Title: Efficient quantum simulations with quDits

Speakers: Christine Muschik

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Subject: Quantum Information

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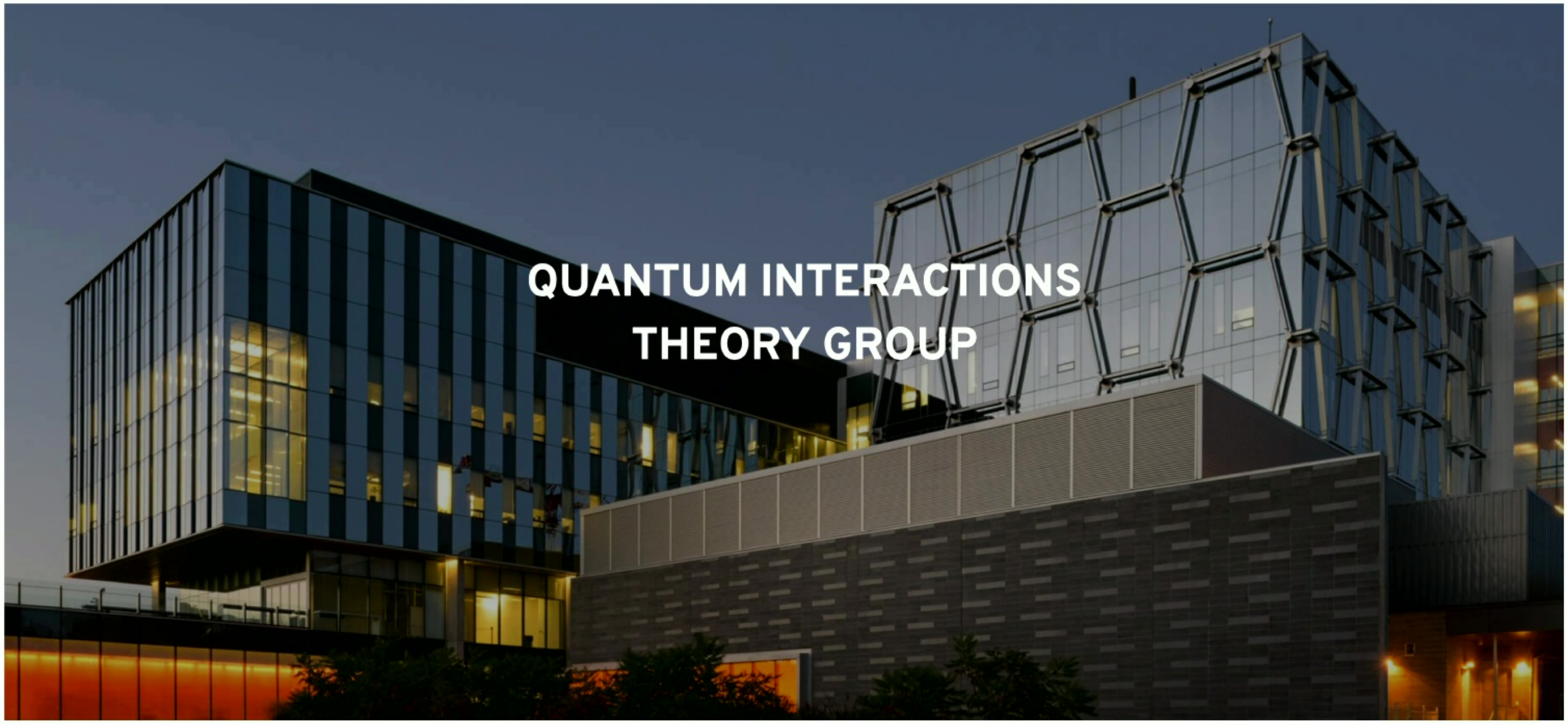
Hardware-efficient quantum computing using qudits

Christine Muschik

Quantum Interactions



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Simulating 2D lattice gauge theories on a qudit quantum computer

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[6] *Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, Technikerstraße 21a, Innsbruck, Austria*
[7] *Alpine Quantum Technologies GmbH, Innsbruck, Austria*
[8] *Universität Innsbruck, Institut für Theoretische Physik, Technikerstraße 21a, Innsbruck, Austria*
[9] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario, N2L 2Y5, Canada*

Particle physics underpins our understanding of the world at a fundamental level by describing the interplay of matter and forces through gauge theories. Yet, despite their unmatched success, the intrinsic quantum mechanical nature of gauge theories makes important problem classes notoriously difficult to address with classical computational techniques. A promising way to overcome these roadblocks is offered by quantum computers, which are based on the same laws that make the classical computations so difficult. Here, we present a quantum computation of the properties of the basic building block of two-dimensional lattice quantum electrodynamics, involving both gauge fields and matter. This computation is made possible by the use of a trapped-ion qudit quantum processor, where quantum information is encoded in d different states per ion, rather than in two states as in qubits. Qudits are ideally suited for describing gauge fields, which are naturally high-dimensional, leading to a dramatic reduction in the quantum register size and circuit complexity. Using a variational quantum eigensolver we find the ground state of the model and observe the interplay between virtual pair creation and quantized magnetic field effects. The qudit approach further allows us to seamlessly observe the effect of different gauge field truncations by controlling the qudit dimension. Our results open the door for hardware-efficient quantum simulations with qudits in near-term quantum devices.

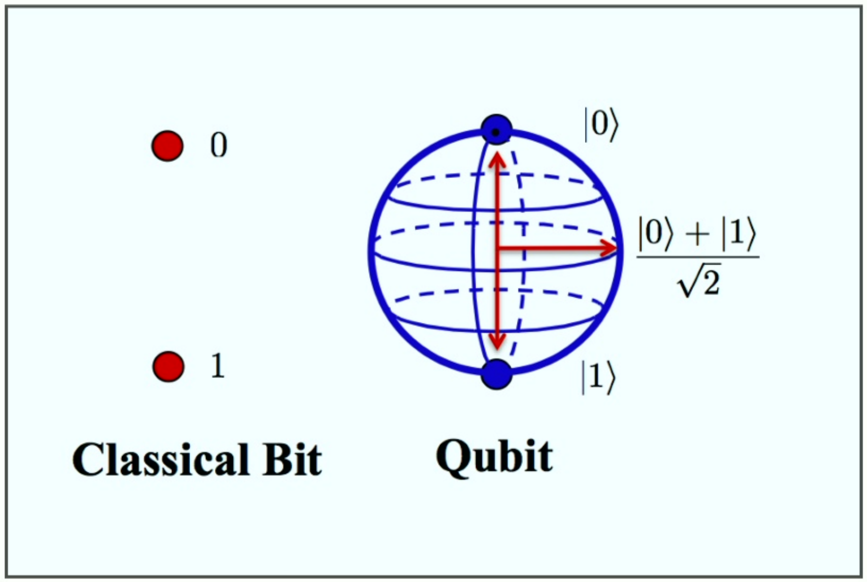
https://arxiv.org/abs/2310.12110

Overview

1. Introduction
2. Using qudits for problems in particle physics
3. Simulating both: gauge fields and matter
4. Increasing d
5. Conclusions

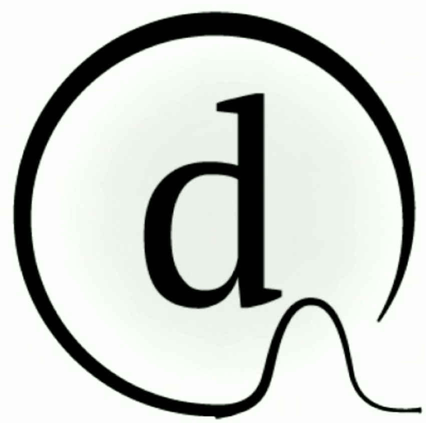
Classical computing today:
Almost exclusively based on binary encoding.

Quantum computing:
Qubit encoding currently dominating



quBelt

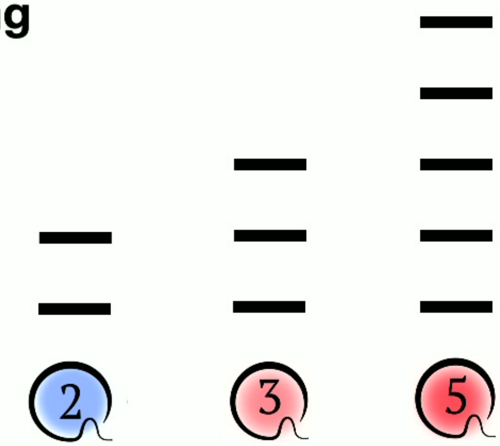
Beyond binary



Beyond binary

Today's quantum hardware: **capable of qudit encoding**

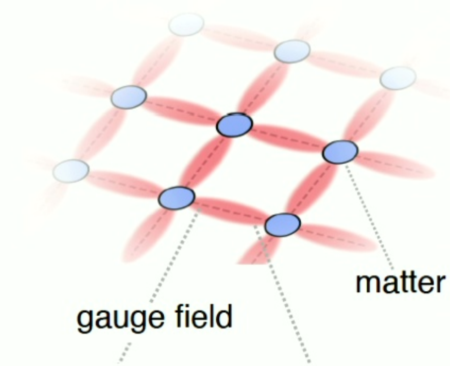
- Trapped ions
- Superconducting architectures
- Rydberg atoms in optical tweezers
- Ultracold atoms in optical lattices
- Nuclear spins
- Photonic systems



Beyond binary

Short-depth qu**d**it circuits

Lattice gauge theories



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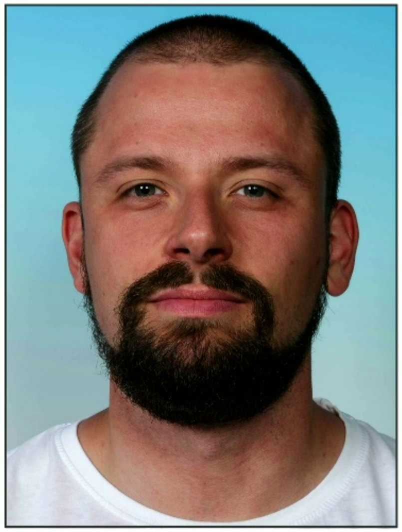
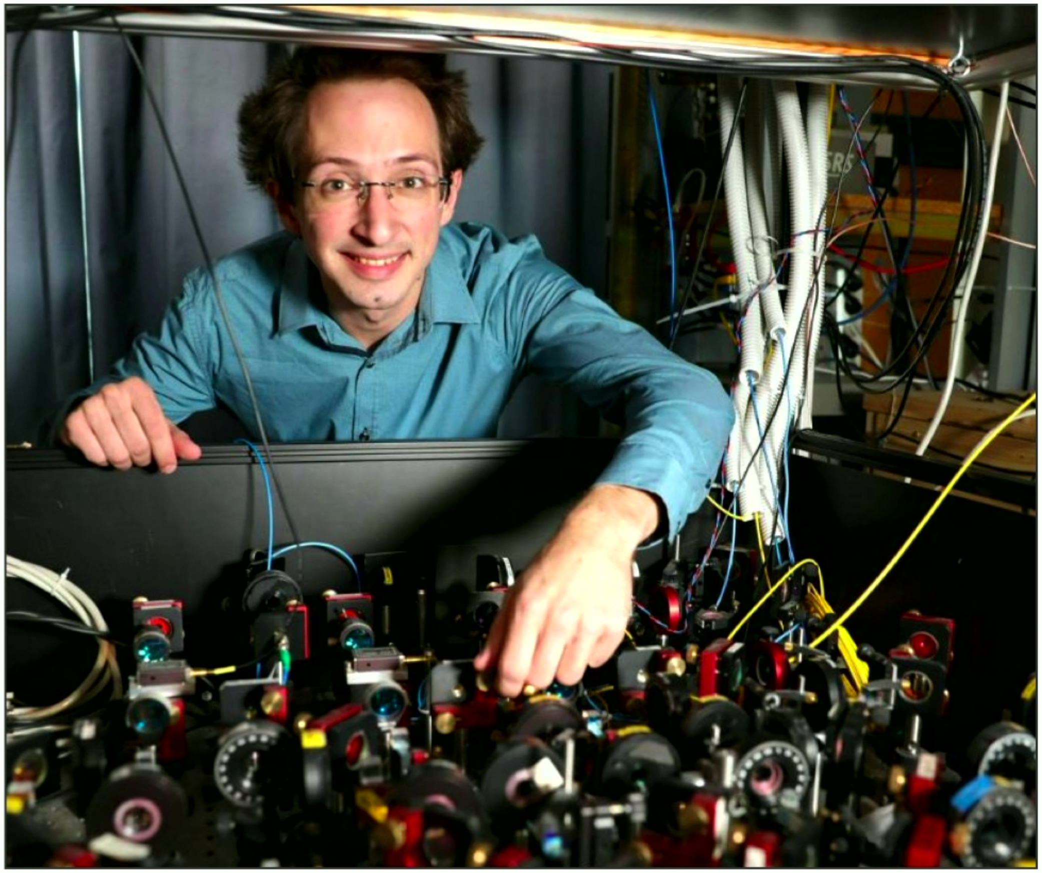


Mike Meth



Martin Ringbauer

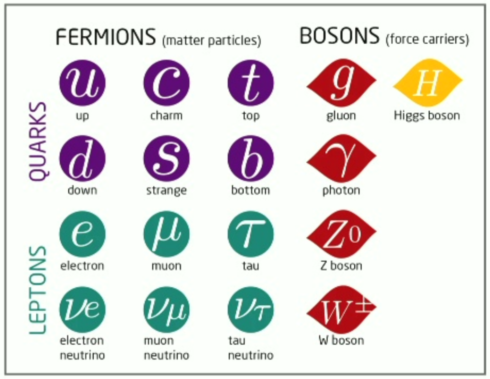
Waterloo & Innsbruck

Theorie	Experiment
Jan Haase	Michael Meth
Jinglei Zhang	Claire Edmunds
Andrew Jena	Lukas Postler
Luca Dellantonio	Thomas Monz
Peter Zoller	Philipp Schindler
Christine Muschik	Rainer Blatt
	Martin Ringbauer

arXiv:2310.12110

Gauge theories

Standard model of
particle physics



Condensed matter systems
(Frustration, topological order)

Gauge theories in particle physics
(Quantum Electrodynamics, Quantum Chromodynamics,…)

Simulations of gauge theories

particle physics

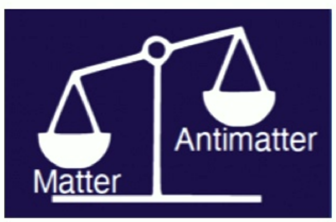
1. Understand the physics within the standard model of particle physics.

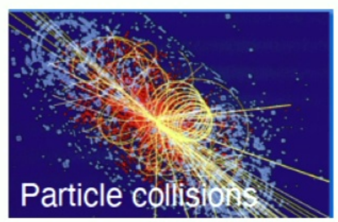
2. Understand where the standard model fails (observations don't agree with simulations) to discover new laws and to develop new theories beyond the standard model.

Inaccessible to traditional lattice gauge theory

Afflicted by sign problems

Topological terms	Real time evolutions	Chemical potentials
Fundamental symmetries	Dynamical processes	High matter density





Two major hurdles in quantum LGT calculations

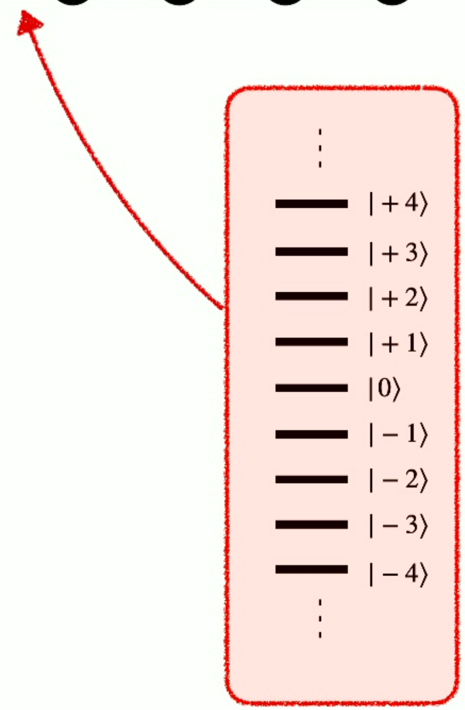
#1: Representing gauge fields

#2: Quantum simulations beyond 1D

Representing gauge fields

Matter: on vertices

Gauge fields: on links

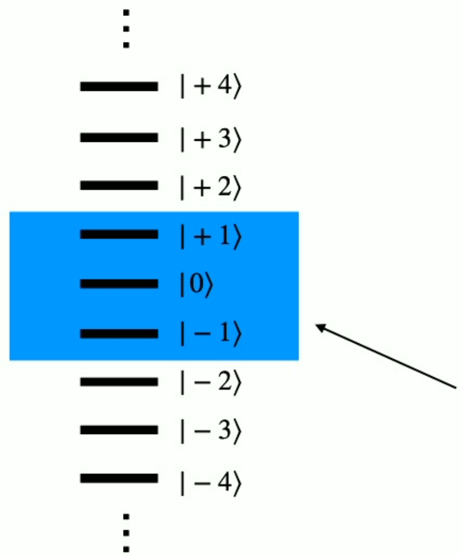


Gauge fields

Example: lattice QED

$$\hat{E}\,|E\rangle = E\,|E\rangle,$$

$$E = 0,\ \pm 1,\ \pm 2, \ldots$$



Classical simulations and finite dimensional quantum hardware require a **truncation**:

Minimal truncation: quTrit
- field in positive direction
- no flux
- field in negative direction

Truncations for bosonic systems:

- Schwinger boson representation
- Holstein-Primakoff-representation
- Dysen-Maleev transformation
- Highly occupied boson model

Truncations for qubit systems:

$$\hat{E} \longmapsto \hat{S}^z, \qquad \hat{U} \longmapsto \hat{V}^-,$$

$$\hat{V}^- \equiv \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ 1 & \cdots & \cdots & 0 \\ 0 & \ddots & \vdots & 0 \\ 0 & \cdots & 1 & 0 \end{bmatrix}$$

Alternative:

$$\hat{V}^- \equiv \hat{S}^- / |l|$$

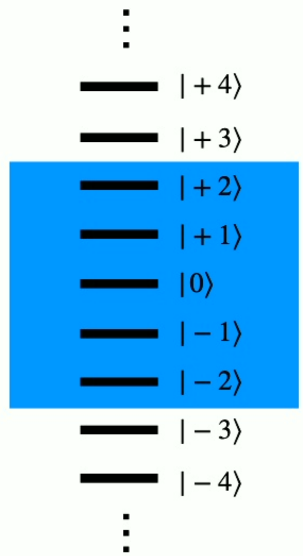
$$\hat{S}^- = \hat{S}^x - i\hat{S}^y$$

Gauge fields

Example: lattice QED

$\hat{E}\,|E\rangle = E\,|E\rangle,$

$E = 0,\ \pm 1,\ \pm 2, \ldots$



Classical simulations and finite dimensional quantum hardware require a **truncation**:

Better truncation: quQuint
- field in positive direction
- no flux
- field in negative direction

Truncations for bosonic systems:

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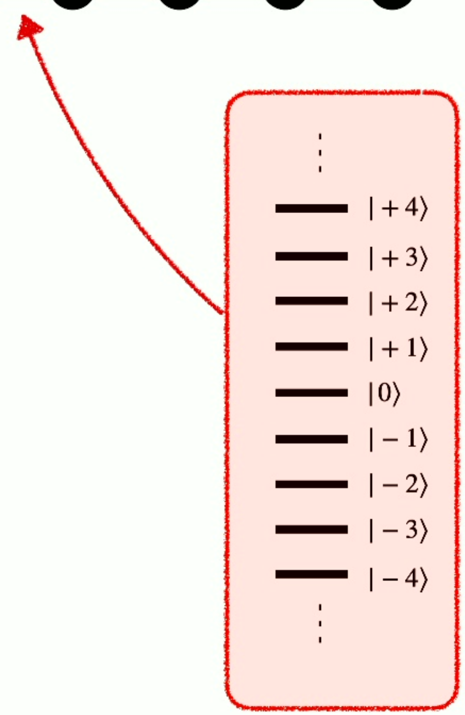
Representing gauge fields

Matter: on vertices

Gauge fields: on links



1D - gauge theory

- Gauge fields can be eliminated

Open boundary conditions

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

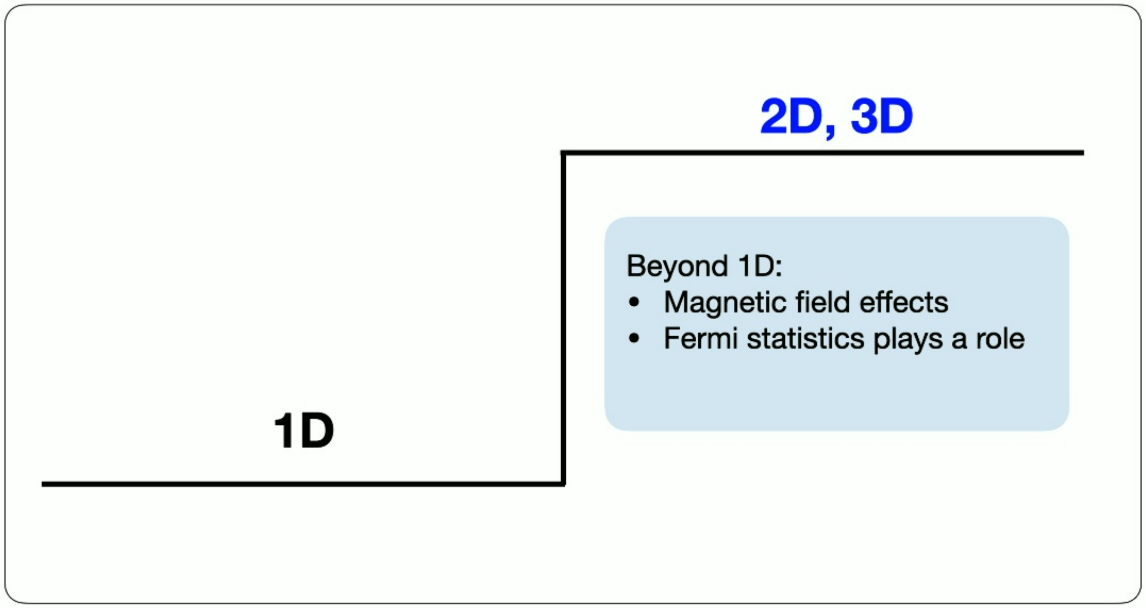
Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard,
Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

Nature **534**, 516–519 (2016)

Theory + Experiment

- Trapped ion quantum computer
- Quantum simulation of 1D-QED
- Real-time dynamics of electron-positron pair creation.

Gauge theories for particle physics beyond 1D

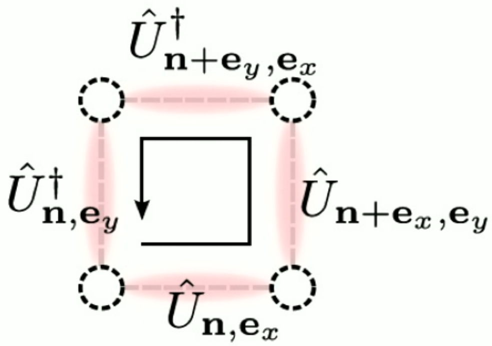


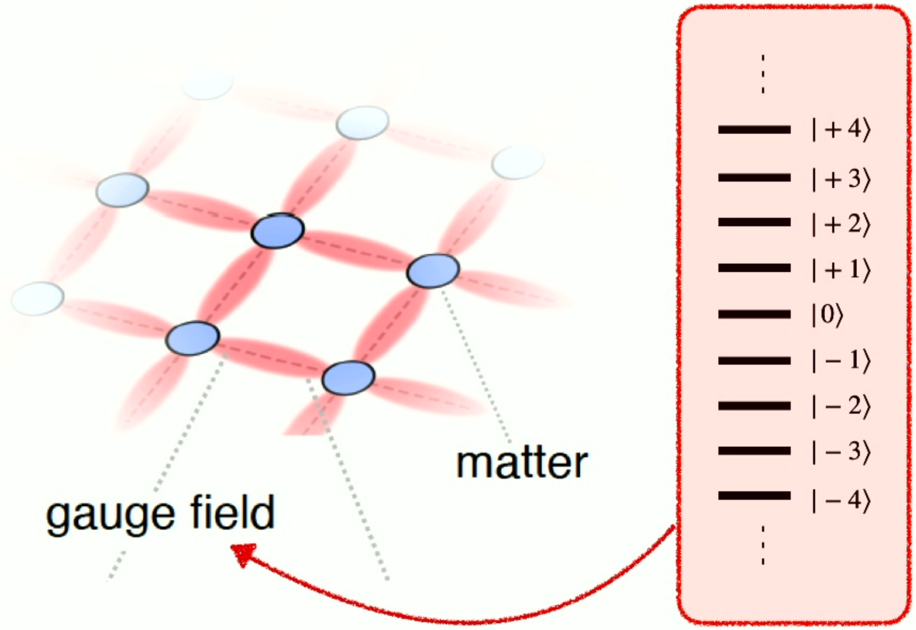
Gauge theories <u>beyond 1D</u>

2D gauge theory

- Not all gauge fields can be eliminated
- i.e. <u>"dynamical" gauge fields</u>

→ Magnetic field effects





SCIENTIFIC
AMERICAN

In a First, Quantum Computer Simulates High-Energy Physics

The technique could allow quantum computers to address otherwise-intractable problems in particle physics

By Davide Castelvecchi, Nature magazine on June 23, 2016

Muschik says that her team is already making plans to use two-dimensional configurations

Teaching quantum computers
to address problems from particle physics

Review article (QTFLAG collaboration):

Simulating Lattice Gauge Theories within Quantum Technologies: arXiv:1911.00003

Experimental demonstrations

Gauge theories for particle physics beyond 1D

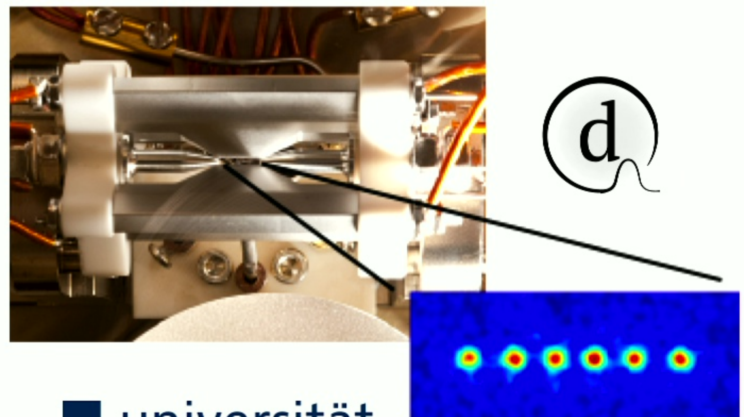
- N. Klco, M. J. Savage, and J. R. Stryker, Phys. Rev. D **101**, 074512 (2020).
- A. Ciavarella, N. Klco, and M. J. Savage, Phys. Rev. D **103**, 094501 (2021).
- S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D **104**, 034501 (2021).
- A. N. Ciavarella and I. A. Chernyshev, Phys. Rev. D **105**, 074504 (2022).
- S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, Phys. Rev. D **106**, 074502 (2022).
- S. A. Rahman, R. Lewis, E. Mendicelli, and S. Powell, "Real time evolution and a traveling excitation in SU(2) pure gauge theory on a quantum computer," (2022), arXiv:2210.11606 [hep-lat].
- A. N. Ciavarella, "Quantum Simulation of Lattice QCD with Improved Hamiltonians," (2023), arXiv:2307.05593 [hep-lat].

Impressive Advances!
(but gauge fields or matter fields are trivial)

Experimental qudit system

Linear ion-trap quantum processor
with all-to-all connectivity

Extending qubit entangling gates
to mixed-dimensional qudit systems



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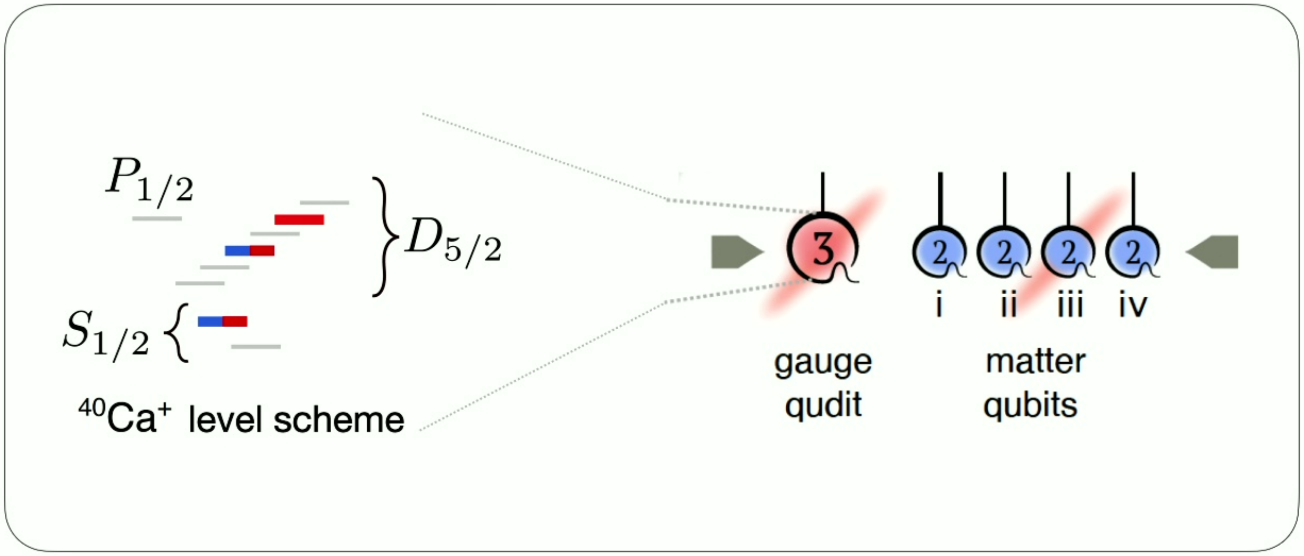
Martin Ringbauer

M. Ringbauer, M. Meth, L. Postler, R. Stricker, R. Blatt, P. Schindler, T. Monz, Nature Physics 18, 1053 (2022)

Encoding qudits in trapped ions



Experimental demonstration

Gauge theories for particle physics beyond 1D

Including both - dynamical gauge and matter fields

Necessary theory steps

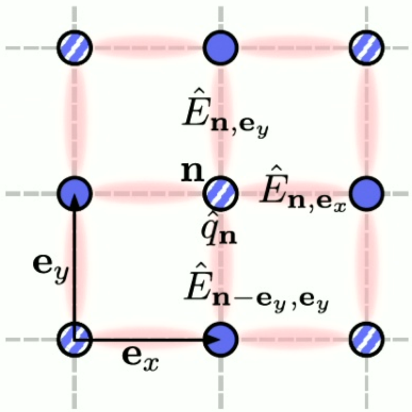
Step A: **Efficient Hamiltonian formulation**

A resource efficient approach for quantum and classical simulations of gauge theories in particle physics
J. Haase, L. Dellantonio, A. Celi, D. Paulson, A. Kan, K. Jansen, C. Muschik
Quantum 5, 393 (2021).

Step B: **Efficient Quantum Simulation scheme**

Towards simulating 2D effects in lattice gauge theories on a quantum computer
D. Paulson, L. Dellantonio, J. Haase, A. Celi, A. Kan, A. Jena, C. Kokail, R. van Bijnen, K. Jansen, P. Zoller, C. Muschik
PRX Quantum 2, 030334 (2021).

2D lattice QED



Even $n_x + n_y$ lattice sites

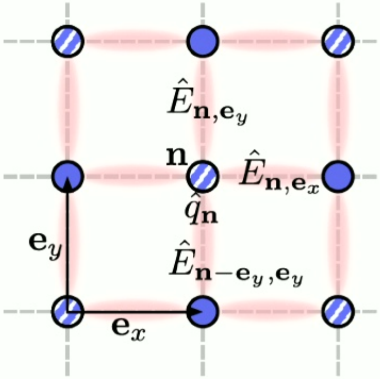
◌ $\cong |\downarrow\rangle \cong |e\rangle\,, \quad \hat{q}_{\mathbf{n}} = -1$

● $\cong |\uparrow\rangle \cong |v\rangle\,, \quad \hat{q}_{\mathbf{n}} = 0$

Odd $n_x + n_y$ lattice sites

◌ $\cong |\downarrow\rangle \cong |v\rangle\,, \quad \hat{q}_{\mathbf{n}} = 0$

● $\cong |\uparrow\rangle \cong |p\rangle\,, \quad \hat{q}_{\mathbf{n}} = +1$



2D lattice QED

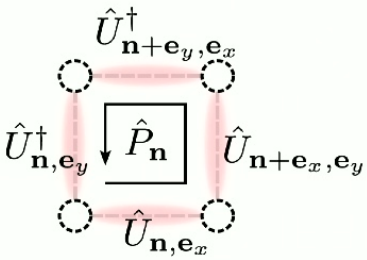
$$\boxed{\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2}\hat{H}_B + m\hat{H}_m + \Omega \hat{H}_k}$$

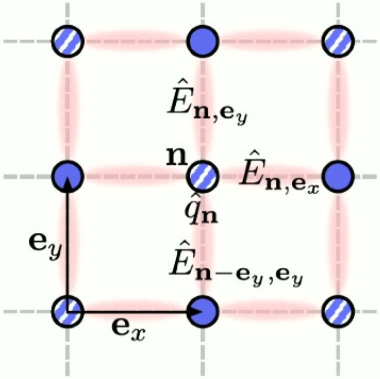
Electric field energy $\longrightarrow$ $\hat{H}_E = \frac{1}{2}\sum_{\mathbf{n}} \left(\hat{E}^2_{\mathbf{n},\mathbf{e}_x} + \hat{E}^2_{\mathbf{n},\mathbf{e}_y}\right),$

Magnetic field energy $\longrightarrow$ $\hat{H}_B = -\frac{1}{2}\sum_{\mathbf{n}} \left(\hat{P}_{\mathbf{n}} + \hat{P}^\dagger_{\mathbf{n}}\right),$ $\quad \hat{P}_{\mathbf{n}} = \hat{U}_{\mathbf{n},\mathbf{e}_x}\hat{U}_{\mathbf{n}+\mathbf{e}_x,\mathbf{e}_y}\hat{U}^\dagger_{\mathbf{n}+\mathbf{e}_y,\mathbf{e}_x}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_y}.$

Particle rest mass $\longrightarrow$ $\hat{H}_m = \sum_{\mathbf{n}} (-1)^{n_x+n_y}\hat{\phi}^\dagger_{\mathbf{n}}\hat{\phi}_{\mathbf{n}},$

Kinetic energy term (pair creation) $\longrightarrow$ $\hat{H}_k = \sum_{\mathbf{n}}\sum_{\mu=x,y}\left(\hat{\phi}_{\mathbf{n}}\hat{U}^\dagger_{\mathbf{n},\mathbf{e}_\mu}\hat{\phi}^\dagger_{\mathbf{n}+\mathbf{e}_\mu} + \text{H.c.}\right).$





2D lattice QED

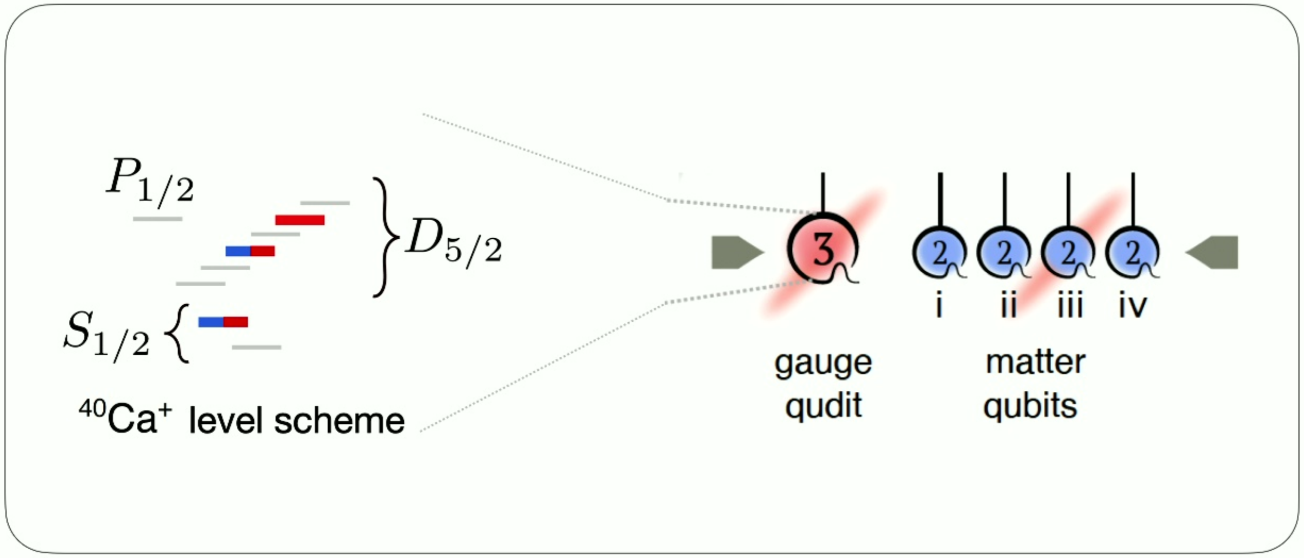
$$\boxed{\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2}\hat{H}_B + m\hat{H}_m + \Omega\hat{H}_k}$$

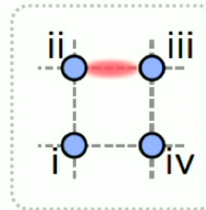
Classical Gauss law: $\nabla \mathbf{E}(\mathbf{r}) - \rho(\mathbf{r}) = 0$

QED Gauss law: $\hat{G}_{\mathbf{n}}|\Psi_{\text{phys}}\rangle = 0,$

$$\hat{G}_{\mathbf{n}} = \sum_{\mu}(\hat{E}_{\mathbf{n},\mathbf{e}_\mu} - \hat{E}_{\mathbf{n}-\mathbf{e}_\mu,\mathbf{e}_\mu}) - \hat{q}_{\mathbf{n}}$$

Experiment





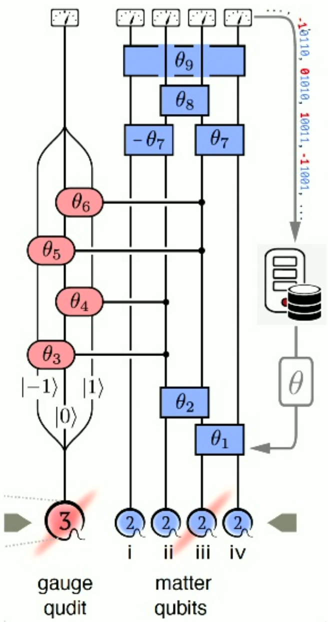
Basic building block: Plaquette

Experiment

Variational Quantum Eigensolver → Ground state preparation → Measure plaquette expectation value

Hybrid qubit-quTrit variational circuit

$\langle \hat{\square} \rangle \propto \langle \hat{H}_B \rangle$



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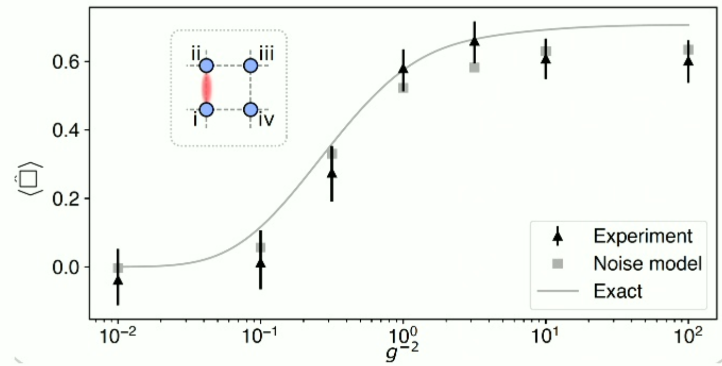
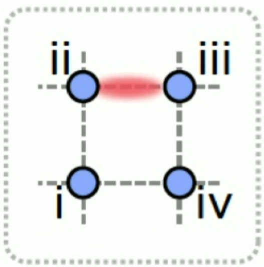


Qudit encoding: short circuits

Basic building block: Single plaquette:



	Qudit enc			Qubit enc.		
dimension d	3	5	7	3	5	7
register size	5	5	5	7	9	11
CNOT count	$\boxed{26}$	34	42	$\boxed{96}$	174	252
CNOT fidelity	99%					
approx. circ. fid.	77%	71%	66%	38%	17%	8%
CNOT fidelity	99.5%					
approx. circ. fid.	88%	84%	81%	62%	42%	28%

QuDit review article:

Y. Wang, Z. Hu, B. C. Sanders, and S. Kais, Front. Phys. 8 (2020).

3.14 reasons why **Barry Sanders** was the worst guest to spend PI day with:

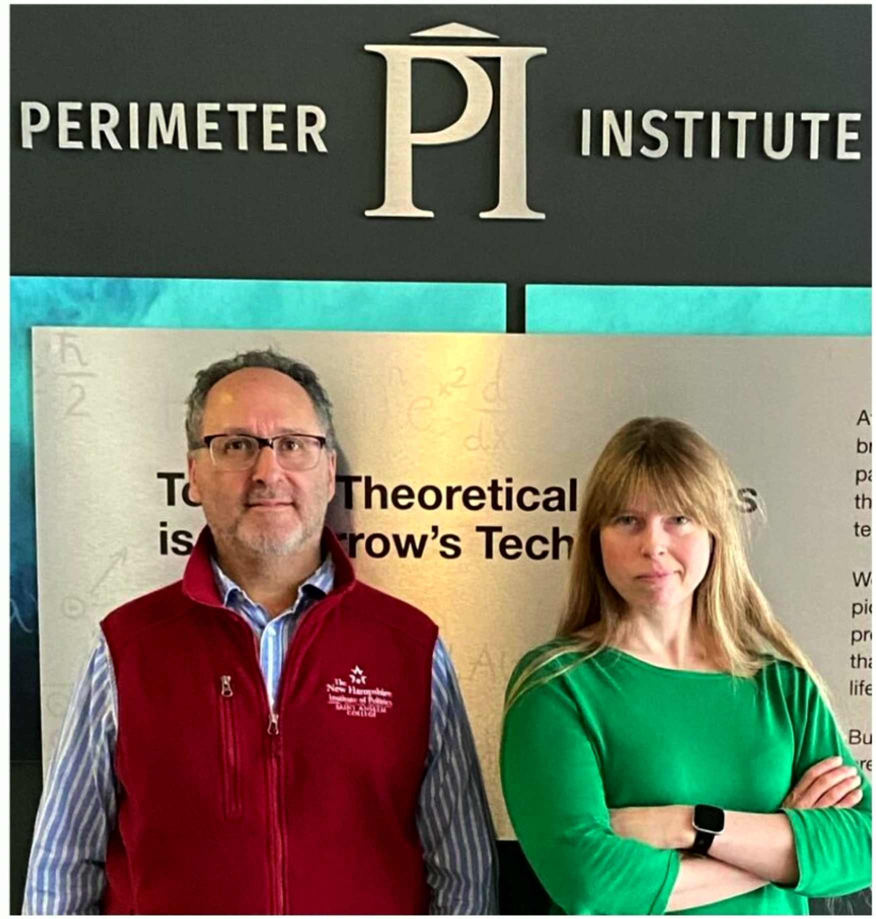
⭕ During his entire stay, I have not heard him say anything remotely irrational.
⭕ He repeated himself several times!
⭕ His visit has, unfortunately, definitely ended.

(and if you watched him moderating the Quantum Computing panel on the Quantum Days 2024, you know that he will never allow a conversation to go in a circle.)

On the bright side: We spent PI day at PI!

QuDit review article:

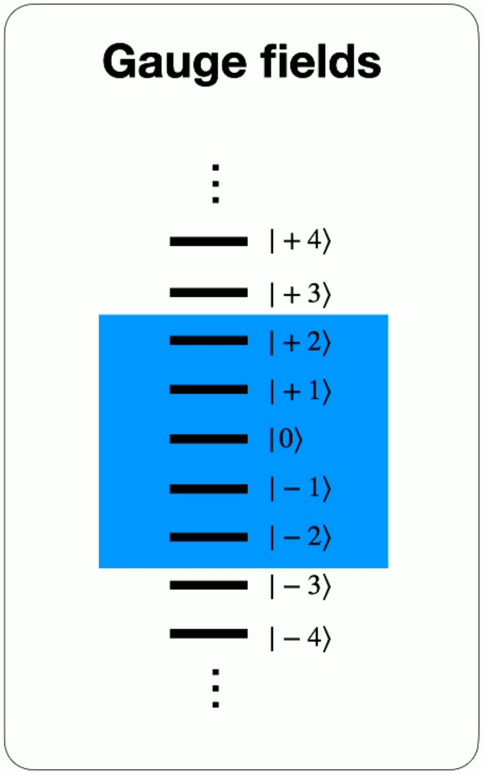
Y. Wang, Z. Hu, B. C. Sanders, and S. Kais, Front. Phys. 8 (2020).



Increasing d

So far:
- Relevant physics could be simulated.
- Using the minimal truncation $d = 3$.

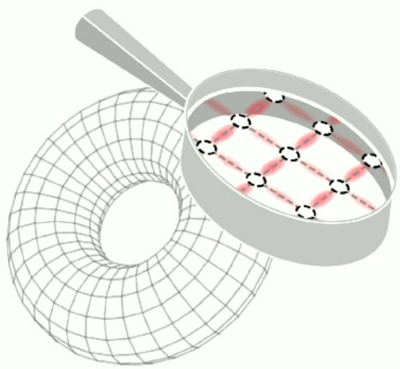
Next step:
- Study how we can improve the truncation.
- Go to $d = 5$.



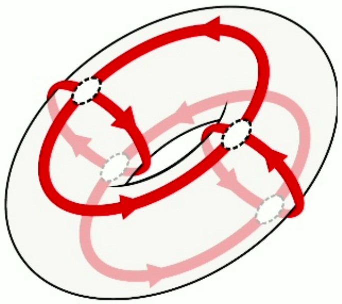
Increasing d

Improving the gauge-field truncation

Concrete example:
- Study the dependence of $\langle \hat{\square} \rangle$ on the bare coupling at different discretizations.
- Consider QED on a 2D lattice with periodic boundary conditions.
- No matter fields.



2D-QED with periodic boundary conditions



One plaquette with periodic boundary conditions

Increasing d

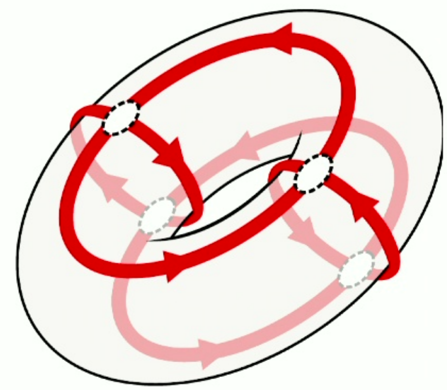
Improving the gauge-field truncation

Pure gauge theory:

$$\hat{H} = g^2 \hat{H}_E + (1/g^2)\hat{H}_B$$

Single plaquette:

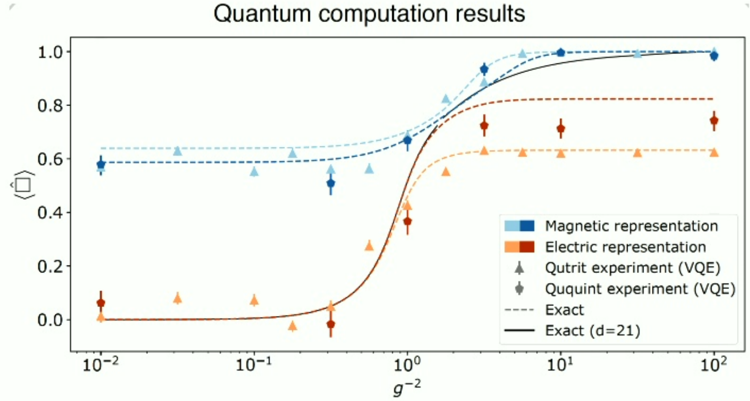
- 4 vertices.
- 8 links = 8 gauge fields.
- Gauss law: 5 independent gauge fields.
- For ground state properties **3 gauge fields**.

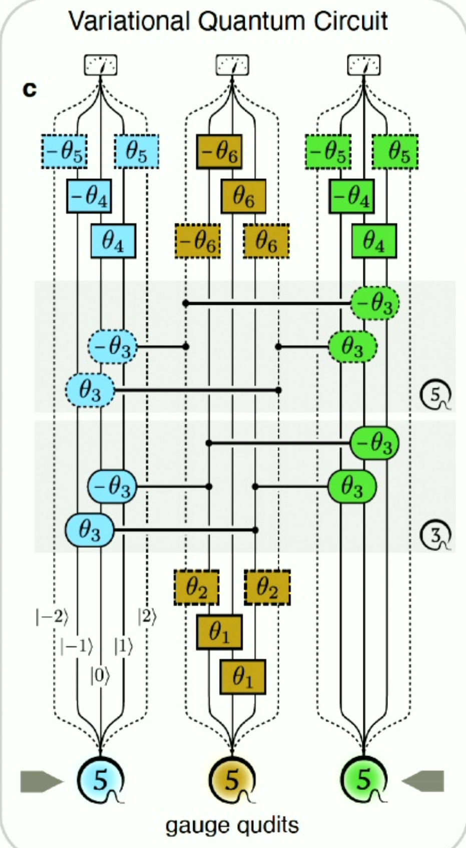


J. F. Haase, L. Dellantonio, A. Celi, D. Paulson, A. Kan, K. Jansen, and C. A. Muschik, Quantum **5**, 393 (2021).

Experiment:

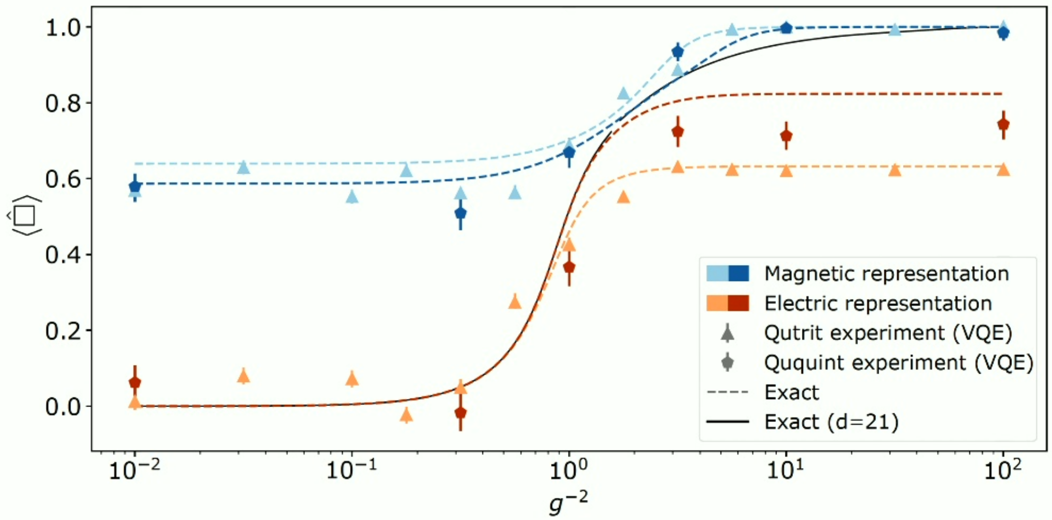
- VQE protocol to prepare the ground state.
- Measurement of $\langle \hat{\square} \rangle$ in the ground state
- for different values of the bare coupling g.





The closing of the gap between the values in the E-representation (red) and the B-representation (blue) corresponds to a better representation of the ground state.

Also: avoid “freezing” for large values of g^{-2}.



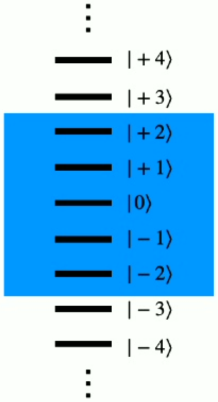
Take home messages:

- Full VQE ground state search for 3-level qutrits and 5-level ququints.
- Qudits allows to seamlessly observe the effect of different gauge field truncations by controlling the qudit dimension.

Experimental demonstration

Gauge theories for particle physics beyond 1D

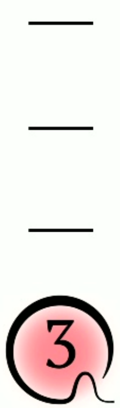
Including both - dynamical gauge and matter fields

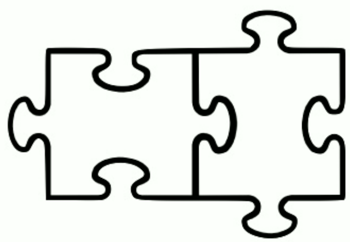


Adjustable discretisation of the gauge fields

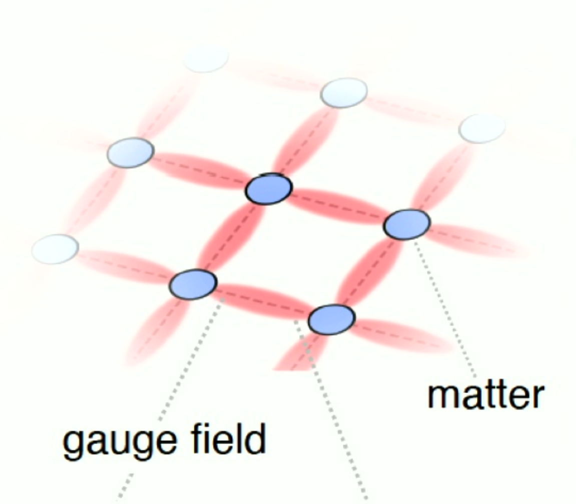
Beyond binary

short-depth qu**d**it circuits

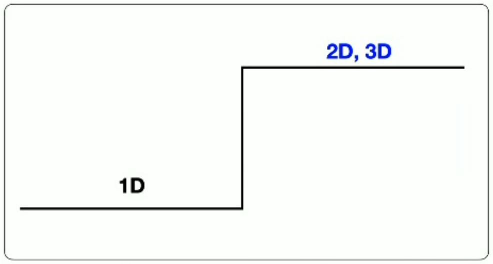




Lattice Gauge theories for particle physics



Outlook



Long-term vision:
- Simulate nature in 3 spatial dimensions
- Scaling up the system sizes
- Both make an efficient gauge field representation even more important

Qudit techniques:
- Can be implemented on many quantum computing platforms
- Are directly adaptable to digital quantum simulations of real-time dynamics
  Exciting perspective: quantum LGT calculations in regimes that are classically intractable due to sign problems
- Applications in chemistry and materials science
  e.g. in the form of exotic large-spin models: see e.g. work at Harvard (N. Maskara et. al.)

Qudit workshop

9-10 July 2024, Waterloo

Quantum Algorithms

Quantum Networks

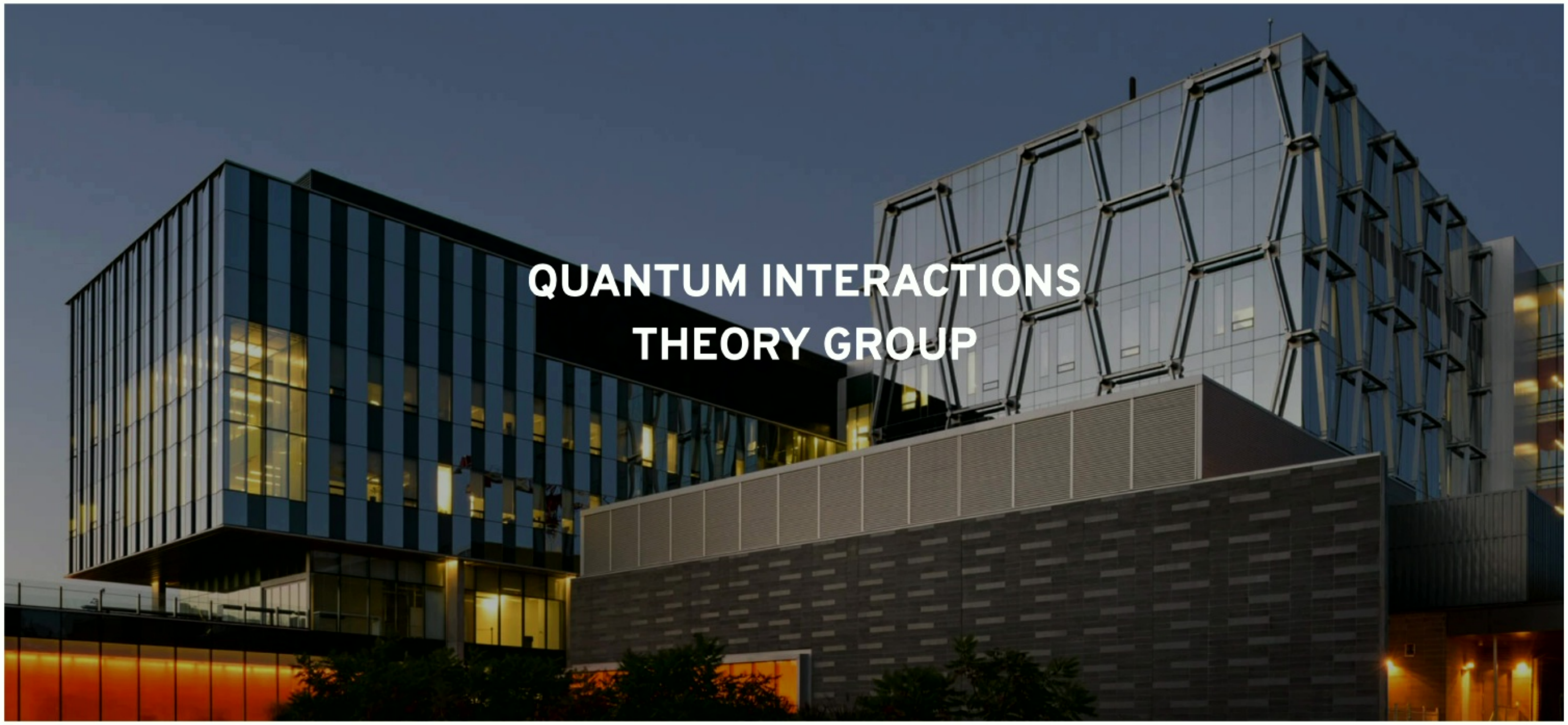
Quantum Simulations

Quantum Sensing

Theory + Experimental

Positions available (Masters/PhD/Postdoc)

www.quantum-interactions.com



UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

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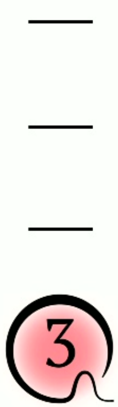
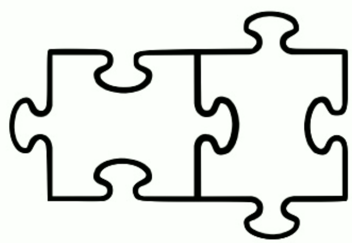
arXiv:2310.12110

Beyond binary

short-depth qudit circuits





Lattice Gauge theories for particle physics