Title: Lecture - Statistical Physics, PHYS 602

Speakers: Emilie Huffman

Collection/Series: Statistical Physics (Core), PHYS 602, October 7 - November 6, 2024

Subject: Condensed Matter, Other

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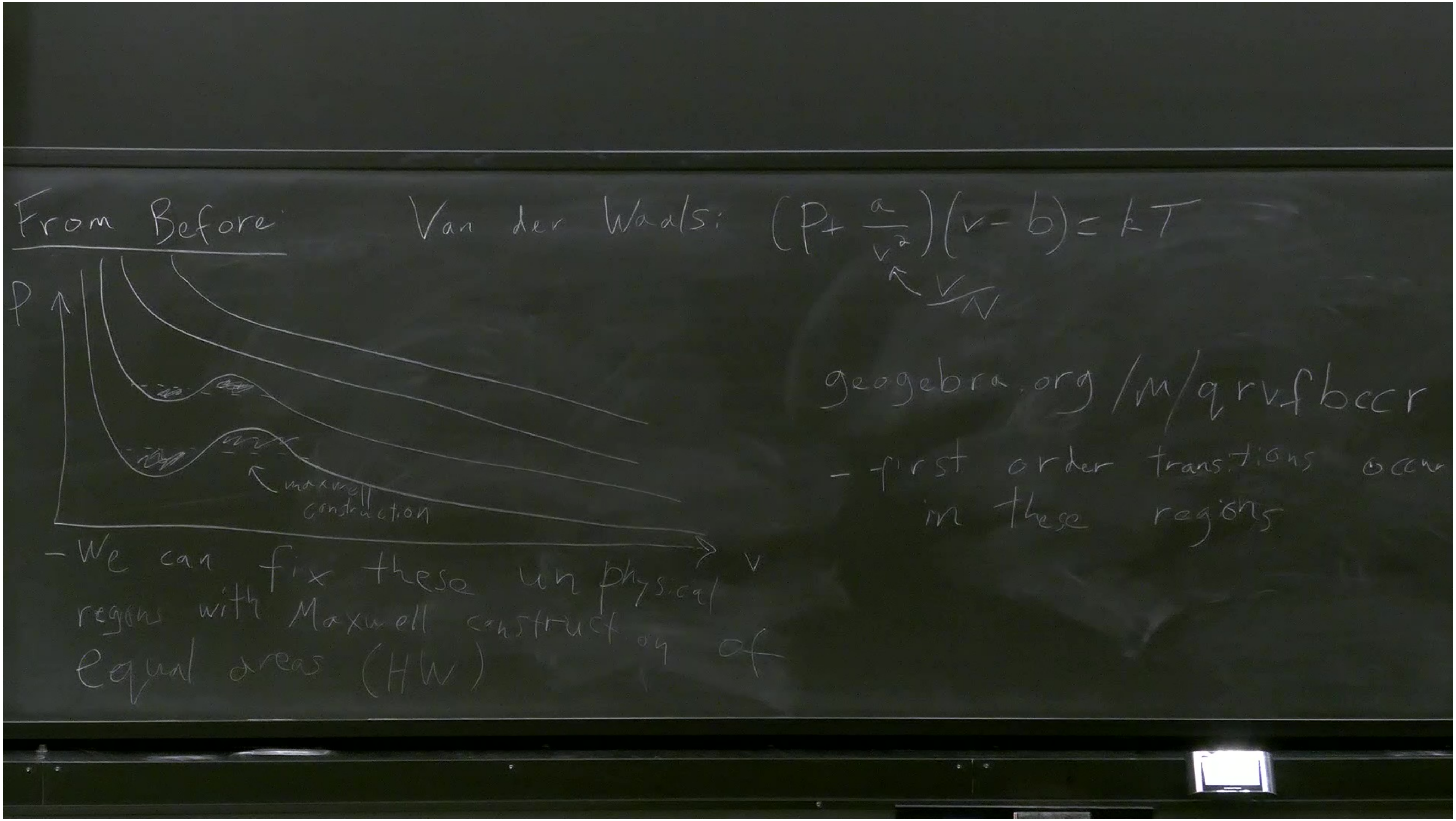
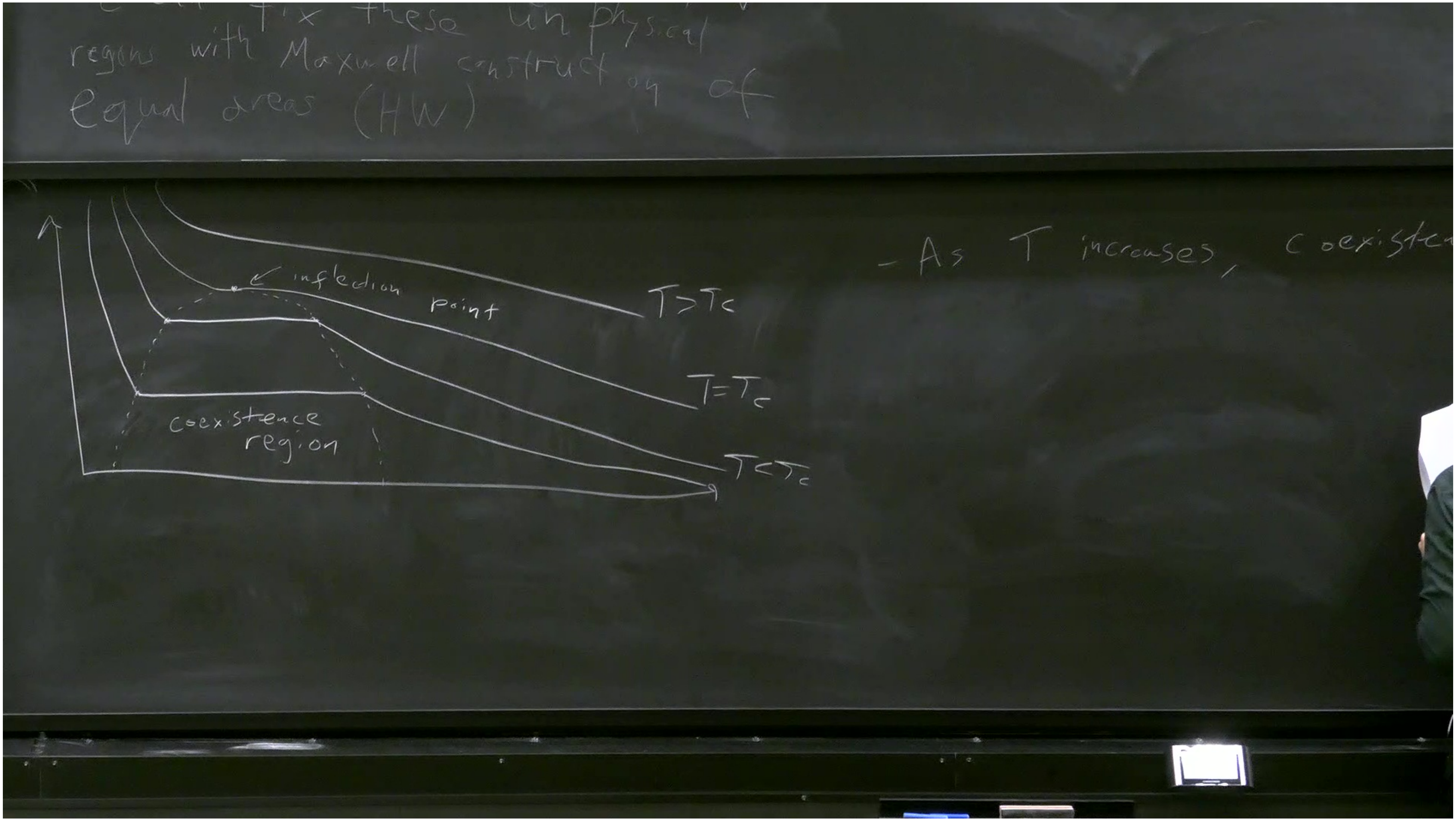

From Before
Van der Waals: (P+ a/v²)(v-b)=kT
V/N
P
maxwell construction
v
- We can fix these unphysical regions with Maxwell construction of equal areas (HW)
geogebra.org/m/qrvfbccr
- first order transitions occur in these regions


fix these unphysical
regions with Maxwell construction of
equal areas (HW)
inflection point
T>Tc
T=Tc
T<Tc
coexistence region
- As T increases, coexisten

- As T increases, coexistence region becomes narrower, collapsing to a single point, T_c

Here we have both $\frac{\partial P}{\partial v} = 0$, $\frac{\partial^2 P}{\partial v^2} = 0$ at T_c

What are T_c, v_c, P_c?

$$\left(P + \frac{a}{v^2}\right)(v-b) = kT$$

$$P = \frac{kT}{v-b} - \frac{a}{v^2} \longrightarrow \frac{\partial P}{\partial v} = \frac{-kT}{(v-b)^2} + \frac{2a}{v^3} = 0 \longrightarrow \frac{kT_c}{(v_c-b)^2} = \frac{2a}{v_c^3} \quad (1)$$

$$\frac{\partial^2 P}{\partial v^2} = \frac{2kT}{(v-b)^3} - \frac{6a}{v^4} = 0 \longrightarrow \frac{2kT_c}{(v_c-b)^3} = \frac{6a}{v_c^4} \quad (2)$$

$$\cdots\frac{\cdots c}{\cdots b)^2} = \frac{2a}{V_c^3} \quad (1)$$

$$\cdots = \frac{6a}{V_c^4} \quad (2)$$

Divide (1) by (2)

$$\frac{v_c - b}{2} = \frac{v_c}{3} \rightarrow \boxed{v_c = 3b}$$

$$\frac{kT_c}{(v_c - b)} = \frac{2a}{v_c^3} \longrightarrow \boxed{T_c = \frac{8a}{27bk}}$$

$$P_c = \frac{kT_c}{v_c - b} - \frac{a}{v_c^2} \longrightarrow \boxed{P_c = \frac{a}{27b^2}}$$

What can we learn <u>near</u> the critical point?

Defining: (reduced variables) $P_r = \frac{P}{P_c}$, $v_r = \frac{v}{v_c}$, $T_r = \frac{T}{T_c}$

$$\left(P + \frac{a}{v^2}\right)(v-b) = kT \rightarrow \left(P_r P_c + \frac{a}{v_r^2 v_c^2}\right)(v_r v_c - b) = k T_r T_c$$

multiply $\frac{27b}{a}$

$$\rightarrow \left(P_r + \frac{3}{v_r^2}\right)(3v_r - 1) = 8T_r$$

We now define $\pi, \psi, t \ll 1$ so that

$$P_r = 1+\pi, \quad v_r = 1+\psi, \quad T_r = 1+t$$

multiply by v_r^2

$$\left((1+\pi)(1+\psi)^2+3\right)\left(3(1+\psi)-1\right) = 8(1+t)(1+\psi)^2$$

multiply, rearange, organize

$$\pi\left(2+7\psi+8\psi^2+3\psi^3\right)+3\psi^3 = 8t\left(1+2\psi+\psi^2\right)$$

- First, with $T=T_c \to \quad (T_r=1,\ t=0)$

$$\pi = -3\psi^3$$

$$\to \left((1+\pi)(1+\psi)+3\right)\left(3(1+\psi)-1\right) = 8(1+t)(1+\psi)$$

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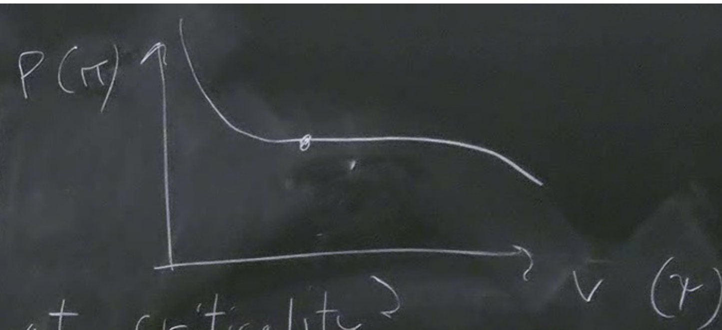
- First, with $T=T_c \to \quad (T_r=1,\ t=0)$

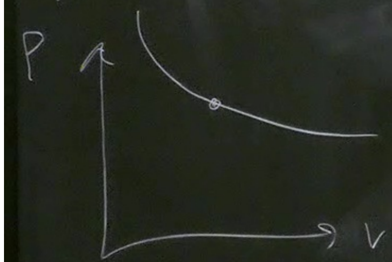
$$\pi = \frac{-3\psi^3}{2+7\psi+8\psi^2+3\psi^3} \qquad (1+x)^{-n} \approx 1-nx$$

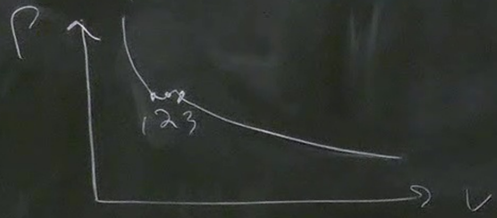
small

$$\Pi = \frac{-3\psi^3}{2\left(1+\frac{3}{2}\psi+4\psi^2+\frac{3}{2}\psi^3\right)} \approx \boxed{-\frac{3}{2}\psi^3}$$



- Now how does ψ depend on t at criticality?





ψ_1, ψ_2, ψ_3

close to zero

ψ_1, ψ_2, ψ_3

close to zero

$$(3+\cancel{3\pi}^{\,0})\psi^3 + (8\pi-8t)\psi^2 + (7\pi-16t)\psi + \underbrace{(2\pi-8t)}_{=0} = 0$$

Assume $|\psi_2| \ll |\psi_{1,3}|$ and $|\psi_1| \simeq |\psi_3|$

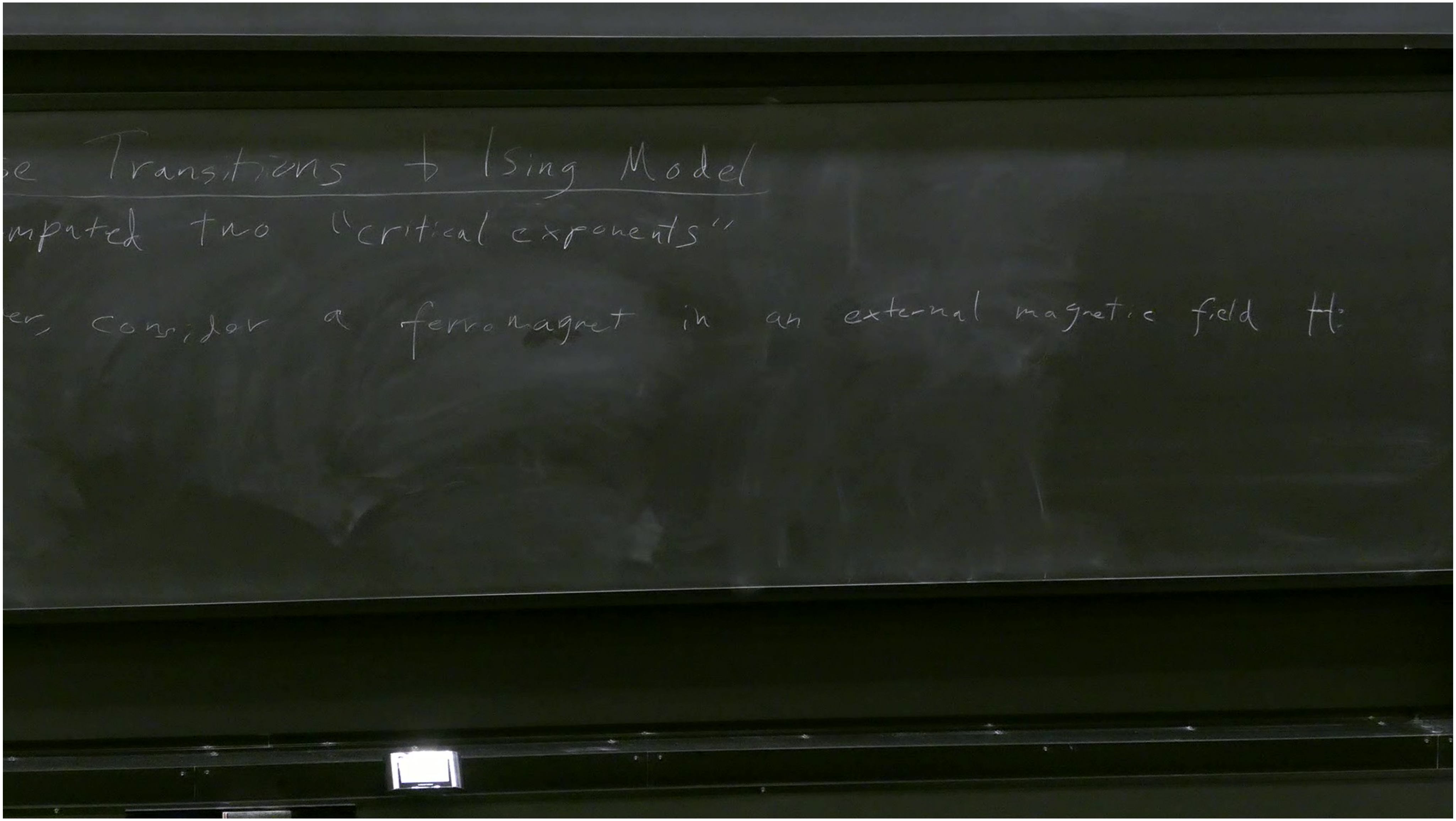
Assume $2\pi - 8t = 0 \rightarrow \pi \simeq 4t$

$$\psi_2 = 0 \rightarrow \boxed{\psi_{1,3} \simeq \pm 2|t|^{\frac{1}{2}}}$$

$$\boxed{\pi \simeq -\frac{3}{2}\psi^3}$$

Microscopic Theory of Phase Transitions + Ising Model

- We have actually just computed two "critical exponents"
- To develop this idea further, consider a ferromagnet in


se Transitions + Ising Model
mputed two "critical exponents"
er, consider a ferromagnet in an external magnetic field H:

Microscopic Theory of Phase Transitions + (

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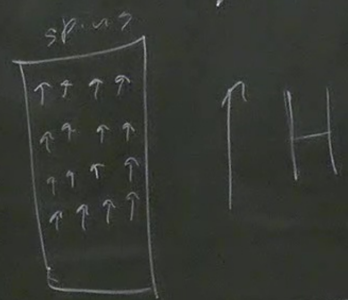
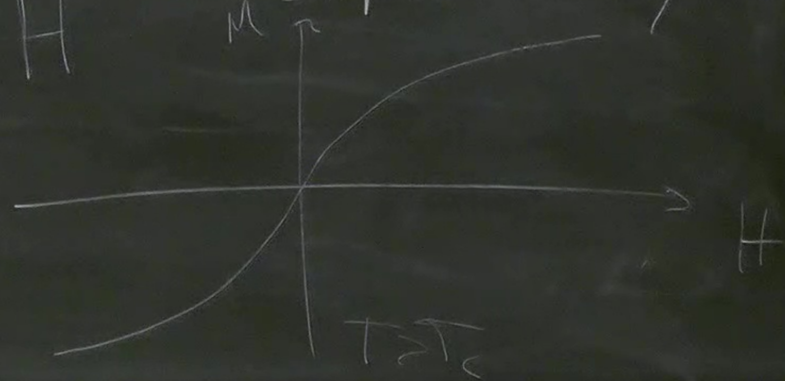
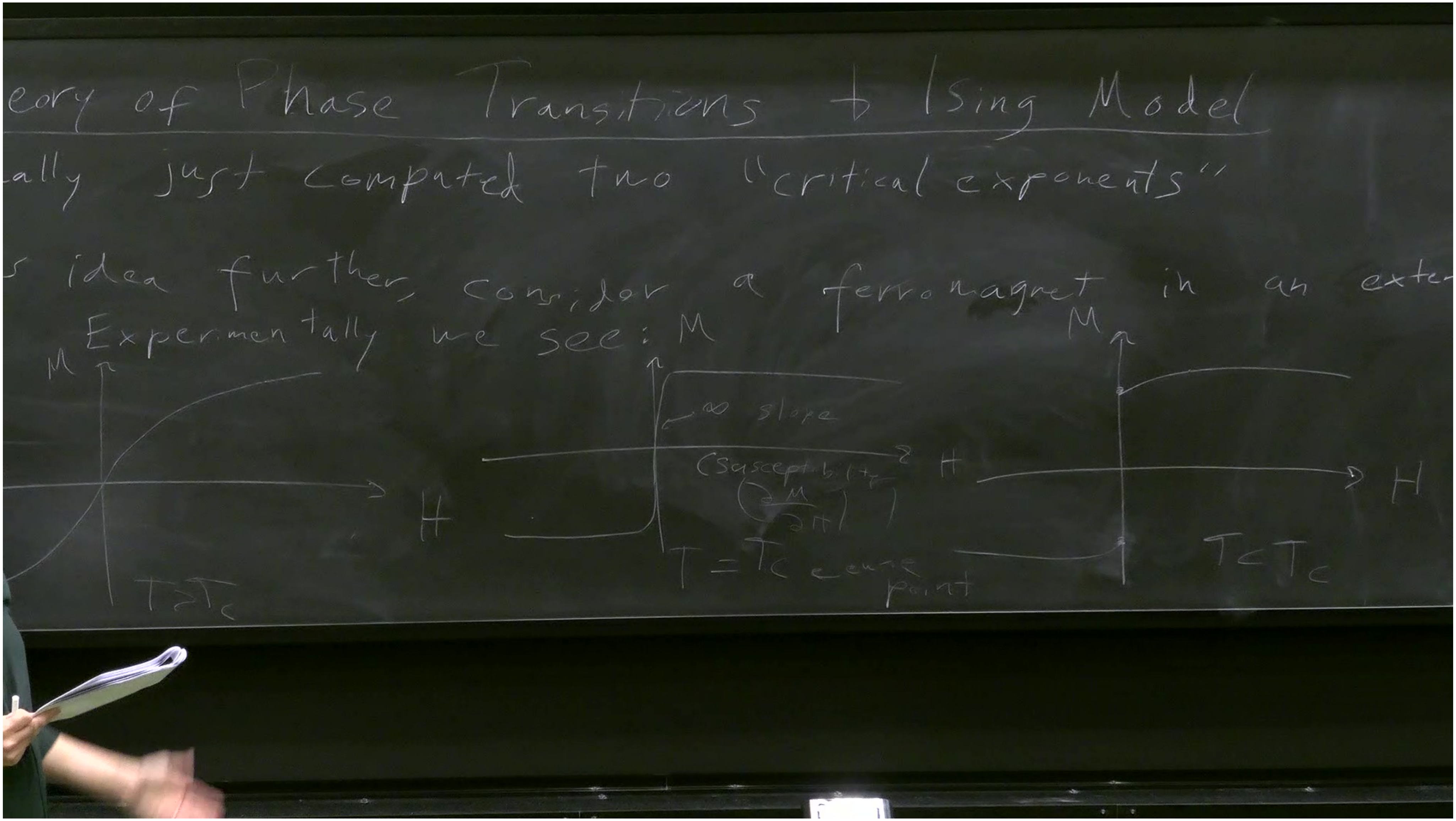


Experimentally we see:



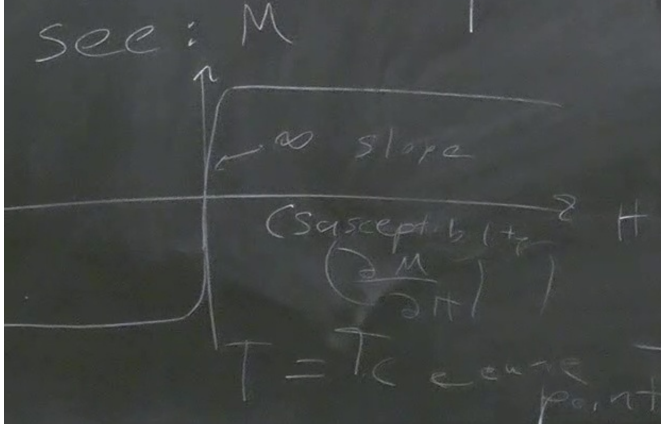

eory of Phase Transitions + Ising Model
ally just computed two "critical exponents"
s idea further, consider a ferromagnet in an exte
Experimentally we see:
M
H
T>Tc
M
∞ slope
(susceptibility
∂M/∂H)
H
T=Tc
point
M
H
T<Tc

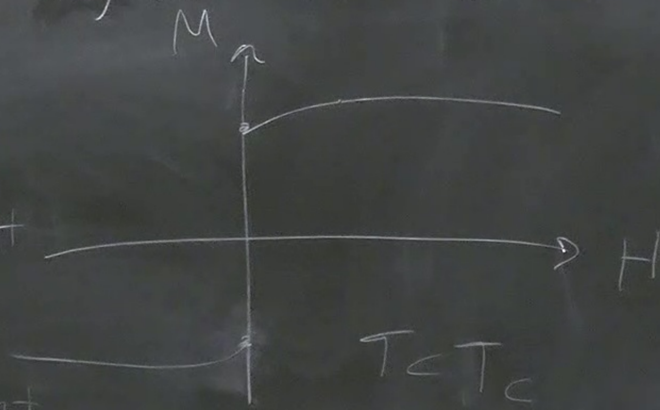
ransitions + Ising Model

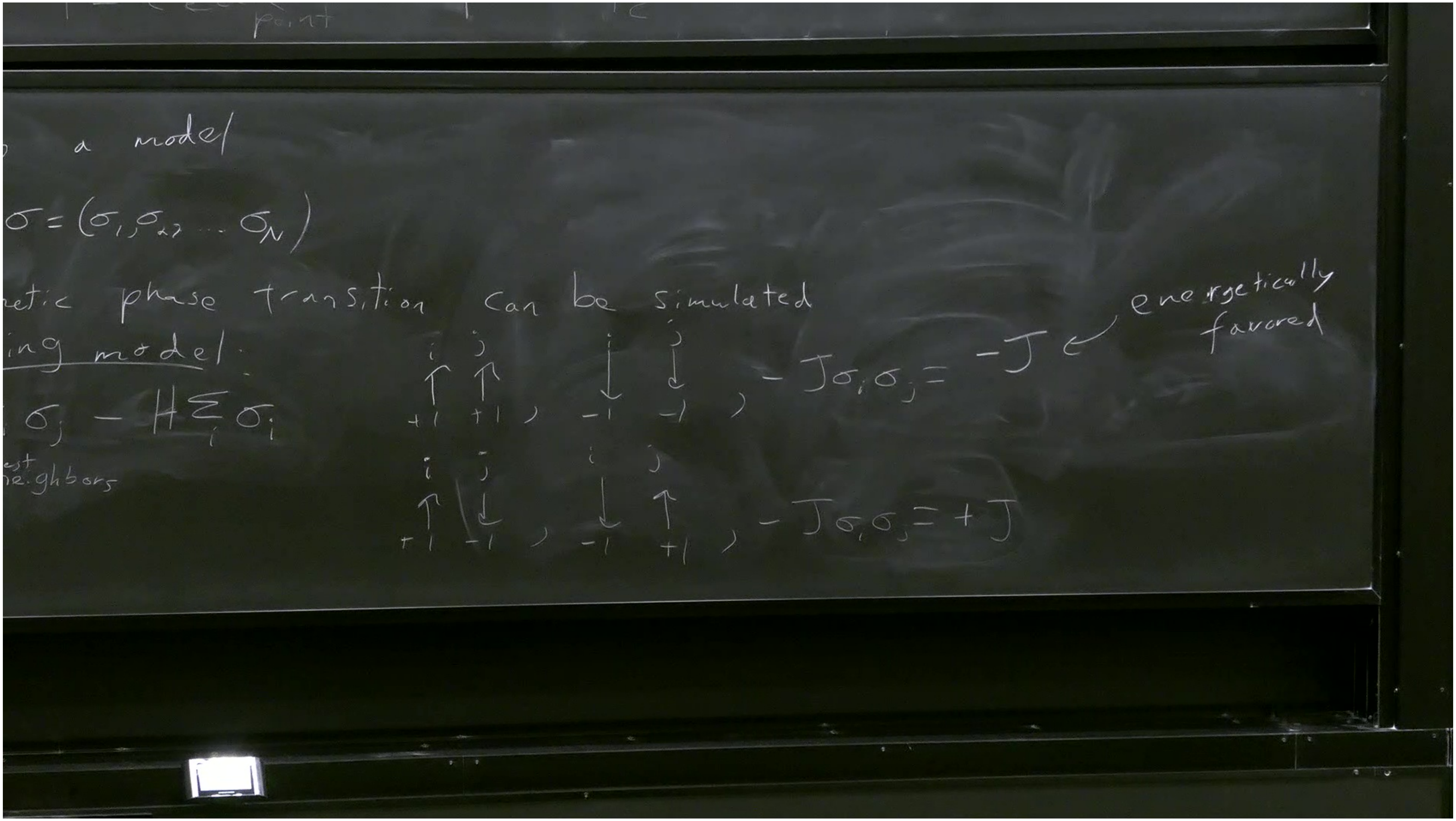
two "critical exponents"

nsider a ferromagnet in an external magnetic field H:

see:






a model
$\sigma = (\sigma_1, \sigma_2, \ldots \sigma_N)$
etic phase transition can be simulated
ing model
$\sigma_j - H \sum_i \sigma_i$
neighbors
$-J\sigma_i\sigma_j = -J$
energetically favored
$-J\sigma_i\sigma_j = +J$

$T > T_c$ $\quad$ $T = T_c$

- At the microscopic level we'll be building a model

Lattice:

$$\begin{array}{cccc} \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \\ \uparrow & \uparrow & \downarrow & \uparrow \\ \uparrow & \uparrow & \downarrow & \downarrow \\ \sigma_5 & \sigma_6 & \sigma_7 & \sigma_8 \end{array}$$

$\sigma_i = \pm 1$, $\quad \sigma = (\sigma_1, \sigma_2, \ldots \sigma_N)$

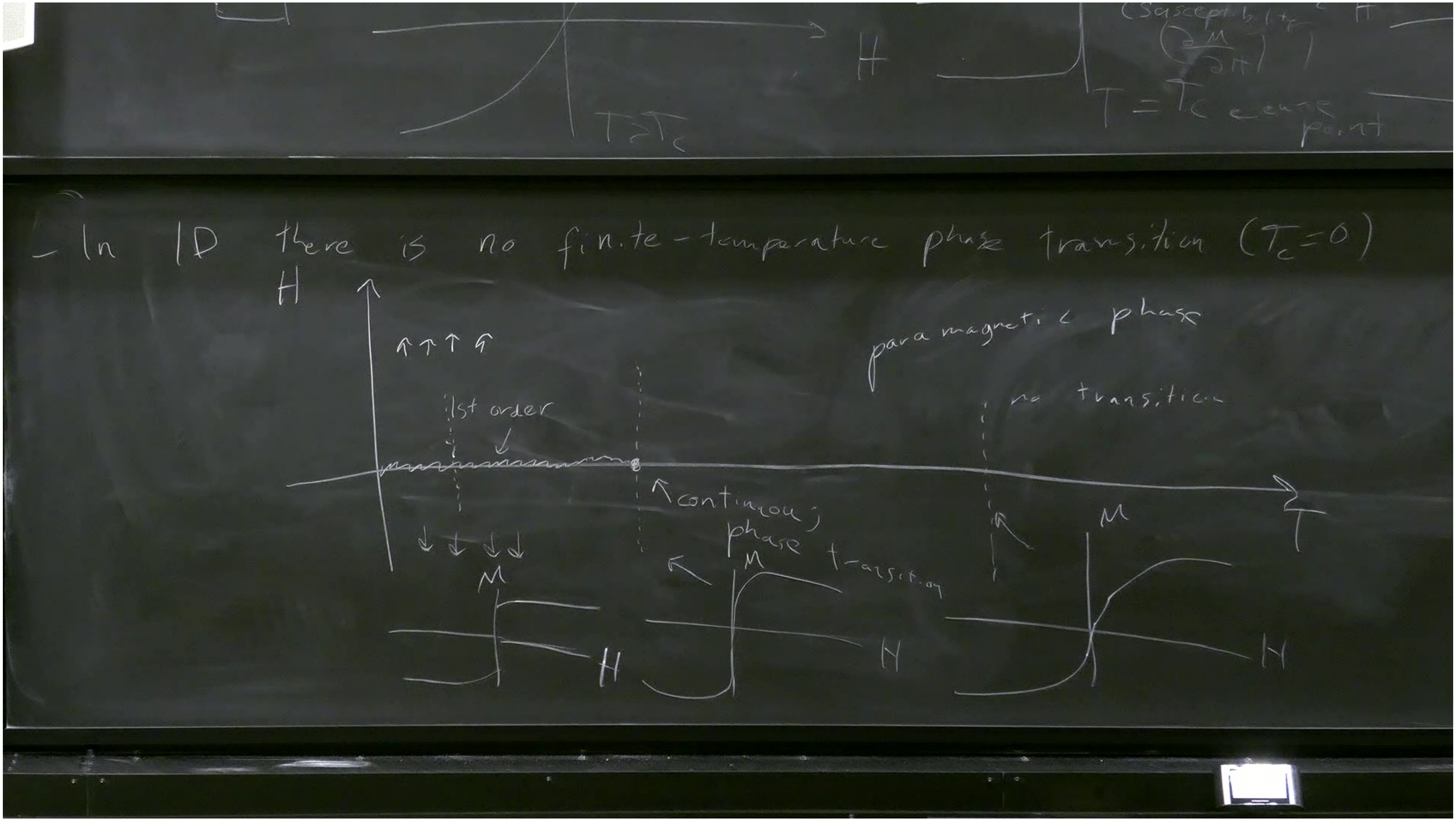
- The ferromagnetic phase t... with the Ising model:

$$E(\sigma) = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - H \sum_i \sigma_i$$

$\langle i,j \rangle$ ← nearest neighbors

$J > 0$

Magnetization (order parameter): $M = \frac{1}{N} \sum_i \langle \sigma_i \rangle$


H
T > Tc
(susceptibility
∂M/∂H)
T = Tc curie point
- In 1D there is no finite-temperature phase transition (Tc = 0)
H
paramagnetic phase
1st order
no transition
continuous phase transition
T
M
H
M
H
M
H

Critical Exponents: $\alpha, \beta, \gamma, \delta$

for a continuous phase transition.

$$M(T, 0) \sim |t|^{\beta} \quad \text{as } t \to 0^-$$

H

$$M(T_c, H) \sim |H|^{1/\delta} \quad \text{as } H \to 0$$

$$\chi(T, 0) \sim |t|^{-\gamma, -\gamma'} \quad \text{as } t \to 0^{+,-}$$

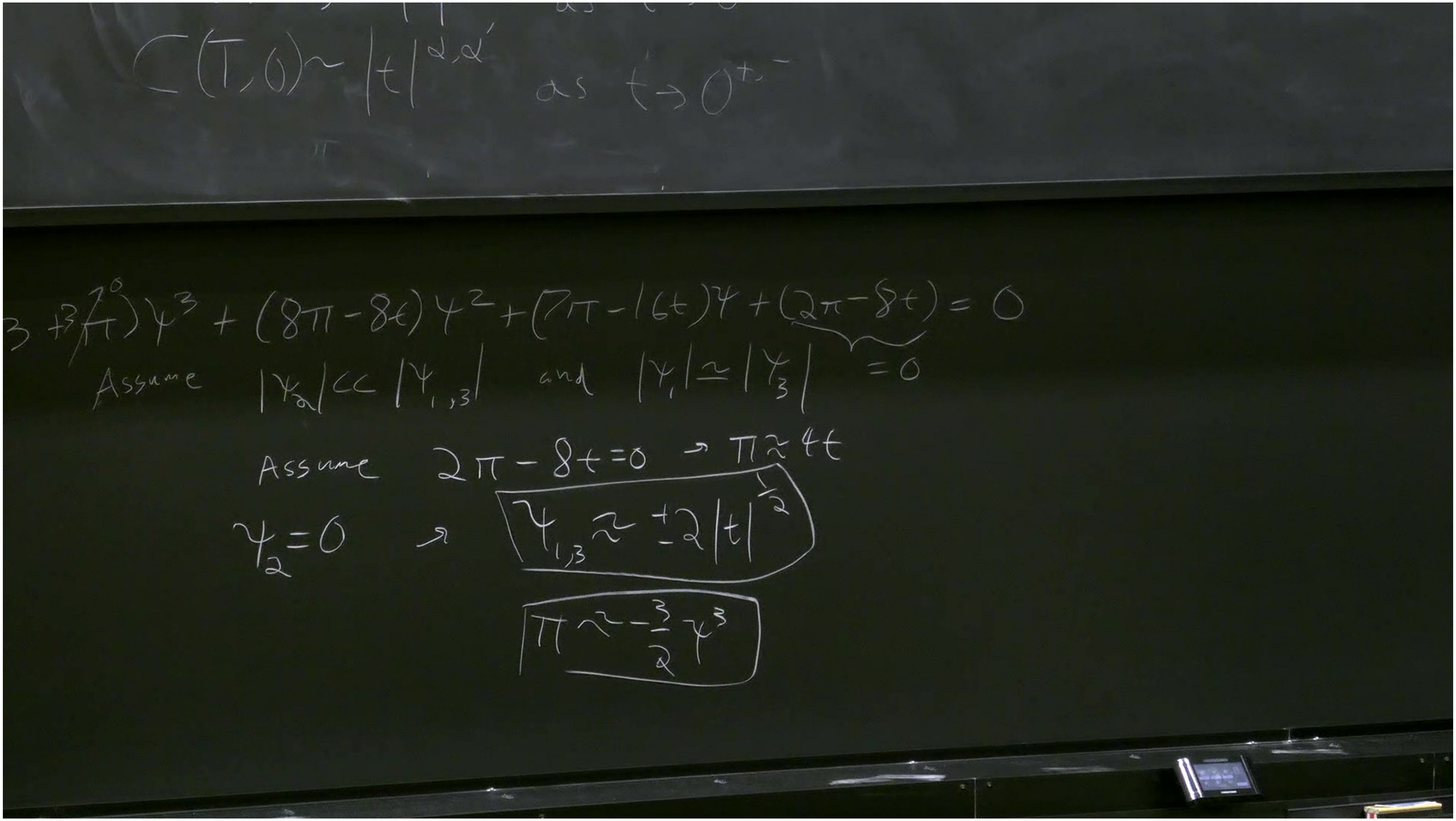
$$C(T, 0) \sim |t|^{\alpha, \alpha'} \quad \text{as } t \to 0^{+,-}$$

In practice we will see

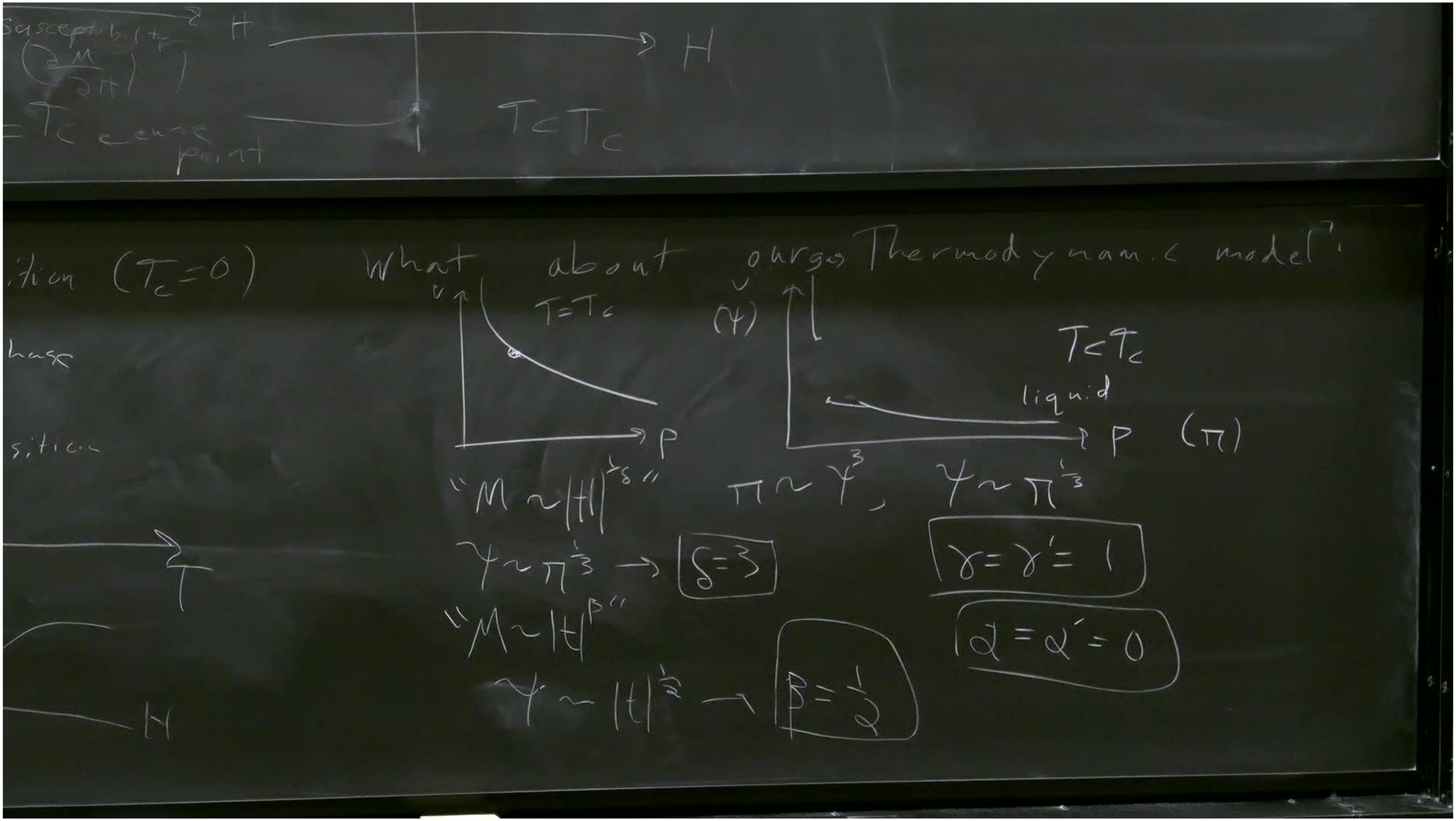
$\gamma = \gamma'$; $\alpha = \alpha'$

Léonard, Delamotte

PRL 115, 200601 (2015)



$C(T,0) \sim |t|^{\alpha,\alpha'}$ as $t \to 0^{+,-}$
$3 + t^3/\pi)\psi^3 + (8\pi - 8t)\psi^2 + (7\pi - 16t)\psi + \underbrace{(2\pi - 8t)}_{=0} = 0$
Assume $|\psi_2| \ll |\psi_{1,3}|$ and $|\psi_1| \simeq |\psi_3|$
Assume $2\pi - 8t = 0 \to \pi \simeq 4t$
$\psi_2 = 0 \to \boxed{\psi_{1,3} \simeq \pm 2|t|^{\frac{1}{2}}}$
$\boxed{\pi \simeq -\frac{3}{2}\psi^3}$


susceptibility
H
H
T<T_c
point
ition (T_c=0)
What about our gas Thermodynamic model?
T=T_c
(ψ)
T<T_c
liquid
P (π)
"M ~ |H|^{1/δ}"
π ~ ψ^3, ψ ~ π^{1/3}
ψ ~ π^{1/3} → δ=3
γ = γ' = 1
"M ~ |t|^β"
α = α' = 0
ψ ~ |t|^{1/2} → β = 1/2

$C(T,\lambda) \sim |t|^{\alpha,\alpha'}$

First-order or higher order transitions (Thermodynamics)

Ehrenfest defn.: (Gibbs, or Helmholtz)

$G = F + PV$ $\qquad \frac{\partial G}{\partial P} = V$

First order: G cont., $\frac{\partial G}{\partial P}$ discontinuous

Second order: $G, \frac{\partial G}{\partial P}$ cont., $\frac{\partial^2 G}{\partial P^2}$ discont.

$\vdots$

nth order $G, \frac{\partial G}{\partial P}$ cont., $\frac{\partial^n G}{\partial P^n}$ discont.