**Title:** Lecture - QFT I, PHYS 601

**Speakers:** Gang Xu

**Collection/Series:** Quantum Field Theory I (Core), PHYS 601, October 7 - November 6, 2024

**Subject:** Condensed Matter, Particle Physics, Quantum Fields and Strings

**Date:** October 18, 2024 - 2:00 PM

**URL:** https://pirsa.org/24100031

Last day of Weinberg (☹, ☺, ^^)

states conflict / Lorentz invariant

$$u^{s'}_{\ell'}(0)\,\vec{J}^{j}_{s's}(R) = \vec{\mathcal{J}}_{\ell'\ell}(R)\,u^{s}_{\ell}(0)$$

plan

pick a rep for the field

↓

solve for $u(0)$ (determine the spin)

↓ causality

determine statistics

in-class exer

Last day of Weinberg (☹, ☺, 😄)

states conflict(( Lorentz invariant

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causality

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for the field

) (determine the spin)

stics

in-class exercise: pick trivial rep

now: spinor representation

$$\underbrace{\vec{J}_\ell = \frac{1}{2}\begin{pmatrix}\vec{\sigma} & \\ & \vec{\sigma}\end{pmatrix}}$$

$$\vec{J}_{\underbrace{m'+}_{\ell'},\,\underbrace{m+}_{\ell}} = \vec{J}_{m'-,\,m-} = \frac{1}{2}\vec{\sigma}_{m'm}$$

$u_\ell \ (\ell = 1, 2, 3, 4)$

$u_{m,\pm}$

$$\begin{pmatrix} u_1(0) \\ u_2(0) \\ u_3(0) \\ u_4(0) \end{pmatrix}$$

$u_1 = u_{\frac{1}{2},+}$

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states conflict ( Lorentz invariant

$$u^{s'}_{\ell'}(0)\, \vec{J}^{\,j}_{s's}(R) = \vec{\mathcal{J}}_{\ell'\ell}(R)\, u^{s}_{\ell}(0)$$

how states actually transform

$|p, s\rangle \rightarrow |p, s'\rangle$

hope field transform

plan

pick a rep f

↓

solve for $u(0)$

↓ causality

determine statist

$$u^{s'}_{m',\pm}(0)\,\vec{J}^{\,j}_{s's}(R) = \underline{\tfrac{1}{2}\vec{\sigma}_{m'm}}\,u^{s}_{m,\pm}(0)$$

$$U^{\dagger}_{m's'}\,\vec{J}_{s's} = \boxed{\tfrac{1}{2}\vec{\sigma}_{m'm}}\,U_{ms} \leftarrow 2\times 2$$

2x2 represtn

$$\underbrace{U^{\pm}\vec{J}^{\,j}(R)}_{su(2)\ \text{rep}} = \underbrace{\tfrac{1}{2}\vec{\sigma}}_{\text{spinor}}\,U^{\pm}$$

spinor rep of Lorentz = dim 2 rep of su(2)

$$\begin{array}{c|cc} & 1 & 2 \\ 1 & & \\ 2 & & \end{array}$$

$$\rightarrow \begin{array}{c|cc} & -\tfrac{1}{2} & \tfrac{1}{2} \\ -\tfrac{1}{2} & & \\ \tfrac{1}{2} & & \end{array}$$

spin $\frac{1}{2}$

$U \cdot$ certain rep = other certain rep $U$

shur: $0$ or square matrix

vanish

$$s = -\tfrac{1}{2},\ \tfrac{1}{2}$$

$$u^{\frac{1}{2}}(0) = \begin{pmatrix} c_+ \\ 0 \\ c_- \\ 0 \end{pmatrix}$$

Part II

KG

$$(\partial_\mu \partial^\mu + m^2)\Phi = 0 \qquad \bar{\Phi}\gamma^\mu\Phi$$

$(\partial^\mu$

Dirac: one time derivative
one space derivative
needs to be solution of KG
Lorentz covariant

$$(\partial^\mu \partial_\mu + m^2)\Phi = 0$$

$$a^2 + b^2 = (-ia-b)(ia-b)$$

$$(i\partial_\mu \gamma^\mu - m)\psi = 0$$

consts

$$(-i\partial_\mu\gamma^\mu - m)(i\partial_\nu\gamma^\nu - m)\psi = 0$$

$$(\partial_\nu\partial_\mu \underbrace{\gamma^\mu\gamma^\nu}_{\eta^{\mu\nu}} + m^2)\psi = 0$$

$$\frac{\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu}{2} = \eta^{\mu\nu}$$

$$(i\partial_\mu\gamma^\mu - m)\psi = 0$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$$