

Title: Verification of quantum simulations and indistinguishability of photons

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Collection/Series: Waterloo-Munich Joint Workshop

Subject: Quantum Information

Date: October 01, 2024 - 9:00 AM

URL: <https://pirsa.org/24100063>



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Verification of quantum computing and simulation & indistinguishability of photons

B. Kraus

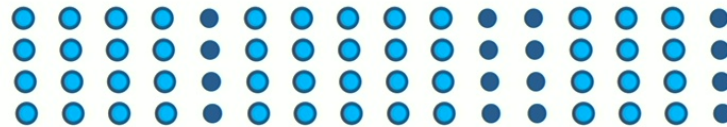


Waterloo-Munich Joint Workshop



VERIFICATION OF QUANTUM DEVICES

NOW & NEAR FUTURE: MEDIUM SIZE/LARGE SYSTEMS



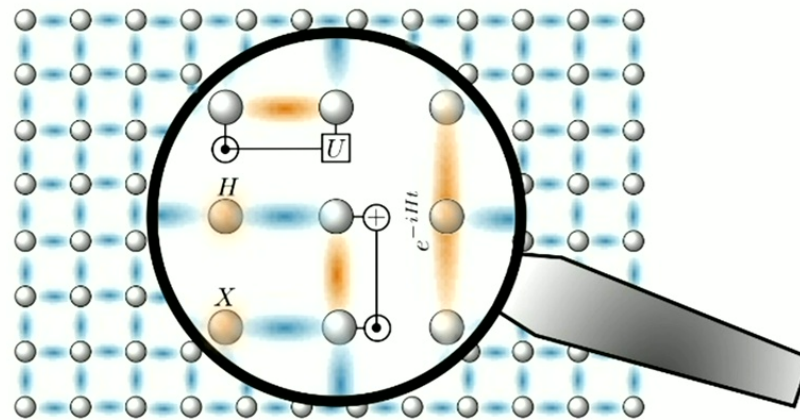
$$\mathcal{H} = 2^n$$
A large red 3D question mark.

HOW?

Use device for e.g. q. computing/ q. simulation

How can we verify it?

VERIFYING QUANTUM COMPUTATION/SIMULATION



WHAT DO WE ACTUALLY WANT TO VERIFY? CORRECT OUTPUT STATE, OR CORRECT OUTPUT FOR LOCAL OBSERVABLES, ...?

DEPENDS STRONGLY ON THE TASK AND ON WHO POSSESSES THE Q. DEVICE

QUANTUM VERIFICATION

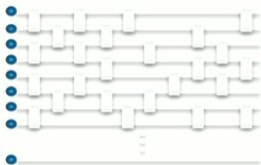


Solve a problem, e.g.
In NP: factorize 25



Easy to check classically

Solve a problem, e.g.
Is output of

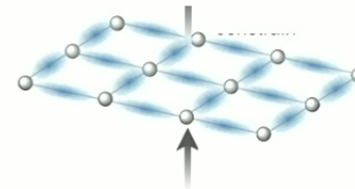


0 or 1 ($p_1 = \|\langle 1|U|0\rangle^{\otimes n}\|^2 \geq \frac{2}{3}$)
(in BQP)

Output:
0



Q. simulation



$$H = \sum_m c_m S_m = \mathbf{c} \cdot \mathbf{S}$$

How should we check?
How should we check q. advantage?

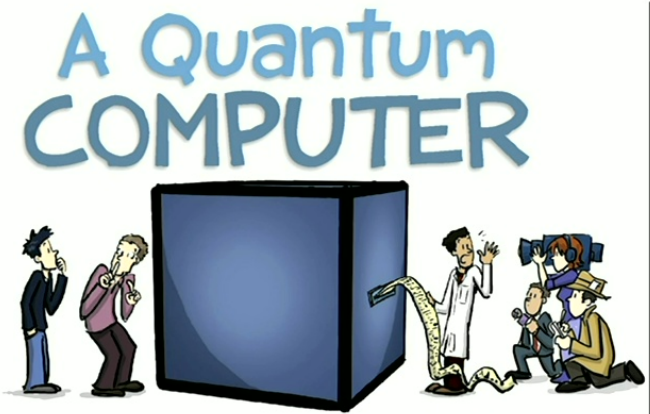
QUANTUM VERIFICATION

HAVING ACCESS TO THE DEVICE



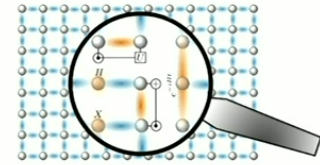
- ⇒ Run tests, obtain error model, estimate error,
- ⇒ improve device, confidence that works

CLOUD COMPUTATION (NO ACCESS /UNTRUSTED DEVICE)



- Why?
- will not all possess Q laptop
- ⇒ Q. cloud computer

OUTLINE



GAINING CONFIDENCE ABOUT THE PERFORMANCE OF Q. DEVICES

J. Carrasco, M. Langer, A. Neven, BK Physical Review Research 2024, See also poster by M. Langer

Hamiltonian and Liouvillian learning in weakly-dissipative quantum many-body systems

T. Olsacher, T. Kraft, CH. Kokail, B.K., P. Zoller (arXiv `24)

Indistinguishability of identical bosons from a quantum information theory perspective

M. Englbrecht, T. Kraft, Ch. Dittel, A. Buchleitner, G. Giedke, BK, PRL'23

QUANTUM VERIFICATION HAVING ACCESS



Check that single qubit and 2-qubit gates can be implemented with high fidelity

⇒ Enough?

⇒ Crosstalking, drifts,...

⇒ Depends on output

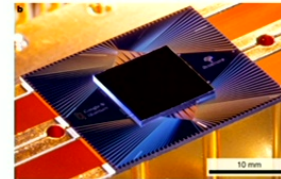
⇒ Run tests, obtain error model, estimate error,

⇒ improve device

VERIFICATION OF Q. COMPUTATION/ SIMULATION OF DEVICE WITH DIRECT ACCESS TO IT

Tracing: ✗
Check that leads to correct outcome for those computations which we can verify ✓
e.g. simulation on small input, simplified q. circuits

NISQ devices



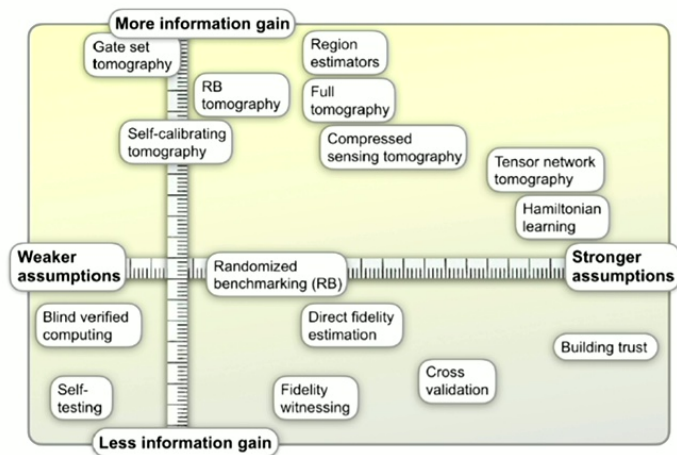
There are errors!

Might not be enough to check whether output is correct or not, need to estimate the error

single & two-qubit gates, use cl. simulable q. circuits to validate error model
need to be able to compare to correct/ predicted outcome

VERIFICATION OF Q. DEVICE WITH DIRECT ACCESS TO IT

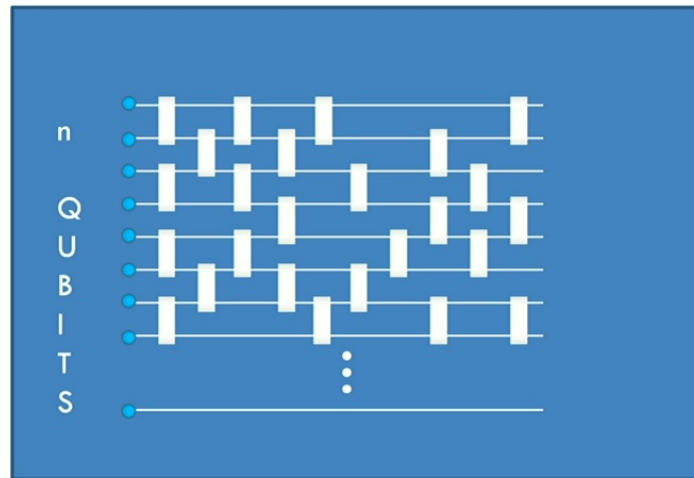
Review: e.g J. Eisert, D. Hangleiter, N. Walk, I. Roth,
D. Markham, R. Parekh, U. Chabaud, E. Kashefi (2019)



Test gate set, ...
Averaged error

Different approach: Given Circuit, what is the error?

Given Circuit C



Ideal: $C|0\rangle^{\otimes n}$
Errors: ρ_{out}

How can we estimate the error? How can we gain confidence in the realization of C up to a certain error? ρ_{out} ?

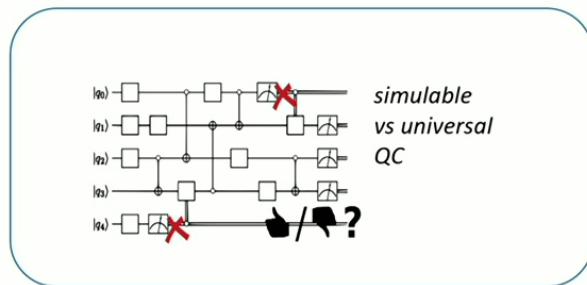
J. Carrasco, M. Langer, A. Neven, BK (2023)

Universal versus efficiently simulable circuits

Some circuits can be simulated efficiently adding additional resources \Rightarrow universal



Use for verification (of arbitrary q. computation)



Match gates

2-qubit gates fulfilling algebraic constraint

n.n. Match gate circuits: cl. efficiently simulable

Clifford circuits:

R. Jozsa and S. Strelchuk (arXiv: 1705.02817 (2017))
S. Ferracin, T. Kapourniotis, and A. Datta, New. J. Phys. 21, 113038 (2019),...

J. Carrasco, M. Langer, A. Neven, BK (2023)

UNIVERSAL VERSUS EFFICIENTLY SIMULABLE CIRCUITS

Main idea:

Some circuits can be simulated cl. efficiently
adding additional resources \Rightarrow universal



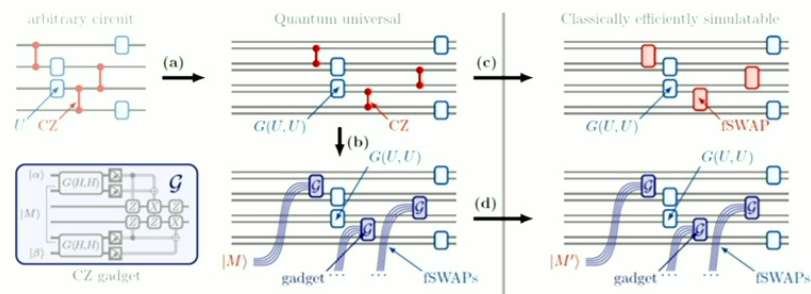
Use for verification (of arbitrary q.
computation)

Construct circuits which are:

- (i) **classically efficiently simulable**
not only without, but also with errors

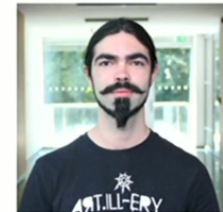
- (ii) **very similar to the original (universal)**
computation

CL. EFFICIENTLY SIMULABLE Q. CIRCUIT



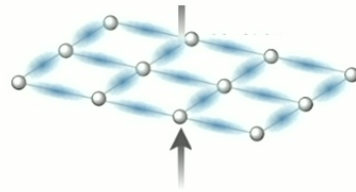
- (i) classically efficiently simulable
(weak simulation, M. Hebenstreit, R. Jozsa, BK, S. Strelchuk, PRA 2020)
- (ii) very similar to the original (universal) computation
in (d) exactly same circuit applied to slightly different input state
- (iii) Errors can be included using weak simulation &
randomized compiling (J. J. Wallman, J. Emerson, PRA (2016))
⇒ cl. simulate output including errors

See poster



Marc Langer

Quantum Simulation



$$H = \sum_m c_m S_m = \mathbf{c} \cdot \mathbf{S}$$

How can we make sure that the evolution is due to some specific H ?

Having access to the device



Hamiltonian learning

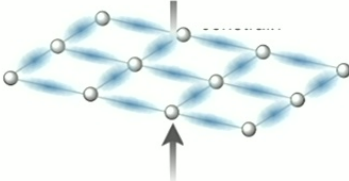
“Learning a local Hamiltonian from local measurements” E. Bairey, I. Arad, N. H. Lindner, PRL (2019),

“Hamiltonian tomography via quantum quench “ Z. Li, L. Zou, and T. H. Hsieh, PRL (2020),...

X.-L. Qi and D. Ranard, Quantum 3, 159 (2019)

E. Bairey, C. Guo, D. Poletti, N. H. Lindner, and I. Arad, New J. Phys. 22, 032001 (2020)

D. Stilck França, L. A. Markovich, V. V. Dobrovitski, A. H. Werner, J. Borregaard (2022),..



The diagram shows a 2D lattice of particles represented by grey spheres connected by blue lines. A central site is highlighted with a grey circle and an upward-pointing arrow. Below the diagram is the equation
$$H = \sum_m c_m S_m = c \cdot S$$

Learning Hamiltonian and Liouvillian in weakly-dissipative quantum many-body systems

T. Olsacher, T. Kraft, Ch. Kokail, B.K., P. Zoller (arXiv '24)

Hamiltonian learning

Assumption: unitary evolution $|\Psi(0)\rangle \rightarrow |\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle$

$$H = \sum_{i=1}^m c_i^H h_i \quad h_i \text{ traceless hermitian operators, e.g. Pauli operators}$$

AIM: determine c_i^H

Main idea: e.g. energy conservation $\langle H \rangle_0 - \langle H \rangle_T = 0$ with $\langle H \rangle_t = \langle \Psi(t) | H | \Psi(t) \rangle$

Ansatz for H: $A(c) = \sum_{i=1}^n c_i h_i$, for some choice of subset of operators

$$\{|\Psi_i(0)\rangle \rightarrow |\Psi_i(t)\rangle = e^{-iHt} |\Psi_i(0)\rangle, i = 1, 2, \dots, p\}$$

$$M_{i,j} = \langle \Psi_i(0) | h_j | \Psi_i(0) \rangle - \langle \Psi_i(t) | h_j | \Psi_i(t) \rangle$$

Measure $M_{i,j} \Rightarrow$ Solve $M\vec{c} = 0 \Rightarrow$ subspace commuting with H

Z. Li, L. Zou, and T. H. Hsieh, Hamiltonian tomography via quantum quench, PRL 124, 160502 (2020).

E. Bairey, I. Arad, N. H. Lindner, Learning a local Hamiltonian from local measurements, PRL (2019),...

Hamiltonian & Liouvillian learning in weakly-dissipative quantum many-body systems

Assumption: evolution governed by master equation

$$\frac{d}{dt}\rho = -i[H, \rho] + \frac{1}{2} \sum_k \gamma_k ([l_k \rho, l_k^\dagger] + [l_k, \rho l_k^\dagger]) \equiv L(\rho)$$

l_k Lindblad operators
 $\gamma_k > 0$ rates
 Weak dissipation: $\gamma_k \ll \|H\|$

e.g. $H = \sum_{i=1}^{N-1} J_{i,i+1}^z \sigma_i^z \sigma_{i+1}^z + \dots \quad l_k \in \{\sigma_-, \sigma_+, \sigma_x\} \quad \gamma_k \ll \|H\|$

For $\gamma_k = 0 \rightarrow H$ -learning

For $\gamma_k > 0 \rightarrow$ **small perturbation** to energy conservation

Aim: determine H and (if possible) $\{\gamma_k, l_k\}$

T. Olsacher, T. Kraft, CH. Kokail, B.K., P. Zoller (arXiv '24)

Hamiltonian & Liouvillian learning (weak dissipation)

Ansatz: for H : $A(c) = \sum c_i h_i$, for dissipation: $\{d_k, a_k\}_k$

Aim: determine c^H and (if possible) $\{\gamma_k, l_k\}_k$

Ehrenfest's theorem:
$$\frac{d}{dt} \langle O \rangle = \langle i[O, H] \rangle + \frac{1}{2} \sum_k \gamma_k (\langle l_k^t [O, l_k] + [l_k^t, O] l_k \rangle)$$

For $O = H \Rightarrow$

$$-\langle H \rangle_0 + \langle H \rangle_T = \frac{1}{2} \sum_k \gamma_k \int_0^T \langle l_k^t [H, l_k] + [l_k^t, H] l_k \rangle_t dt \quad \text{for any state (pure and mixed)}$$

Hamiltonian & Liouvillian learning (weak dissipation)

Inserting Ansatz: for H: $A(c) = \sum c_i h_i$, for dissipation: $\{d_k, a_k\}_k$

$$\Rightarrow [M_H + M_D(\vec{d})] \vec{c} = 0$$

$$M_H = (M)_{ij} = \langle h_j \rangle_{i,0} - \langle h_j \rangle_{i,T}$$

$$M_D(d) = \sum_k d_k M^{(k)} \quad [M^{(k)}]_{ij} = \int_0^T \langle a_k^t [h_j, a_k] + [a_k^t, h_j] a_k \rangle_{\Psi_i(t)} dt$$

$$(c^{rec}, d^{rec}) = \arg \min_{(c,d): \|c\|=1, d_k \geq 0} \|[\tilde{M}_H + \tilde{M}_D(d)] c\|$$

Taking errors into account

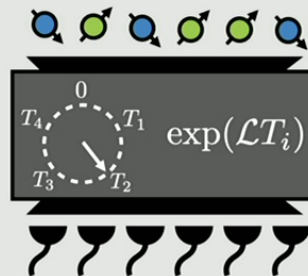
Solve by finding d^{rec} which leads to smallest singular value, λ_1
 c^{rec} is right eigenvector to this singular value

Measurements for Hamiltonian & Liouvillian learning

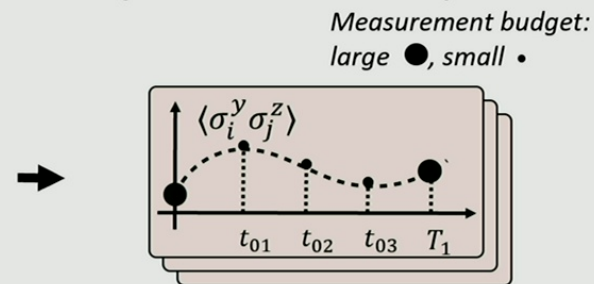
Aim: measure elements of constraint matrix $-\langle H \rangle_0 + \langle H \rangle_T = \frac{1}{2} \sum_k \gamma_k \int_0^T \langle [l_k^t, H] + [l_k^t, H] l_k \rangle_t dt$

We consider simple quench experiments (with product states and product measurements):

1. Prepare product state.
2. Apply \mathcal{L} for time t .
3. Measure product basis, e.g., in the Pauli basis.



Typically, many of these can be measured in parallel!



$$\langle X \rangle_t = \text{tr}[X \exp\{(\mathcal{L}_H + \mathcal{L}_D)t\} \rho(0)]$$

AIM: experimentally feasible
 \Rightarrow small number of measurements (10^6)

How to assess the ansatz

$$(c^{rec}, d^{rec}) = \arg \min_{(c, d): \|c\| = 1, d_k \geq 0} \|[\tilde{M}_H + \tilde{M}_D(d)] c\| \Rightarrow d^{rec}, c^{rec}$$

λ_1, λ_2 smallest singular values of $\tilde{M}_H + \tilde{M}_D(d)$

2 types of errors: (1) **finite** # of measurements \rightarrow shot-noise (statistical errors): $M_H + M_D \rightarrow \tilde{M}_H + \tilde{M}_D$
(2) Ansatz does not include H or Liouvillian (systematic errors)

systematic errors $\Rightarrow \lambda_1 > 0$ (even if ∞ many measurements)

$\frac{\lambda_1}{\lambda_2}$ plays important role in finding right strategy

Protocol for H & L learning (weak dissipation)

Hamiltonian and Liouvillian learning

(1) Choose ansatz

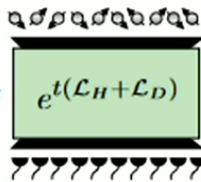
quasi-local, few-body terms

$$H \rightarrow \Lambda(\mathbf{c}) = \sum_j c_j h_j$$

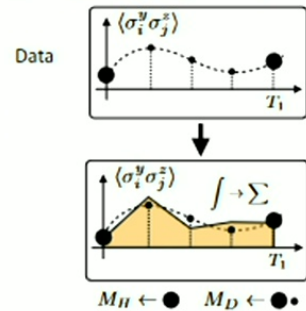
$$\mathcal{L}_D \rightarrow \mathcal{D}(\mathbf{d}) = \frac{1}{2} \sum_k d_k ([a_k \rho, a_k^\dagger] + \text{h.c.})$$

(2) Experiment

1. Prepare product states
2. Evolve system for time t
3. Measure output in product basis

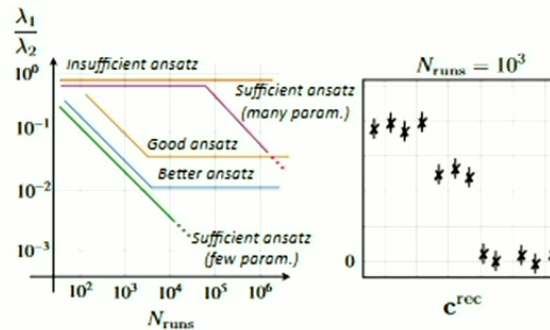


(3) Compute M_H, M_D from exp. data



Reconstruct Hamiltonian and Liouvillian parameters via Eq. (9)

(4) Ansatz assessment via λ_1/λ_2 and \mathbf{c}^{rec}



→ Update ansatz accordingly (via cl. post-processing, recycling data, additional measurements)

- (Re-)Parametrize ansatz
 - Reduce number of parameters, and obtain better estimate for a fixed N_{runs} ✓
- Correct dissipation via M_D
 - discover terms in H "hidden" by dissipation ✓
- Employ additional constraints
 - determine overall scale ✓
 - exclude cons. quantities ✓
 - resolve entire Liouvillian ✓

Use Ehrenfest Theorem to derive constraints on parameters of H (and dissipation)

Use $\frac{\lambda_1}{\lambda_2}$ to update the Ansatz: whenever plateau:

- either reparameterize the Ansatz $\vec{c} \rightarrow G\vec{c}$ (e.g. to reduce # of parameters)
- or focus on dominante terms
- or include dissipation,...

Good ansatz: early shot-noise scaling, low plateau

H & L learning (weak dissipation)

$$(c^{rec}, d^{rec}) = \arg \min ||[\tilde{M}_H + \tilde{M}_D(d)] c|| \Rightarrow$$

without dissipation \rightarrow lin. optimization;
with dissipation \rightarrow nonlinear optimization

Learn H **without** learning the complete Liouvillian
Dissipation: does not need to be uniquely defined (e.g depends only on $d_1 + d_2$).

Learn H up to **overall scale** & possible **conserved quantities** of H.

Additional constraints \rightarrow learning H and complete Liouvillian

Illustration

Example Longitudinal & transverse-field Ising model with dominant nearest neighbor & sub-dominant next-nearest neighbor interaction (small disorder).

$$H = \sum_{i=1}^{N-1} J_{i,i+1}^z \sigma_i^z \sigma_{i+1}^z + \sum_{i=1}^{N-2} J_{i,i+2}^z \sigma_i^z \sigma_{i+2}^z + B_x \sum_{i=1}^N \sigma_i^x + B_z \sum_{i=1}^N \sigma_i^z \quad + \text{weak dissipation: } \gamma_k \ll \|H\|$$

Assumption (not required) : Know type of dissipation, but not the rates

Dissipation: spatially homogeneous: σ_+ , σ_- , σ_x

For $n=10$ qubits

Goal: Learn operator structure of H and dissipation rates. Use only appx. 10^6 runs.

Illustration

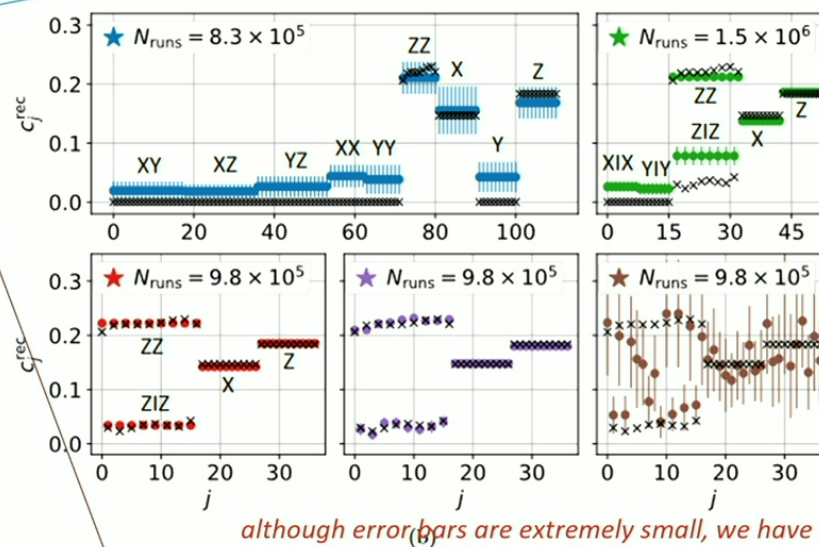
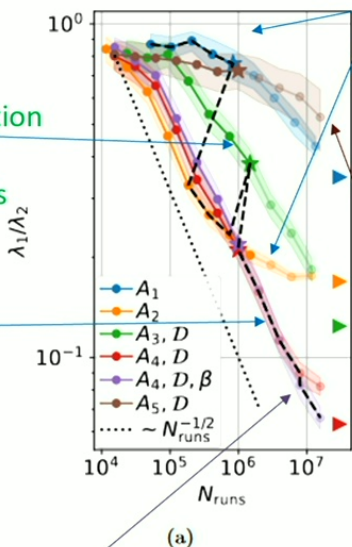
plateau → extend ansatz

A2: $\frac{\lambda_1}{\lambda_2}$ much smaller/ plateau

A1: all n.n. & local terms
identify 3 dominante terms

A3: after Including dissipation
Can learn n.n.n. coefficients

A4: including Dominant nnn Term



keep Z1Z

although error bars are extremely small, we have not yet reached a plateau. But, can search for solutions taking spatial disorder into account

A5: reparameterization

A6: naive approach (unparametrized ansatz of the form)

$$A_6 = A_{nnZZ}^{\text{free}} + A_{nnnZZ}^{\text{free}} + A_X^{\text{free}} + A_Z^{\text{free}}$$

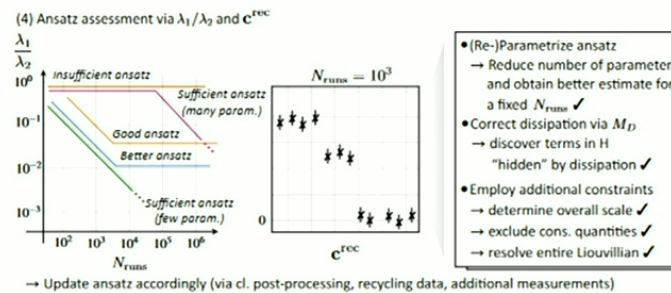
Matches behaviour of error

Conclusion and Outlook

Gaining confidence about realization of arbitrary q. circuit

Derive cl. efficiently simulable circuits, which are very similar to the original (encoded) circuit
Circuit can be weakly simulated including errors

H & L learning in presence of weak dissipation



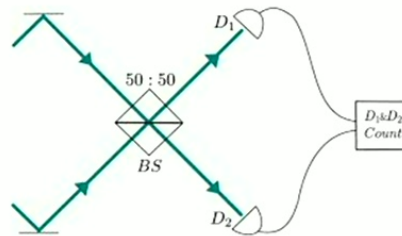
Outlook: experiment
combine with classical shadow techniques and other H-learning protocols

Indistinguishability of identical bosons

Consider N bosons: When are they indistinguishable? How can one measure indistinguishability?

e.g. 2 photons:

Hong–Ou–Mandel



If perfectly indistinguishable
would always exit in the same mode

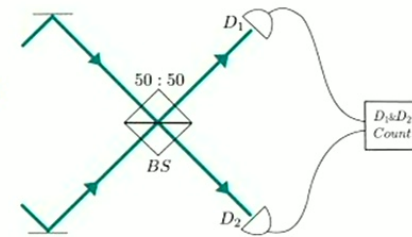
When are N photons indistinguishable?

Why do we care? Indistinguishability is a resource, e.g. boson sampling, quantum computing

Indistinguishability of identical particles (bosons)

How to define (in)distinguishability:

- E.g. Hong Ou mandel experiment \Rightarrow only partial info for N particles
- For states with fixed particle number **per external mode**:
entanglement between external and internal DOF



\Rightarrow **What about general states?**

How to quantify (in)distinguishability? E.g suppression laws, correlations between output of detectors, entanglement measures ...

\Rightarrow **Arbitrary states? How can we efficiently measure it experimentally?**

- Dittel, C., Dufour, G., Walschaers, M., Weihs, G., Buchleitner, A., & Keil, R. (2018). *Physical review letters*, 120(24), 240404.
Walschaers, M. (2020). *Journal of Physics B: Atomic, Molecular and Optical Physics*, 53(4), 043001.
Dittel, C., Dufour, G., Weihs, G., & Buchleitner, A. (2021) *Physical Review X*, 11(3), 031041.
Shchesnovich, V. S. *Physical Review A* 91.1 (2015): 013844.

Indistinguishability of identical bosons from a quantum information theory perspective

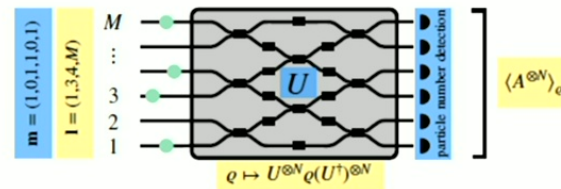
- Definition of (in)distinguishability for arbitrary states with fixed total particle number
- Measure of indistinguishability and its operational meaning
- Tight upper and lower bounds on measure (experiment)
- Simple characterization of perfectly distinguishable states (representation theory)
- Perfect distinguishability not preserved under convex combinations

M. Englbrecht, T. Kraft, Ch. Dittel, A. Buchleitner, G. Giedke, BK, PRL'23

Setting

Bosons: indistinguishable particles whose total state is symmetric

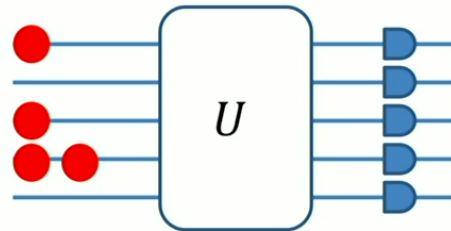
N bosons in M external modes of a passive linear interferometer + particle # measurement



Experiments: only a subset of degrees of freedom are accessible = external dofs.
indistinguishable particles become (partially) distinguishable

Inaccessible dofs (internal dofs): e.g. polarization, time of arrival,...

Indistinguishability depends on experimental setting



Single photon Hilbert space:

$$H_{ex} \otimes H_{int}$$

DOF which are exp. accessible, i.e are manipulated and measured

- Unresolved, e.g. polarization, time of arrival,...
- makes particles distinguishable

Only

$$\rho_{ex} = Tr_{in}(|\psi\rangle\langle\psi|)$$

is accessible \Rightarrow contains all the information about distinguishability

Dittel, C., Dufour, G., Weihs, G., & Buchleitner, A. (2021) *Physical Review X*, 11(3), 031041.
 Shchesnovich, V. S. *Physical Review A* 91.1 (2015): 013844.

Description of experiment

Bosons \Rightarrow symmetric states

$$\pi \otimes \tilde{\pi} |\Psi\rangle_{ext,int} = |\Psi\rangle_{ext,int} \text{ for any permutation } \pi \in S_N$$

$\pi(\tilde{\pi})$ representation of a permutation π in the external (internal) Hilbert space

projector onto the N-particle symmetric subspace of H_N : $P_{tot}^{(N)} = \sum_{\pi \in S_N} \pi \otimes \tilde{\pi}$

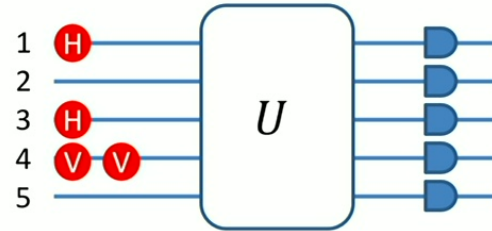
And for ext Dof: $P^{(N)} = \sum_{\pi \in S_N} \pi$

$$\Rightarrow \pi \rho_{ext} \pi^t = \rho_{ext}$$

Bosonic states:

$$Inv(\mathcal{H}) = \{\rho: \pi \rho \pi^t = \rho \ \forall \pi \in S_N, \rho \geq 0, tr(\rho) = 1\}$$

Example



$$a_{1,H}^\dagger a_{3,H}^\dagger a_{4,V}^\dagger a_{5,V}^\dagger |vac\rangle = \sum_{\pi \in S_4} \pi \otimes \pi |1344\rangle |HHVV\rangle$$

interferometer acts as $U \otimes U \otimes \dots \otimes U = U^{\otimes 4}$

Example for 2 distinguishable particles:

$$|\psi\rangle \propto \sum_{\pi \in S_2} \pi \otimes \pi |12\rangle |12\rangle$$

$$\rightarrow \rho_{ex} = |12\rangle\langle 12| + |21\rangle\langle 21|$$

$$H_{ex} = \text{span}\{|1\rangle, |2\rangle\}$$

$$H_{in} = \text{span}\{|1\rangle, |2\rangle\}$$

Example for 2 indistinguishable particles:

$$|\psi\rangle \propto \sum_{\pi \in S_2} \pi \otimes \pi |12\rangle |11\rangle$$

$$\rightarrow \rho_{ex} = (|12\rangle + |21\rangle)(\langle 12| + \langle 21|)$$

Indistinguishable particles

DEF: Indistinguishable particles if externally, particles behave like bosons, i.e identical particles described by a state $\rho \in \text{Inv}(\mathcal{H})$ are called perfectly indistinguishable if

$$\pi_N \rho_{ext} = \rho_{ext}$$

REMARK: Does not make use of the internal d.o.f. (as they are inaccessible)
e.g., two bosons in the state

$$|\Psi\rangle = |11\rangle_{ext} \otimes |11\rangle_{int} \text{ as well as}$$

$$|\Psi\rangle = |11\rangle_{ext} \otimes (|12\rangle + |21\rangle)_{int} \text{ are indistinguishable}$$

$$\text{As } \pi_N \rho_{ext} = \rho_{ext}$$

Quantifying indistinguishability

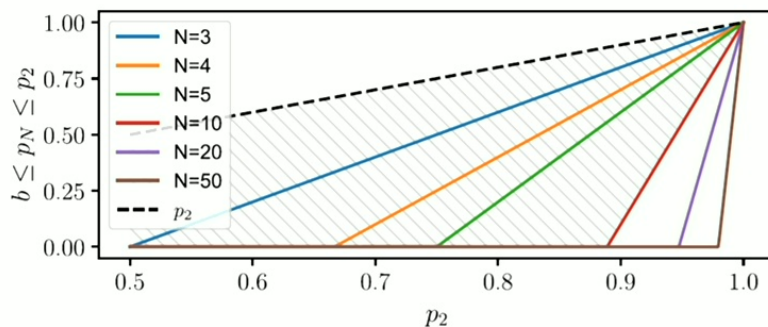
$$P_N = \langle P_N \rangle = \text{tr}(P_N \rho_{ext})$$

- $P_N = 1$ perfectly indistinguishable (independent of internal states)
- Generalizes result for fixed mode occupations to arbitrary states
- Gives upper bound on trace distance to indistinguishable part of ρ_{ext} , i.e. $P_N \rho_{ext} P_N$

Bounds on quantifier of indistinguishability

Note if $P_2 = 1 \Rightarrow P_N = 1$

A large value of $P_2 \Rightarrow$ strong conditions on $\rho \in \text{Inv}(\mathcal{H}) \Rightarrow$ leads to good bounds on P_N :



$$b = \begin{cases} 0 & \text{if } 0 \leq p_2 \leq (N - 2/N - 1) \\ p_2(N - 1) - (N - 2) & \text{else} \end{cases}$$

$$b \leq p_N \leq p_2$$

Similar with p_k for any $k \geq 3$.

p_2 can be measured efficiently \Rightarrow if N particles are sufficiently indistinguishable, this can be verified efficiently

$P^{(2)} \otimes id^{(N-2)}$ is linear combination of $\sim M^4$ operators of the form $A \otimes A \otimes id^{(N-2)}$, which can be measured efficiently (interferometer U , such that UAU^\dagger is diagonal).

Quantifying indistinguishability

$$P_N = \langle P_N \rangle = \text{tr}(P_N \rho_{ext})$$

How can we measure P_N ?

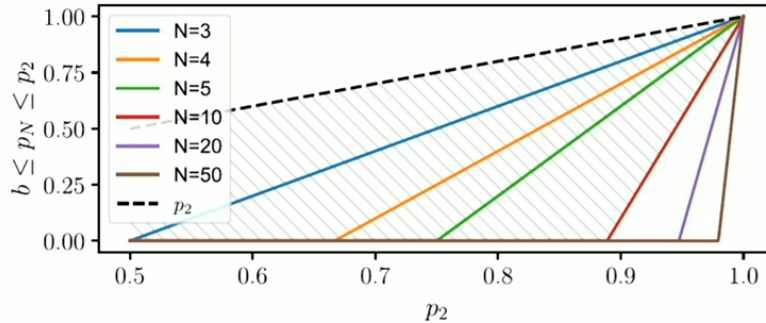
Other approaches to quantify indistinguishability (also P_N) rely on measuring probabilities that become exponentially small with N (e.g. (V. S. Shchesnovich, Phys. Rev. A 91, 063842 (2015))).

Here: tight bound via simple measurements via $P_2 = \langle P^{(2)} \rangle = \text{tr}(P_2 \otimes Id^{(N-2)})\rho_{ext}$ measures how symmetric the two-particle reduced state is.

Bounds on quantifier of indistinguishability

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Perfectly distinguishable particles

perfectly distinguishable particles if maximally violating indistinguishability condition, $\pi_N \rho_{ext} = \rho_{ext}$

Characterization of perfectly distinguishable states:

States whose eigenspaces are spanned (up to a permutationally invariant unitary) by computational basis vectors with distinct entries (e.g. $|1,2,3\rangle$) and their permutations (only possible if $M \geq N$).

Example: the fixed mode occupation state $\rho = \sum_{\pi \in S_N} \pi |1, \dots, N\rangle \langle 1, \dots, N| \pi^t$ is perfectly distinguishable, also any state $\tilde{\rho} = U^{\otimes N} \rho (U^{\otimes N})^t$

Conclusion and Outlook

Gaining confidence about realization of arbitrary q. circuit

H & L learning in presence of weak dissipation

Indistinguishability of photons:

- Definition of (in)distinguishability for arbitrary states with fixed total particle number
- Measure of indistinguishability + bounds on measure (experiment)

Outlook: Using $P^{(k)}$ to learn more about indistinguishability; relation to entanglement/interference? How can we characterize the source producing the photons?



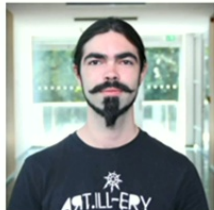
Thanks to

T. Olsacher, CH. Kokail, P. Zoller

Jose Carrasco

Antoine Neven

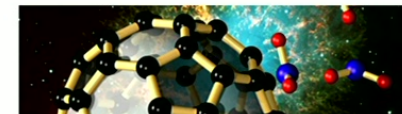
Tristan Kraft



Marc Langer



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