Title: Lecture - Statistical Physics, PHYS 602

Speakers: Emilie Huffman

Collection/Series: Statistical Physics (Core), PHYS 602, October 7 - November 6, 2024

Subject: Condensed Matter, Other

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Overview

- Statistical physics involves systems of many particles
  - it's how we scale up to 10^{23} particles

Statistical physics

Classical phenom

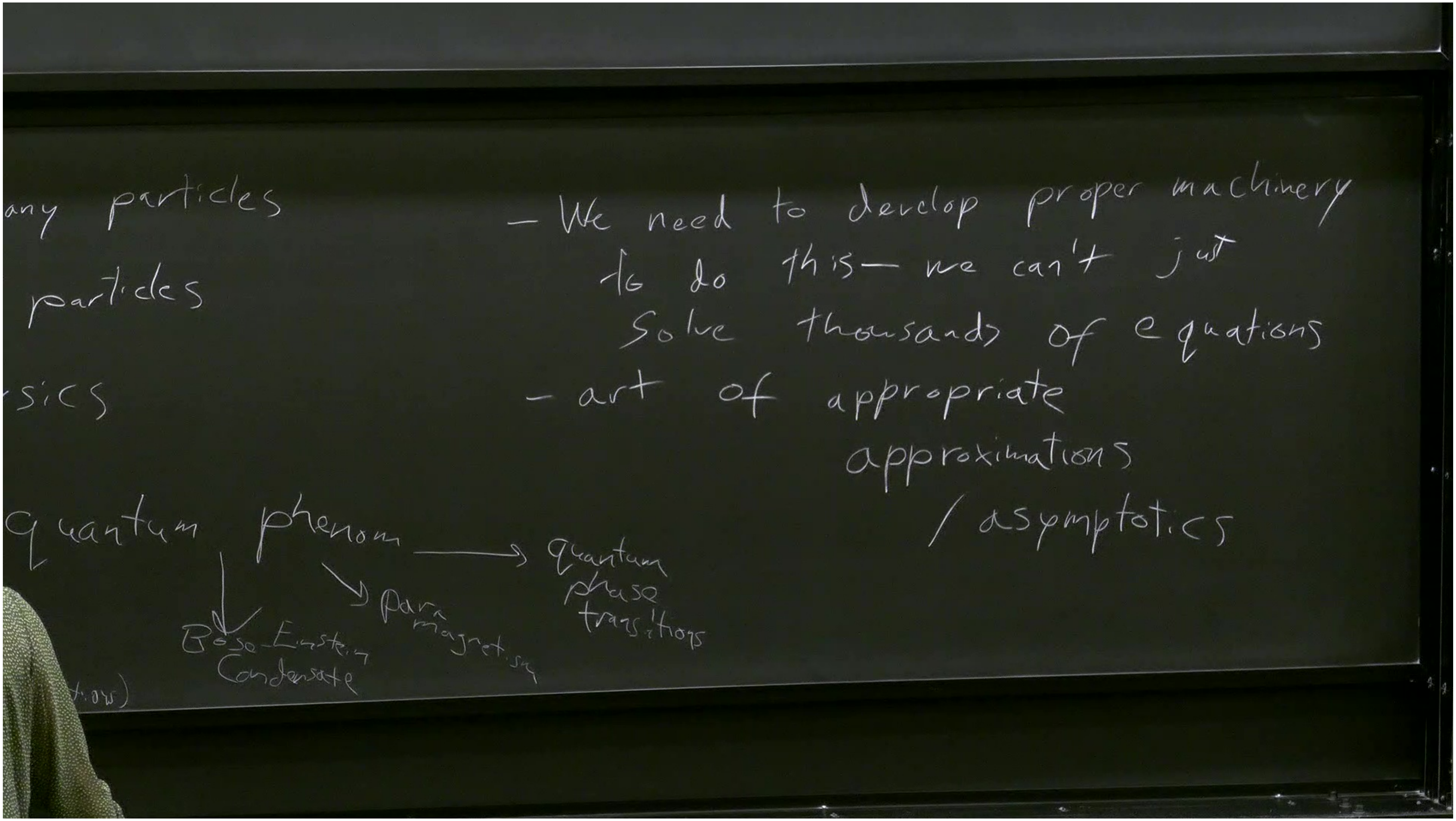
ideal gases

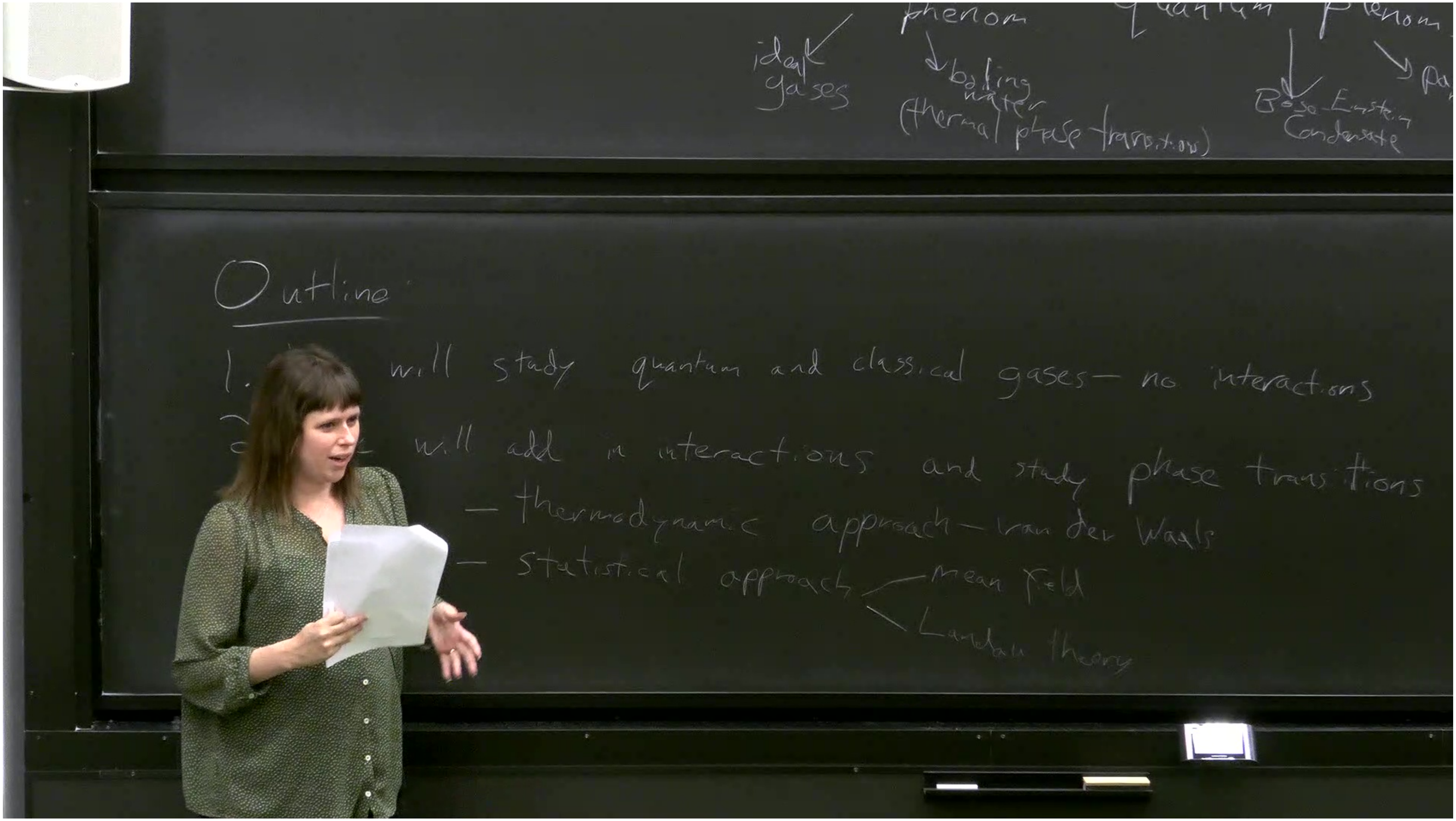
boiling water (thermal phase transitions)

quantum phenom → quantum phase transitions

Bose-Einstein Condensate

para magnetism


any particles
particles
sics
quantum phenom
quantum phase transitions
para magnetism
Bose-Einstein Condensate
- We need to develop proper machinery to do this — we can't just solve thousands of equations
- art of appropriate approximations / asymptotics


phenom
quantum
phenom
ideal gases
boiling water
(thermal phase transition)
Bose-Einstein Condensate
Outline:
will study quantum and classical gases — no interactions
will add in interactions and study phase transitions
— thermodynamic approach — van der Waals
— statistical approach
mean field
Landau theory

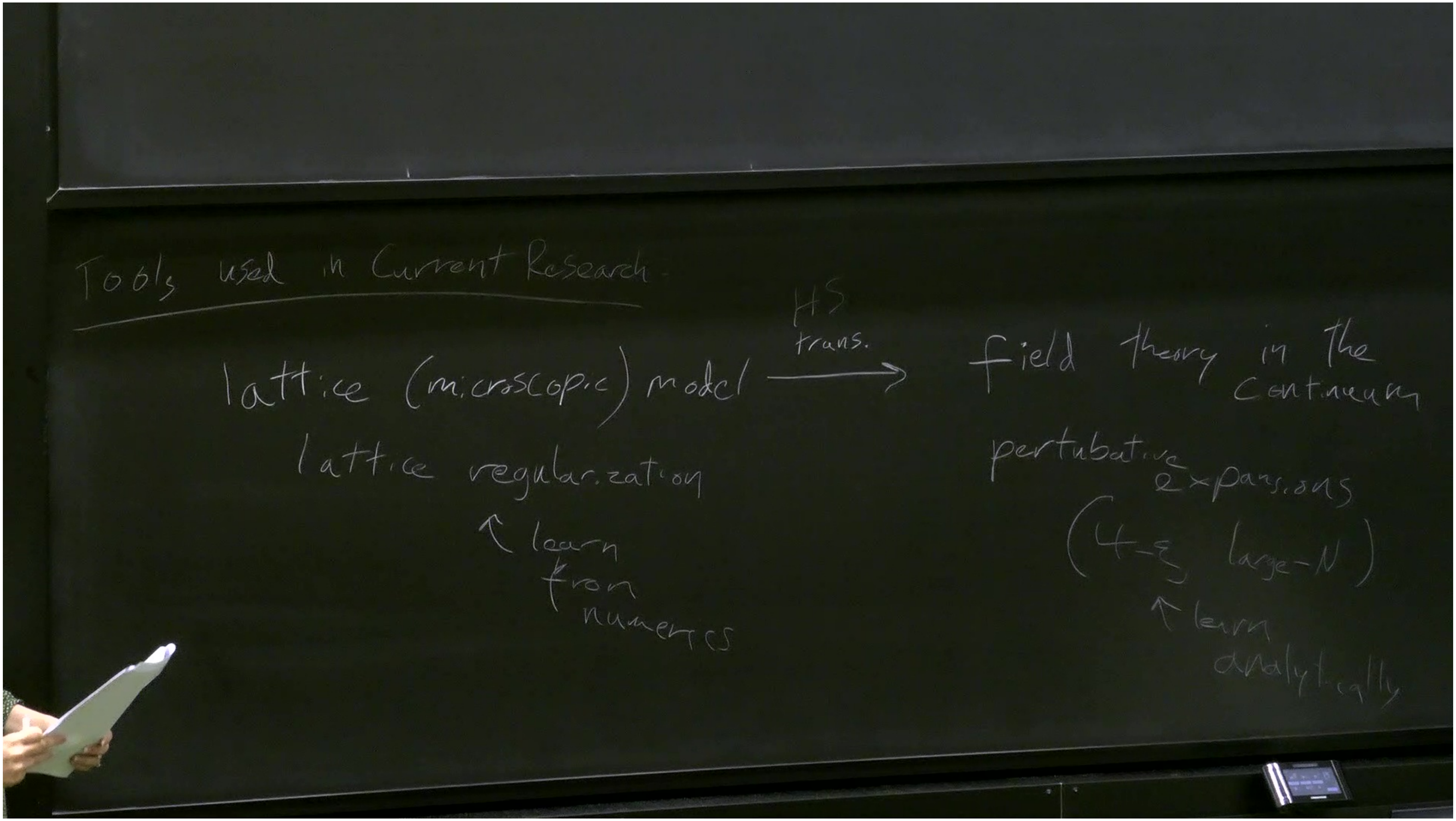
2. We will add in interactions and study phase transitions
   - thermodynamic approach — van der Waals
   - statistical approach — mean field / Landau theory

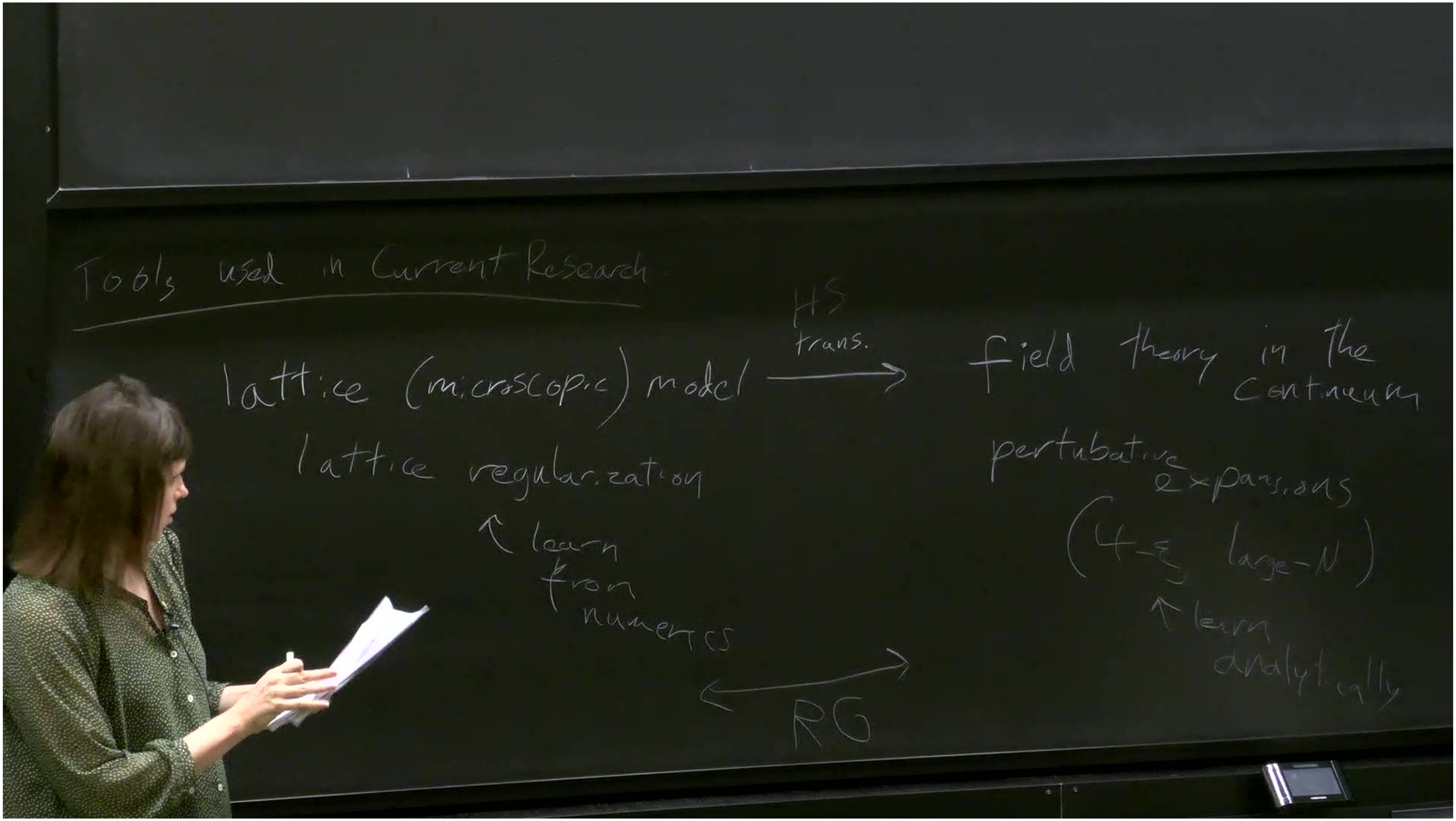
3. From here we add in fluctuations of particles and build to Statistical field theory

SFT: $Z = \int D\phi \, e^{-\frac{1}{T}\int d^d x \, F(\phi)}$

QFT: $Z = \int D\phi \, e^{\frac{i}{\hbar}\int d^d x \, \mathcal{L}(\phi)}$

4. Quantum-Classical mapping   Wick rotation, $\tau = it$
   - Quantum phase transition


Tools used in Current Research
HS trans.
lattice (microscopic) model
field theory in the continuum
lattice regularization
learn from numerics
perturbative expansions
(4-ε, large-N)
learn analytically


Tools used in Current Research
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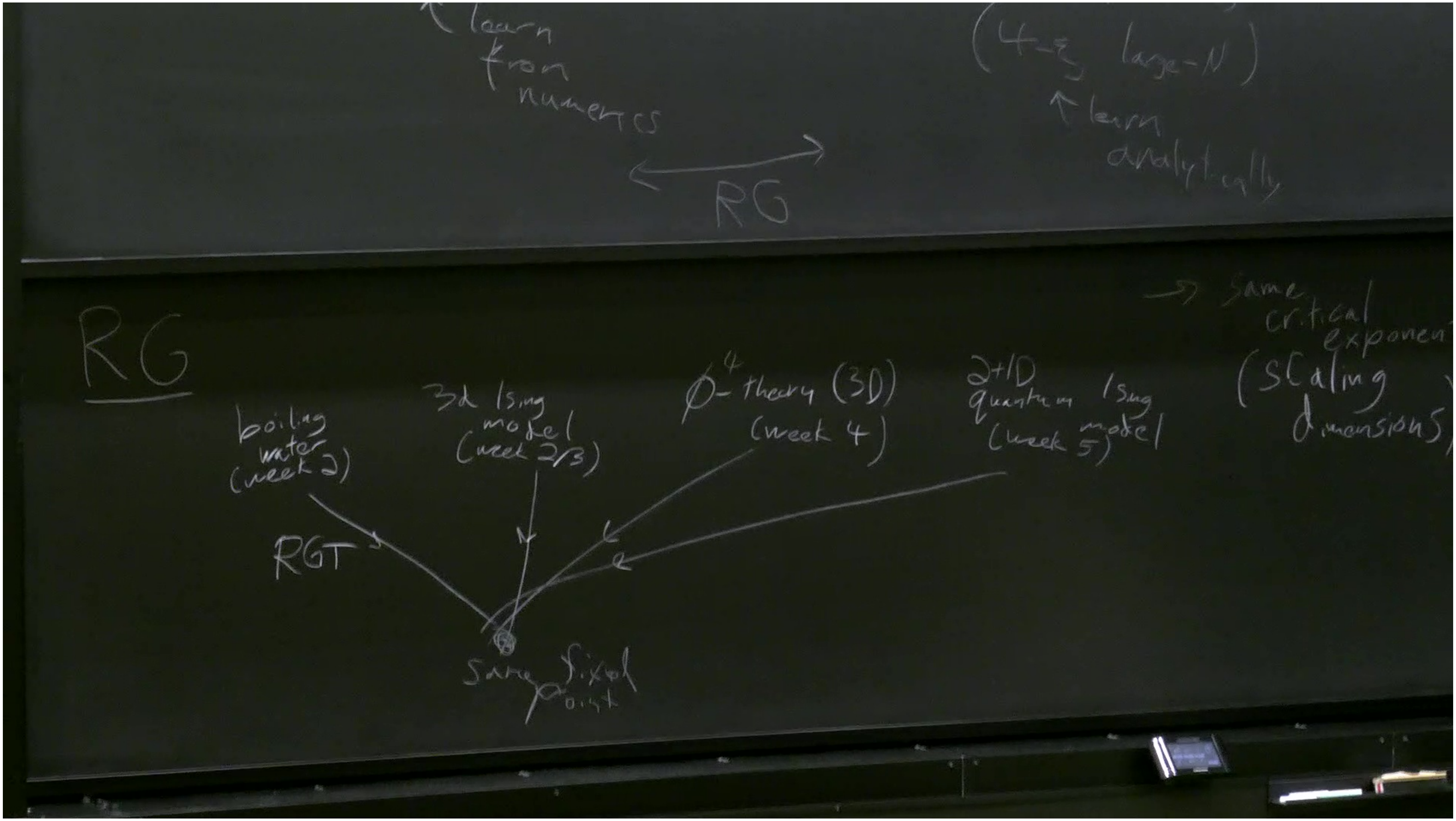
RG


learn from numerics
RG
(4-ε, large-N)
learn analytically
RG
boiling water (week 2)
3d Ising model (week 2/3)
ϕ^4 theory (3D) (week 4)
2+1D quantum Ising model (week 5)
→ same critical exponents (scaling dimensions)
RGT
same fixed point

Phase Space Formalism

(q^5, q^6) (p_5, p_6)

(q^1, q^2) (p_1, p_2)

(q^3, q^4) (p_3, p_4)

N particles (two spatial dimensions)

state (configuration)

$$(q, p) \equiv (q^1, q^2 \ldots q^{2N}, p_1, p_2 \ldots p_{2N})$$

Dynamics:

— specified by Hamiltonian, $H(q, p, t)$

dimensions)

$p_2 \ldots p_{2N})$

$H(q,p,t)$

Hamilton's Equations

$$\dot{q}^i(t) = \frac{\partial}{\partial p_i} H(q(t), p(t), t)$$

$$\dot{p}_i(t) = -\frac{\partial H}{\partial q^i}(q(t), p(t), t)$$

i.c.: $q(0) = q^0 \qquad p(0) = p_0$

This means computing averages for systems of interest.

$H(q,p,t)$ This means computing averages for systems of interest.

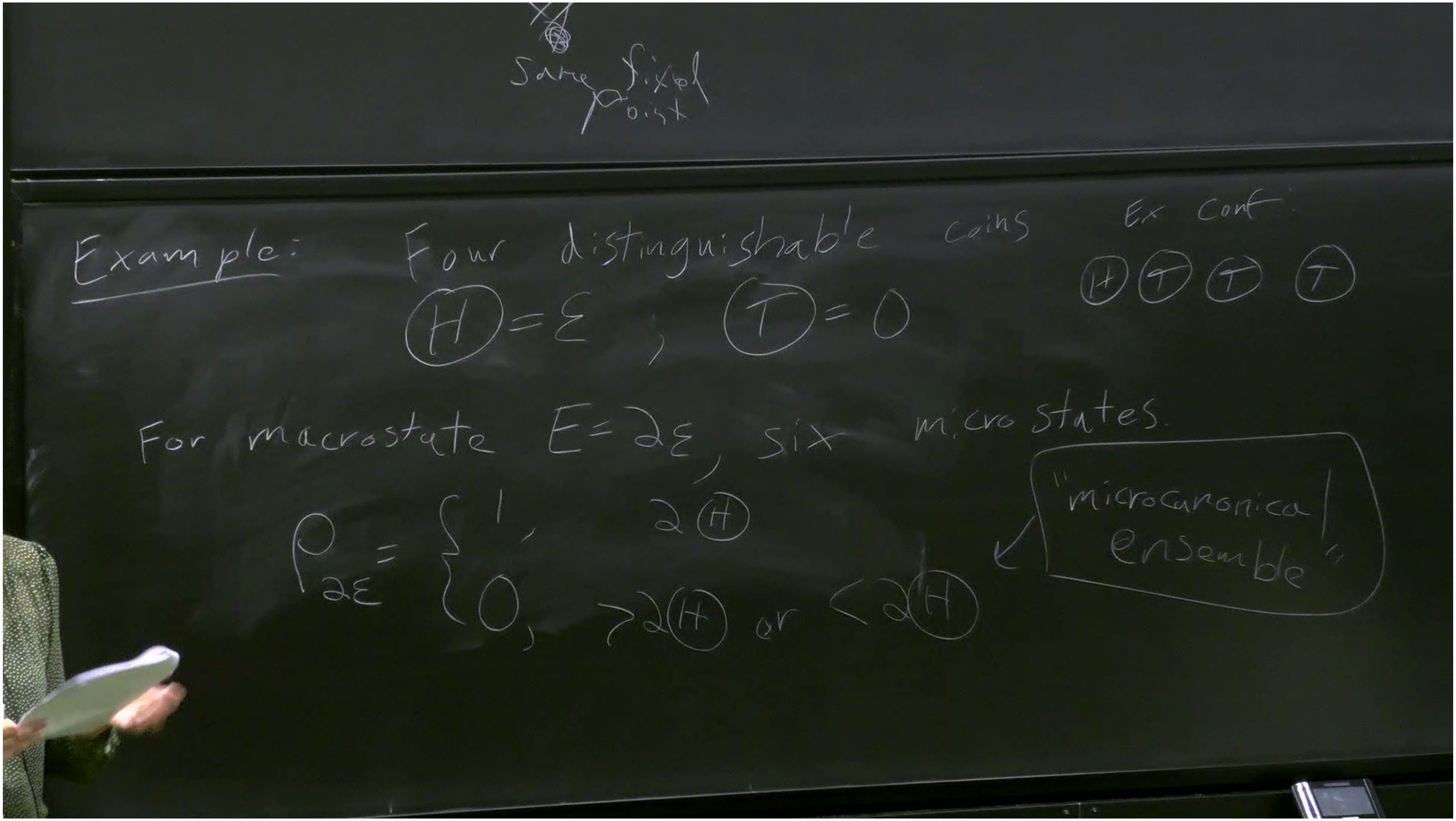
How do we think of macrostates?

phase space, M

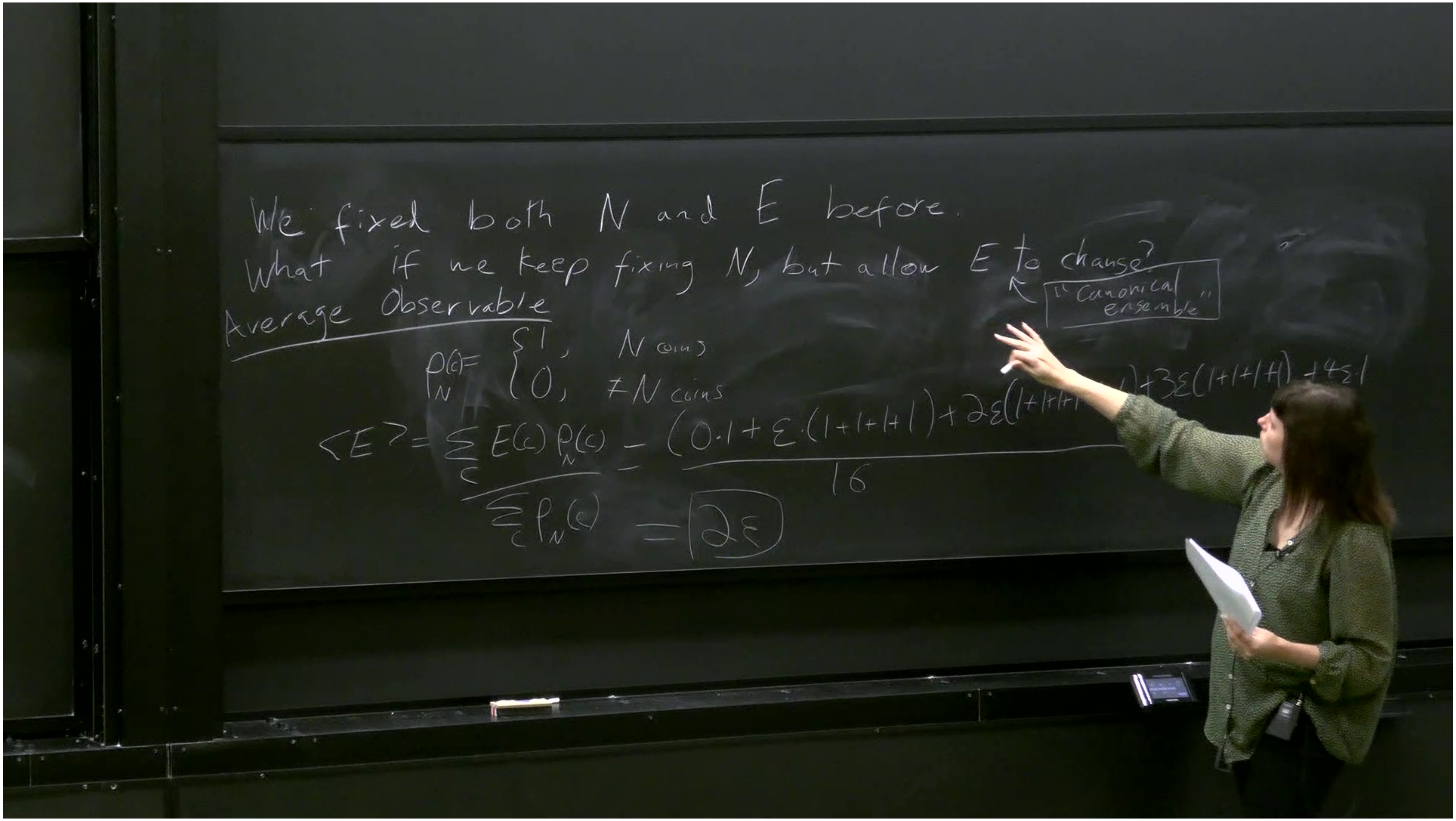
microstate (q,p)

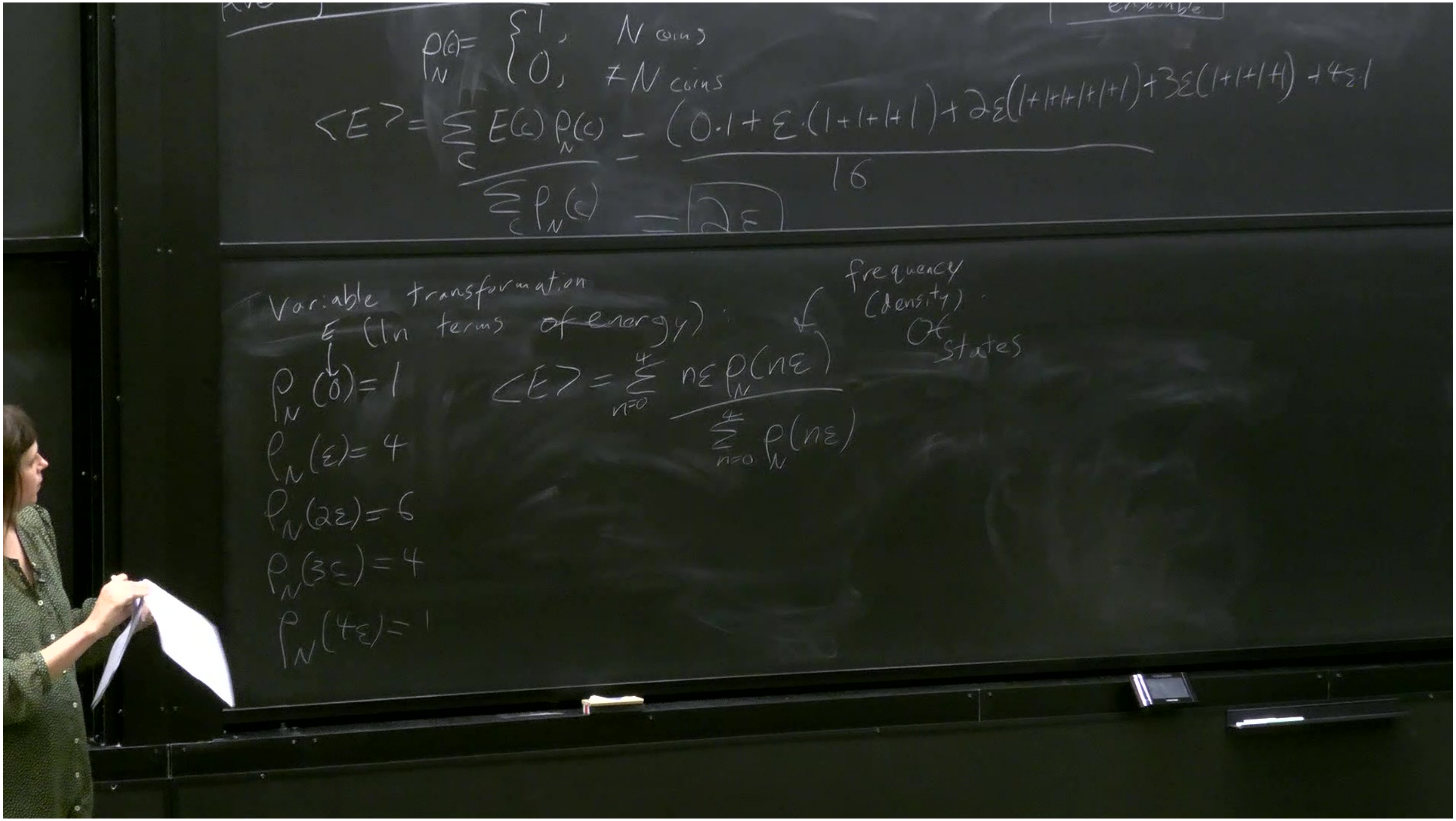
quantities

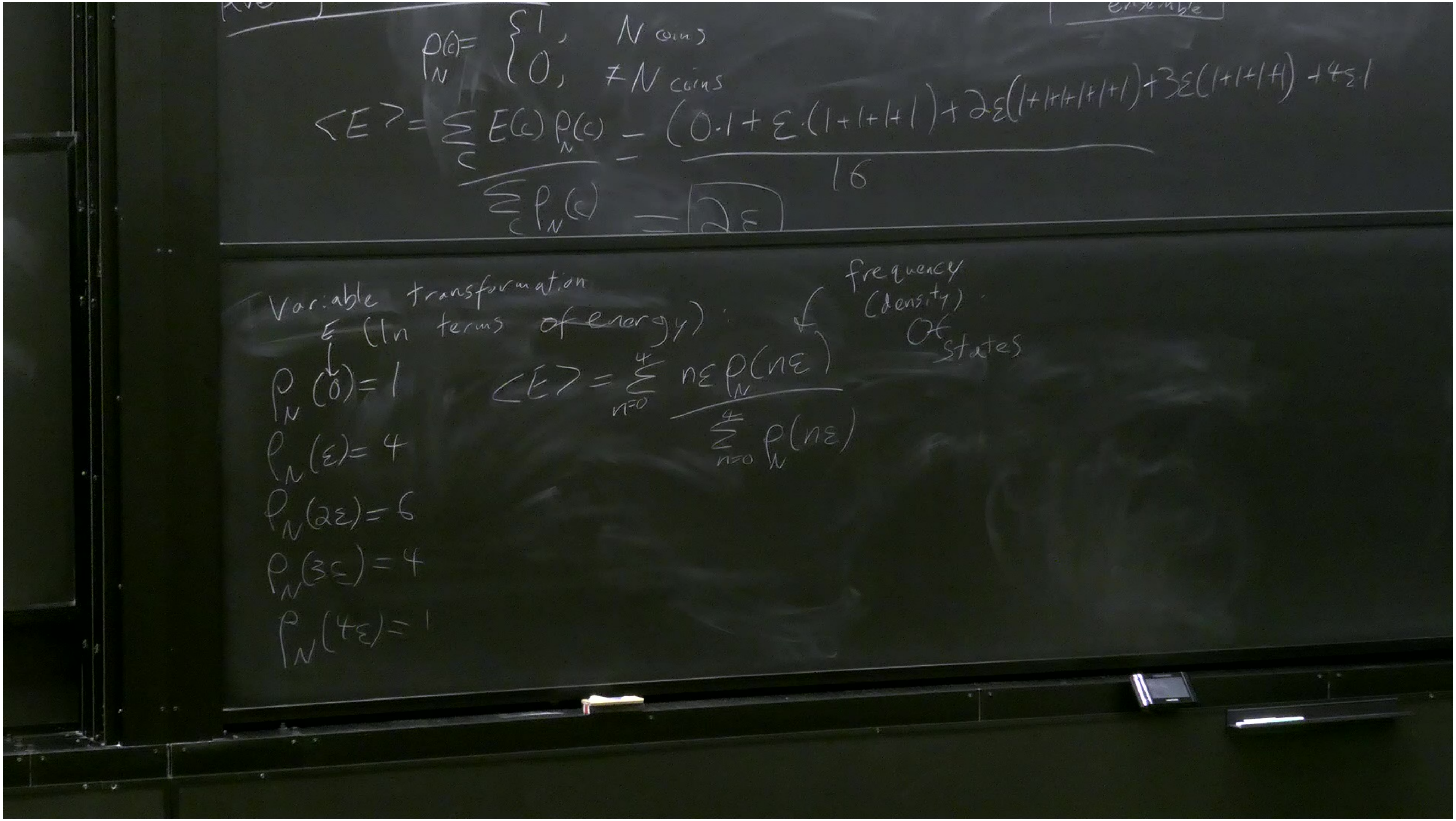
One way: We can assign weights to each microstate (q,p), given a macrostate S. We use $\rho_S(p,q)$ for this



Same fixed point
Example: Four distinguishable coins
H = ε ; T = 0
Ex Conf:
H T T T
For macrostate E = 2ε, six microstates.
ρ_{2ε} = { 1, 2H ; 0, >2H or <2H
"microcanonical ensemble"


We fixed both N and E before.
What if we keep fixing N, but allow E to change?
"Canonical ensemble"
Average Observable
ρN(c) = 1, N coins
0, ≠ N coins
<E> = Σc E(c) ρN(c) / Σc ρN(c)
= (0·1 + ε·(1+1+1+1) + 2ε(1+1+1+1 …) + 3ε(1+1+1+1) + 4ε·1) / 16
= 2ε


ρ_N(c) = { 1, N coins ; 0, ≠ N coins
⟨E⟩ = Σ_c E(c) ρ_N(c) / Σ_c ρ_N(c) = (0·1 + ε·(1+1+1+1) + 2ε(1+1+1+1+1+1) + 3ε(1+1+1+1) + 4ε·1) / 16 = 2ε
Variable transformation
ε (In terms of energy)
ρ_N(0) = 1
ρ_N(ε) = 4
ρ_N(2ε) = 6
ρ_N(3ε) = 4
ρ_N(4ε) = 1
⟨E⟩ = Σ_{n=0}^{4} nε ρ_N(nε) / Σ_{n=0}^{4} ρ_N(nε)
frequency
(density)
of states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frequency (density) of states

magnetization

macros

Continuous variables:

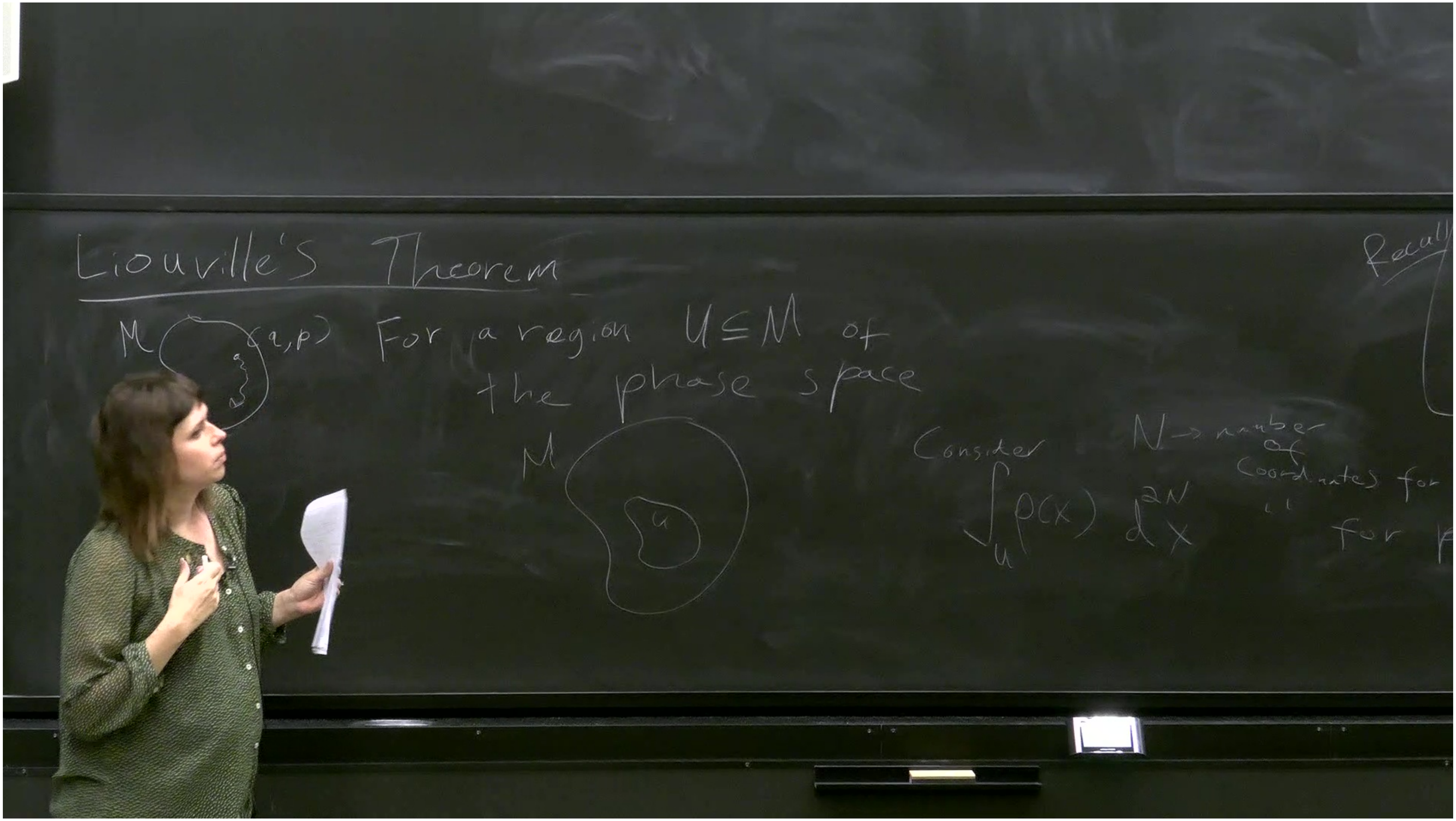
$$d\mu = \rho(q,p)\, d^N q\, d^N p$$

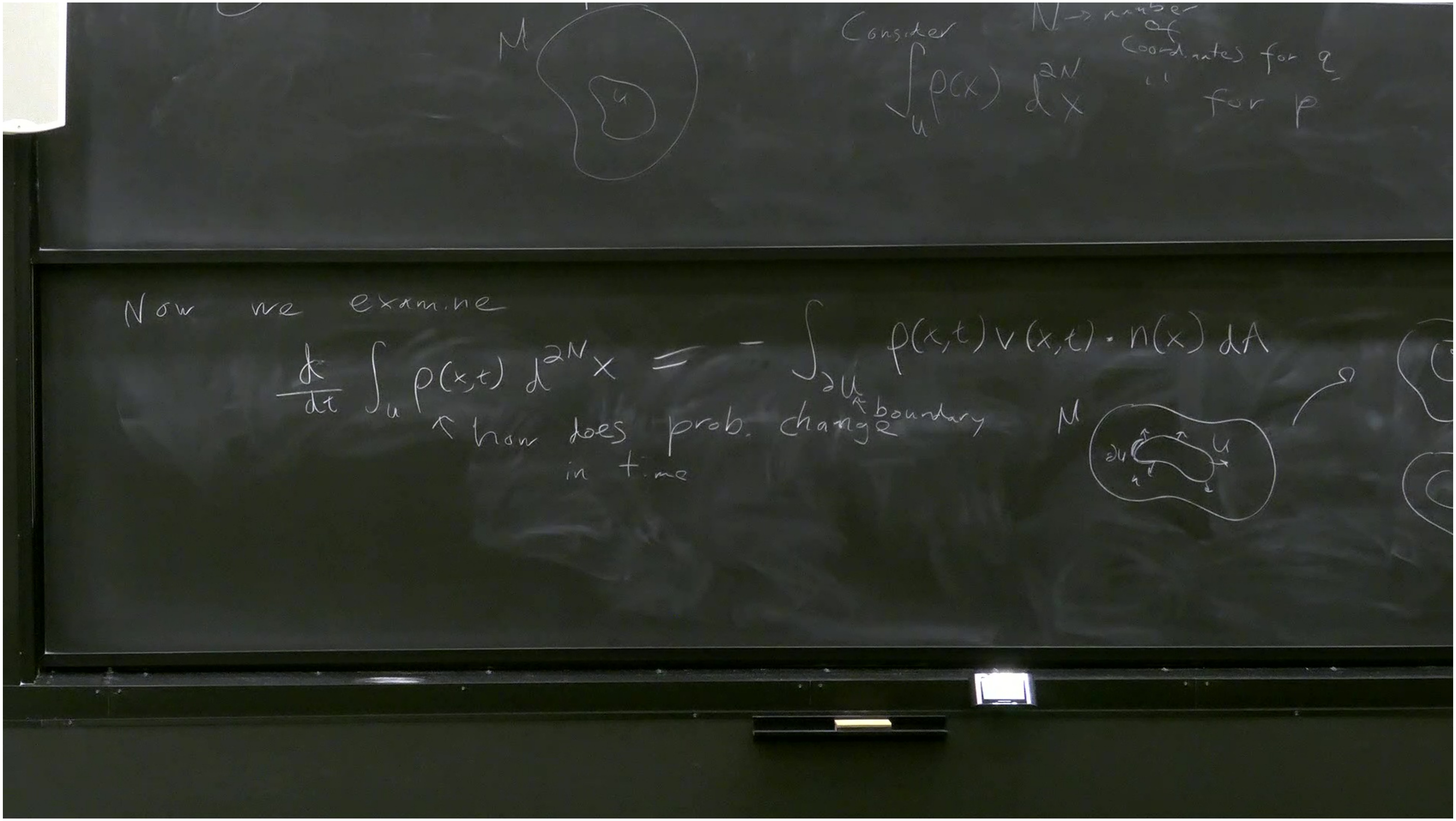
measure

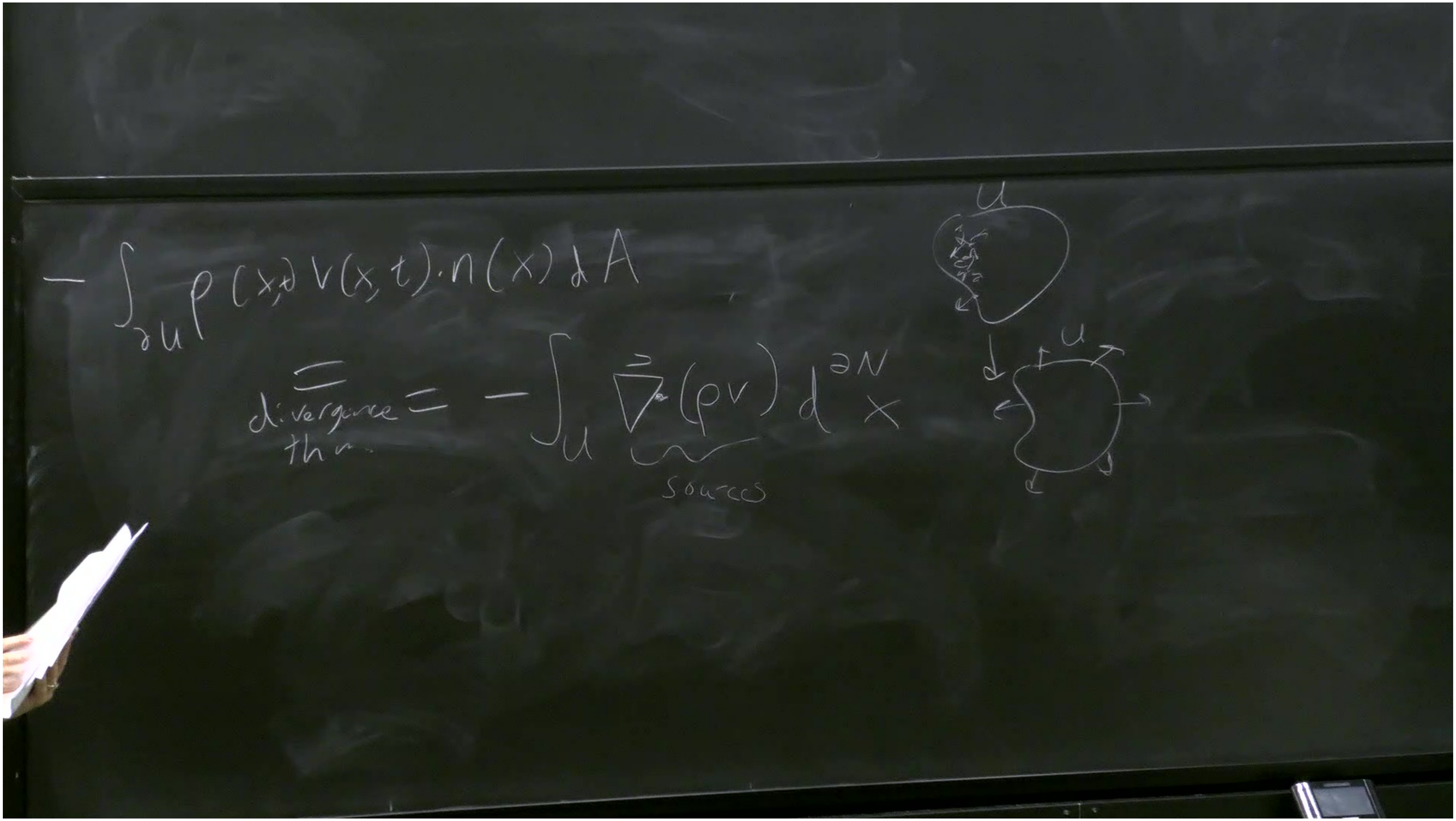
probability density

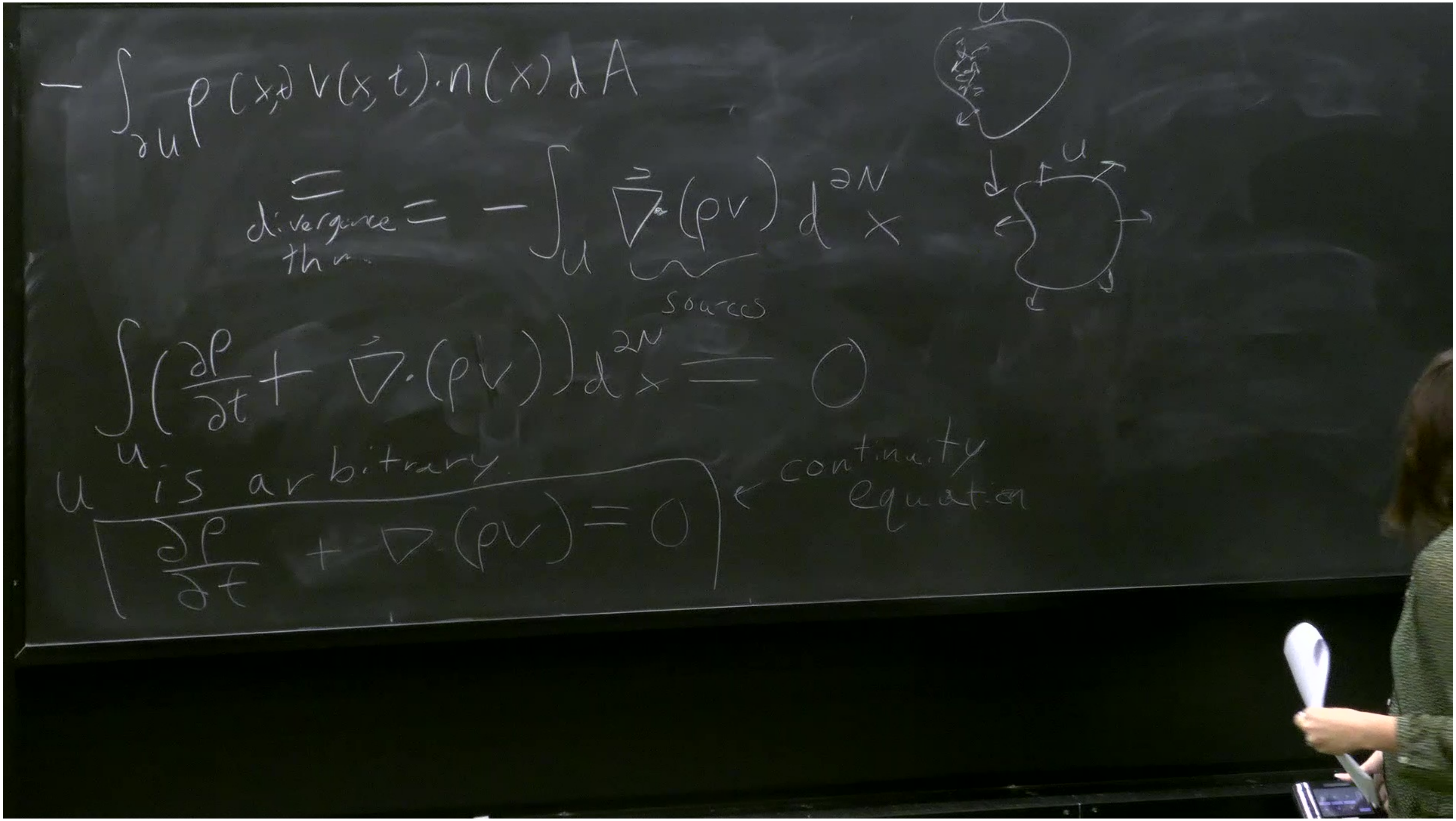
Averages:

$$\langle A \rangle = \frac{\int_\mu A(q,p)\, d\mu}{\int_\mu d\mu}$$


Liouville's Theorem
M (q,p)
For a region U⊆M of the phase space
M
U
Consider
∫U ρ(x) d2N x
N → number of coordinates for
for
Recall


M
U
Consider
$N \to$ number of coordinates for q
$\int_U \rho(x)\, d^{2N}x$
" for p
Now we examine
$\frac{d}{dt}\int_U \rho(x,t)\, d^{2N}x = -\int_{\partial U} \rho(x,t)\, v(x,t)\cdot n(x)\, dA$
how does prob. change in time
boundary
M
∂U
U


$-\int_{\partial U} \rho(x,t)\, v(x,t)\cdot n(x)\, dA$
= divergence thm = $-\int_U \vec{\nabla}\cdot(\rho v)\, d^N x$
sources
U
U



$-\int_{\partial U} \rho(x,t)\, v(x,t) \cdot n(x)\, dA$
divergence thm
$= -\int_U \vec{\nabla} \cdot (\rho v)\, d^{2N}x$
sources
$\int_U \left(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho v) \right) d^{2N}x = 0$
U is arbitrary
$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$
continuity equation
U

u^a is arbitrary

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho v) = 0} \leftarrow \text{continuity equation}$$

So what is v?

$$v^a = \dot{x}^a = \sum_b J^{ab} \frac{\partial H}{\partial x^b} \quad \text{where} \quad J = \begin{pmatrix} 0 & \mathbb{1}_N \\ -\mathbb{1}_N & 0 \end{pmatrix} \qquad J^{ab} = -J^{ba}$$

$$\nabla\cdot v = \sum_a \frac{\partial v^a}{\partial x^a} = \sum_{a,b} \frac{\partial}{\partial x^a}\left[J^{ab}\frac{\partial H(x,t)}{\partial x^b}\right]$$

$$= \sum_{a,b} \frac{\partial}{\partial x^b}\left[-J^{ba}\frac{\partial H(x,t)}{\partial x^a}\right] = 0$$

$$\nabla\cdot(\rho v) = \nabla\rho\, v = \sum_{ab} \underbrace{\frac{\partial \rho}{\partial x^a} J^{ab} \frac{\partial H}{\partial x^b}}_{\frac{\partial \rho}{\partial q^i}\frac{\partial H}{\partial p_i} - \frac{\partial \rho}{\partial p_i}\frac{\partial H}{\partial q^i}} = \{\rho, H\}$$

in trajectories

$\{\rho, H\}$

Liouville's Equation:

$$\boxed{\frac{\partial \rho}{\partial t} + \{\rho, H\} = 0}$$

In equilibrium: $\frac{\partial \rho}{\partial t}$

$$\therefore \boxed{\{\rho, H\} = 0}$$

microcanonical

$\rho = \text{const.}$

canonical

$\rho \propto e^{-\frac{1}{kT}H}$

$$\int_u \left(\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho v)\right) d^{2N}x = 0$$

u is arbitrary

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho v) = 0} \leftarrow \text{continuity equation}$$

Ergodic hypothesis $\boxed{\langle A_{ens}\rangle = \langle A_{time}\rangle}$

$\langle A \rangle$

Exp: $$\langle A\rangle_{time} = \frac{1}{n}\sum_{l=1}^{n} A(q(t_l), p(t_l))$$

Theory: $$\langle A\rangle_{ens} = \frac{\sum_c A(c)\,\rho(c)}{\sum_c \rho(c)}$$