Title: Lecture - Beautiful Papers

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Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

Subject: Other

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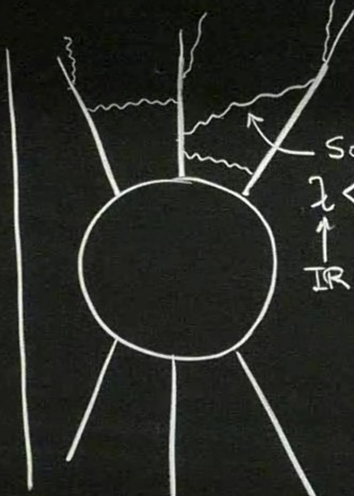
URL: https://pirsa.org/24100066

$$\sum_{\text{soft}}$$



$$\Big|\;\cdots\;\Big|^2 = \underbrace{\underbrace{\left(\frac{\lambda}{\Lambda}\right)^{A}}_{0} \underbrace{\left(\frac{E}{\lambda}\right)^{A}}_{\infty}}_{\text{div go away}} b(E)\,\Gamma_{\text{hard}}$$

in pert th

A is infinitesimal

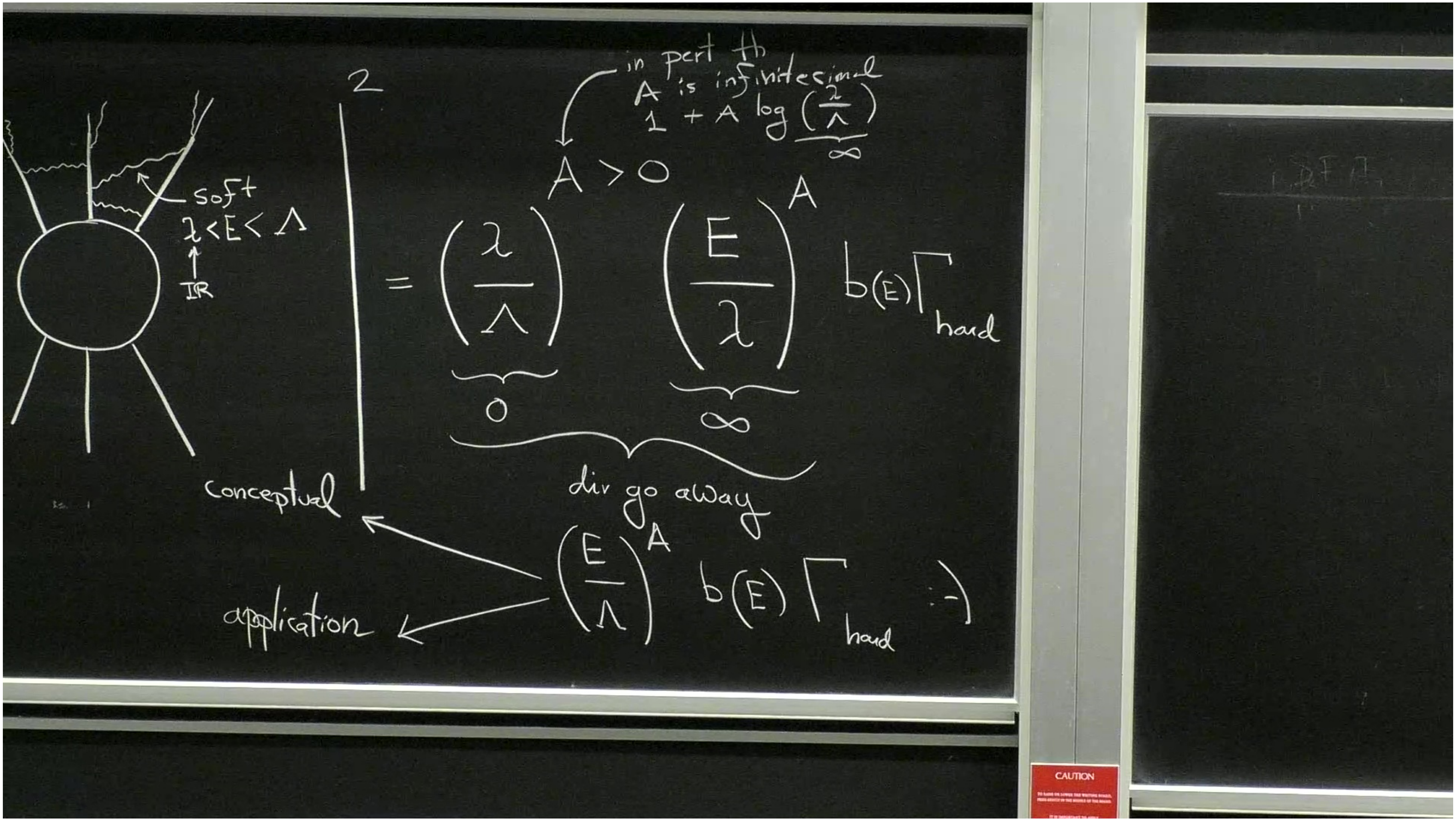
$$1 + A \log \underbrace{\left(\frac{\lambda}{\Lambda}\right)}_{\infty}$$

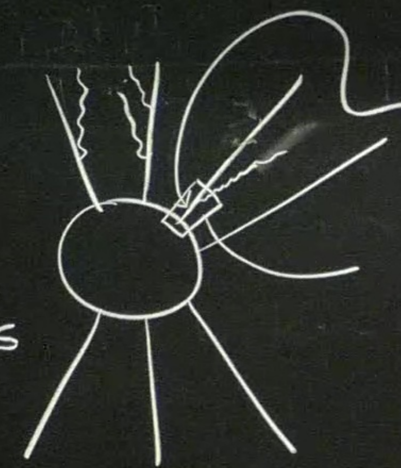
$A > 0$

conceptual

$$\left(\frac{E}{\Lambda}\right)^{A} b(E)\,\Gamma_{\text{hard}} \quad :-)$$

application


in pert th
A is infinitesimal
$1 + A \log\left(\frac{\lambda}{\Lambda}\right)$
∞
2
$A > 0$
soft
$\lambda < E < \Lambda$
IR
$= \underbrace{\left(\frac{\lambda}{\Lambda}\right)^A}_{0} \underbrace{\left(\frac{E}{\lambda}\right)^A}_{\infty} b(E)\,\Gamma_{\text{hard}}$
div go away
conceptual
$\left(\frac{E}{\Lambda}\right)^A b(E)\,\Gamma_{\text{hard}}$:-)
application
CAUTION



± 1 out/in

$$\frac{e_n\, p_n^{\mu}\, \eta_n}{p_n \cdot q - i\varepsilon\, \eta_n} \quad \leftarrow S \oplus \text{soft insertion}$$

$$\underbrace{\prod_{k=1}^{N} \sum_{n=1} \frac{e_n\, \eta_n\, p_n^{\mu}}{p_n \cdot q^{(k)} - i\varepsilon\, \eta_n}}_{\text{section 2}} \qquad (2.3)$$

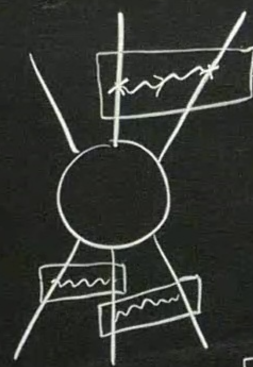
factorized soft factor $\longrightarrow$ exponentiation soft insertion

P

iation

Virtual

$$\frac{1}{x-i0} = P\frac{1}{x} + i\pi\,\delta(x)$$



$$\underbrace{\boxed{\frac{-i}{q^2 - i\epsilon}}}_{\to\ \delta(q^2)} g^{\mu\nu} \sum_{n,m} \frac{e_n \eta_n p_n^\mu \; e_m \eta_m p_m^\mu}{(p_n\cdot q - i\epsilon\eta_n)(-p_m\cdot q - i\epsilon\eta_m)}$$

(2.9) $$\frac{1}{N!}\left[\frac{1}{2}\int_\lambda^\Lambda d^4q\, A(q)\right]^N, \quad A =$$

$$\int \mathrm{Re}\,A \quad (2.12)$$

$$\Gamma = \Gamma_{hard}\, e^{\frac{1}{2}\int A}$$

$$S = S_{hard}\, e \quad (2.11)$$

$$\mathrm{Re}\int A \underset{\substack{\uparrow\\ q\ \text{on-shell}}}{=} \int_{\lambda}^{\Lambda} \frac{d\omega}{\omega} \int d^2\Omega\ \underbrace{A(\hat{q}) = \#\sum_{n,m} e_n e_m \eta_n \eta_m (p_n\cdot p_m)\, f(\beta_{n,m})}_{\left(-\log\frac{\Lambda}{\lambda}\right)}$$

$$= \left(\log\frac{\lambda}{\Lambda}\right) \times (A > 0)$$

q on-shell

$$q = \omega(1, \hat{q})$$

unit vector

$$\frac{1}{\omega} \qquad \underbrace{\frac{1}{\omega}\times\frac{1}{\omega}}_{\text{there in } A} \qquad \underbrace{\omega^2 d\omega}_{d^3\vec{q}} = \frac{d\omega}{\omega}$$

$\delta(q^2)$ there in A $\quad d^3\vec{q}$

$$\frac{1}{\omega} \qquad \underbrace{\frac{1}{\omega}\times\frac{1}{\omega}}_{\text{there in A}} \qquad \underbrace{\omega^2 d\omega}_{d^3\vec{q}} = \frac{d\omega}{\omega} = d(\log\omega)$$

$\delta(q^2)$

in 4d !

vector

(2.18) ✓

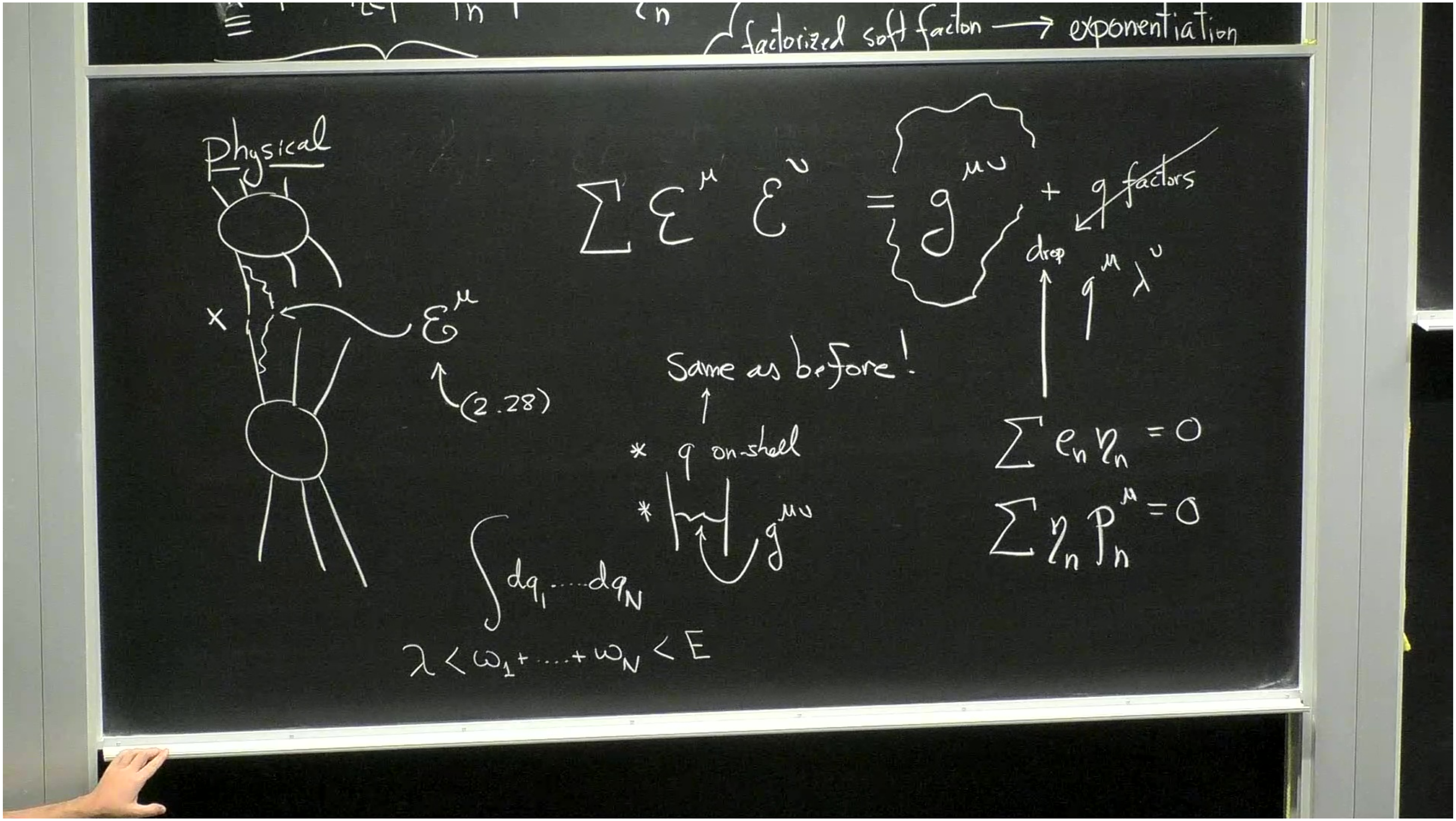
(2.27) $A \to B$

$$\underbrace{\underbrace{\quad}_{0} \qquad \underbrace{\quad}_{\infty}}_{\text{div go away}}$$

conceptual

application

$$\left(\frac{E}{\Lambda}\right)^{A} b(E)\, \Gamma_{\text{hard}} \quad :-)$$


factorized soft factor → exponentiation
Physical
(2.28)
Same as before!
* q on-shell
*
g^{μν}
∫dq_1 ..... dq_N
λ < ω_1 + ... + ω_N < E
Σ ε^μ ε^ν = g^{μν} + q factors
drop
q^μ λ^ν
Σ e_n η_n = 0
Σ η_n p_n^μ = 0

(2.44)

$$\sum_{N=0}^{\infty} \frac{A^N}{N!} \Gamma_{\text{BEFORE}} \int_\lambda^E \frac{d\omega_1}{\omega_1}\int_\lambda^E \frac{d\omega_2}{\omega_2}\cdots\cdots\int_\lambda^E \frac{d\omega_N}{\omega_N}\;\Theta(\omega_1+\dots+\omega_N<E)$$

$$\Theta = \int_{-\infty}^{+\infty} dx\; \underbrace{e^{ix(\omega_1+\dots+\omega_N-E)}}_{\mathrm{Sin}(x(\dots))} \qquad \frac{1}{x}$$

$$\log\frac{E}{\lambda}$$

$$(2.48) \longleftarrow \int dx\, \frac{e^{-ixE}}{x}\left[\prod\left(\int_\lambda^E \frac{d\omega}{\omega}\left(e^{i\omega x}-1+1\right)\right)\right]$$

$$\Gamma_{\text{phys}}(\text{soft} \lesssim E) = \left(\frac{E}{\cancel{\hbar}}\right)^A \underbrace{b(A)}_{\simeq 1 \text{ for } A \ll 1} \left(\frac{\cancel{\hbar}}{\Lambda}\right)^A \underbrace{\Gamma_{\text{hard}}}_{\text{no soft stuff}}$$

$$b(x) = \int \frac{\sin\sigma}{\sigma} \exp\left[x \int_0^1 \frac{d\omega}{\omega}\left(e^{i\omega\sigma} - 1\right)\right] = 1 - \frac{\pi^2}{32} x^2 + \dots$$

Section 6

$$\boxed{\Gamma = \left(\frac{E}{\Lambda}\right)^A \hat{\Gamma}_{\text{hard}}}$$

, Same for grav $A \to B$

(2.18)

$(2.27)\ A \to B$

$$\lambda < \omega_1 + \dots + \omega_N < E$$

$$\Theta(\omega_1 + \dots + \omega_N < E)$$

$\log \frac{E}{\lambda}$

$+1$

$$\Gamma_{phys}(\text{soft} \le E) = \left(\frac{E}{\cancel{\lambda}}\right)^A \underbrace{b(A)}_{\simeq 1 \text{ for } A \ll 1} \left(\frac{\cancel{\lambda}}{\Lambda}\right)^A \underbrace{\Gamma_{hard}}_{\text{no soft stuff}}$$

$$b(x) = \int \frac{\sin\sigma}{\sigma} \exp\left[x \int_0^1 \frac{d\omega}{\omega}\left(e^{i\omega\sigma} - 1\right)\right] = 1 - \frac{\pi^2}{32}x^2 + \dots$$

Section 6 $\leftarrow$ $\boxed{\Gamma_{\le E} = \left(\frac{E}{\Lambda}\right)^A \hat{\Gamma}_{hard}}$, same for grav $A \to B$

$$\Gamma = E^A \times \text{rest}$$

$$f_{\text{photons}}(\beta_{nm}) = -\frac{1}{8\pi^2}\frac{1}{\beta_{nm}}\log\frac{1+\beta_{nm}}{1-\beta_{nm}} = \int\frac{d^2\Omega}{8\pi^2}\frac{1}{(E_n-\vec{p}_n\cdot\hat{q})(E_m-\vec{p}_m\cdot\hat{q})}$$

relative velocity

$$\begin{cases} ct & \beta\to 0 \\ \log(1-\beta) & \beta\to 1 \end{cases}$$

very light

$$\beta_{nm} = \sqrt{1-\frac{m_n^2\, m_m^2}{(p_n\cdot p_m)^2}} \underbrace{\to 1}_{m\to 0}$$

$x^2 + \dots$

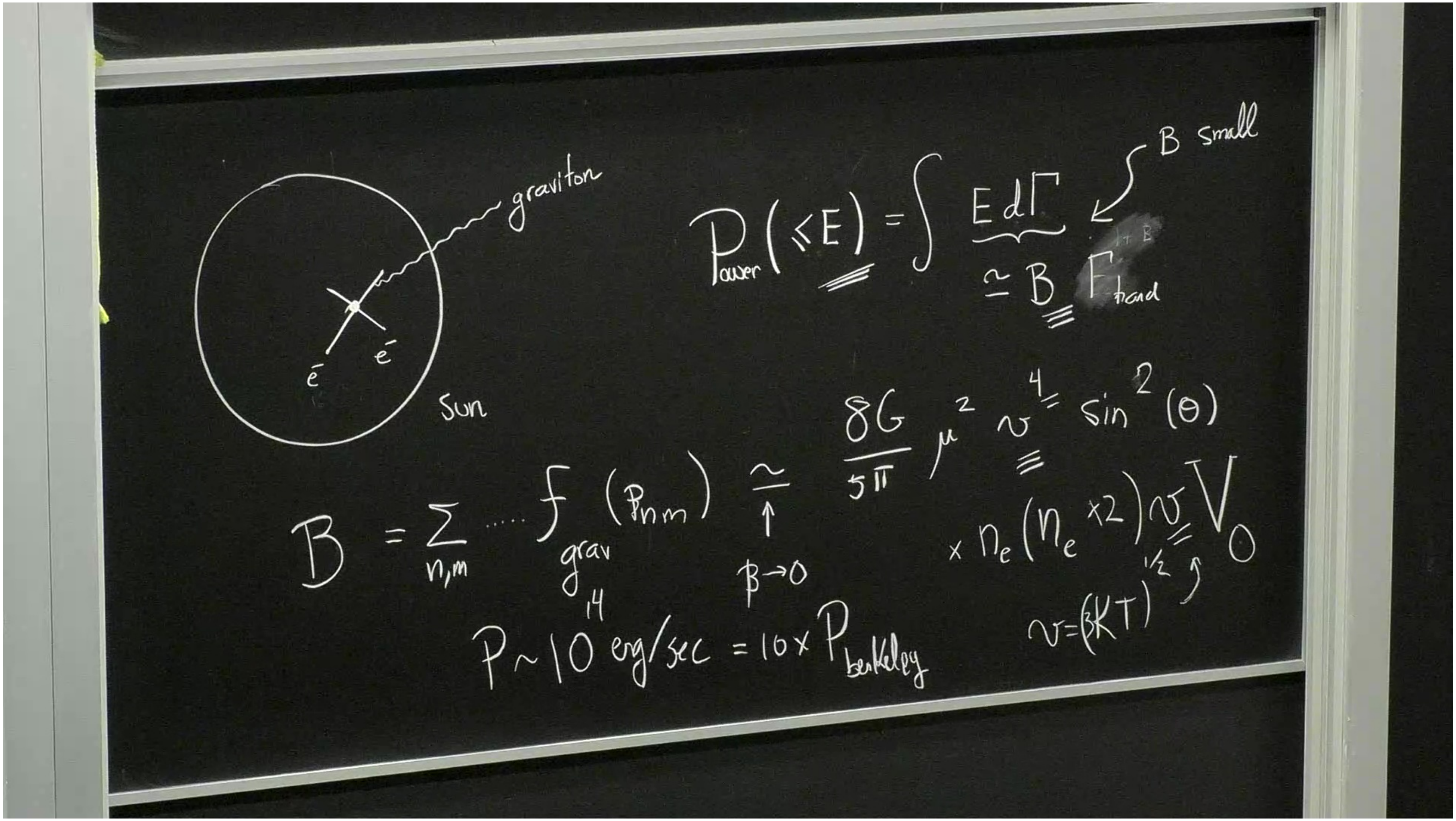
$$f_{\text{gravitons}}(\beta) = \frac{1+\beta^2}{4\beta(1-\beta^2)^{1/2}}\log\frac{1+\beta}{1-\beta}$$

$$A = \sum_{n,m}\eta_n\eta_m e_n e_m\, p\cdot p\, f > 0$$

photons: $\sum_{n\neq 1} \log m_1 \; e_n\cdot e_1\, \eta_1\eta_n \simeq -e_1^2 \log m_1$

massless

hard


graviton
e^-
e^-
Sun
$P_{power}(\leq E) = \int \underbrace{E\,d\Gamma}_{\simeq B\, F_{band}}$
B small
$B = \sum_{n,m} \ldots f_{grav}(P_{nm}) \underset{\beta \to 0}{\simeq} \frac{8G}{5\pi} \mu^2 v^4 \sin^2(\theta)$
$\times\, n_e (n_e \times 2)\, v = V_0$
$v = (3kT)^{1/2}$
$P \sim 10^{14}$ erg/sec $= 10 \times P_{berkeley}$