Title: Lecture - QFT I, PHYS 601

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Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

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Canonical Quantization

1) Weyl Lagrangian

The interaction triology

1) what kind of interaction?
2) LSZ - Dyson - Wick
3) Rules

1) Quantization

Dan's recipe

1) $\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$ 4 ψ: complex × 4

2) $\Pi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = i\psi^\dagger$

1 d classical particle

phase space

d.o.f. $= \frac{\overset{x}{1} + \overset{p}{1}}{2} = 1$

$$\frac{\overset{\psi}{8}+0}{2}=4$$

$$\mathcal{H}=\psi^\dagger\underline{(-i\gamma^0\gamma^i\partial_i+m\gamma^0)}\psi$$

$\not{\partial}-m)\psi$

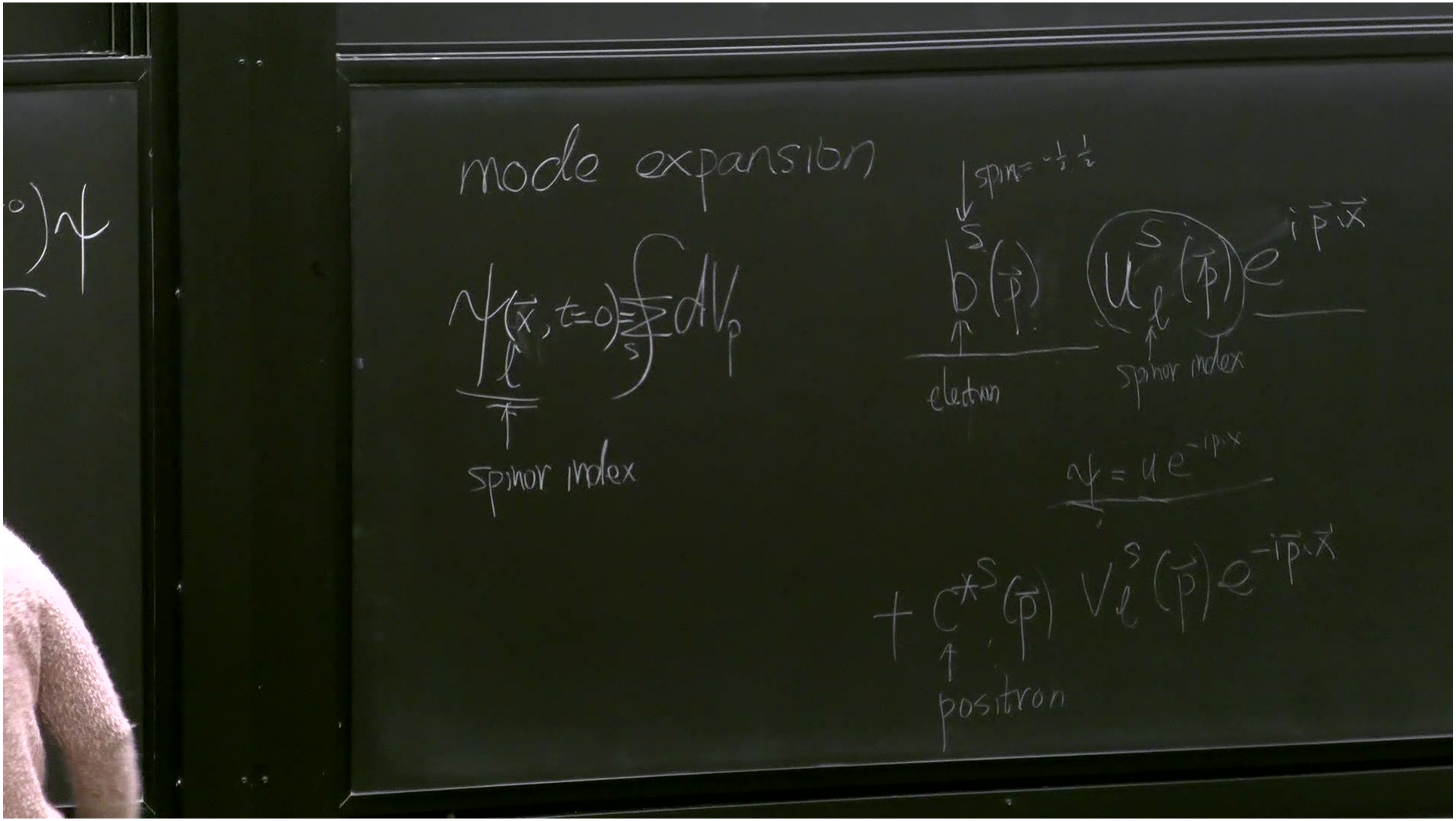
$=i\psi^\dagger$

④ $\underline{\psi: \text{complex} \times 4}$

1 d classical particle

phase space

$\text{d.o.f.} = \frac{\overset{x}{1}+\overset{p}{1}}{2}=1$


mode expansion
$\psi_\ell(\vec{x}, t=0) = \sum_s \int dV_p$
spinor index
spin = $-\frac{1}{2}, \frac{1}{2}$
$b^s(\vec{p})\, u^s_\ell(\vec{p})\, e^{i\vec{p}\cdot\vec{x}}$
electron
spinor index
$\psi = u e^{-ip\cdot x}$
$+\, c^{*s}(\vec{p})\, V^s_\ell(\vec{p})\, e^{-i\vec{p}\cdot\vec{x}}$
positron

$$H = \int dV_{\vec p}\, \epsilon_{\vec p} \left(b^{*}(\vec p)\, b^{s}(\vec p) - c^{s}(\vec p)\, c^{s*}(\vec p) \right)$$

↓ promotion

$$\hat H = \int dV_p\, \epsilon_{\vec p} \left(b^{+s}_{\vec p}\, b^{s}_{\vec p} - c^{s}(\vec p)\, c^{s+}_{\vec p} \right)$$

$$- \left(\left[c^{s}_{\vec p}, c^{s+}_{\vec p} \right] + c^{s+}_{\vec p}\, c^{s}_{\vec p} \right)$$

$$H = \int dV_p\, \epsilon_p \left(b^{+s}_{\vec p}\, b^{s}_{\vec p} - c^{s+}_{\vec p}\, c^{s}_{\vec p} \right) = \int dV_{\vec p}\, \epsilon_{\vec p} \left(N_b - N_c \right)$$

$$H = \int dV_p E_{\vec{p}} \left(\underbrace{b^{+s}_{\vec{p}} b^{s}_{\vec{p}} + c^{+s}_{\vec{p}} c^{s}_{\vec{p}}} - c^{+s}_{\vec{p}} c^{s}_{\vec{p}} - c^{s}_{\vec{p}} c^{s+}_{\vec{p}} \right)$$

$$= \int dV_p E_{\vec{p}} (N_b + N_c)$$

$$+ \int dV_{\vec{p}} E_{\vec{p}} \left(- \{ c^{s}_{\vec{p}}, c^{s+}_{\vec{p}} \} \right)$$

Dirac field is fermion

normal ordering

$$:c_p^s c_p^{\dagger s}: = -c_p^{\dagger s} c_p^s$$

Causality

$$O_1 = \bar{\psi}_a \Gamma_{ab} \psi_b$$

spacelike $[O_1 , O_2] = 0$ $\quad \bar{\psi}_a \{\psi_b, \bar{\psi}_c\} \psi_d$

$\{b, b^\dagger\}$ $i\not\partial D(x-y)$