Title: Quantum metrology

Speakers: Sisi Zhou

Collection/Series: Waterloo-Munich Joint Workshop

Subject: Quantum Information

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Limits of noisy quantum metrology with restricted quantum controls

Sisi Zhou

arXiv: 2402.18765, to appear in PRL

Oct 01, 2024

Waterloo-Munich Joint Workshop @ Perimeter Institute

Institute for Quantum Computing

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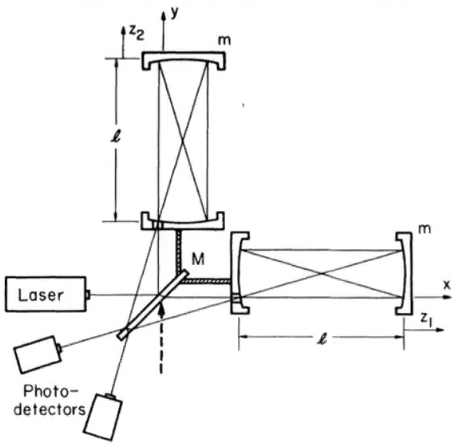
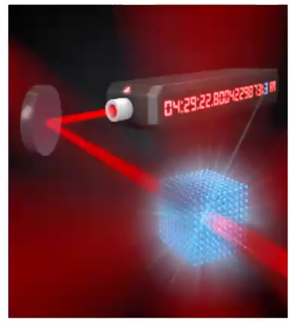
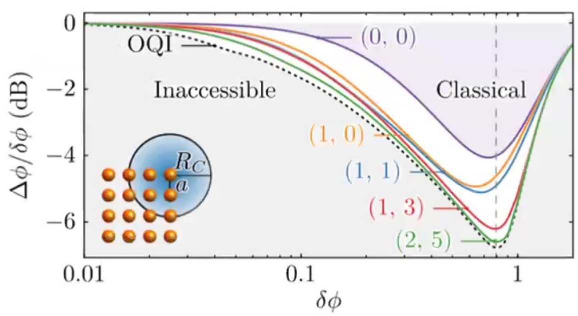
Quantum metrology is the science of estimation in quantum systems.

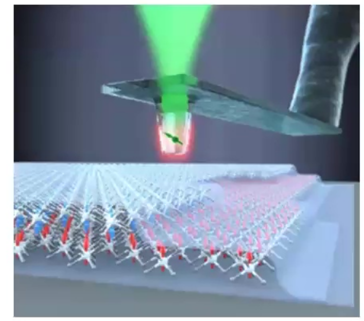


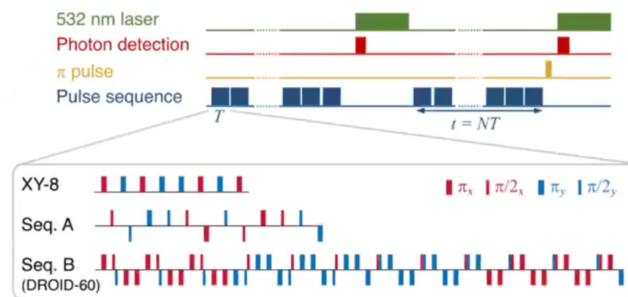
Optical interferometry





Atomic clock





Nitrogen-vacancy centers

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Images from: https://www.ligo.caltech.edu/news/ligo20191015, https://www.photonicsviews.com/atomic-clock-with-a-3d-optical-lattice/ https://physicsworld.com/a/diamond-quantum-microscope-images-nanoscale-features-in-2d-magnets/, PhysRevD.23.1693, PhysRevX.11.041045, PhysRevX.10.031003

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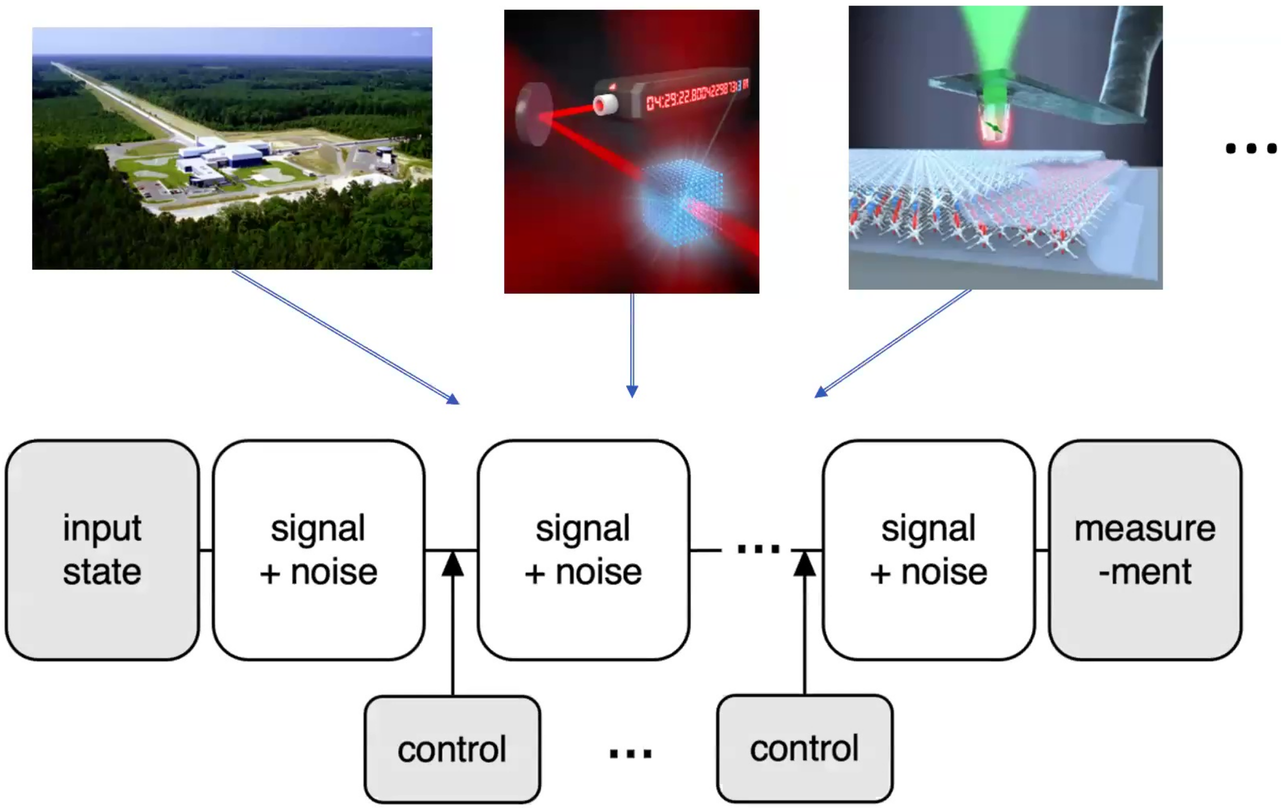
Quantum metrology enhanced by quantum controls?



Images from: https://www.ligo.caltech.edu/news/ligo20191015, https://www.photonicsviews.com/atomic-clock-with-a-3d-optical-lattice/ https://physicsworld.com/a/diamond-quantum-microscope-images-nanoscale-features-in-2d-magnets/, PhysRevD.23.1693, PhysRevX.11.041045, PhysRevX.10.031003

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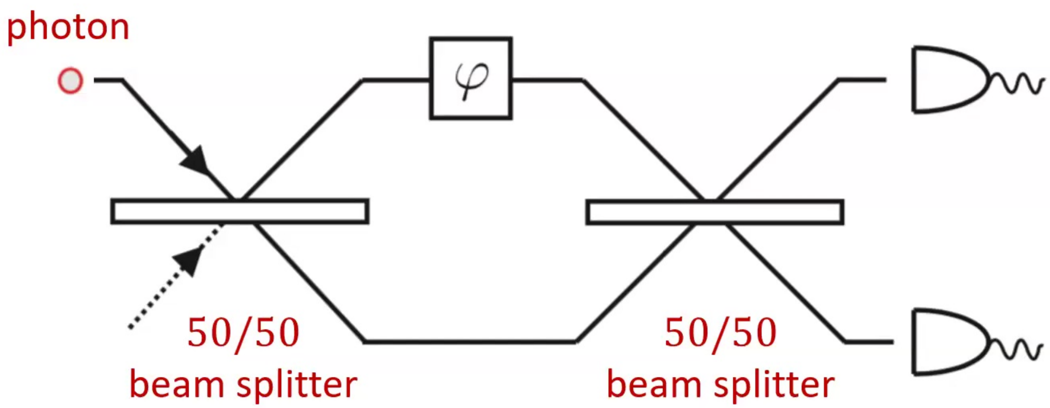
Mach-Zehnder Interferometry



$$p_\uparrow = \cos^2\frac{\varphi}{2}$$

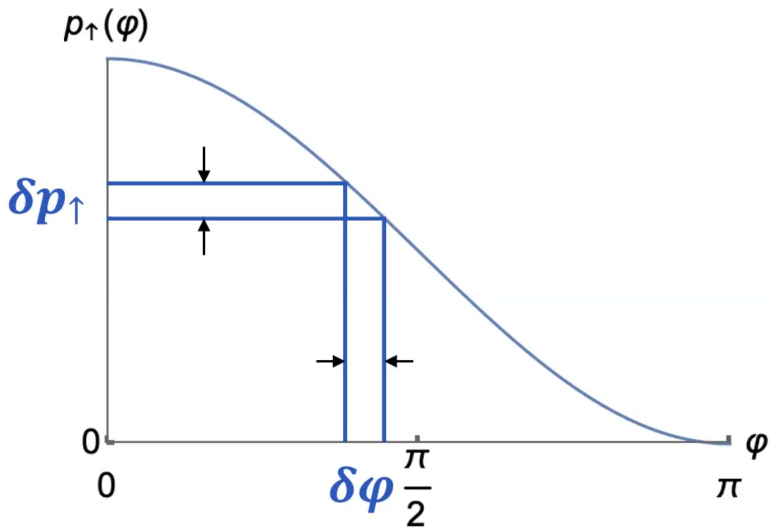
$$p_\downarrow = \sin^2\frac{\varphi}{2}$$

$$|1{,}0\rangle \;\to\; \frac{|1{,}0\rangle + |0{,}1\rangle}{\sqrt{2}} \;\to\; \frac{e^{i\varphi}|1{,}0\rangle + |0{,}1\rangle}{\sqrt{2}} \;\to\; \cos\frac{\varphi}{2}|1{,}0\rangle + \sin\frac{\varphi}{2}|0{,}1\rangle$$

The probability of detecting the photon in the upper (lower) port is $p_\uparrow$ ($p_\downarrow$), like in the biased-coin-tossing experiment where the probability of getting heads is $p_\uparrow = \cos^2\frac{\varphi}{2}$ and the probability of getting tails is $p_\downarrow = \sin^2\frac{\varphi}{2}$.

Image Credit: Kolodynski, PhDThesis (2014)

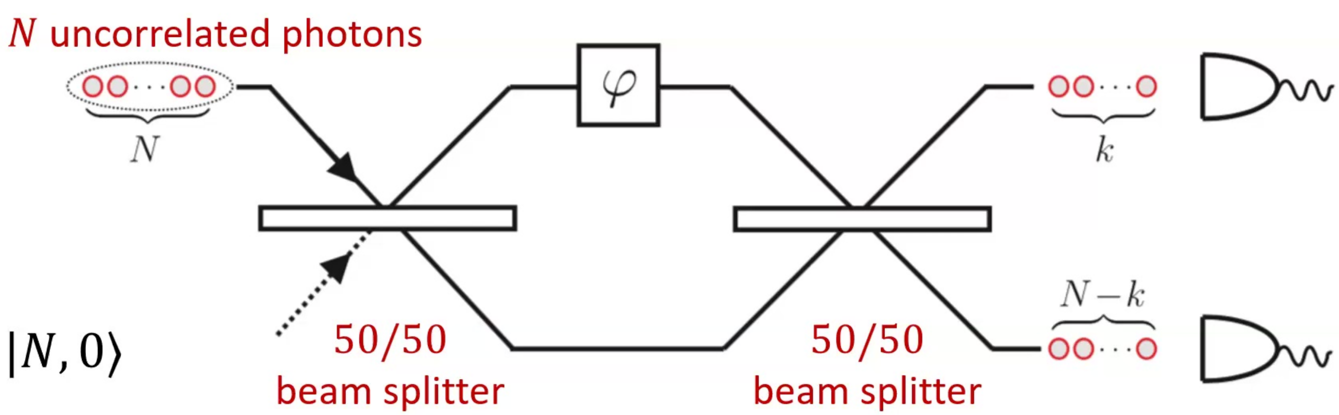
Mach-Zehnder Interferometry



The probability of detecting the photon in the upper (lower) port is $p_\uparrow$ ($p_\downarrow$), like in the biased-coin-tossing experiment where the probability of getting heads is $p_\uparrow = \cos^2 \frac{\varphi}{2}$ and the probability of getting tails is $p_\downarrow = \sin^2 \frac{\varphi}{2}$.

Image Credit: Kolodynski, PhDThesis (2014)

Mach-Zehnder Interferometry with Uncorrelated Photons



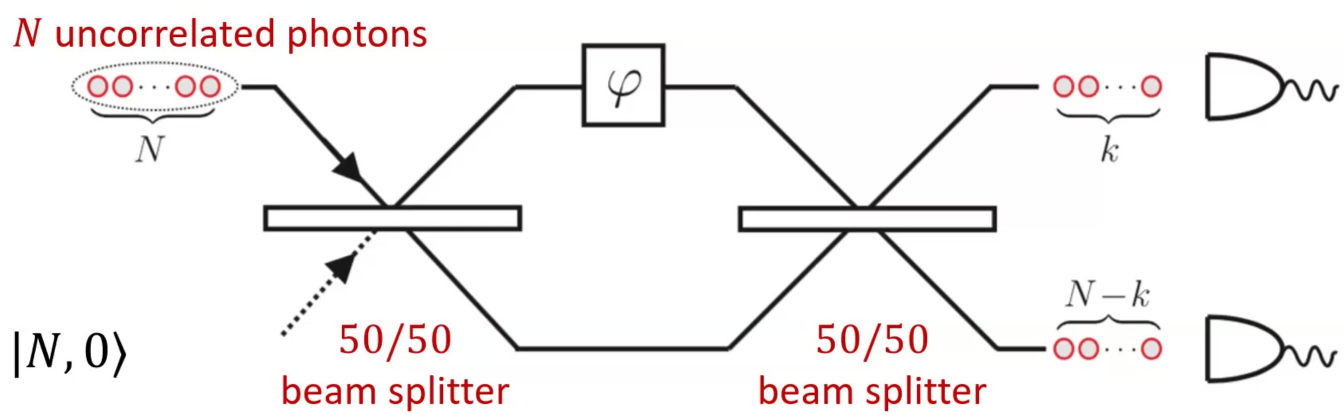
Probability of detecting k photons in the upper port:

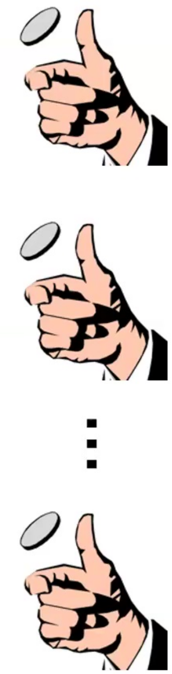
$$p_{\uparrow}^{N}(k) = \binom{N}{k} {p_{\uparrow}}^{k} (1 - p_{\uparrow})^{N-k} \text{ with } p_{\uparrow} = \cos^2 \frac{\varphi}{2},$$

equivalent to the outcome of k repeated coin-tossing experiments.

Image Credit: Kolodynski, PhDThesis (2014)

Mach-Zehnder Interferometry with Uncorrelated Photons



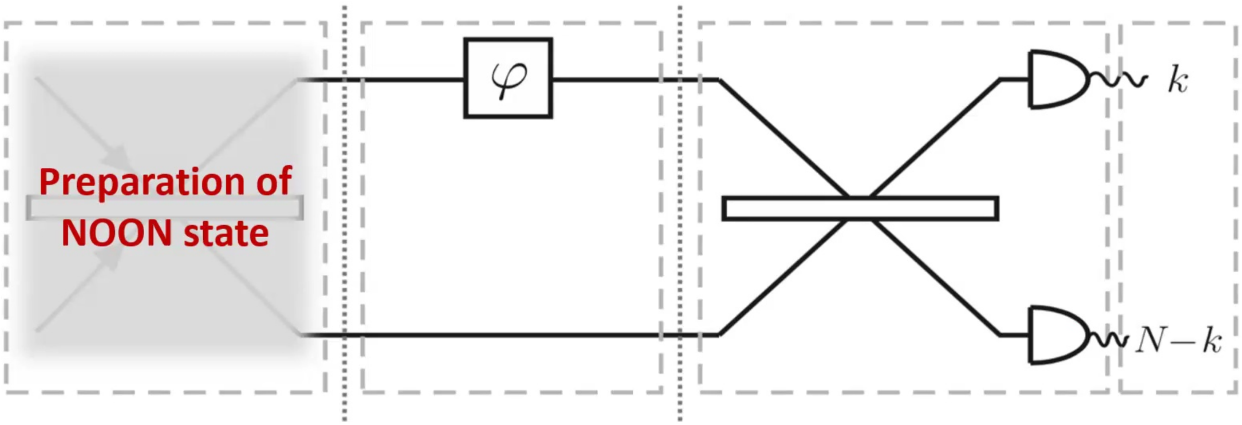


According to the central limit theorem,
the estimation error $\delta\varphi = \mathbb{E}[(\hat{\varphi} - \varphi)^2]^{1/2}$ is proportional to $1/\sqrt{N}$.

"Standard quantum limit"

Image Credit: Kolodynski, PhDThesis (2014)

Mach-Zehnder Interferometry with NOON States

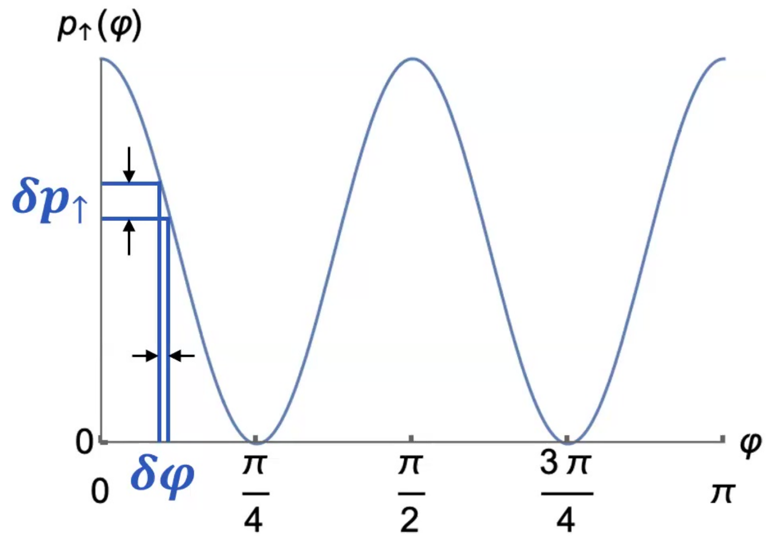


NOON state: (N photons in the upper port, or N photons in the lower port):

$$\frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle) \rightarrow \frac{1}{\sqrt{2}}\left(e^{iN\varphi}|N,0\rangle + |0,N\rangle\right).$$

Image Credit: Kolodynski, PhDThesis (2014)

Mach-Zehnder Interferometry with NOON States



Probability of detecting even/odd number of photons in the upper port:

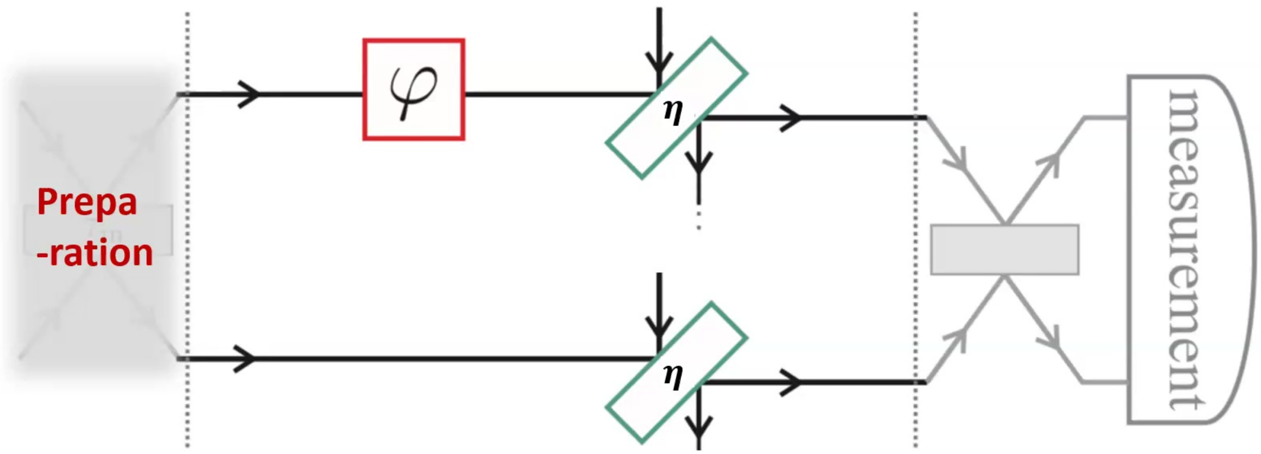
$$p_\uparrow^N(\text{even}) = \cos^2\frac{N\varphi}{2}, \qquad p_\uparrow^N(\text{odd}) = \sin^2\frac{N\varphi}{2}.$$

The estimation error $\delta\varphi$ is proportional to $1/N$. ***"Heisenberg limit"***

Image Credit: Kolodynski, PhDThesis (2014)

Mach-Zehnder Interferometry with Photon Losses

Losses are modeled by fictitious beam splitters of transmissivity η.



$$\rho_\varphi = \eta^N \left(\frac{e^{iN\varphi}|N,0\rangle + |0,N\rangle}{\sqrt{2}}\right)\left(\frac{e^{-iN\varphi}\langle N,0| + \langle 0,N|}{\sqrt{2}}\right) + (1-\eta^N)\rho_0$$

The estimation error $\delta\varphi$ grows exponentially with N, due to quantum noise.

Image Credit: Kolodynski, PhDThesis (2014)

Mach-Zehnder Interferometry with Photon Losses

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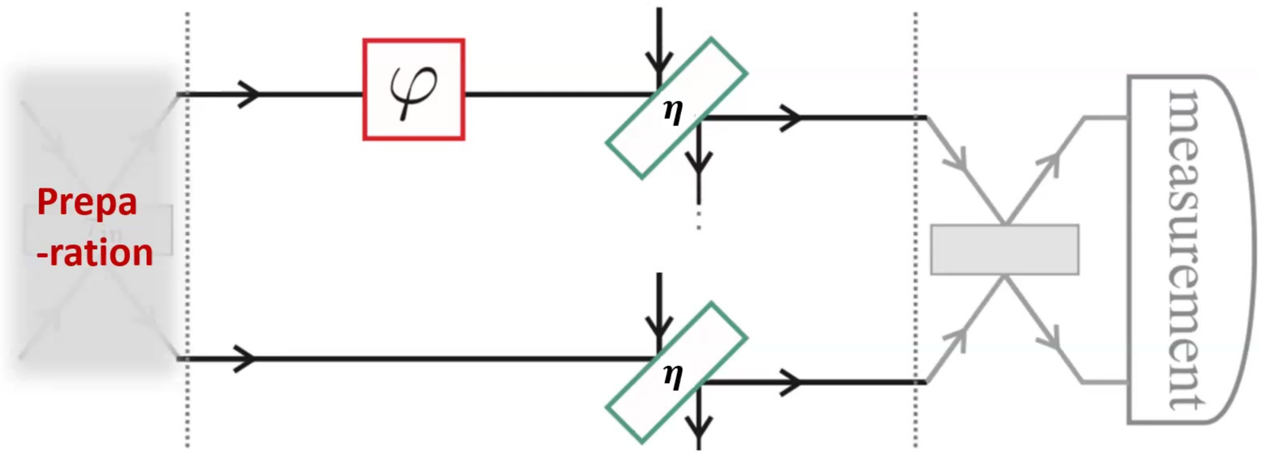


$$\rho_\varphi = \eta^N \left(\frac{e^{iN\varphi}|N,0\rangle + |0,N\rangle}{\sqrt{2}}\right)\left(\frac{e^{-iN\varphi}\langle N,0| + \langle 0,N|}{\sqrt{2}}\right) + (1-\eta^N)\rho_0$$

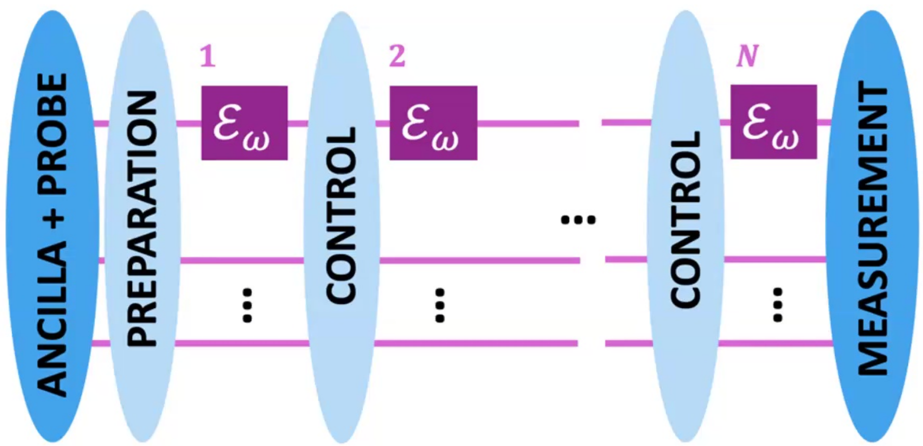
The estimation error $\delta\varphi$ grows exponentially with N, due to quantum noise.

Can quantum controls, e.g., quantum error correction, help?

Image Credit: Kolodynski, PhDThesis (2014)

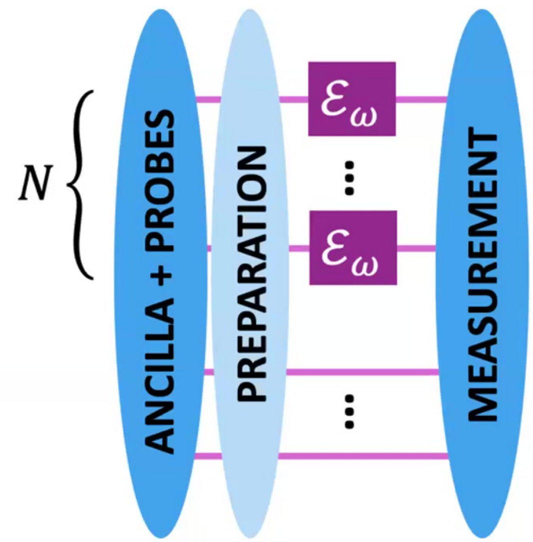
Quantum Channel Estimation & Quantum Fisher Information

- General quantum channel: $\mathcal{E}_\omega(\rho) = \sum_{i=1}^{r} K_{\omega,i} \rho K_{\omega,i}^\dagger$, $\omega \approx 0$.
- **Heisenberg limit (HL)**: $\delta\omega \propto 1/N$
- **Standard quantum limit (SQL)**: $\delta\omega \propto 1/\sqrt{N}$



Sequential strategy

$\supseteq$



Parallel strategy

Quantum Channel Estimation & Quantum Fisher Information

Cramér–Rao bound: $\delta\omega \gtrsim 1/\sqrt{\mathrm{QFI}}$

- General quantum channel: $\mathcal{E}_\omega(\rho) = \sum_{i=1}^{r} K_{\omega,i}\rho K_{\omega,i}^\dagger,\ \omega \approx 0.$
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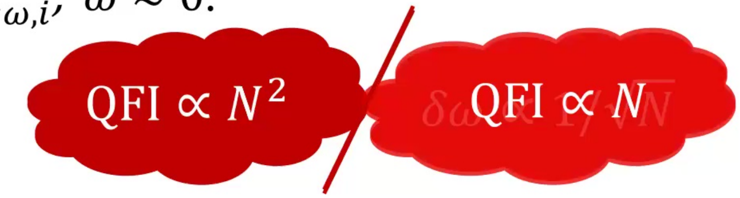
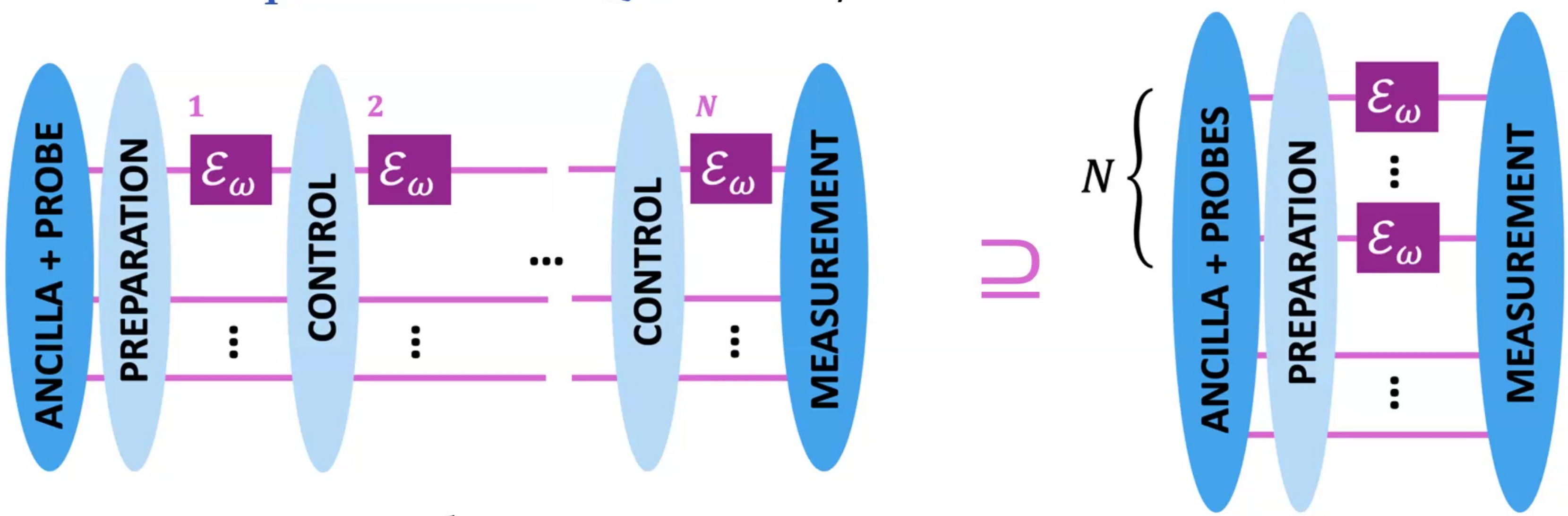




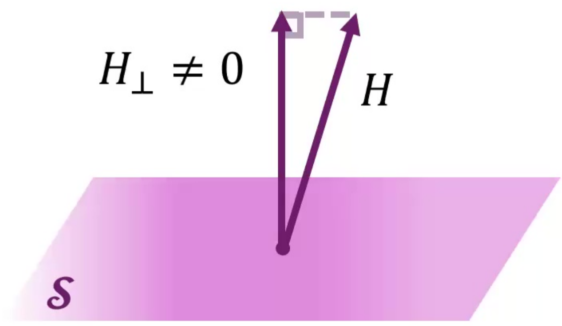
Sequential strategy

Parallel strategy

"Hamiltonian-not-in-Kraus-span" (HNKS) Condition

- General quantum channel: $\mathcal{E}_\omega(\rho) = \sum_{i=1}^{r} K_{\omega,i}\rho K_{\omega,i}^\dagger$
- The Heisenberg limit (QFI $\propto N^2$) is achievable using sequential/parallel strategies **IF AND ONLY IF**

Hamiltonian $H \notin$ **Kraus Span** $\mathcal{S}$,



where

Hamiltonian (signal): $H(\mathcal{E}_\omega) = i\sum_i K_i^\dagger \partial_\omega K_i$,

Kraus span (noise): $\mathcal{S}(\mathcal{E}_\omega) = \text{span}\{K_i^\dagger K_j, \forall i,j\}$.

SZ & Jiang, PRX Quantum 2, 010343 (2021)
SZ, Zhang, Preskill, Jiang, Nature Communications 9:78 (2018); Demkowicz-Dobrzański *et al.* PRX 7, 041009 (2017)

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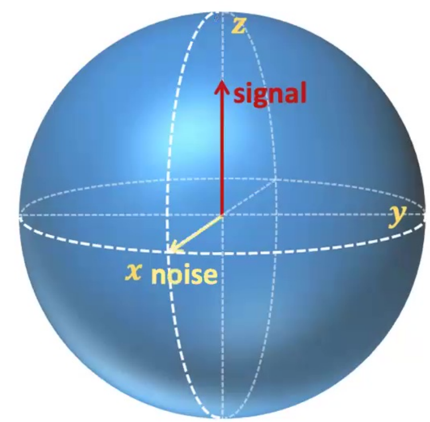


Example:

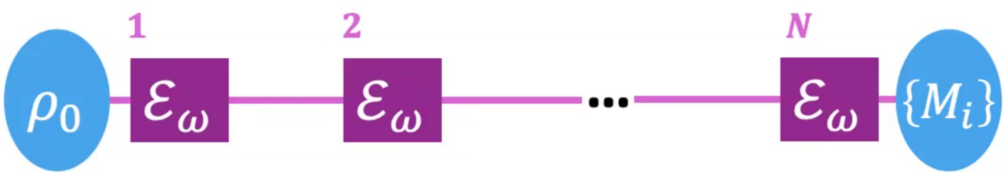
$$\boldsymbol{\mathcal{E}_\omega(\rho) = e^{-i\omega H}\left((1-p)(\cdot) + p\,E\,(\cdot)\,E^\dagger\right)e^{i\omega H}}$$

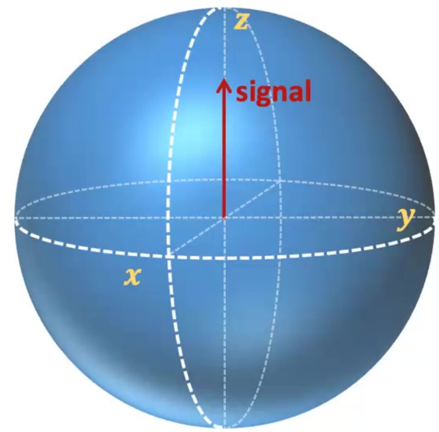
- Hamiltonian $H = Z$, Error $E = Z$, Kraus span = span$\{I, Z\}$. The HL is not achievable.
- Hamiltonian $H = Z$, Error $E = X$, Kraus span = span$\{I, X\}$. Quantum error correction (QEC) can recover the HL.

SZ & Jiang, PRX Quantum 2, 010343 (2021)
SZ, Zhang, Preskill, Jiang, Nature Communications 9:78 (2018); Demkowicz-Dobrzański *et al.* PRX 7, 041009 (2017)

Example: Pauli-Z Hamiltonian

$$\mathcal{E}_\omega(\cdot) = e^{-i\omega Z}(\cdot)e^{i\omega Z}$$





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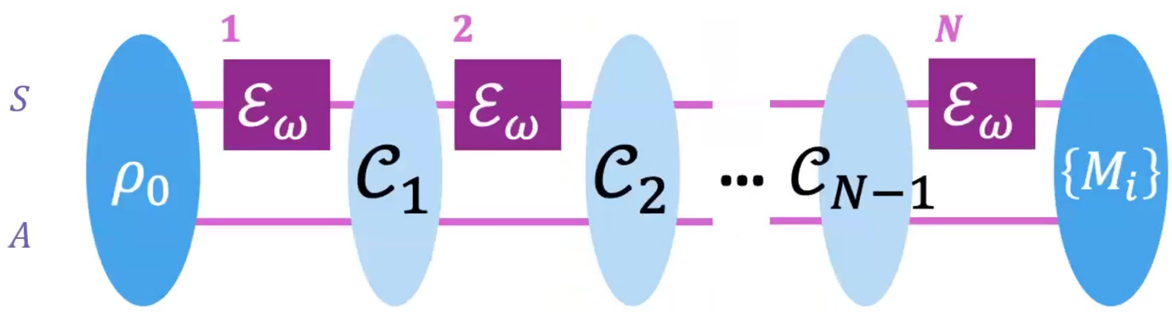
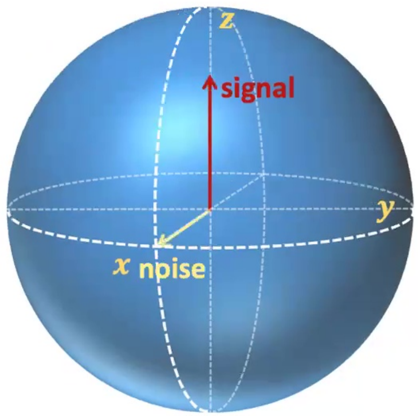
Example: Pauli-Z Hamiltonian + Bit-flip noise

$$\mathcal{E}_\omega(\rho) = e^{-i\omega Z}\left((1-p)(\cdot) + p\,X\,(\cdot)\,X^\dagger\right)e^{i\omega Z}$$





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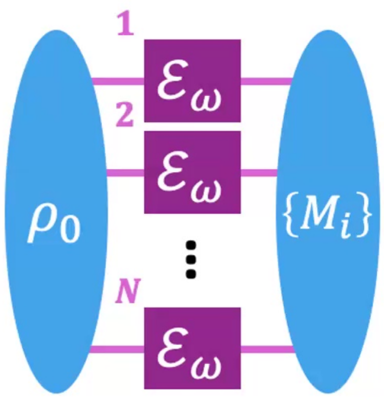
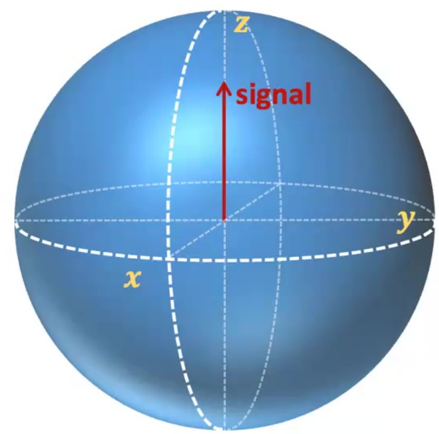
Dur *et al.* PRL 112, 080801 (2014); Kessler *et al.* PRL 112, 080802 (2014); Arrad *et al.* PRL 112, 150801 (2014)

Example: Pauli-Z Hamiltonian

$$\mathcal{E}_\omega(\cdot) = e^{-i\omega Z}(\cdot)e^{i\omega Z}$$



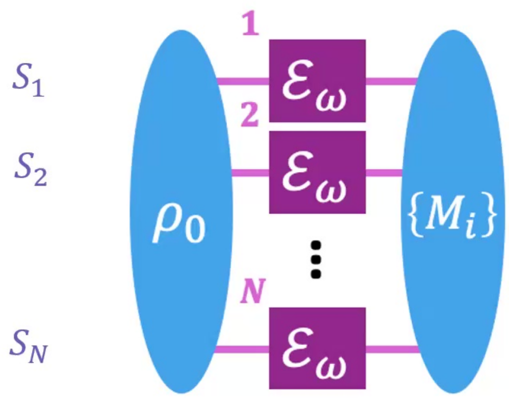


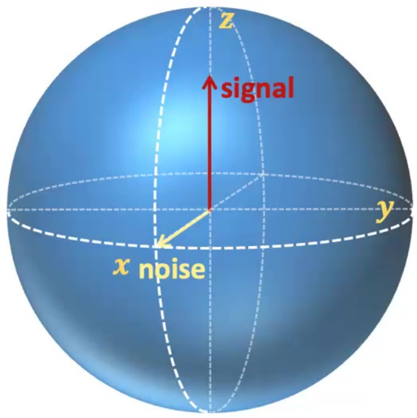
$$|\psi_0\rangle = \frac{|0^{\otimes N}\rangle + |1^{\otimes N}\rangle}{\sqrt{2}} \Rightarrow |\psi_\omega\rangle = \frac{e^{-i\omega N}|0^{\otimes N}\rangle + e^{i\omega N}|1^{\otimes N}\rangle}{\sqrt{2}}$$

Heisenberg limit: $\delta\omega \propto 1/N$

Example: Pauli-Z Hamiltonian + Bit-flip noise

$$\mathcal{E}_\omega(\rho) = e^{-i\omega Z}\left((1-p)(\cdot) + p\, X\,(\cdot)\, X^\dagger\right)e^{i\omega Z}$$





$$|\psi_0\rangle = \frac{|0_L\rangle + |1_L\rangle}{\sqrt{2}}$$
$$= \frac{|00\cdots 0\rangle_{S_1\cdots S_N} + |11\cdots 1\rangle_{S_1\cdots S_N}}{\sqrt{2}}$$

Heisenberg limit: $\delta\omega \propto 1/N$

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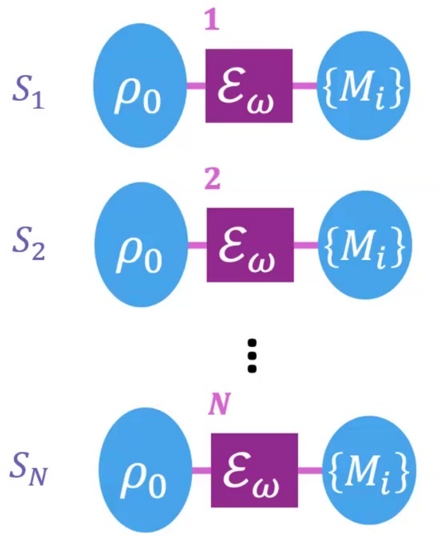
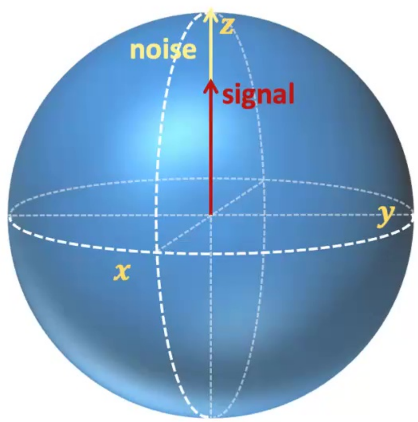
Dur *et al.* PRL 112, 080801 (2014); Kessler *et al.* PRL 112, 080802 (2014); Arrad *et al.* PRL 112, 150801 (2014)

Example: Pauli-Z Hamiltonian + Dephasing noise

$$\mathcal{E}_\omega(\rho) = e^{-i\omega Z}\left((1-p)(\cdot) + p\, Z\,(\cdot)\, Z^\dagger\right)e^{i\omega Z}$$





$$|\psi_0\rangle = \left(\frac{|0_L\rangle + |1_L\rangle}{\sqrt{2}}\right)^{\otimes N}$$

Standard quantum limit: $\delta\omega \propto 1/\sqrt{N}$

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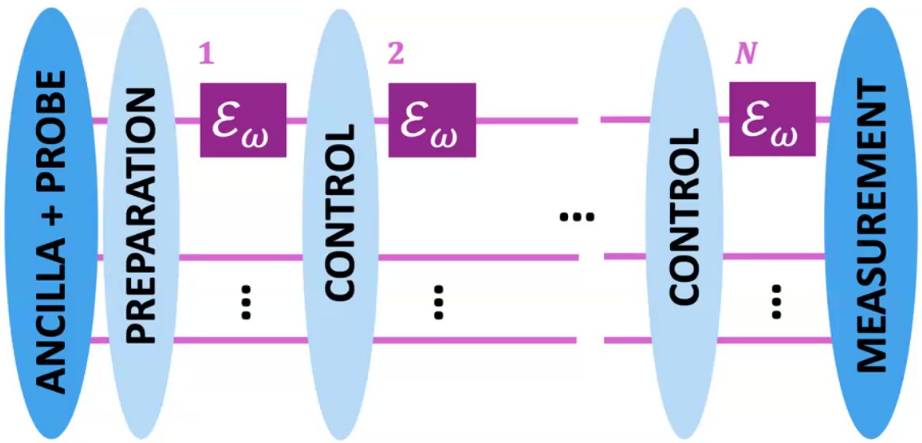
Dur *et al.* PRL 112, 080801 (2014); Kessler *et al.* PRL 112, 080802 (2014); Arrad *et al.* PRL 112, 150801 (2014)

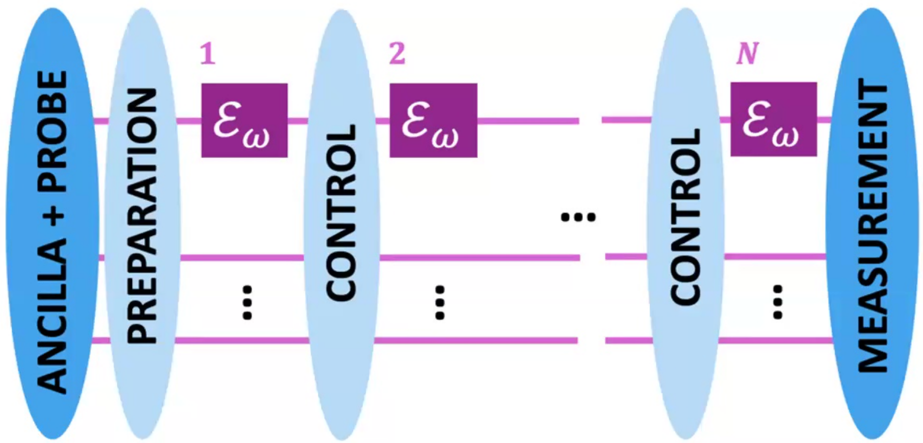
Metrological Limits with Restricted Controls



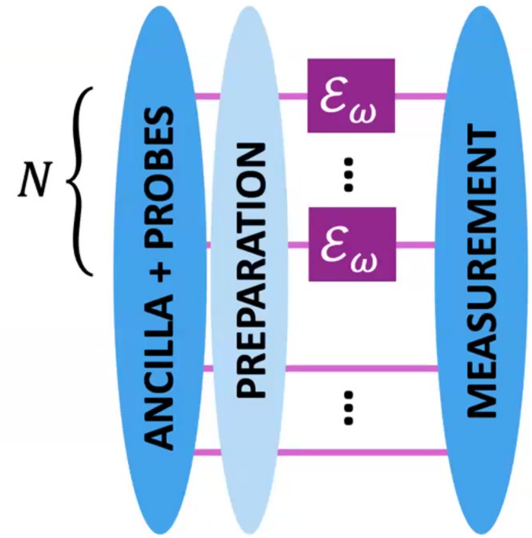
Sequential strategy

- Noiseless ancilla
- CPTP controls (mid-circuit measurement)

Metrological Limits with Restricted Controls



Sequential strategy



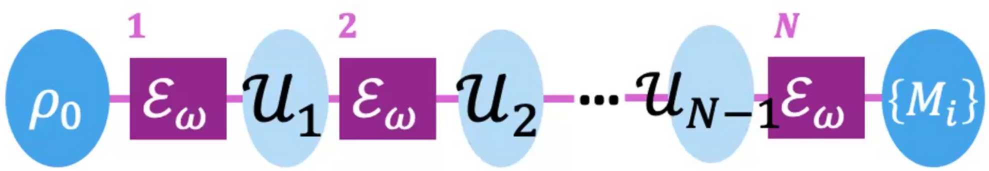
Parallel strategy

- Noiseless ancilla
- CPTP controls (mid-circuit measurement)

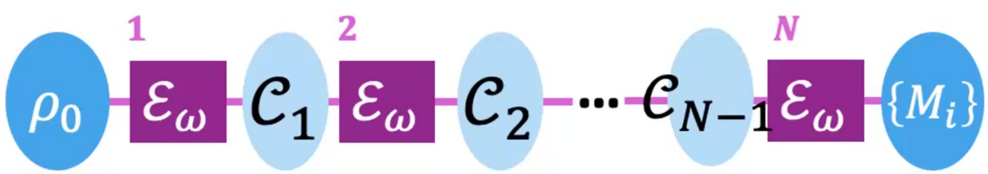
- Large system size
- Long-range entanglement

Metrological Limits with Restricted Controls

- Ancilla-free sequential strategy, unital controls

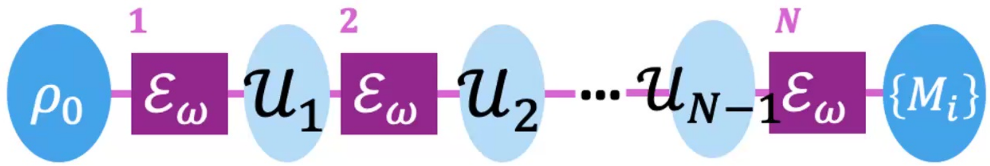


- Ancilla-free sequential strategy, CPTP controls

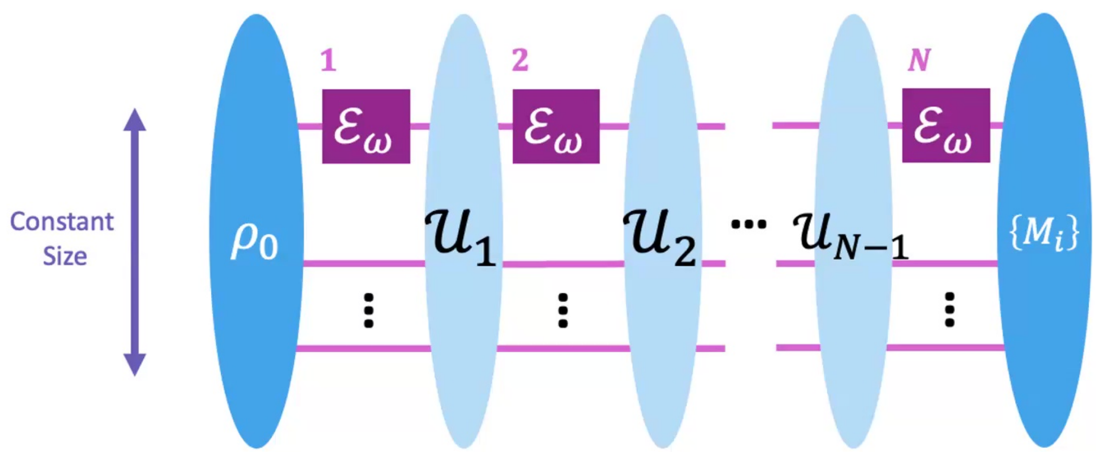


Metrological Limits with Restricted Controls

- Ancilla-free sequential strategy, unital controls



- Bounded-ancilla strategy, unital controls



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Classification of Qubit Channels

- **Unitary channels**

$$\mathcal{E}_\omega(\cdot) = V_\omega(\cdot)V_\omega^\dagger,$$

$$\text{Hamiltonian: } H = iV_\omega^\dagger \partial_\omega V_\omega$$

- **Dephasing-class channels** (Dephasing channels up to unitary rotations)

$$\mathcal{E}_\omega(\cdot) = \boldsymbol{V_\omega}\Big((1-p_\omega)\boldsymbol{U_\omega}(\cdot)\boldsymbol{U_\omega^\dagger} + p_\omega\, \boldsymbol{Z}\, \boldsymbol{U_\omega}(\cdot)\boldsymbol{U_\omega^\dagger}\, \boldsymbol{Z}\Big)\boldsymbol{V_\omega^\dagger},$$

$$\text{Unitary rotation generators: } \boldsymbol{H_0 = iV_\omega^\dagger \partial_\omega V_\omega},\ \boldsymbol{H_1 = iU_\omega^\dagger \partial_\omega U_\omega}$$

- **Strictly contractive channels**

$$\|\mathcal{E}_\omega(\rho) - \mathcal{E}_\omega(\sigma)\|_1 < \|\rho - \sigma\|_1$$

King & Ruskai IEEE-TIT (2001)

Results: Scalings of QFI

Strategies	Sequential, or Parallel	Sequential, Ancilla-free, Unital controls	Sequential, Ancilla-free, CPTP controls	Sequential, Bounded-ancilla, Unital controls
Unitary	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$
Dephasing-class (H_0 or $H_1 \notin \mathcal{S}$)				
Dephasing-class ($H_{0,1} \in \mathcal{S}$)				
Strictly Contractive				

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SZ, arXiv: 2402.18765

Results: Scalings of QFI

Strategies	Sequential, or Parallel	Sequential, Ancilla-free, Unital controls	Sequential, Ancilla-free, CPTP controls	Sequential, Bounded-ancilla, Unital controls
Unitary	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$
Dephasing-class (H_0 or $H_1 \notin \mathcal{S}$)	← e.g. Pauli-Z signal and Pauli-X noise			
Dephasing-class ($H_{0,1} \in \mathcal{S}$)	← e.g. Pauli-Z signal and Pauli-Z noise			
Strictly Contractive				

SZ, arXiv: 2402.18765

Results: Scalings of QFI

HNKS condition

Strategies	Sequential, or Parallel	Sequential, Ancilla-free, Unital controls	Sequential, Ancilla-free, CPTP controls	Sequential, Bounded-ancilla, Unital controls
Unitary	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$	$\Theta(N^2)$
Dephasing-class (H_0 or $H_1 \notin \mathcal{S}$)	$\Theta(N^2)$ (HNKS holds) $\Theta(N)$ (HNKS fails)	$\Theta(N)$	$O(N^{3/2}), \Omega(N)$	$\Theta(N)$
Dephasing-class ($H_{0,1} \in \mathcal{S}$)	$\Theta(N)$	$O(1)$	$O(N)$	$O(N)$
Strictly Contractive	$\Theta(N)$	$O(1)$	$\Theta(1)$	$O(N)$

SZ, arXiv: 2402.18765

Linear Upper bound: **Modified Channel Extension Method**
Linear Lower bound: **Single-qubit Unitary Control Sequence**

Results: Scalings of QFI

HNKS condition

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SZ, arXiv: 2402.18765

Channel Extension Method

- Sequential/Parallel strategy:

$$\mathrm{QFI} \le 4N\|\alpha\| + 4N(N-1)(\|\beta\|^2 + o(1))$$

where $\alpha = \sum_i \partial_\omega K_{\omega,i}^\dagger \partial_\omega K_{\omega,i}$ and $\beta = i \sum_i K_{\omega,i}^\dagger \partial_\omega K_{\omega,i}$. $\beta \neq 0 \Leftrightarrow$ HNKS.

Fujiwara & Imai, JPA (2008), Escher *et al.* NPhys (2011), Demkowicz-Dobrzanski, *et al.* NComms (2012),
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$$\boldsymbol{\mathcal{E}_\omega(\cdot) = e^{-i\omega Z}\left((1-p)(\cdot) + p\,X\,(\cdot)\,X^\dagger\right)e^{i\omega Z}}$$

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$$\mathrm{QFI} \leq 4\sum_{k=1}^{N} \mathrm{Tr}(\alpha_k) + 4\sum_{k=1}^{N} \mathrm{Tr}(\boldsymbol{\gamma_k}\beta_k)$$

$$\alpha_k = O(1), \quad \beta_k = O(1), \quad \gamma_k = O(k)$$

Fujiwara & Imai, JPA (2008), Escher *et al.* NPhys (2011), Demkowicz-Dobrzanski, *et al.* NComms (2012), SZ, arXiv: 2402.18765

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Fujiwara & Imai, JPA (2008), Escher *et al.* NPhys (2011), Demkowicz-Dobrzanski, *et al.* NComms (2012),
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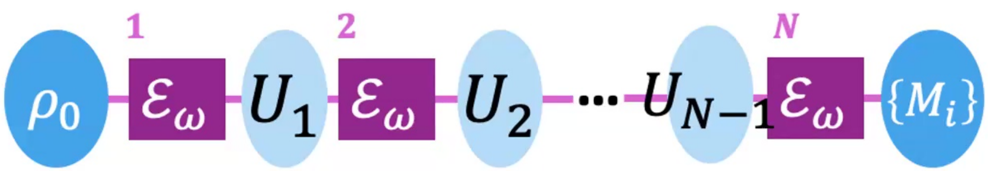
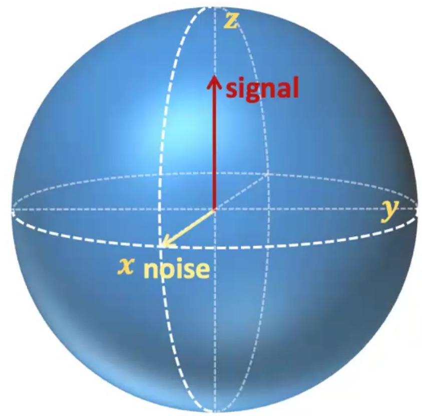
Unitary Control to Achieve the SQL

$$\mathcal{E}_\omega(\rho) = e^{-i\omega Z}\left((1-p)(\cdot) + p\,X\,(\cdot)\,X^\dagger\right)e^{i\omega Z}$$





Unitary control sequence:

$$U_1 = U_2 = \cdots = U_{N-1} = e^{-i\sqrt{\frac{w}{4N}}Z}$$

w is a small constant

Standard quantum limit: $\delta\omega \propto 1/\sqrt{N}$

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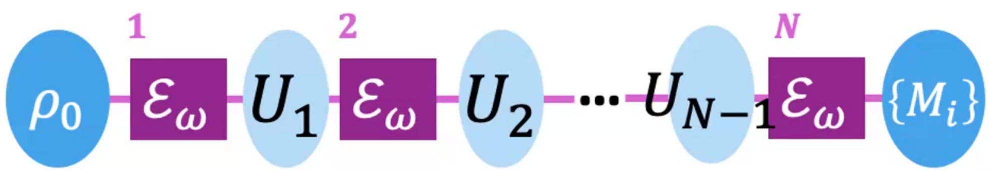

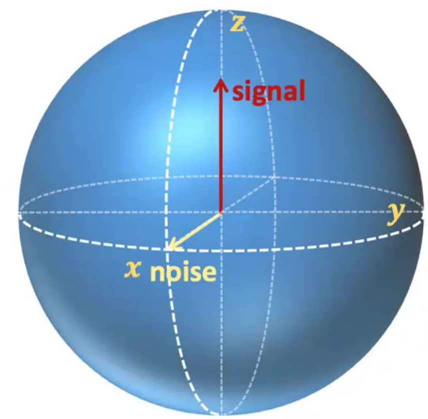

State evolution:

$$\rho_\omega : (1,0,0) \Rightarrow \left(1 - \Theta\left(\frac{1}{\sqrt{N}}\right), \Theta\left(\frac{1}{\sqrt{N}}\right), 0\right)$$

$$\partial_\omega \rho_\omega : (0,0,0) \Rightarrow \left(\Theta(\sqrt{N}), 0,0\right)$$

Standard quantum limit: $\delta\omega \propto 1/\sqrt{N}$

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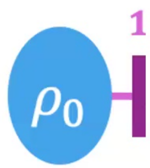
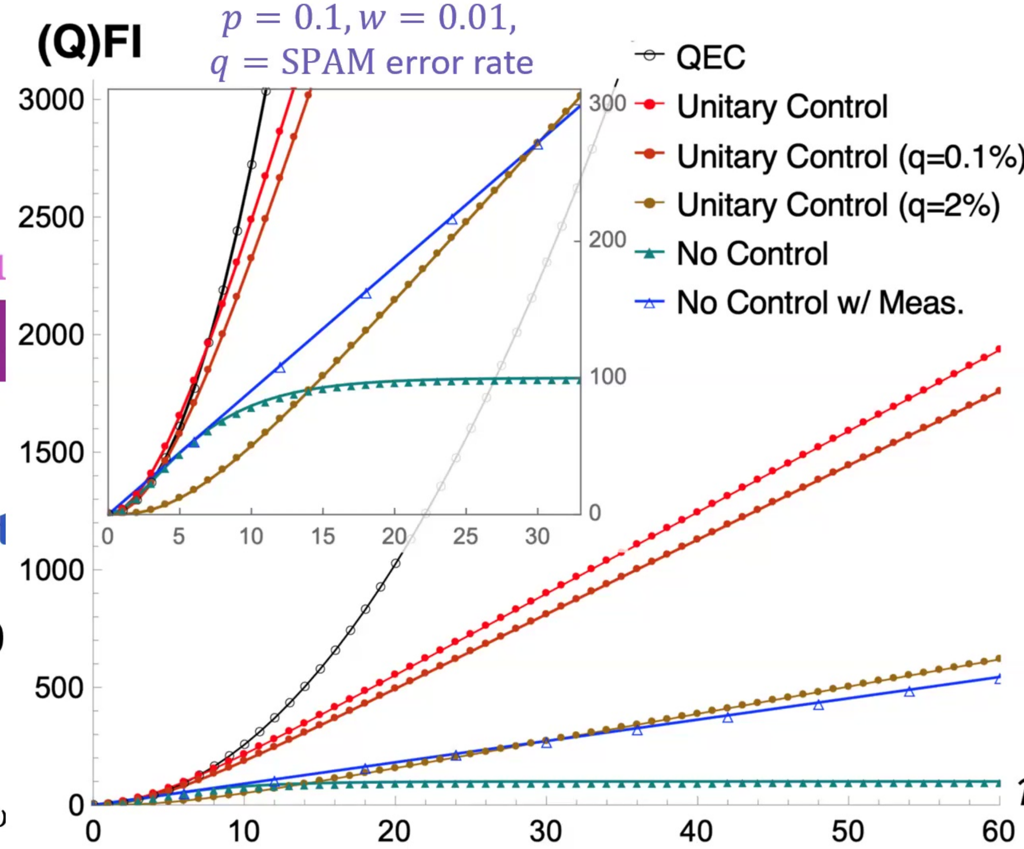
Unitary Control to Achieve the SQL

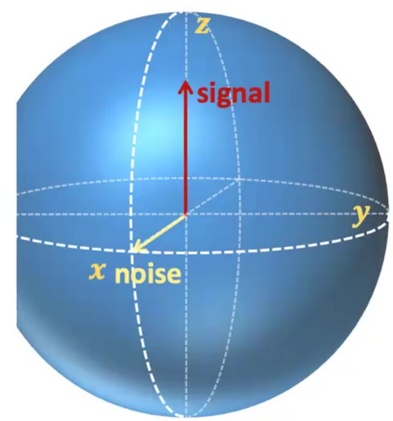


State evolut

ρ_ω: (1,0

∂_ω





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Results: Scalings of QFI

Linear Upper bound: **Modified Channel Extension Method**
Linear Lower bound: **Single-qubit Unitary Control Sequence**
Constant Upper bound: **Bloch Sphere Vector Analysis**

HNKS condition

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Linear Upper bound: **Modified Channel Extension Method**
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Constant Upper bound: **Contraction Coefficient wrt QFI**

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Results: Scalings of QFI

HL vs. SQL **New Provable Separations**

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Results: Scalings of QFI

HL vs. SQL | New Provable Separations | No Separation Proved

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Summary and Outlook

- The structure of the noise and the signal determines the estimation precision limit in quantum metrology.
- Sensing limits are compromised with restricted quantum controls---the HL is no longer achievable under noise; and the achievability of the SQL has a dichotomous behavior.
- **Future directions:**
  - Generalization to qudit systems
  - Measurement and feedforward protocols
  - QFI as a function of size of ancilla

Thank you!