**Title:** Lecture - QFT I, PHYS 601

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# Dirac equation

in-class exercise

$$(i\partial_\mu \gamma^\mu - m)\psi = 0 \rightarrow \text{is covariant}$$

constant coefficients

if $\psi \rightarrow D(\Lambda)\psi$

spinor rep

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$$

in-class exercise

find Dirac Lagrangian

Historical also important

$$i\partial_0\psi = H\psi$$

$$H = -i\gamma^0\gamma^i\partial_i + m\gamma^0$$

Hermitian

$$(\gamma^0)^\dagger = \gamma^0 \qquad (\gamma^i)^\dagger = \gamma^0\gamma^i\gamma^0$$

$$(-i\gamma^0\gamma^i\partial_i)^\dagger$$

$$= -i\partial_i(\gamma^i)^\dagger\gamma^0$$

$$= -i\partial_i\gamma^0\gamma^i$$

xercise

invariant
Lagrangian

also important

$\partial_i + m\gamma^0$

$(\gamma^0)^\dagger = \gamma^0$

$(-i\gamma^0\gamma^i\partial_i)^\dagger$

$= -i\partial_i(\gamma^i)^\dagger\gamma^0$

$= -i\partial_i\gamma^0\gamma^i$

$(\gamma^i)^\dagger = \gamma^0\gamma^i\gamma^0$

$\boxed{(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0}$

Weyl spinor

spinor rep

$$D(\Lambda_{rot}) = \begin{pmatrix} e^{\frac{i\vec{\theta}\vec{\sigma}}{2}} & \\ & e^{\frac{i}{2}\vec{\theta}\cdot\vec{\sigma}} \end{pmatrix}$$

$$D(\Lambda_{boost}) = \begin{pmatrix} e^{\frac{1}{2}\vec{\chi}\cdot\vec{\sigma}} & \\ & e^{-\frac{1}{2}\vec{\chi}\cdot\vec{\sigma}} \end{pmatrix}$$

sis

$$D(\Lambda_{\text{boost}}) = \begin{pmatrix} e^{\frac{1}{2}\vec{\chi}\cdot\vec{\sigma}} & \\ & e^{-\frac{1}{2}\vec{\chi}\cdot\vec{\sigma}} \end{pmatrix} \qquad \psi = \begin{pmatrix} W_+ \\ W_- \end{pmatrix}$$

One idea my weyl spinor commutes with Lorentz

$$[\text{Projection}, J_{\mu\nu}] = 0 \qquad J_{\mu\nu} \longrightarrow [\gamma^\mu, \gamma^\nu]$$

$$[\text{Proj}, \gamma^\mu] = 0 \quad \text{or} \quad \{\text{Proj}, \gamma^\mu\} = 0$$

$$\text{Proj} = \mathbb{1}$$

$$\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$$

ex $\{\gamma^5, \gamma^\mu\} = 0$

$$(\gamma^5)^2 = 1 \qquad (\gamma^5)^\dagger = \gamma^5$$

$$P_\pm^2 = P_\pm \qquad P_+ + P_- = 1$$

$$P_\pm = \frac{1 \pm \gamma^5}{2} \qquad \underline{\psi_\pm = P_\pm \psi}$$