Title: Lecture - QFT I, PHYS 601

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Collection/Series: Quantum Field Theory I (Core), PHYS 601, October 7 - November 6, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: October 10, 2024 - 9:00 AM

URL: https://pirsa.org/24100027

Big picture

SR $|\psi_B\rangle = U_{T(\Lambda,b)}\,|\psi_A\rangle$

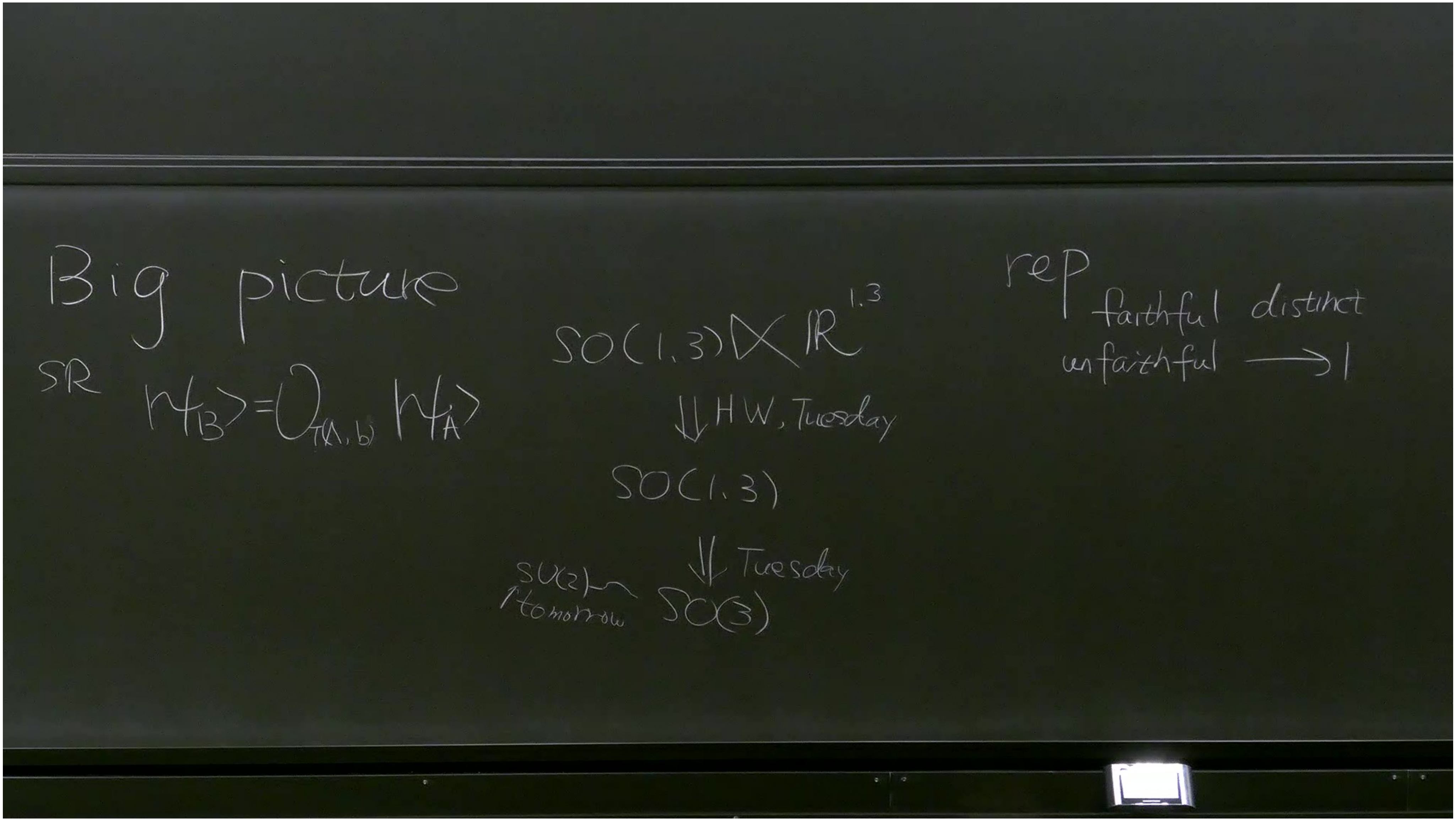
$SO(1,3) \ltimes \mathbb{R}^{1,3}$

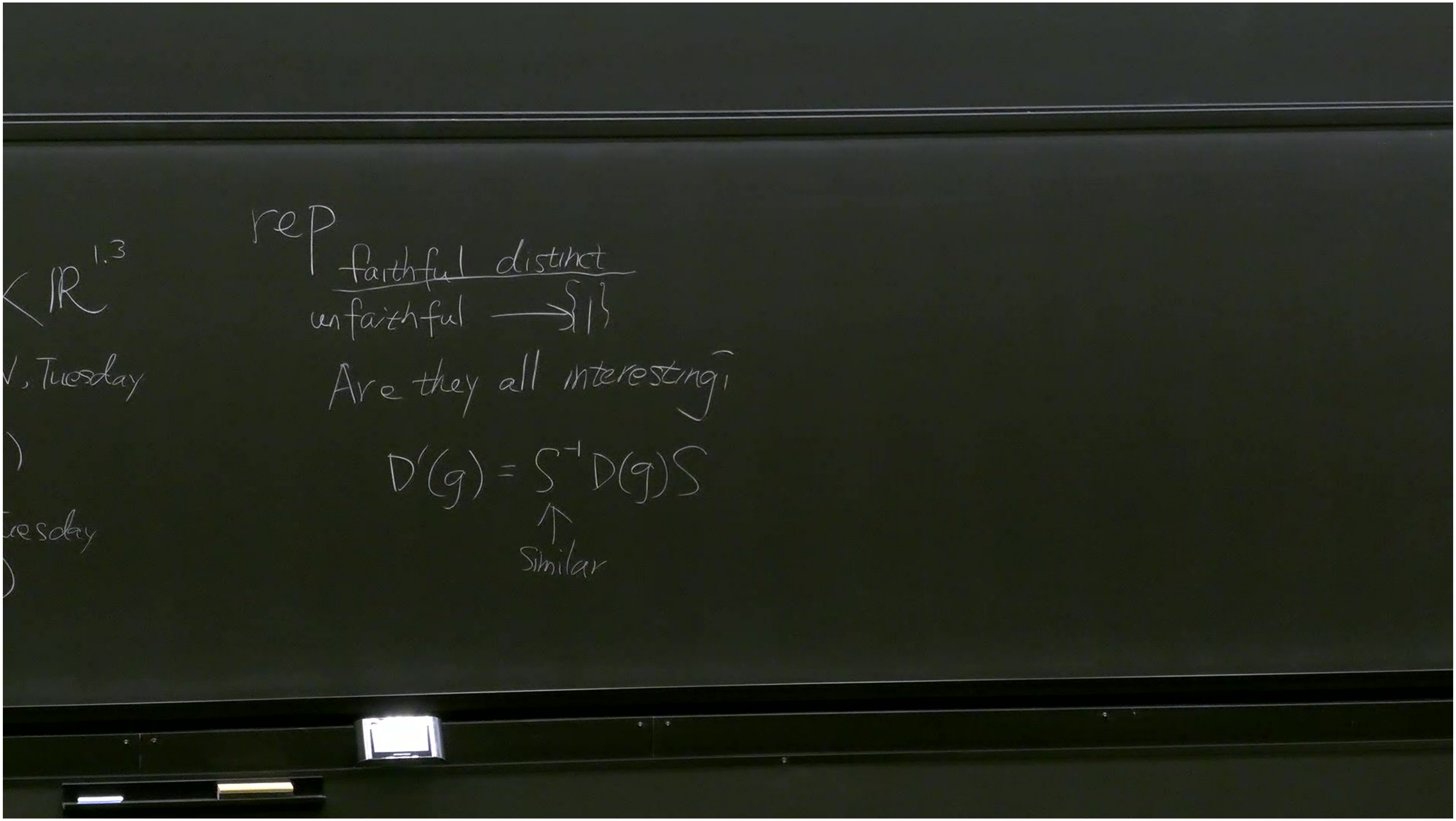
$\Downarrow$ HW, Tuesday

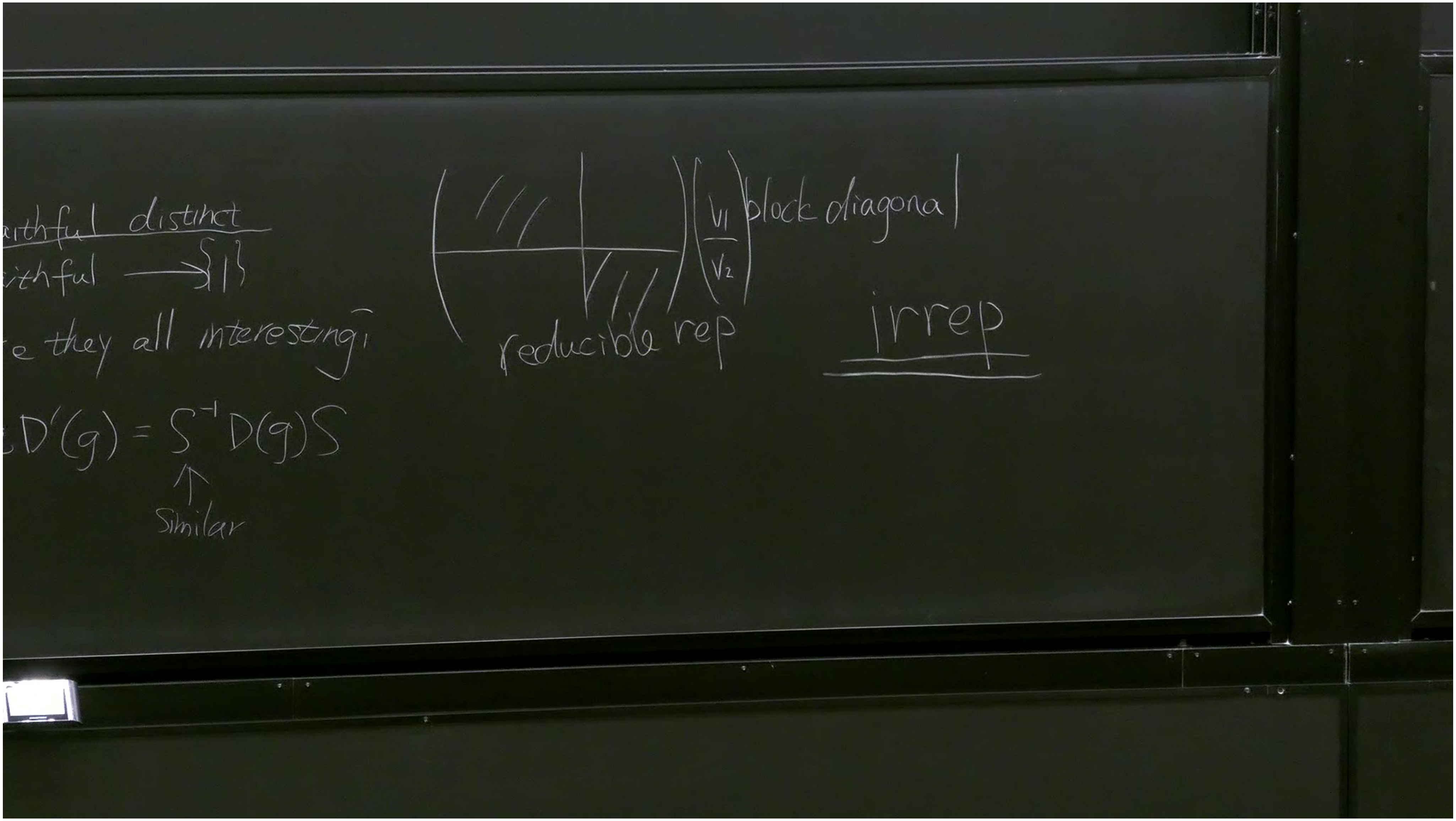
$SO(1,3)$

$\Downarrow$ Tuesday

$SU(2) \rightsquigarrow SO(3)$
tomorrow


Big picture
SR |ψB> = O T(Λ, b) |ψA>
SO(1,3) ⋉ ℝ 1,3
⇓ HW, Tuesday
SO(1,3)
⇓ Tuesday
SU(2)
↑tomorrow
SO(3)
rep faithful distinct
unfaithful → 1


rep
faithful distinct
unfaithful → {1}
Are they all interesting?
D'(g) = S^-1 D(g) S
similar
Tuesday


aithful distinct
ithful
e they all interesting
D'(g) = S^-1 D(g) S
similar
V1
V2
block diagonal
reducible rep
irrep

$$U(\tilde{\epsilon})_{M\times M} = \mathbb{1}_{M\times M} + \sum_a i\epsilon_a (T_a)_{M\times M} + O(\epsilon^2)$$

$$e^x = \lim_{N\to\infty}\left(1+\frac{x}{N}\right)^N$$

$$U(\tilde{\varphi}) = \lim_{N\to\infty}\left(1+\frac{\sum_a \varphi_a T_a}{N}\right)^N \equiv e^{\sum_a i\varphi_a T_a}$$

structure constant

$$[T_a, T_b] = i\sum_c f_{abc}\, T_c$$

3 matrix (

$O(\epsilon^2)$

structure constant

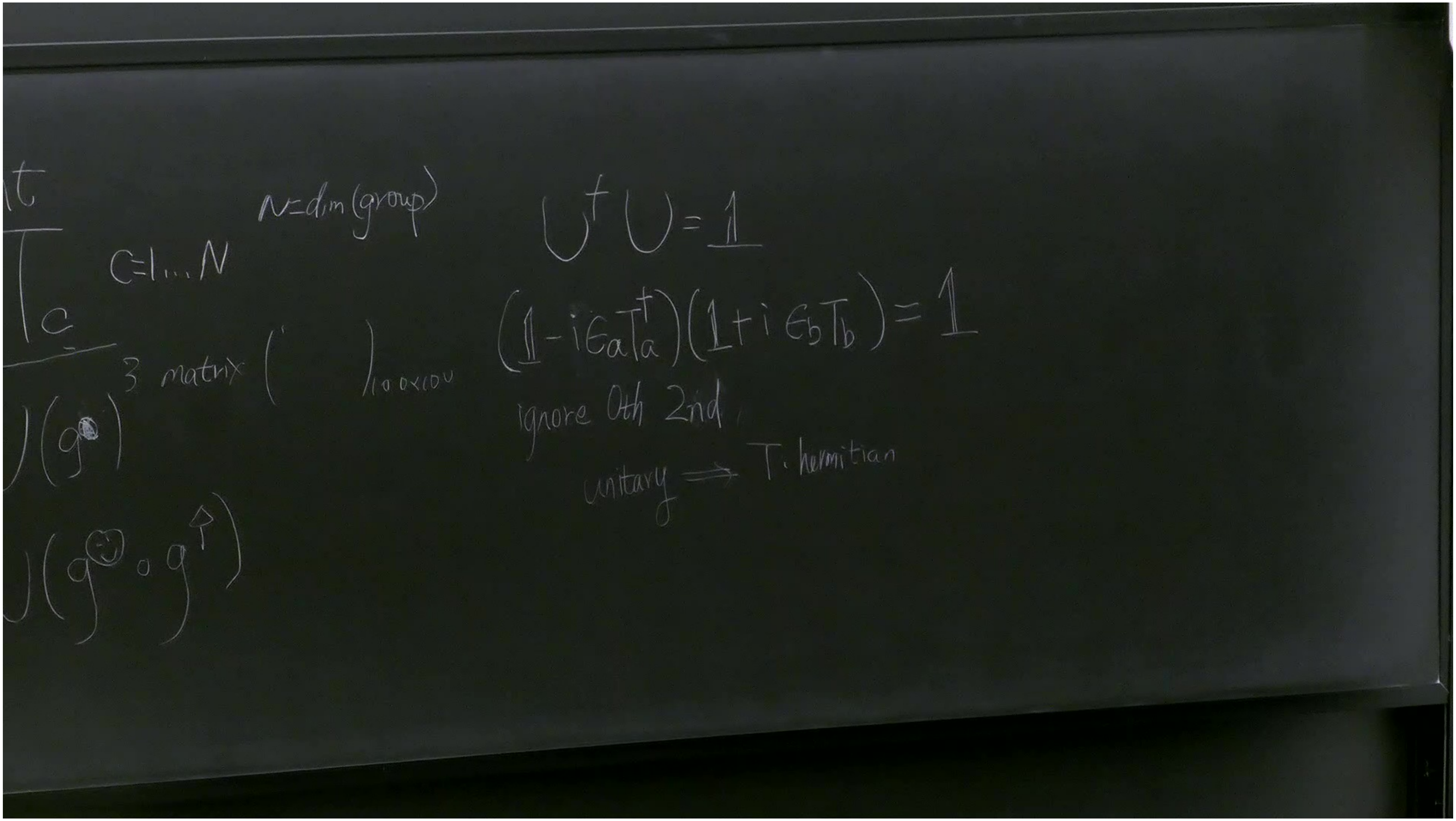
$N = \dim(\text{group})$

$$[T_a, T_b] = i \sum_c f_{abc} T_c \qquad c = 1 \ldots N$$

3 matrix $\left(\quad \right)_{100 \times 100}$

$$U(g^{☺})\,U(g^{\uparrow}) = U(g^{\bullet})$$

$$\tilde{\varphi}(g^{\bullet}) = \tilde{f}(\varphi^{☺}, \varphi^{\uparrow}) = U(g^{☺} \circ g^{\uparrow})$$


N=dim(group)
$U^\dagger U = 1$
$a=1 \ldots N$
$(1 - i\epsilon_a T_a^\dagger)(1 + i\epsilon_b T_b) = 1$
3 matrix
100×100
ignore 0th 2nd
unitary ⟹ T: hermitian

Adjoint rep

$m = n$

$\dim(\text{rep}) = \dim(\text{group})$

ex. $\left\{ \begin{array}{l} \left(T_a^{adj}\right)_{bc} = -i f_{abc} \\ \left[[T_a, T_b], T_c\right] + \text{cyclic} = 0 \end{array} \right.$

$\left(T_a^{adj}\right)$ ↙ rep = matrix : act on vector

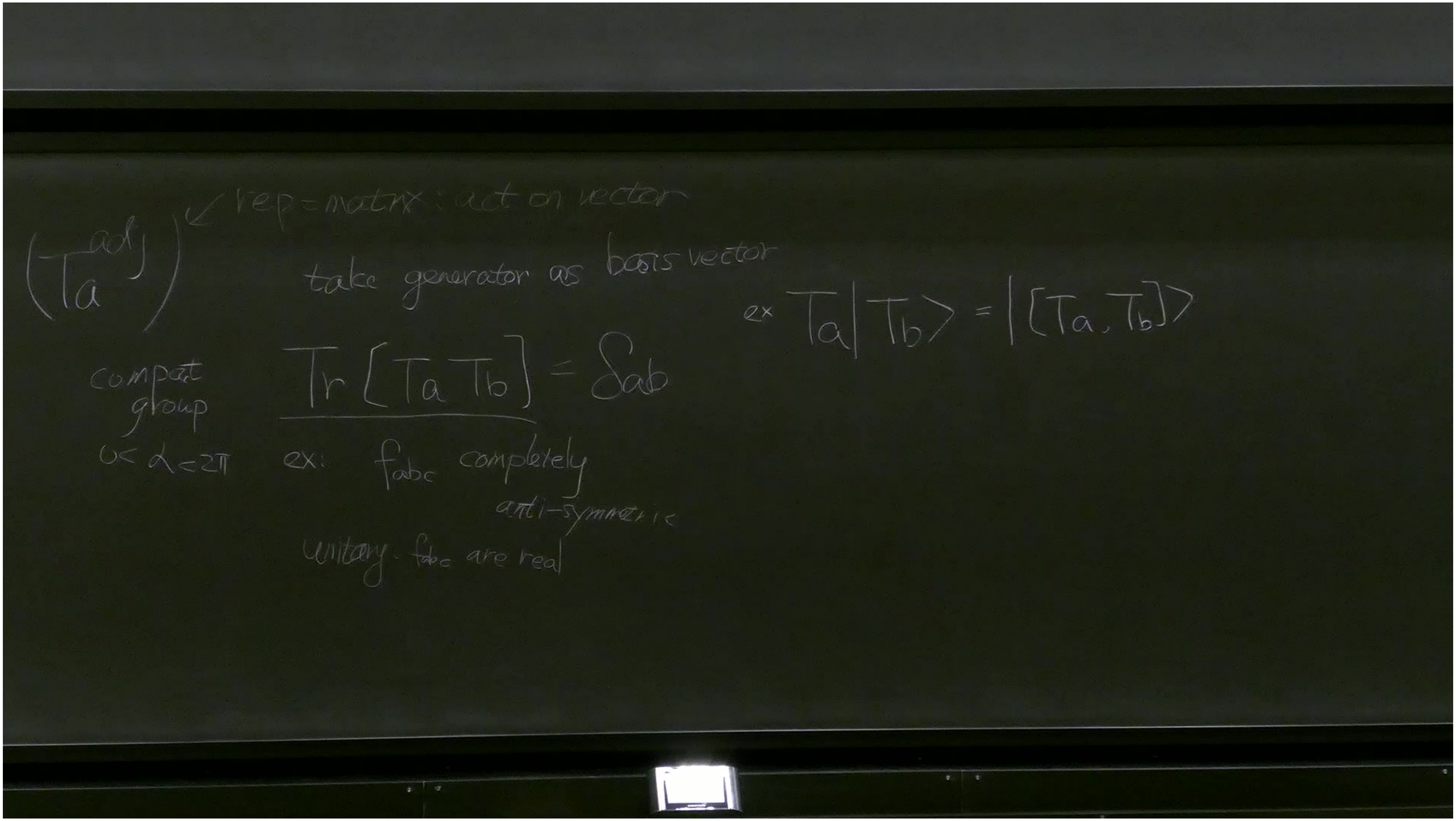
take generator as basis vector

compact group

$0 < \alpha < 2\pi$

$\mathrm{Tr}\left[T_a T_b\right] = \delta_{ab}$

ex: f_{abc} completely anti-symmetric


rep = matrix : act on vector
(T_a^{adj})
take generator as basis vector
ex $T_a|T_b\rangle = |[T_a, T_b]\rangle$
compact group
$Tr[T_a T_b] = \delta_{ab}$
$0 < \alpha < 2\pi$
ex: f_{abc} completely anti-symmetric
unitary. f_{abc} are real