Title: Entanglement Distillation in Holography

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Subject: Quantum Information

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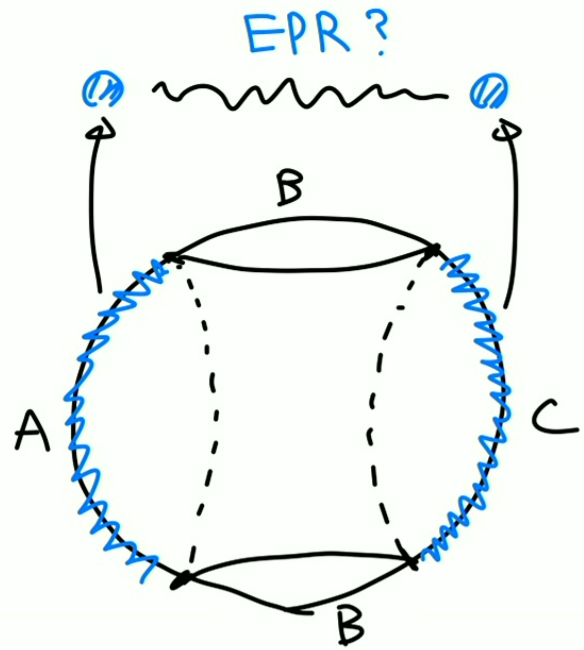
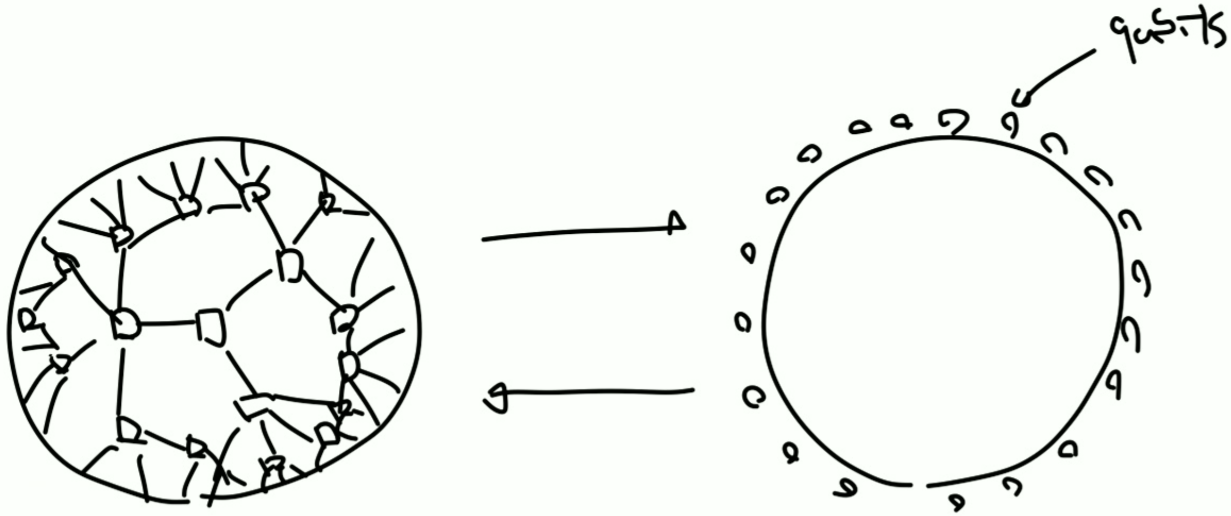
Entanglement Distillation in Holography



Beni Yoshida (Perimeter)

with Takato Mori

AdS/CFT correspondence

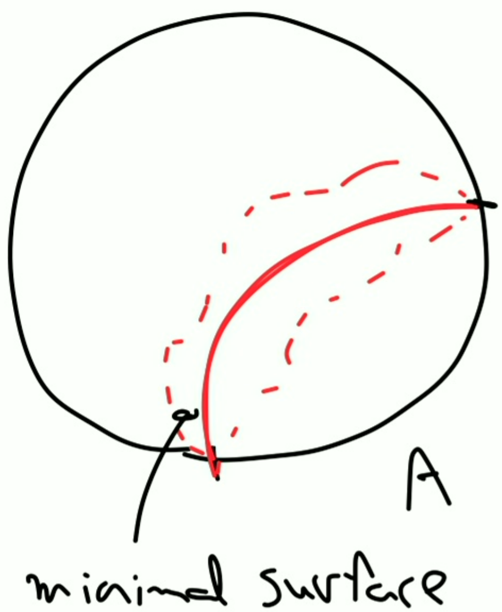


gravity in AdS space

qubits on boundary

Ryu–Takayanagi formula

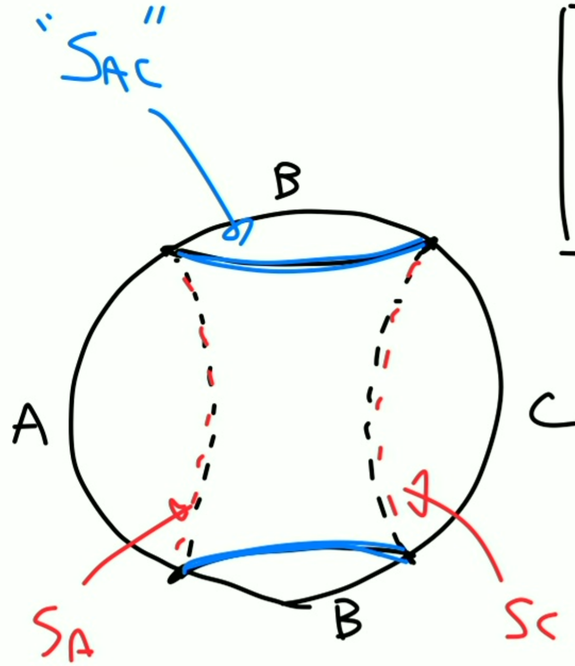
Entanglement entropy S_A



$$\boxed{S_A = \frac{1}{4G_N} \min_{\gamma_A} \mathrm{Area}(\gamma_A) + \cdots}$$

Very very big

How are A and C entangled ?



Mutual Info

$$I(A,C) \equiv S_A + S_C - S_{AC} = O(1/G_N)$$

- Not classical correlation
- Mostly bipartite (EPR) ?

No ... !! (Akers et al)

(One-shot) Distillable Entanglement

of EPR pairs distillable from ρ_{AC}.

single-copy

$E_D^{[LOCC]}$ v.s. $E_D^{[LO]}$

LO : Local operation

CC : Classical communication

(e.g. Measurement outcome)

Known Results

$$\boxed{\begin{array}{l} \mathrm{hash}(A,C) \lesssim E_D^{[LOCC]} \leq \frac{1}{2} I(A,C) \\ E_D^{[LO]} \quad ??? \end{array}}$$

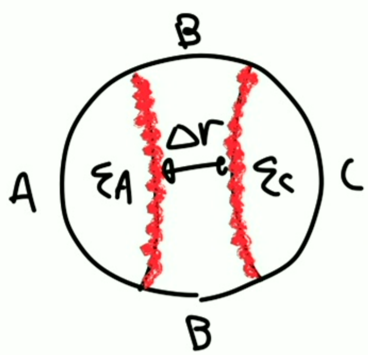
hashing bound

$$\mathrm{hash} \equiv \max(S_A - S_{AC},\ S_C - S_{AC})$$

Main Claims (1)

1). $\boxed{E_D^{[LO]} \simeq 0.}$ Mostly non-bipartite

if minimal surfaces are separated.



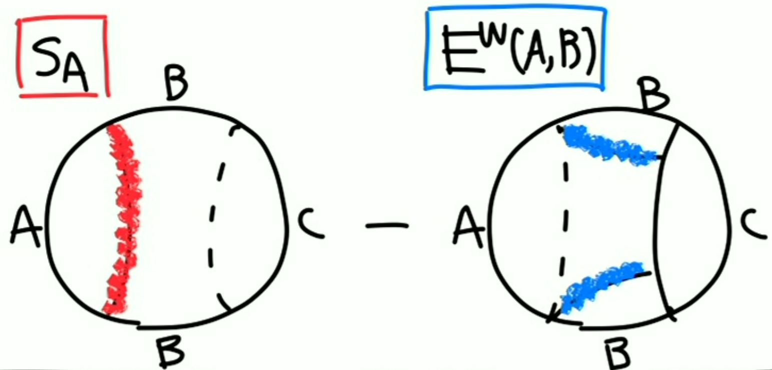
($\Delta r \gg l_{Planck}$).

Main Claims (2)

2). $E_D^{[LOCC]} \simeq J(A:C) \equiv \max\left(J(A|C), J(C|A)\right)$

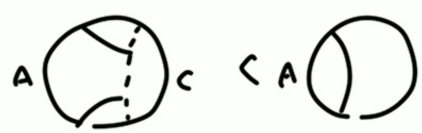
Locally Accessible Information

$J(A|C) \simeq$

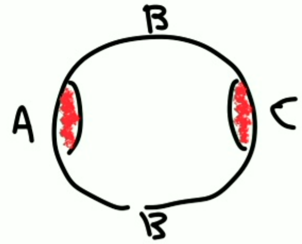
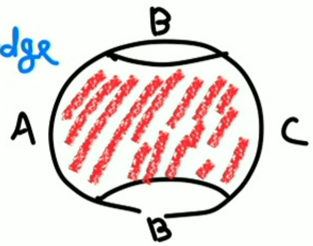
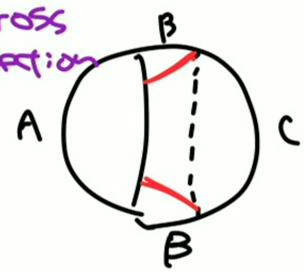


Connected wedge is NOT enough ...

$J(A|C), J(C|A)$ is needed !!



Three regimes

	E_D^{LO}	E_D^{LOCC}	$\frac{1}{2} I(A,C)$
	×	×	×
wedge 	×	×	$O(1/G_N)$
cross-section 	×	$O(1/G_N)$	$O(1/G_N)$

bound entanglement ?

separation

Main Claims (3)

3). Subleading Effects

$E_D^{[LO]} \simeq 0$ at leading order in $O(1/G_N)$.

$E_D^{[LOCC]} \simeq J(A:C)$

Violations at subleading order

* Traversable wormhole

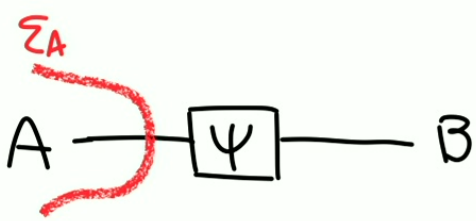
* Holographic Scattering.

* Planck-scale effect

Haar as Holography (1)

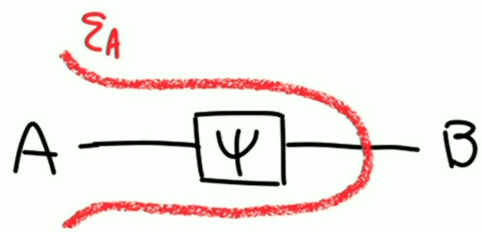
n-qubit Haar random state in bipartition

$$\boxed{S_A \simeq \min(|A|,\ n-|A|) \quad \text{"RT"-formula}}$$



$S_A = n_A$

$(A < B)$



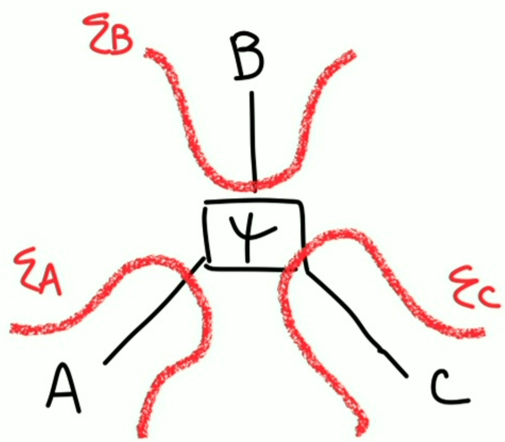
$S_A = n_B$

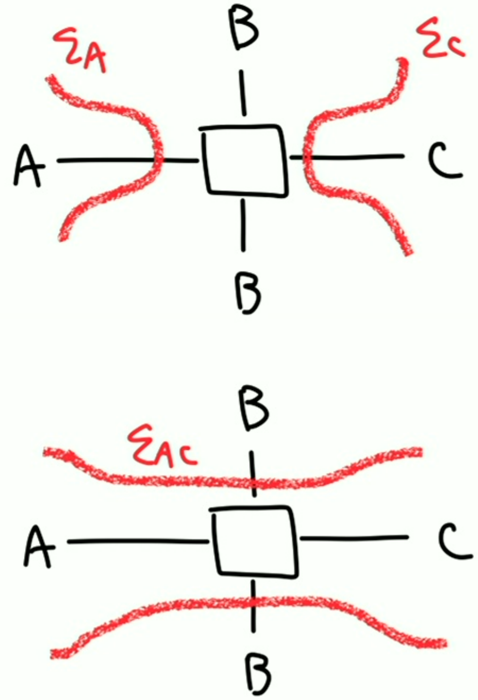
$(A > B)$

Haar as Holography (2)

Tripartition $\left(A, B, C \leq \frac{n}{2} \right)$

$$I(A,C) \simeq O(n)$$





Counting Argument (1)

n-qubit Hilbert space has

* 2^n orthogonal states $|1\rangle, |2\rangle, \cdots, |2^n\rangle$

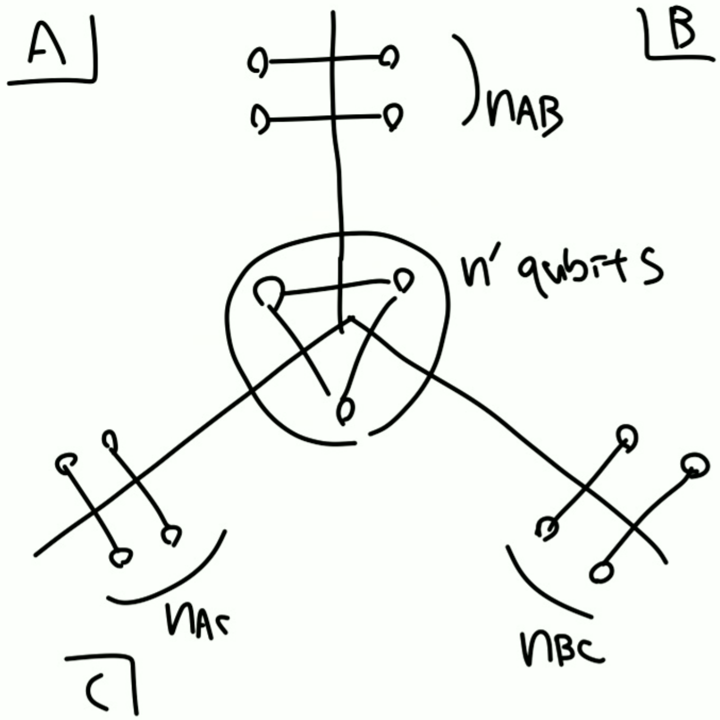
* 2^{2^n} nearly orthogonal states

e.g.

$$|\psi\rangle = \frac{1}{\sqrt{2^n}}\Big(\underbrace{c_1}_{\pm 1} |1\rangle + c_2 |2\rangle + \cdots\cdots + c_{2^n} |2^n\rangle \Big)$$

$$|\langle\psi|\psi'\rangle| \simeq 0 .$$

Counting Argument (2)



$$U_A \otimes U_B \otimes U_C |\Psi\rangle$$

How many states?

$$n' = n - 2(n_{AB} + n_{BC} + n_{AC})$$

Petz Recovery map

$$D_{\mathrm{Petz}}(\cdot) = \rho^{1/2} \mathcal{N}^{\dagger}\left[\mathcal{N}(\rho)^{-1/2}(\cdot)\,\mathcal{N}(\rho)^{-1/2}\right]\rho^{1/2}.$$

Barnum & Knill theorem

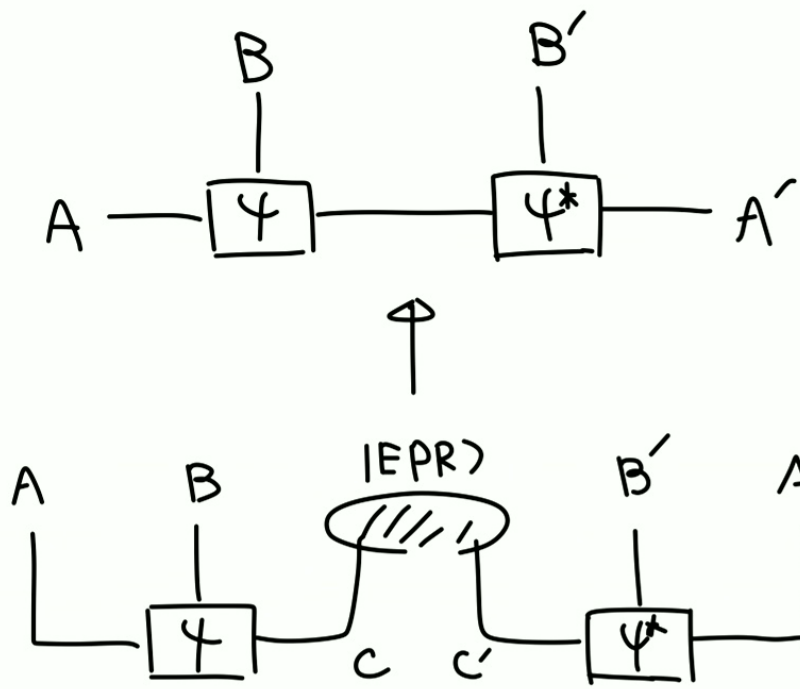
If there exists a good decoder D_{best}, then Petz map D_{Petz} also decodes well.

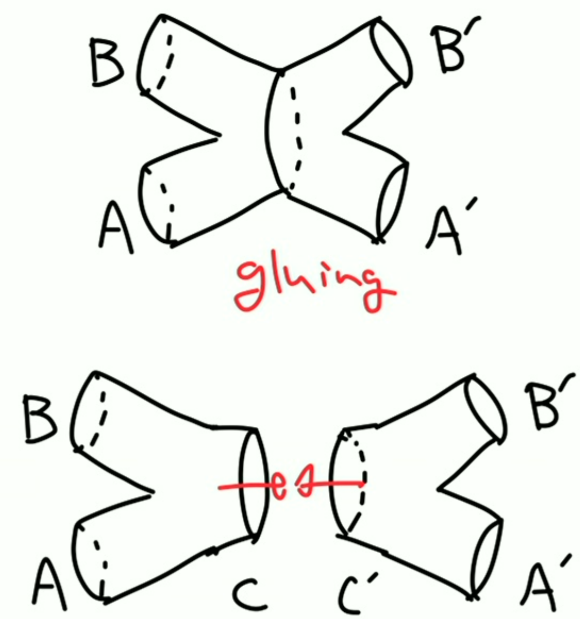
Entanglement Fidelity ($\simeq$ LO entanglement distillation)

$$\boxed{\underbrace{F_{D_{\mathrm{Petz}}}}_{1-2\varepsilon} \geq \Big(\underbrace{F_{D_{\mathrm{best}}}}_{1-\varepsilon}\Big)^{2}}$$

Double-copy state (1)

$\rho_{AA'}$ is entangled?

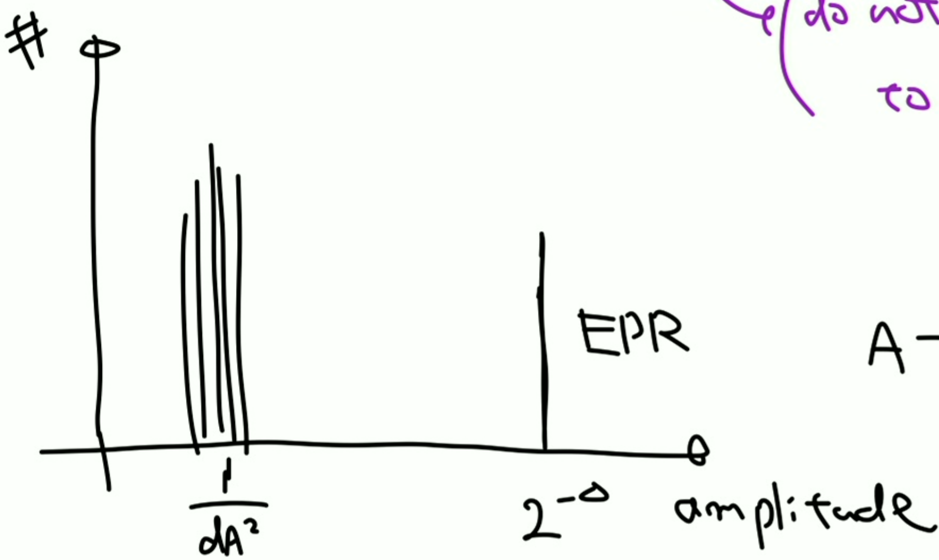


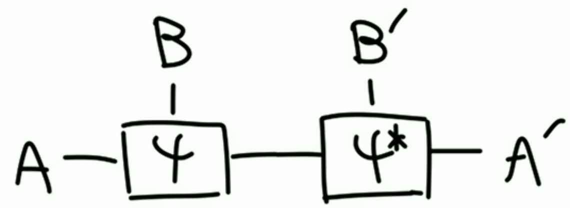


Double-copy state (2) $\quad (C,\ B > A)$

$$\rho_{AA'} \simeq \underbrace{2^{-\Delta}\,|EPR\rangle\langle EPR|}_{\text{(do not contribute to } E_D^{[LO]}\text{)}} + (1-2^{-\Delta})\,\underbrace{\mathbb{1}_{max}}_{\text{max-mixed state}}$$

$$\Delta \equiv n_A + n_B - n_C$$





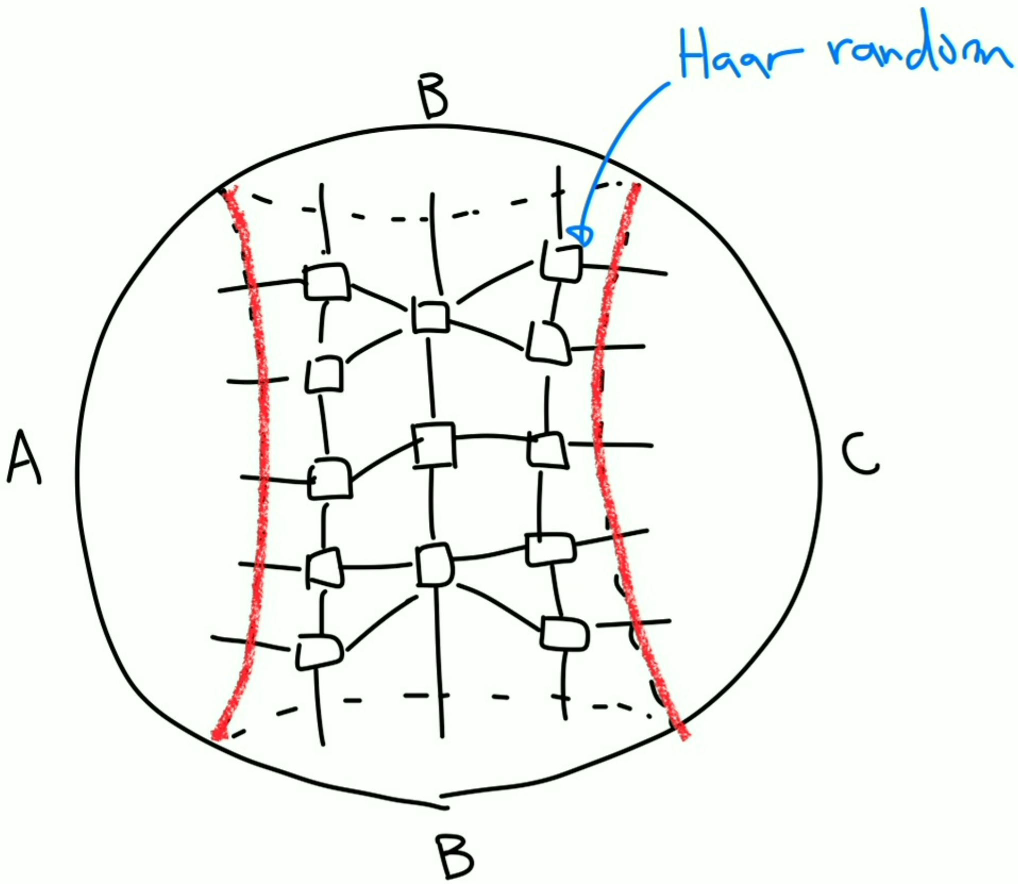
Double-copy state (2) $\quad (C, B > A)$

$$\boxed{\rho_{AA'} \simeq \underbrace{2^{-\Delta} |EPR\rangle\langle EPR|}_{\text{(do not contribute to } E_D^{[LO]})} + (1-2^{-\Delta})\, \underset{\substack{\uparrow \\ \text{max-mixed state}}}{\mathbb{1}_{max}} \qquad \Delta \equiv n_A + n_B - n_C}$$

$$I(A:A') \approx 0 \qquad (C, B > A)$$

$$\Rightarrow \boxed{E_D^{[LO]}(A:C) \approx 0} \qquad \begin{pmatrix} B, C > A \\ A, B > C \end{pmatrix}.$$

Holographic Tensor Network

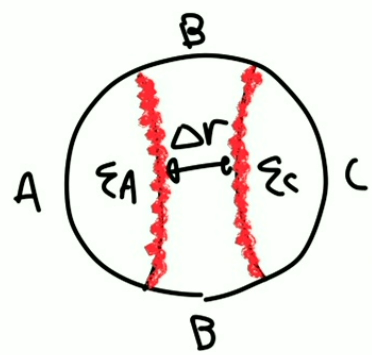


LO distillable entanglement

$\boxed{E_D^{[LO]} \simeq 0.}$

Mostly non-bipartite

if minimal surfaces are separated.



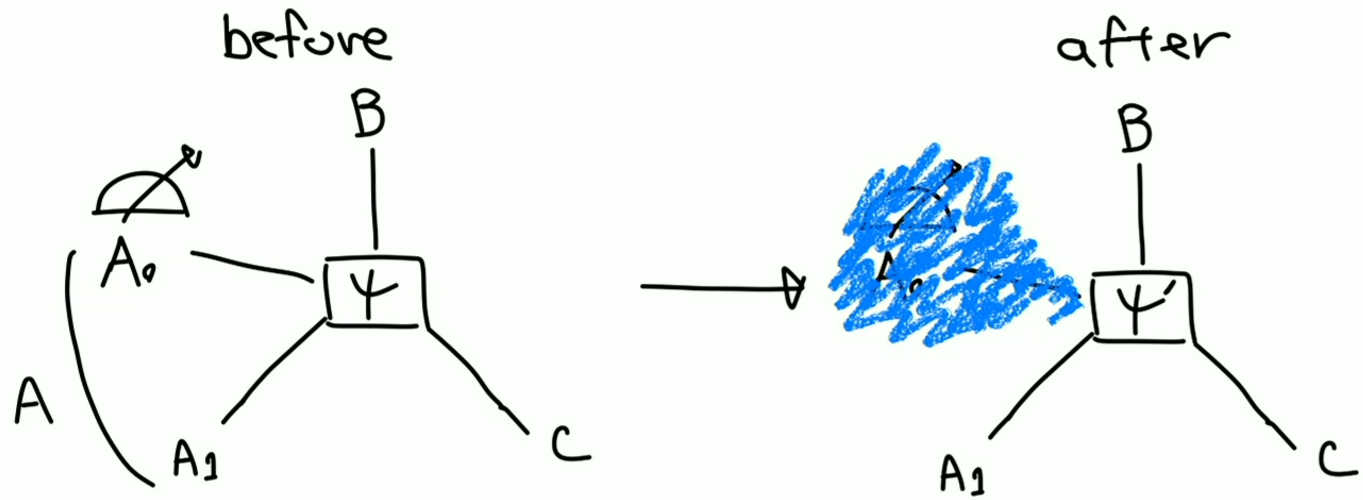
$(\Delta r \gg l_{Planck}).$

LOCC distillable entanglement

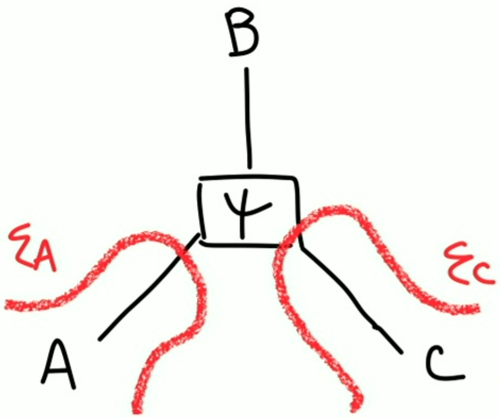
* $\mathrm{hash}(A,C) \equiv \max(S_A - S_{AC}, S_C - S_{AC}) \leq E_D^{[LOCC]}$

* $|A_1|$ EPR pairs if $|C| \geq |A_1| + |B|$

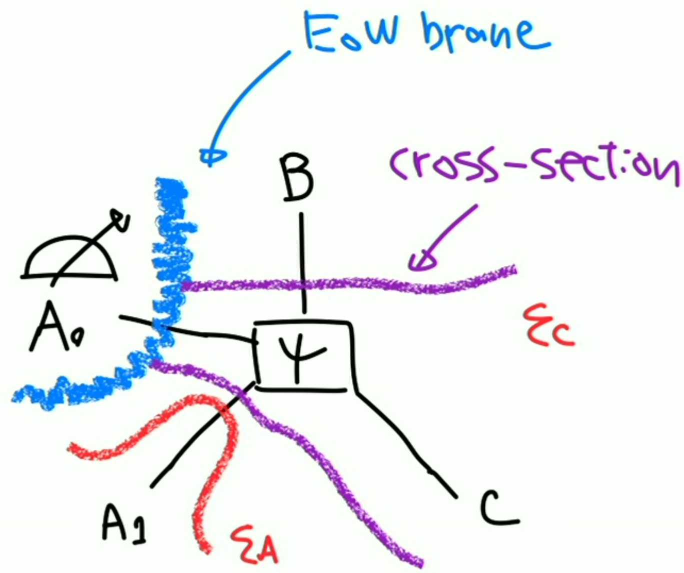
$$|A_1| \simeq S_C - S_B = S_C - S_{AC}$$



Measurement and EoW brane



Not LO-distillable



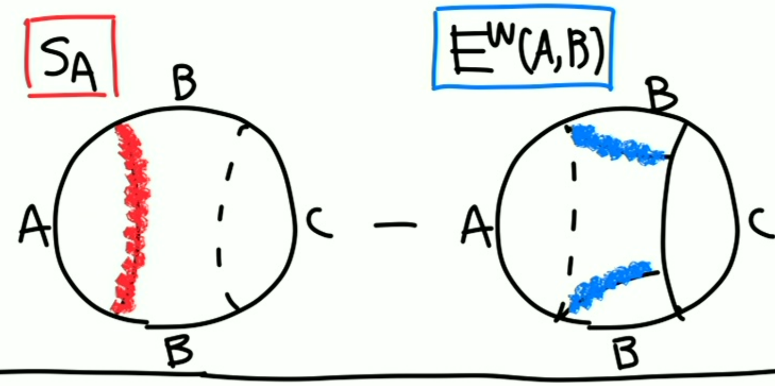
LOCC-distillable

Main Claims (2)

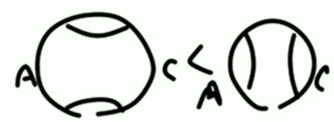
2). $E_D^{[LOCC]} \simeq J(A:C) \equiv \max\left(J(A|C), J(C|A)\right)$

Locally Accessible Information

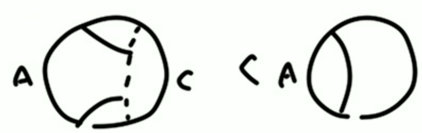
$J(A|C) \simeq$



Connected wedge is NOT enough ...



$J(A|C), J(C|A)$ is needed !!

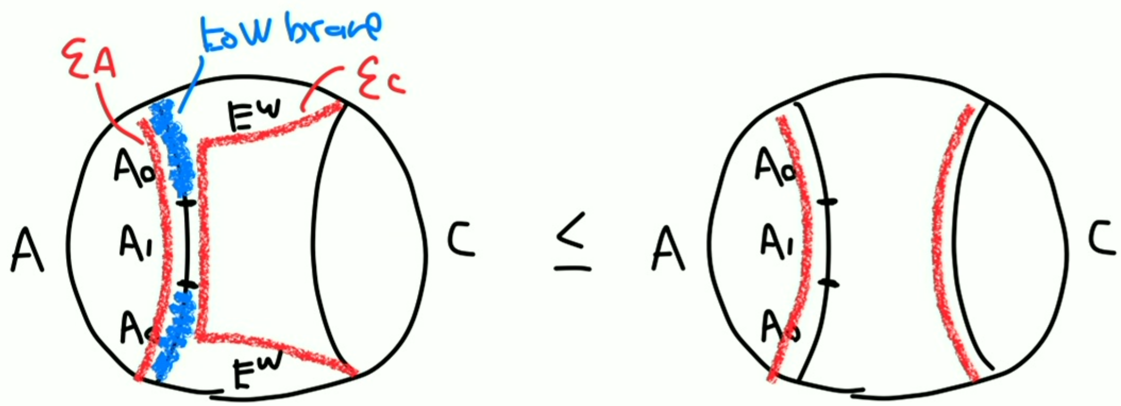


LOCC distillation protocol

S_{A_1} EPR pairs if

$$S_{A_1} + E^W(B,C) \leq S_C$$

$$\Rightarrow \quad \boxed{S_C - E^W(B,C) \quad \text{distillable}} = J(C|A)$$



What we can show

- Focusing on "gravitational" LOCCs (G-LOCCs) whose post-measurement states have semiclassical duals, we have

$$\boxed{E_D^{(G-LOCC)}(A:C) \simeq S_A - E^W(A:B) \qquad S_C - E^W(B:C)}$$

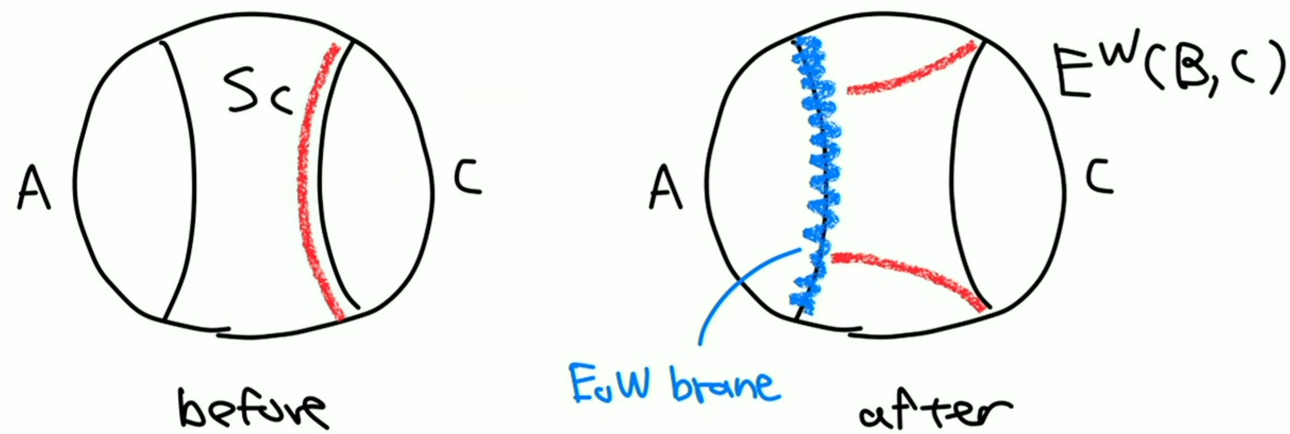
- Optimality follows from "monotonicity" relations

Locally Accessible Information

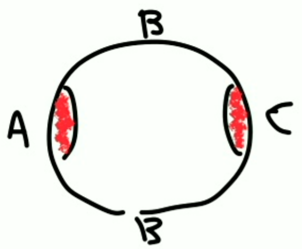
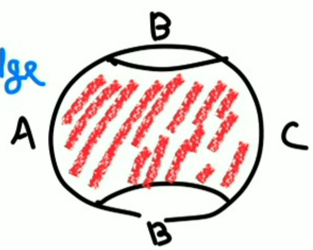
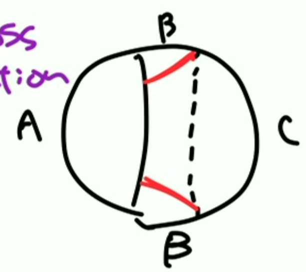
Entropy drop ΔS_C due to POVM in A

$$J(C|A) \equiv \max_{\text{POVM}} S_C^{\text{before}} - S_C^{\text{after}}$$

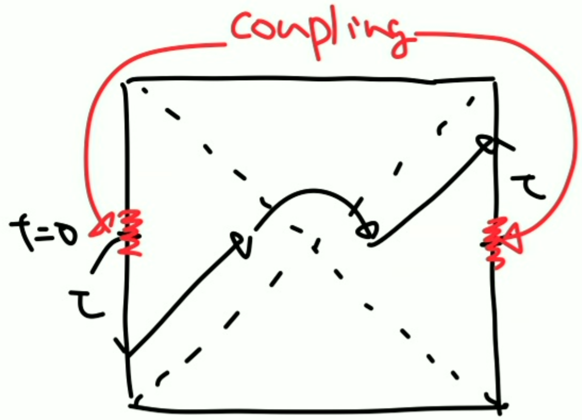
$$= S_C - E^W(B,C)$$ If POVM $\in$ EoW insertions

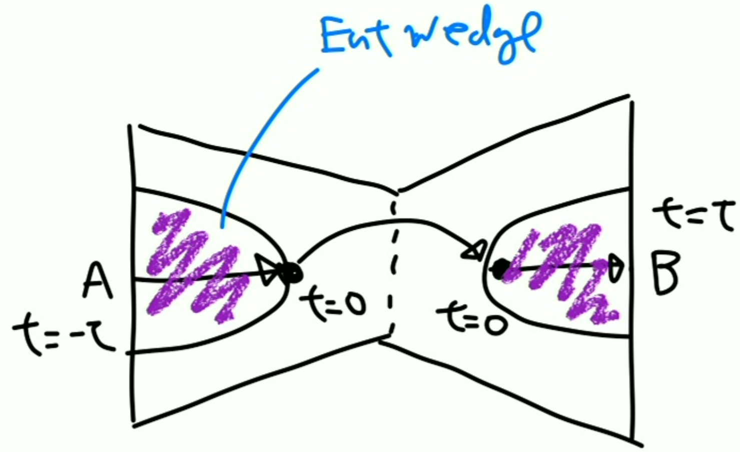


Summary of results

	E_D^{LO}	E_D^{LOCC}	$\frac{1}{2} I(A,C)$
	×	×	×
wedge 	×	×	$O\left(\frac{1}{G_N}\right)$
cross-section 	×	$J(C\vert A)$ $J(A\vert C)$ Optimal ?	$O\left(\frac{1}{G_N}\right)$

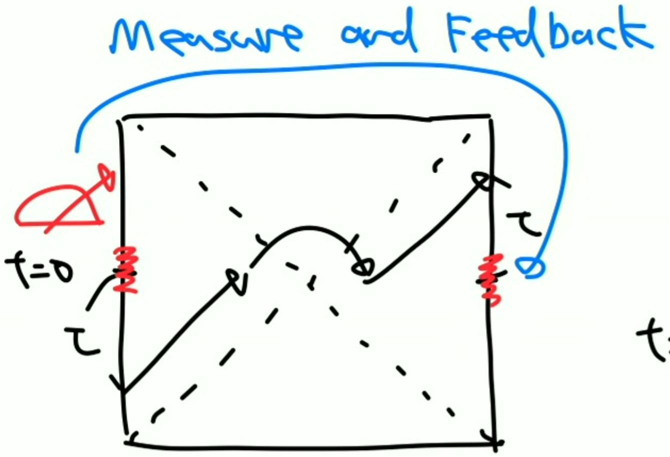
Traversable Wormhole

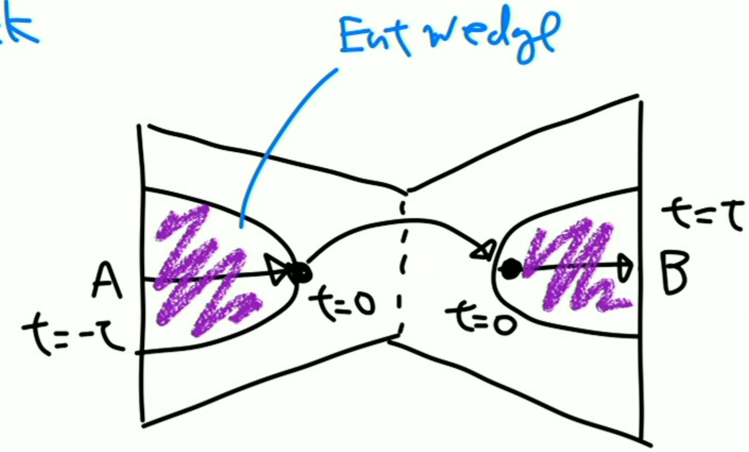




$$\exp(i\theta\, O_L \otimes O_R)$$

Quantum Teleportation through wormhole



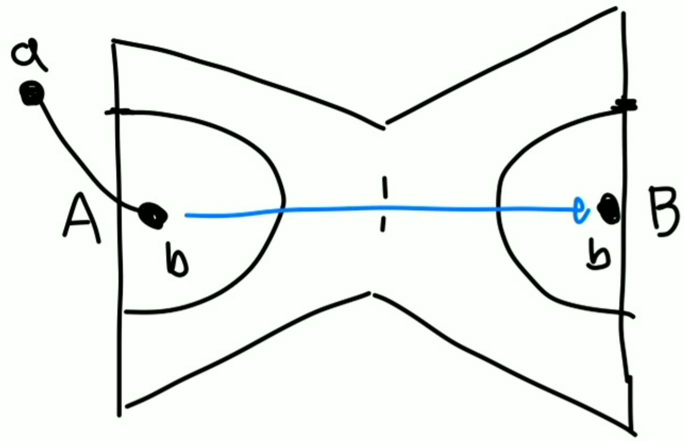


$$\exp(i\Theta\, O_L \otimes O_R) \longrightarrow \exp(\underbrace{\pm}_{} i\Theta\, O_R)$$

depends on measurement

Entanglement Distillation from wormhole

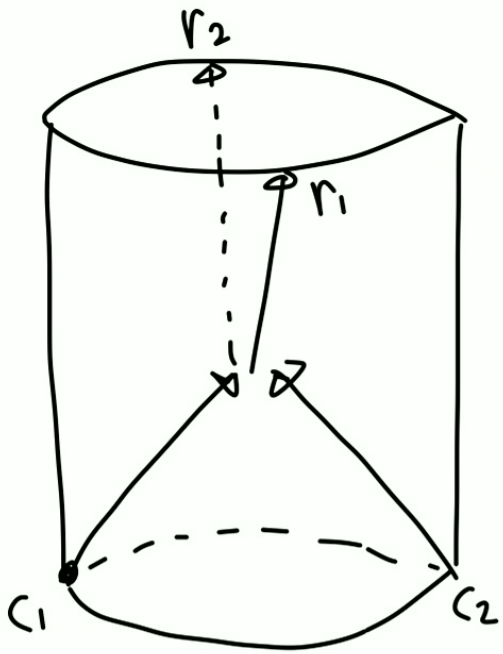
$E_D^{LOCC} \neq 0$!! $\leftarrow \rho_{AB}$



Prepare EPR on A

Send one part to B.

Holographic Scattering

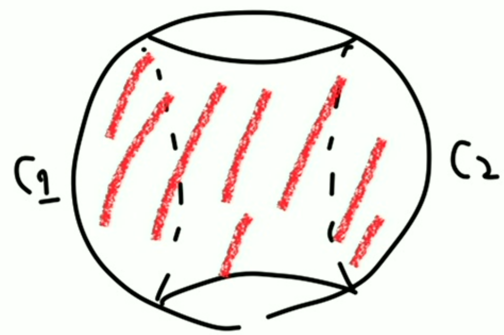


Quantum entanglement

$\downarrow$

Interaction

$E_D^{[LOCC]} \neq 0$?



Remark on Entanglement of Formation (2)

Umemoto showed

$$E_F \neq E^W \quad \cdots \; ?$$

Resolution

$$\boxed{E_F \simeq E^W} \quad \text{at leading order}$$

Subleading violation occurs ... !

Main claim

E_D and E_F at leading order

$$\boxed{LAI \simeq E_D^{[LOCC]} < \frac{1}{2} I(A,C) < E_F \simeq E^W .}$$

Monogamy relation

holographic dual of Koashi-Winter

$$\boxed{E_D^{LOCC}(A,C) = S_A - E_F(A,B) \quad (\text{if } J(A|C) \geq J(C|A))}$$