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Soft Factorization in QED from 2D Kac-Moody Symmetry

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Some Celestial Definitions

Large gauge transformations constitute physical symmetries. In the case of QED with some charged matter

$$\delta_\varepsilon A_z^{(0)} = \partial_z \varepsilon(z,\bar{z})$$
$$\delta_\varepsilon \Psi = i\varepsilon\,\Psi$$

These symmetries are generated by a current supported in the celestial sphere $\mathcal{C}S^2$

$$J_z = \frac{4\pi}{e^2}\left(A_z^{(0)}\Big|_{\mathcal{I}_-^+} + A_z^{(0)}\Big|_{\mathcal{I}_-^-} + A_z^{(0)}\Big|_{\mathcal{I}_+^-} + A_z^{(0)}\Big|_{\mathcal{I}_+^+}\right)$$

These symmetries don't preserve the vacuum however, that is, they change the boundary condition $A_z^{(0)} = 0$. Correspondingly, there is an associated goldstone boson.

Some Celestial Definitions

They call this goldstone boson ϕ and it transforms as

$$\delta_\varepsilon \phi = \varepsilon(z, \bar{z})$$

Associated to this operator there is an operator they refer to as the goldstone current

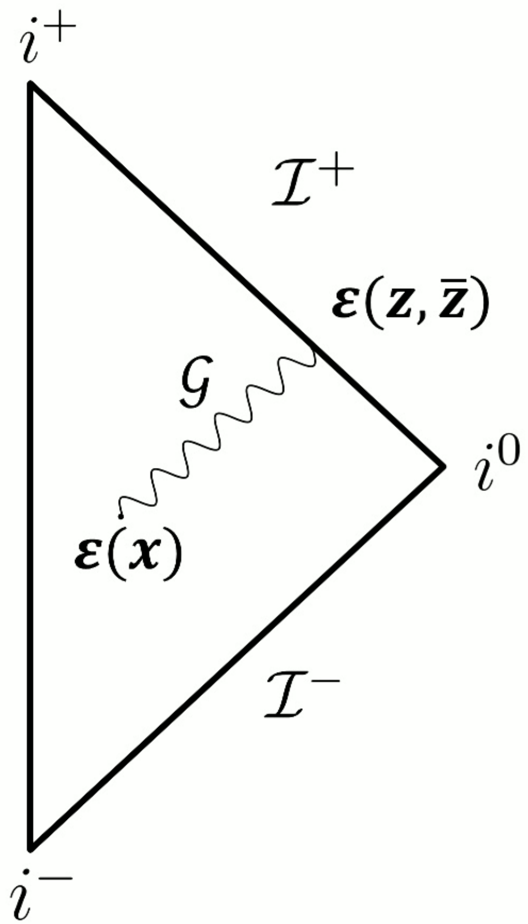
$$S_z = \frac{i}{4}\left(A_z^{(0)}\Big|_{\mathcal{I}_-^+} + A_z^{(0)}\Big|_{\mathcal{I}_-^-} + A_z^{(0)}\Big|_{\mathcal{I}_+^-} + A_z^{(0)}\Big|_{\mathcal{I}_+^+}\right) = i\partial\phi$$

All of these have OPEs

$$J_z(z)\,J_z(w) \sim 0$$

$$J_z(z)S_z(w) \sim \frac{1}{(z-w)^2}$$

$$S_z(z)S_z(w) \sim \frac{k}{(z-w)^2}$$



Large Gauge Transformations for Massive Fields

We have large gauge transformations on $\mathcal{I}^+$ paramatrized by $\varepsilon(z,\bar{z})$.

We may extend these to large gauge transformations to the bulk $\varepsilon(x)$ using a suitable propagator $\mathcal{G}(x,z,\bar{z})$

$$\varepsilon(x) = \int d^2z\, \mathcal{G}(x,z,\bar{z})\, \varepsilon(z,\bar{z})$$

$\mathcal{G}$ is chosen such that $\Box\varepsilon = 0$ to preserve Lorentz gauge in the bulk.

Wilson Lines

In the paper they seek to describe the coupling to the gauge field of timelike Wilson lines on asymptotic trajectories $x^{\mu}(s) = p^{\mu}s$ describing a massive charged particle. They argue that massive particles in momentum eigenstates do not reach $\mathcal{I}^{+}$ and cannot be associated with local operators at unique points on $\mathcal{C}S^2$. Instead $\frac{p}{m}$, which obeys $\frac{p^2}{m^2} = -1$, labels a point on the asymptotic hyperbola $\mathbb{H}^3$ describing future timelike infinity.

In the paper they argue a the Wilson line of a particle with charge Q is described by the smeared operator

$$\mathcal{W}(p) = \exp\left(\mathrm{iQ} \int d^2w\, \mathcal{G}(p; w, \bar{w})\, \phi(w, \bar{w})\right)$$

Wilson Lines

They argue the $\mathcal{W}(p)$ operators themselves do not create physical states. Rather operators creating charged particles $O(p)$, decompose into a hard piece, which is invariant under large gauge transformations, and a soft piece which transforms non trivially:

$$O(p) = \mathcal{W}(p)\tilde{O}(p)$$

Two Point Function of Wilson Lines

$$\langle \mathcal{W}_{Q_1}(p_1)\mathcal{W}_{Q_2}(p_2)\rangle = \delta_{Q_1+Q_2=0}\,\exp\left(\mathrm{kQ_1Q_2}\int d^2w_1 d^2w_2\;\mathcal{G}_1\mathcal{G}_2 \ln|w_1-w_2|^2\right)$$

Which after substituting

$$\mathcal{G}(p,w,\overline{w}) = \frac{1}{2\pi}\partial_z\partial_{\bar{z}}\log(p\cdot\hat{q}(w,\overline{w})$$

and after some manipulations becomes

$$\langle \mathcal{W}_{Q_1}(p_1)\mathcal{W}_{Q_2}(p_2)\rangle = \delta_{Q_1+Q_2=0}\,\exp(k\mathrm{Q_1Q_2}(\gamma_{12}\coth(\gamma_{12})-1))$$

With $\gamma_{12} = \frac{p_1\cdot p_2}{m_1 m_2}$. It is a standard result in QFT that the anomalous dimension of a pair of timelike Wilson lines is given by the previous expression if one identifies the level k with the cusp anomalous dimension of the Wilson line

$$k = \Gamma_{\mathrm{cusp}} = \frac{e^2}{4\pi}$$

Factorization of 4d Scattering

In QED, one introduces a somewhat arbitrary energy scale, λ, that separates hard from soft physics. QED scattering amplitudes then factorize, to leading order in λ, into a hard and a soft part:

$$\langle \dots q_m; \dots p_n | \mathcal{S} | q_1 \dots; p_1 \dots \rangle$$

$$= \underbrace{\langle \dots \hat{p}_n | \hat{p}_1 \dots \rangle}_{\text{hard}} \underbrace{\langle \dots q_m | \mathcal{W}(p_n) \dots \mathcal{W}(p_1) | q_1 \rangle}_{\text{soft}}$$

Factorization of 2d Correlation Functions

In celestial holography such scattering amplitudes are matched to correlation functions on the celestial sphere. Soft photon insertions correspond to insertions of the J_z current

$$\langle J(q_1) \dots J(q_r) O(p_1) \dots O(p_n) \rangle$$

$$= \underbrace{\left\langle \tilde{O}(p_1) \dots \tilde{O}(p_n) \right\rangle}_{\text{hard}} \underbrace{\langle J(q_1) \dots J(q_r) \mathcal{W}(p_1) \dots \mathcal{W}(p_n) \rangle}_{\text{soft}}$$

Factorization of 4d Scattering

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$A_r = 0 \qquad A_u \sim O(1/r)$

$F_{uz} \sim F_{u\bar z} \sim F_{z\bar z} \sim O(1)$

$F_{rz} \sim F_{r\bar z} \sim F_{ur} \sim O(1/r^2)$

$F_{z\bar z} = 0$ at $\partial \mathcal{I}^+$

$A_z \xrightarrow[u \to \pm\infty]{} e^2 \partial_z \phi^\pm$

$N \equiv \phi^+ - \phi^- \sim \lim_{\omega \to 0} \omega a^+(\omega \hat q)$

$\varepsilon(z, \bar z)$: CS $\to U(1) \in \mathcal{U}(1)^+$

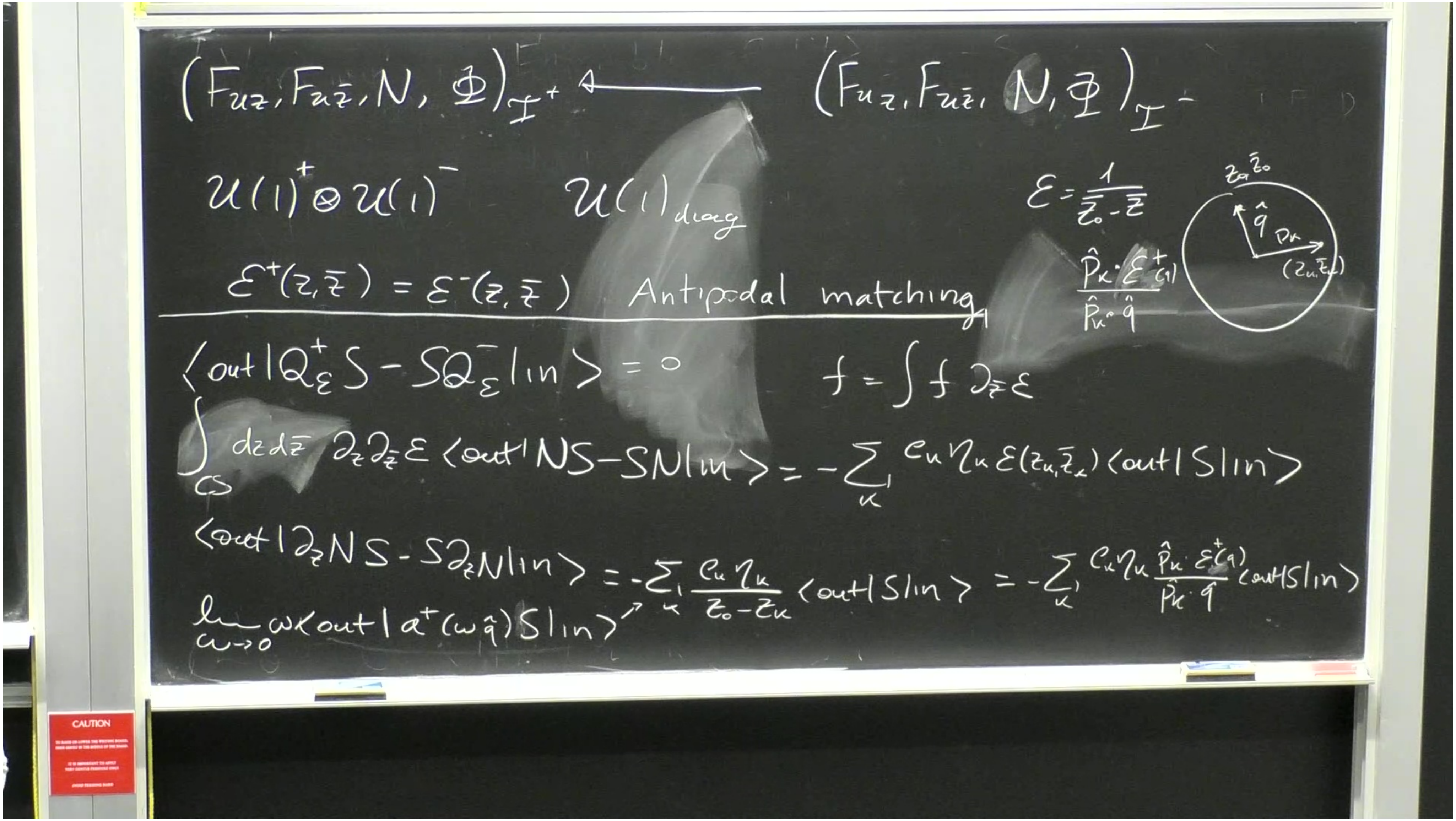
$$Q^+_\varepsilon = -\frac{1}{e^2}\int du\, d^2z \left(\partial_z \varepsilon^+ F_{u\bar z} + \partial_{\bar z} \varepsilon^+ F_{uz}\right)$$

$\downarrow$ soft

$$2\int d^2z\, \partial_z \partial_{\bar z} \varepsilon\, N$$

$$+ \int du\, d^2z\, \gamma_{z\bar z}\, \varepsilon\, j_u$$

$$\Phi \equiv e^2 \frac{\phi^+ + \phi^-}{2}$$

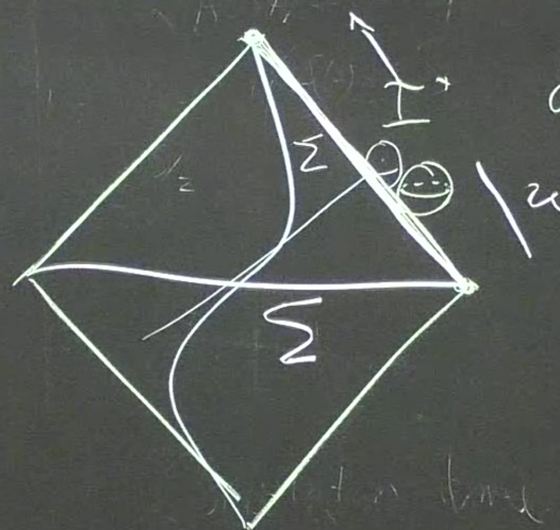

(F_{uz}, F_{uz̄}, N, Φ)_{I+} ⟵ (F_{uz}, F_{uz̄}, N, Φ)_{I−}
U(1)^+ ⊗ U(1)^−   U(1)_{diag}
ε^+(z, z̄) = ε^−(z, z̄) Antipodal matching
ε = 1/(z̄_0 − z̄)
p̂_k · ε^+(q) / p̂_k · q̂
z_0 z̄_0
q̂
p_k
(z_k, z̄_k)
⟨out| Q^+_ε S − S Q^−_ε |in⟩ = 0   f = ∫ f ∂_z̄ ε
∫_{CS} dz dz̄ ∂_z ∂_z̄ ε ⟨out| NS − SN |in⟩ = − Σ_k e_k η_k ε(z_k, z̄_k) ⟨out| S |in⟩
⟨out| ∂_z N S − S ∂_z N |in⟩ = − Σ_k e_k η_k / (z_0 − z_k) ⟨out| S |in⟩ = − Σ_k e_k η_k (p̂_k · ε^+(q)) / (p̂_k · q̂) ⟨out| S |in⟩
lim_{ω→0} ω ⟨out| a^+(ω q̂) S |in⟩
CAUTION

1407.3789

Asymptotic Symmetries

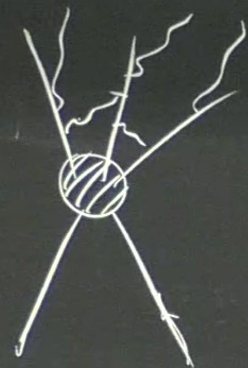
$\Updownarrow$

Soft theorem



$$ds^2 = -du^2 - 2du\,dr + r^2\gamma_{z\bar z}\,dz\,d\bar z$$

$$\delta_\varepsilon S = 0 \Rightarrow J_\varepsilon \hat{=} d\,q_\varepsilon \Rightarrow Q_\varepsilon = \int_\Sigma J_\varepsilon = \oint_{\partial\Sigma} q_\varepsilon$$



inclusive cross-sections
↓
sum over soft stuff

dress the hard pts with "clouds" of soft photons?

$$V = -e\int d^3x\ \bar{\Psi}(x)\,\not{A}(x)\,\Psi(x)$$

$$\Psi(x) = \int (dp)\,\left(b_r(p)\,u^r(p)\,e^{ipx} + d^\dagger_r(p)\,v_r(p)\,e^{-ipx} \right)$$

$$A^\mu(x) = \int (dp)\,\left(a_r(p)\,\epsilon^\mu_r(p)\,e^{ipx} + \dots \right)$$

$$\int d^3x\, (dp)(dp')(dk)\,\left(b(p)\,b(p')\,a(k)\ \cdots\right)\, e^{-i(p+p'+k)\cdot x + i(p_0+p_0'+k)t}$$

$$= \int (dp)(dk)\, e^{i\left(\sqrt{p^2+m^2}+\sqrt{(p-k)^2+m^2}+|k|\right)t}\,\left(\ \cdots\ \right)$$

$$+ \int (dp)(dk)\,\left(b\, b^{+}\, a(k)\right)\, e^{i\left(\sqrt{p^2+m^2}-\sqrt{(p-k)^2+m^2}+|k|\right)t}$$

$$V^{as}(t) = -\int (dk)(dp)\, p^{\mu}\, \rho(p)\left(a_{\mu}(k)\, e^{i\frac{p\cdot k}{p_0}t} + a_{\mu}^{+}(k)\, e^{-i\frac{p\cdot k}{p_0}t}\right)$$

$$\rho(p) = \sum_{r} d_r^{+}(p)\, d_r(p) - b_r^{+}(p)\, b_r(p)$$

$$Q(t_1,t_2) = [V^{as}(t_1), V^{as}(t_2)]$$

$$[Q(t_1,t_2), V^{as}(t_3)] = 0$$

$$U(t) = \boxed{e^{R(t)}}\, e^{i\Phi(t)}$$

$$R(t) = -i\int^t V^{as}(x)\,dx \sim e^{(\alpha a - \alpha^* a^+)}$$

$$\Phi(t) = -\frac{1}{2}\int^t dx \int^x dy\; Q(x,y)$$

$$|Fk\rangle = e^{R(t)} e^{i\Phi(t)} |Fock\rangle$$

$$H = H_0 + V$$

$$= (H_0 + V^{as}) + (V - V^{as})$$

$$|\text{Fock}\rangle \longrightarrow U^{-1}(t)\,|FK\rangle$$

$$U(t) = T\exp\left(-i\int^{t} V^{as}(x)\,dx\right)$$

$$Q^{+}_{\varepsilon}\;|FK\rangle = (\quad)\,|FK\rangle$$

$$\{Q^{+}_{\varepsilon}, S\}$$