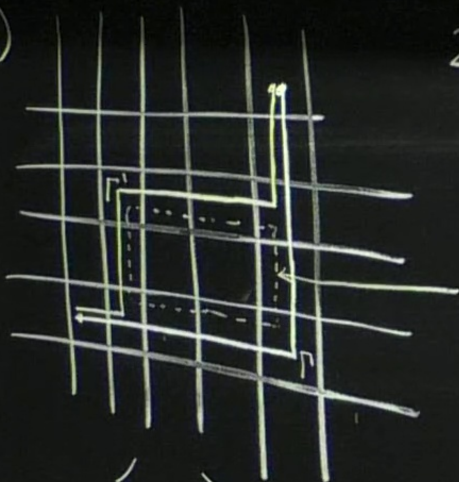
3) $\langle \sigma\sigma\rangle_\Gamma = \dfrac{Z(K \to K^*)}{Z(K)}$ (also, Γ independent)


lattice
1)
Γ'
Γ
2) T→∞
K=J/T→0 , K=-K ⇒ ⟨μ μ⟩ = 1
σ→-σ inside ⟺ K→-K
@ boundary of
topological
What if ∃ obstructions ?
indeed , σ's

in the dual lattice

faces $\longleftrightarrow$ vertices

1)



$\langle\ \ \rangle_{\Gamma} = \langle\ \ \rangle_{\Gamma'}$ topological

2) $T \to \infty$

$K = J/T \to 0$, $K = -K$ $\Rightarrow$ $\langle \mu \mu \rangle = \boxed{1}$

$\underline{T < T_c}$

"+" Γ $2J/T$ cost "+"

$\langle \mu \mu \rangle \sim e^{-|\vec{x} - \vec{x}'| t}$

$\sigma \to -\sigma$ inside $\Big\} \Longleftrightarrow K \to -K$ @ boundary of

topological $\leftarrow$ ⚠ What if $\exists$ obstructions?

indeed, σ's ↺

disorder operator

$$Z(K) \leftarrow J/T$$

CLAIMS:

1) Γ independent!

2) $\langle \mu(\vec{r})\mu(\vec{r}')\rangle$ = Same as $\langle \sigma\sigma\rangle$ with $T \gtrless T_c$ swapped with $T \lessgtr T_c$

$$= \frac{Z(K \to K')}{Z(K)}$$

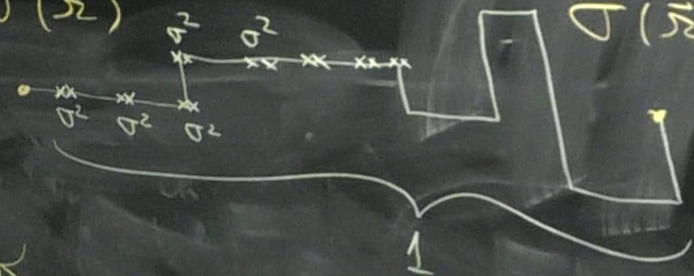
$\langle \mu \rangle \neq 0$ @ $T > T_c$

3) $\langle \sigma\sigma\rangle_\Gamma = $ also, Γ independent

$$i\,\sigma_i\sigma_j\, e^{K\sigma_i\sigma_j} = e^{K'\sigma_i\sigma_j}$$

$$K' = K + i\,\pi/2$$

$$\langle \sigma(\vec{r}) \quad \sigma(\vec{r}\,')$$



$$\langle \sigma\sigma \rangle = \frac{Z(K\to K' \text{ in } \Gamma)}{Z(K)}$$

$$e^{2K} = \operatorname{th} K^*$$

$$K \to -K \iff K^* \to K^* + i\pi/2$$

$$K \to K + i\pi/2 \qquad K^* \to -K^*$$

$$e^{2K} = \tanh K^*$$

$$K \to -K \quad K \to K + i\pi/2$$

$$\Longleftrightarrow$$

$$K \to K + i\pi/2 \qquad K^* \to -K^*$$

Review

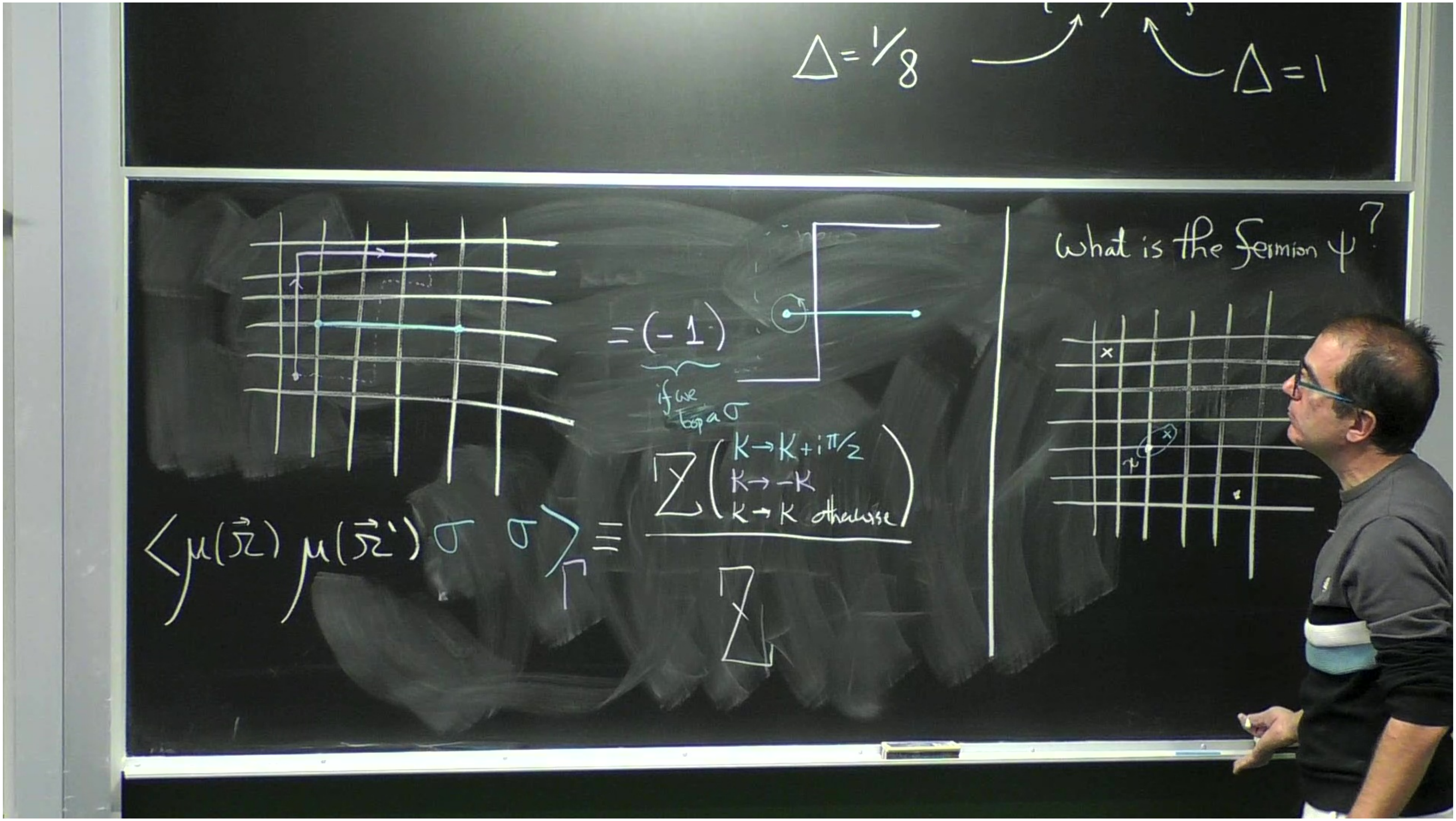
$$\langle \sigma^\mu(\vec{r})\, \sigma^\mu(\vec{r}\,') \rangle \simeq \begin{cases} \langle \sigma^\mu \rangle^2 + O(\exp(-t|\vec{r}-\vec{r}\,'|)) & T < T_c \\ \dfrac{C}{|\vec{r}-\vec{r}\,'|^{1/4}} & T = T_c \\ C\, \dfrac{e^{-t|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|^{1/2}} & T > T_c \end{cases}$$

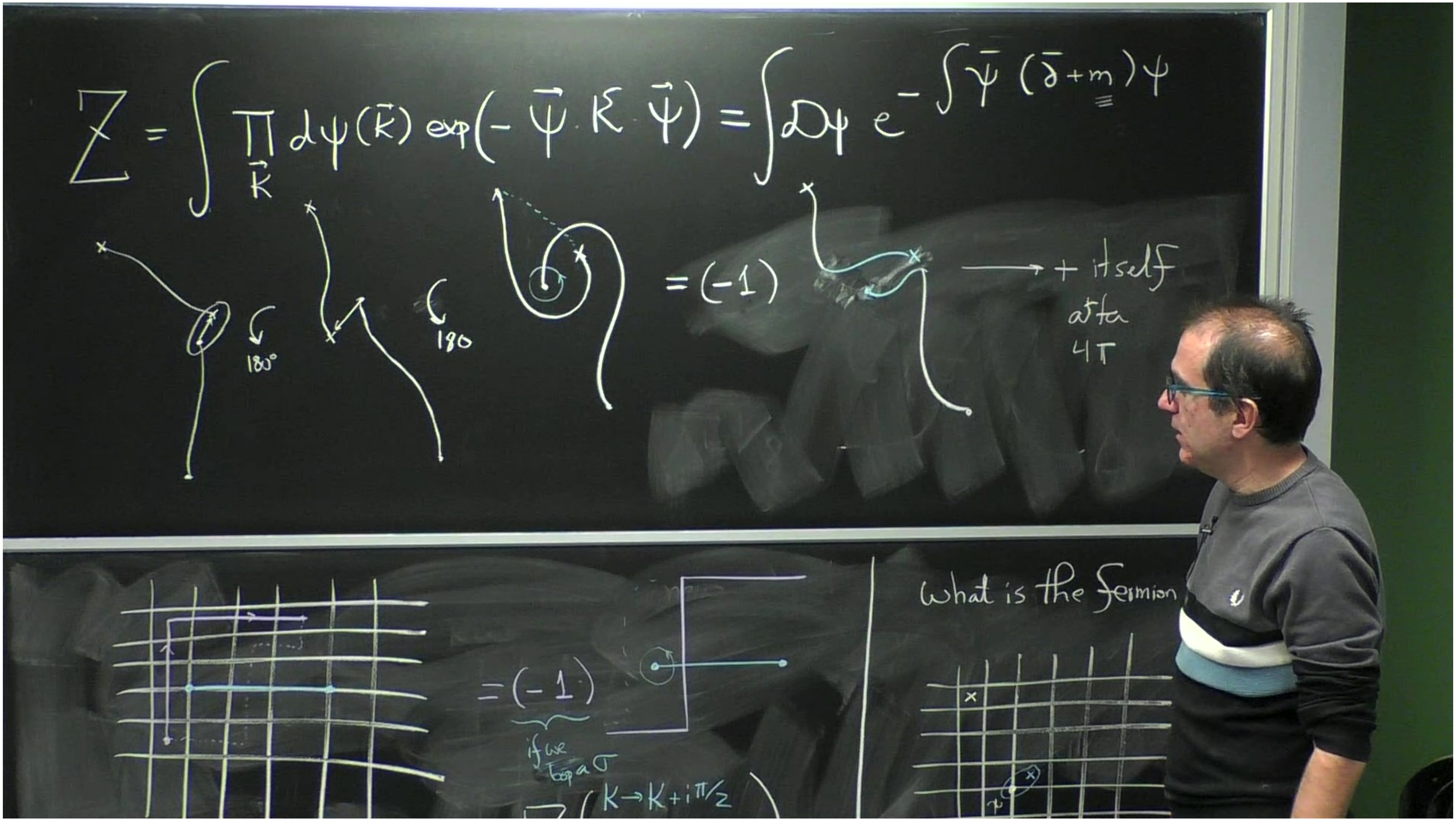
$|\vec{r}-\vec{r}\,'| \gg$ lattice spacing

$\langle \sigma^\mu \rangle$: $\alpha(T_c - T)^{1/8}$; order @ low T; $\boxed{1}$ @ $T=0$

$T_c = 2J/\log(1+\sqrt{2})$

disorder @ high T


Δ=1/8
Δ=1
=(-1)
if we loop a σ
Z(K→K+iπ/2, K→-K, K→K otherwise) / Z
⟨μ(x⃗) μ(x⃗') σ σ⟩_Γ ≡
What is the Fermion ψ?
ψ



$Z = \int \prod_{\vec{K}} d\psi(\vec{K}) \exp(-\vec{\bar{\psi}} \cdot K \cdot \vec{\psi}) = \int \mathcal{D}\psi \, e^{-\int \bar{\psi}(\bar{\partial}+m)\psi}$
180°
180
= (−1)
+ itself
after
4π
= (−1)
if we
loop a σ
K → K + iπ/2
What is the Fermion