Title: Lecture - QFT I, PHYS 601

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Collection/Series: Quantum Field Theory I (Core), PHYS 601, October 7 - November 6, 2024

Subject: Condensed Matter, Particle Physics, Quantum Fields and Strings

Date: October 08, 2024 - 9:00 AM

URL: https://pirsa.org/24100026

Quantum Field Theory I

who ordered fields

1. When QM meets SR
2. Dirac field and interactions
3. a hint of renormalization

symmetry: Lecture One (Group theory. why? what? how?)

s

QM

states. operators

Alice x $|\Psi\rangle$

SR

observers Lorentz transform

Bob $x' = \underline{\Lambda x}$ $|\Psi'\rangle$

big question $|\Psi'\rangle = O_\Lambda |\Psi\rangle$

QM   Alice   Bob

$|\Psi\rangle \quad |\Psi'\rangle$

$|\Phi\rangle \quad |\Phi'\rangle$

Φ

$$|\langle\Psi|\Phi\rangle|^2 = |\langle\Psi'|\Phi'\rangle|^2$$

$$\langle\psi|\Phi\rangle = \langle\Psi'|\Phi'\rangle = \langle U\psi|U\Phi\rangle = \langle\psi|U^\dagger U|\Phi\rangle$$

$$U^\dagger U = 1$$

Alice Bob Charlie

$$|\psi\rangle \xrightarrow{U_1} |\psi'\rangle \xrightarrow{U_2} |\psi''\rangle$$

$$|\psi\rangle \xrightarrow[U_3]{} |\psi''\rangle$$

$$\boxed{U_3 = U_1 U_2}$$

part of group theory def.

$$|\psi'\rangle = U_1|\psi\rangle$$

$$|\psi''\rangle = U_2|\psi'\rangle = U_2 U_1|\psi\rangle$$

$$|\psi''\rangle = U_3|\psi\rangle$$

What is group?

symmetry (invariance)

Group (set of sym operation)

① Identity exsists

② Inverse exsists

③ closeness

④ associative

examples: ① cyclic group

$\{a^n = e\}$ ← identity

② polygon group

□ D_n unigon

R (+, −, ×, ÷)

$e = 0$

1) subgroup

$\{e\}$ whole

2) abelian-commutative

$\mathbb{R}, +$

3) similar homomorphic

G G'

map f

$f(g) = g'$

$f(g_1) \circ' f(g_2) = f(g_1 \circ g_2)$

4) same f one to one

5) direct product

$G \otimes G'$ (g, g')

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

Lie group

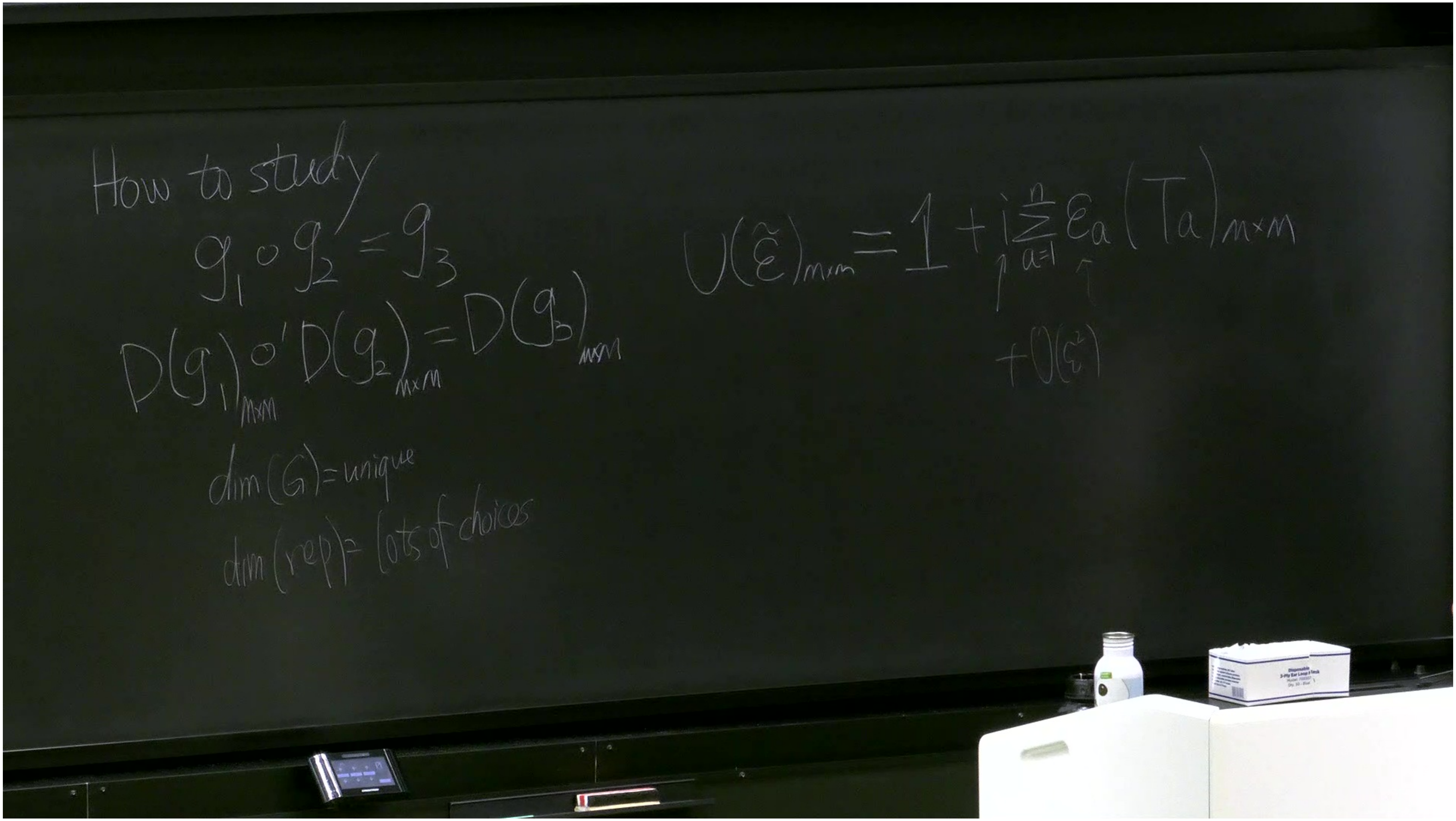
$$g(\vec{\varphi}) \qquad \vec{\varphi} = \underbrace{(\varphi_1, \varphi_2, \cdots)}_{\dim(G)}$$

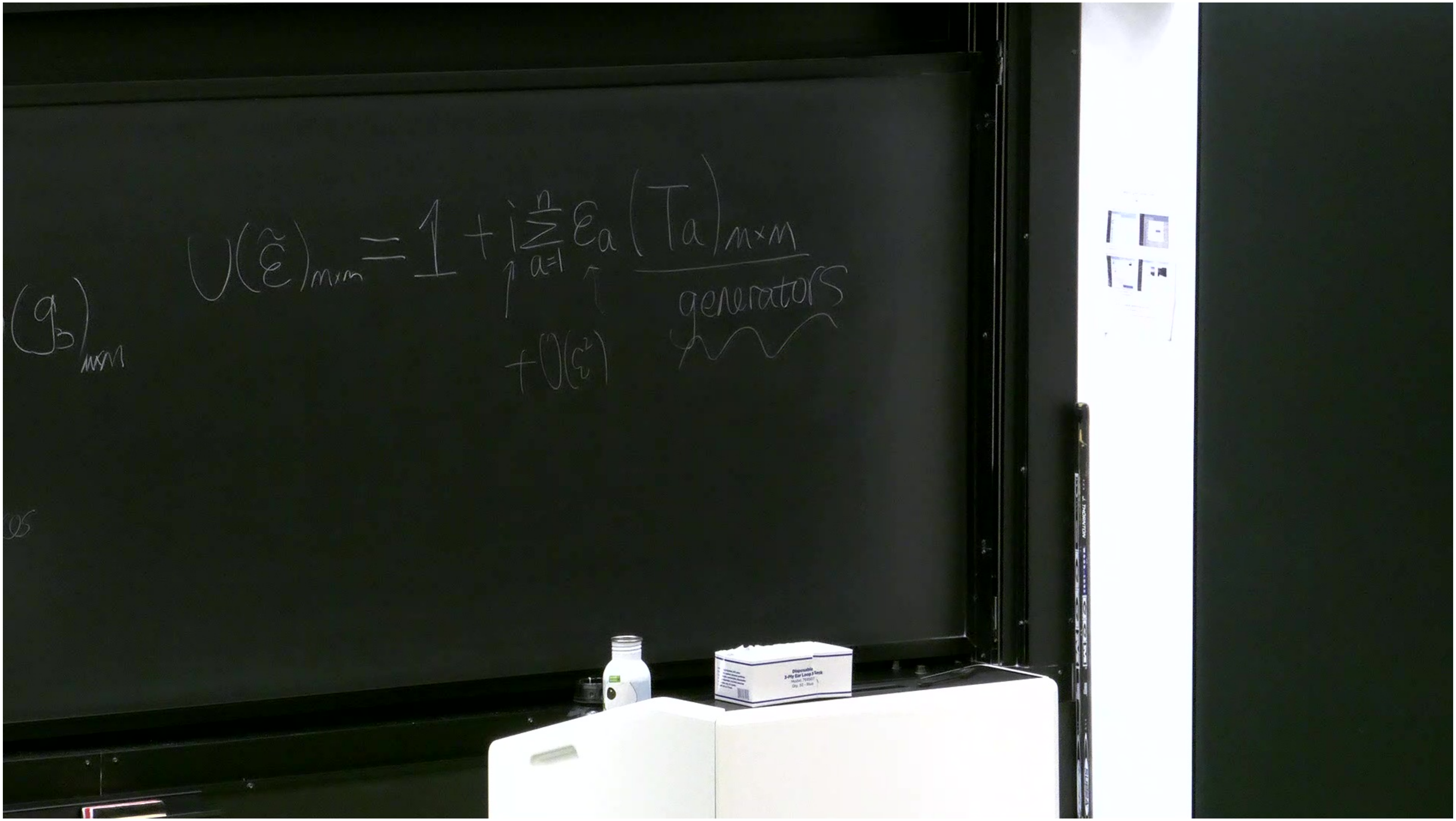
$$g(0) = e$$

$$\langle x | x \rangle = \langle Rx | Rx \rangle = \langle x | R^T R | x \rangle$$

$$R^T R = 1$$

Special $\det R = 1$


How to study
$g_1 \circ g_2 = g_3$
$D(g_1)_{m\times m} \circ' D(g_2)_{m\times m} = D(g_3)_{m\times m}$
$\dim(G) =$ unique
$\dim(\text{rep}) =$ lots of choices
$U(\tilde{\varepsilon})_{m\times m} = 1 + i \sum_{a=1}^{n} \varepsilon_a (T_a)_{m\times m} + O(\varepsilon^2)$


$U(\tilde{\varepsilon})_{m\times m} = 1 + i\sum_{a=1}^{n} \varepsilon_a (T_a)_{m\times m}$
$+O(\varepsilon^2)$
generators
$(g_3)_{m\times m}$