Title: Lecture - QFT I, PHYS 601

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Collection/Series: Quantum Field Theory I (Core), PHYS 601, October 7 - November 6, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

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So far

resolution of first conflict $U(\Lambda,b)|p\,s\rangle = e^{i b\cdot\Lambda p}\sum_{s'} D^{j}_{s's}(W(\Lambda,p))\,|\Lambda p,\, s'\rangle$

$$W(\Lambda,p) = L^{-1}(\Lambda p)\,\Lambda\, L(p)$$

second conflict $S_{\alpha\beta}$ Lorentz invariant

$\int dt$ X

$\mathcal{H}_I \Leftarrow$ Lorentz invariant

$$a^{\dagger s'}_{\Lambda p}|0\rangle$$

$$D^{j}_{s's}(W(\Lambda,p))\,|\Lambda p, s'\rangle$$

$$W(\Lambda,p) = L^{-1}(\Lambda p)\,\Lambda L(p)$$

invariant

$H_I \Leftarrow$ Lorentz invariant

$H(x) \ni$ coefficient $a^{\dagger s_1}_{p_1} \cdots a^{\dagger s_n}_{p_n}\, a \cdots a$

$p_1 \quad p_n \quad p_1' \quad p_n'$

depends on Λ, b, p, s

$$H(x) \sim \sum_{N,M} \underbrace{\text{coefficient}}_{\text{depends on } \Lambda,\, \hbar,\, p,\, s} \; a^{\dagger s_1}_{p_1} \cdots a^{\dagger s_n}_{p_n} \, a \cdots a$$

$p_1 \quad p_n \quad p_1' \quad p_n'$

$p)$

momentum dependent $a^{\dagger} \ldots a$ failed us.

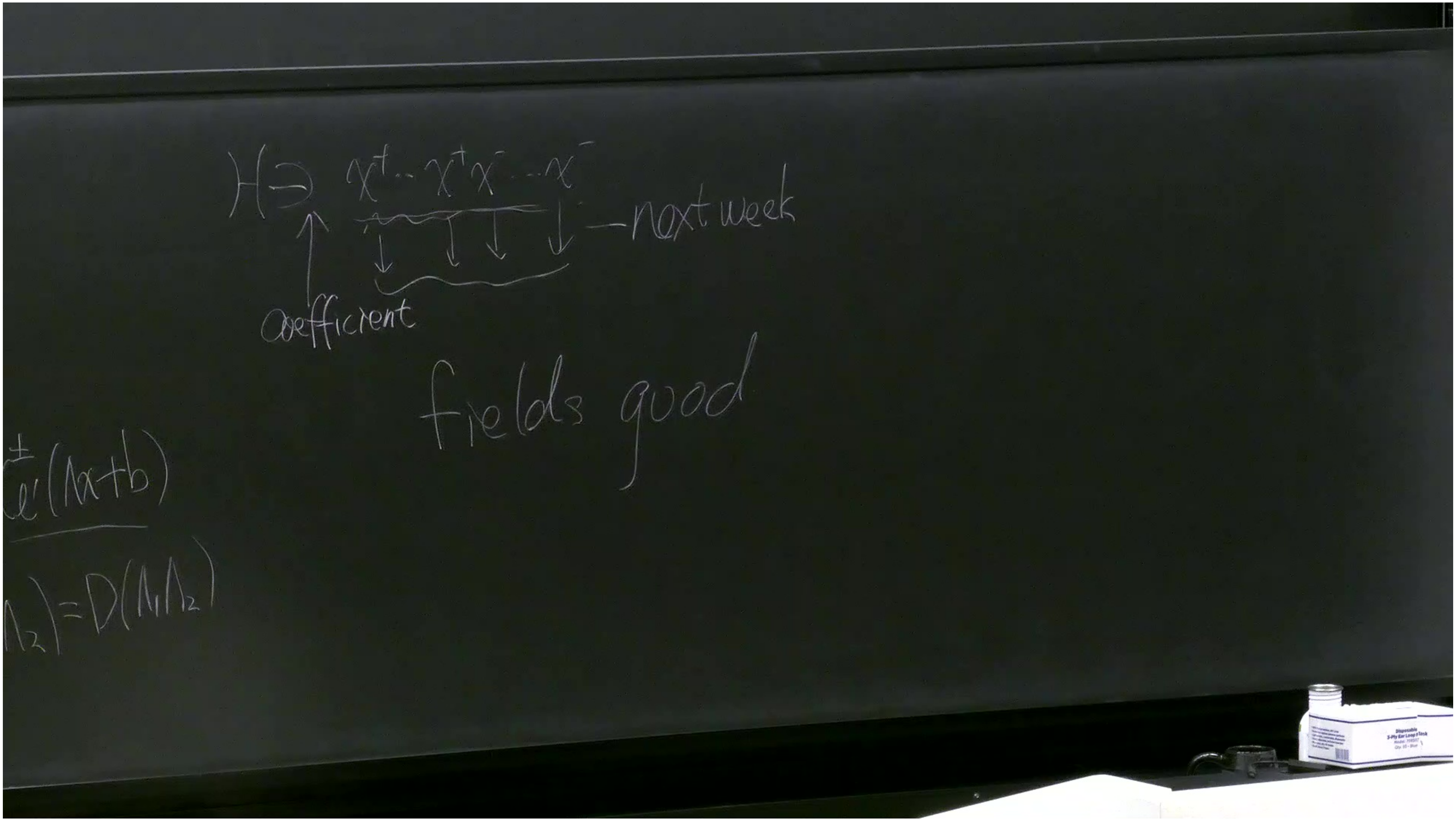
maybe position dependent = field can help

Hope: $U X U^{-1} = \underbrace{\text{only depend on } \Lambda}_{\text{matrix}} X(\Lambda x + b)$

getting real: $+$: creation $-$ annihilation field

$$U(\Lambda, b)\, \chi^{\pm}_{\ell}(x)\, U^{-1}(\Lambda, b) = \sum_{\ell'} D_{\ell\ell'}(\Lambda^{-1})\, \underline{\chi^{\pm}_{\ell'}(\Lambda x + b)}$$

$$D(\Lambda_1) D(\Lambda_2) = D(\Lambda_1 \Lambda_2)$$



H ⊃ χ+ ... χ+ χ- ... χ-
— next week
coefficient
fields good
(Λx+b)
Λ2) = D(Λ1Λ2)

fields

→ equation → field solution $\xrightarrow{?}$ $a, a^\dagger$ → states

Lagrangian ← e.o.m. ← field $\xleftarrow{\text{maybe}}$ $a, a^\dagger$ ← states

QM + Poincare

Weinberg

what is field?

$$dV_p \equiv \frac{d^3p}{(2\pi)^3(2E_p)}$$

ex $\sum_{s'} u^{s'}_{\ell'}(\Lambda x+b, \Lambda p)\,\underbrace{D_{s's}(W(\Lambda,p))}_{\text{little g}} = \sum_{\ell} D$

$$\chi^-_\ell(x) = \int \sum dV_p\, u^s_\ell(x,p)\, a^s_p$$

$$A = B \cdot C$$

what is field?

$$dV_p \equiv \frac{d^3p}{(2\pi)^3(2E_p)}$$

$$\chi_\ell(x) = \int \sum dV_p\, u_\ell^s(x,p)\, a_p^s$$

$$A = B \cdot C$$

ex $\sum_{s'} u_{\ell'}^{s'}(\Lambda x + b, \Lambda p)\, \underbrace{D_{s's}^{j}(W(\Lambda,p))}_{\text{little g}} = \sum_{\ell} \underbrace{D_{\ell'\ell}(\Lambda)}_{\text{Lorentz rep}}\, u_\ell^s(x,p)\, \underbrace{e^{-ib\cdot\Lambda p}}_{\text{little g}}$

step 1:

$\sim p)$

$u_{\ell}^{s}(x,p)\, a_p^{s}$

ex $$\sum_{s'} \underbrace{u_{\ell'}^{s'}(\Lambda x+b, \Lambda p)\, D_{s's}(W(\Lambda,p))}_{\text{little g}} = \sum_{\ell} \underbrace{D_{\ell'\ell}(\Lambda)}_{\text{Lorentz rep}}\, u_{\ell}^{s}(x,p)\, \underbrace{e^{-i\Lambda b\cdot p}}_{\text{little g}}$$

step 1: $W(\Lambda,p) = L^{-1}(\Lambda p)\,\Lambda\, L(p)$

$(\Lambda, b) = (\delta, b)$ $\qquad W \to \mathbb{1}$

$$u(x+b, \vec{p}) = u(x,\vec{p})\, e^{-ib\cdot p}$$

$x \to 0$, $\quad b \to 0$

$$u(x,\vec{p}) = u(0,\vec{p})\, e^{-ix\cdot p}$$

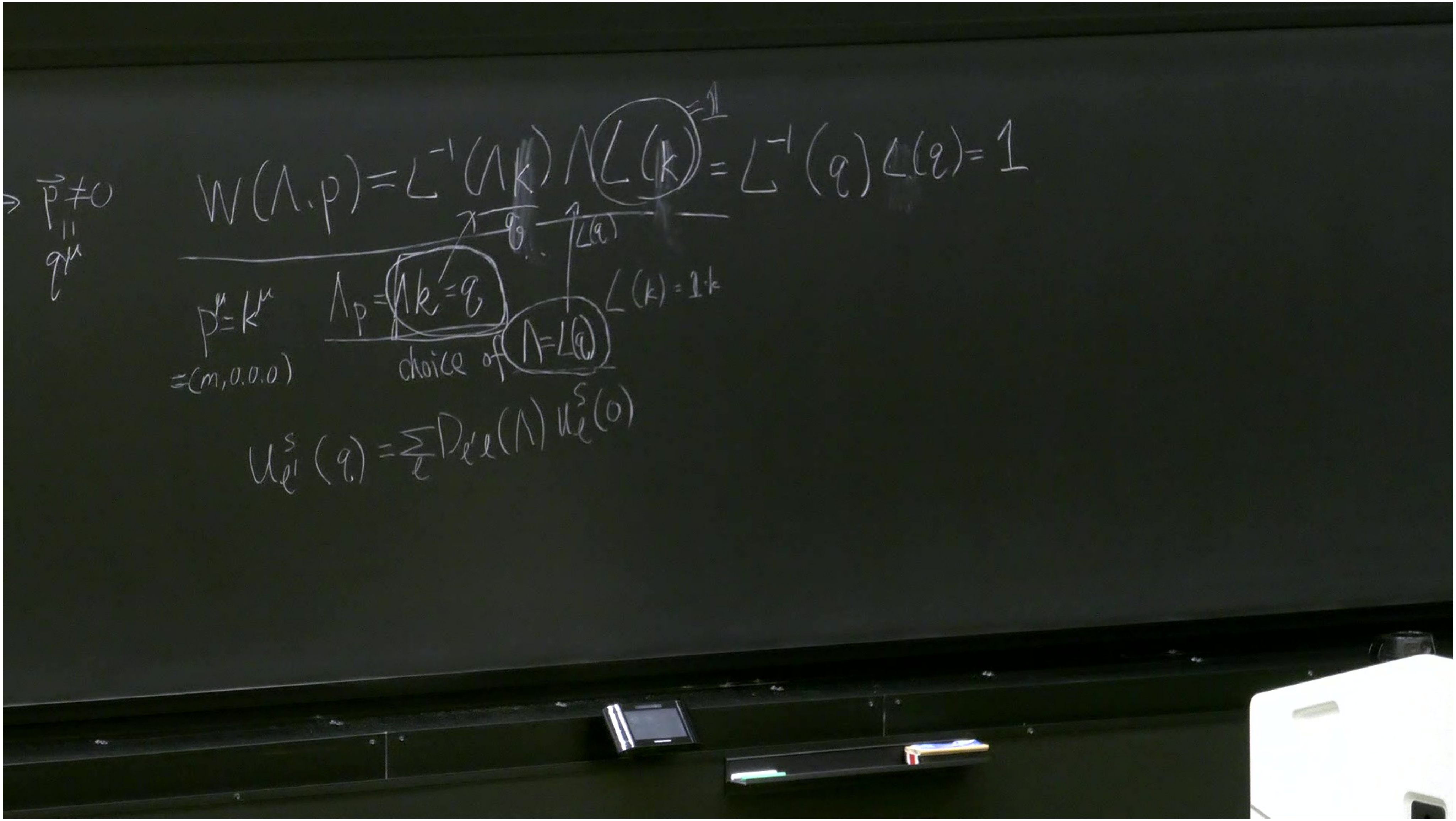
step 2 boost $\vec{p} = 0 \longrightarrow \vec{p} \neq 0$

$(m,0,0,0) \qquad \vec{p} = q^\mu$

$$W(\Lambda, p) = L^{-1}(\Lambda p)\,\Lambda\, L(p)$$

$p^\mu = k^\mu = (m,0,0,0)$

$\Lambda p = \Lambda k = q$



$\Rightarrow \vec{p} \neq 0$
$= q^\mu$
$W(\Lambda,p) = L^{-1}(\Lambda k)\,\Lambda\, L(k) = L^{-1}(q)\, L(q) = 1$
$= 1$
q
$L(q)$
$p^\mu = k^\mu$
$= (m,0.0.0)$
$\Lambda_p = \Lambda k = q$
choice of $\Lambda = L(q)$
$L(k) = 1 \cdot k$
$u^s_{\ell'}(q) = \sum_\ell D_{\ell'\ell}(\Lambda)\, u^s_\ell(0)$

$$u(x,\vec{p}) = u(0,\vec{p})\, e^{-ipx}$$

$$= L^{-1}(\Lambda k)\,\Lambda\, L(k) = L^{-1}(q)\, L(q) = 1$$

$\Lambda k = q$ $\quad L(k) = 1\cdot k$

choice of $\Lambda = L(q)$

$$= \sum_{\ell} D_{\ell'\ell}(\Lambda)\, u_\ell^s(0)$$

Λ = rotation = R $\quad W = R$

$$\boxed{\sum_{s'} u_{\ell'}^{s'}(0)\, D^{j}_{s's}(\underset{W}{R}) = \sum_{\ell} D_{\ell'\ell}(\underset{\Lambda}{R})\, u_\ell^s(0)}$$

trivial rep

$\Lambda k = q$

$\Lambda = L(q)$ choice of

$L(k) = 1 k$

$\sum_{\ell} D_{\ell'\ell}(\Lambda)\, u_{\ell}^{s}(0)$

$\Lambda = \text{rotation} = R$

$$\sum_{s'} u_{\ell'}^{s'}(0)\, J_{s's}^{j}(\overset{W}{\overset{\|}{R}}) = \sum_{\ell} \mathcal{D}_{\ell'\ell}(\overset{\Lambda}{\overset{\|}{R}})\, u_{\ell}^{s}(0)$$

$\| \; 0 \quad (D = 1)$

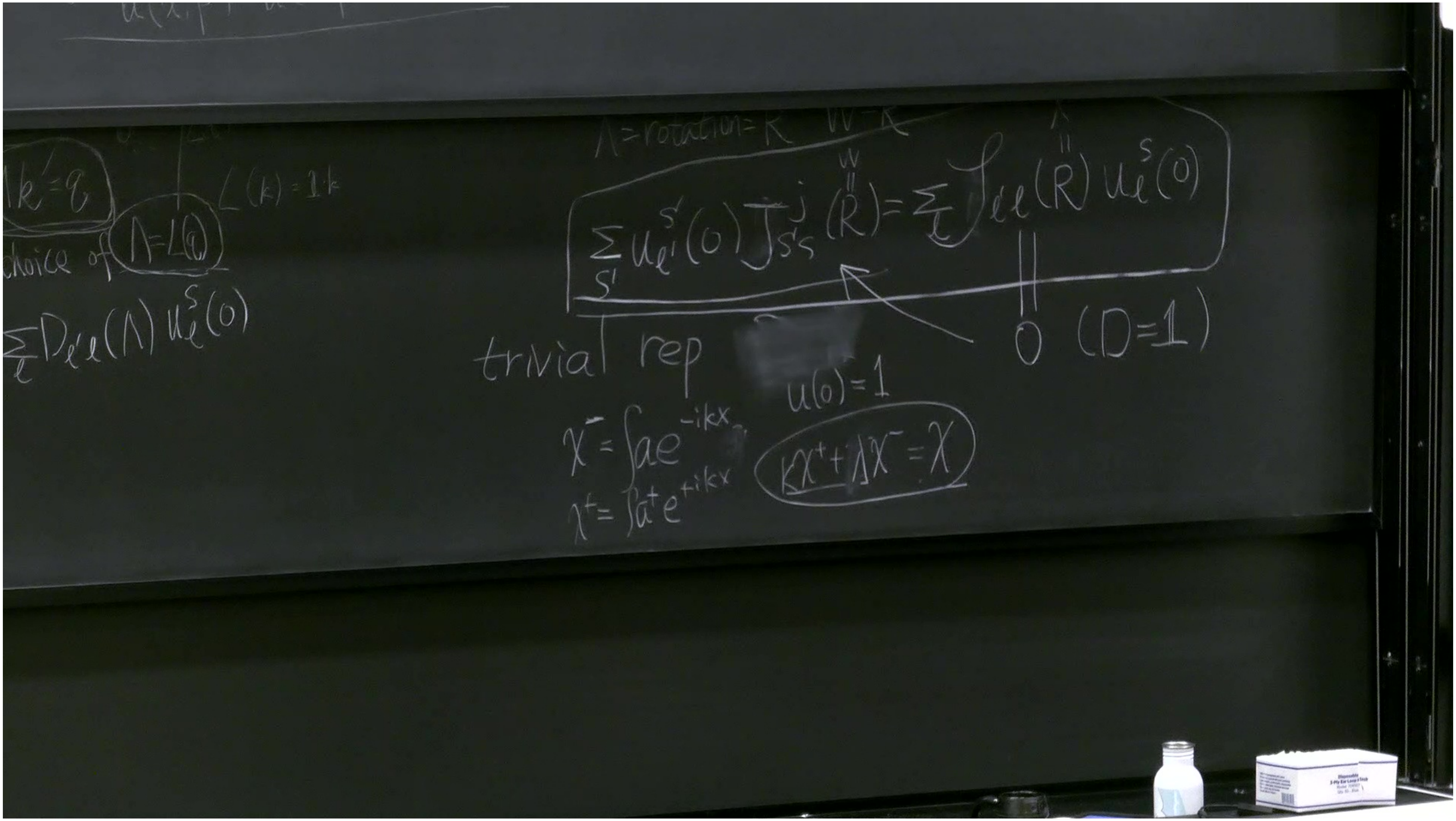
trivial rep

$u(0) = 1$

$\chi^- = \int a e^{-ikx}$

$\chi^+ = \int a^+ e^{+ikx}$

$\kappa\chi^+ + \lambda\chi^- = \chi$


Λ = rotation = R
$\sum_{s'} u_{\ell'}^{s'}(0) J_{s's}^{j}(R) = \sum_{\ell} D_{\ell'\ell}(R)\, u_{\ell}^{s}(0)$
choice of $\Lambda = L(q)$
$L(k) = 1 \cdot k$
$\sum_{\ell} D_{\ell'\ell}(\Lambda)\, u_{\ell}^{s}(0)$
trivial rep
0 (D=1)
u(0) = 1
$\chi^- = \int a e^{-ikx}$
$\chi^+ = \int a^+ e^{+ikx}$
$\kappa\chi^+ + \lambda\chi^- = \chi$