Title: Energy cost of maximal entanglement extraction in QFT

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Collection/Series: Perimeter Institute Graduate Students' Conference 2024

Date: September 12, 2024 - 10:20 AM

URL: https://pirsa.org/24090196

Abstract:

We present a study of the relationship between energy and entanglement in finite regions of possibly arbitrary shape in QFT. We show how one can quantify the entanglement avoiding divergences by using techniques inspired by the formalism of particle detectors in relativistic quantum information. We also show how the energy cost of entanglement extraction varies with the shape and size of the regions, as well as analyze the energy density of the quantum field after this entanglement has been extracted.

Optimal entanglement extraction in QFT

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Quantum fields are highly spatially entangled

- A quantum field contains correlations between points not in causal contact
  - ➡ Entanglement harvesting: two spacelike-separated atoms can become entangled simply by interacting with the field locally!

- We are interested in **entanglement structure**, e.g. {
  - Distribution in space
  - Distribution in DoFs
  - Time evolution

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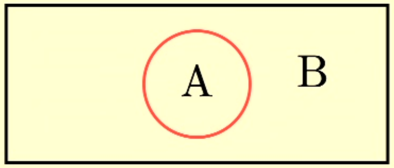
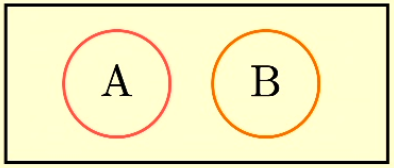
- Natural question: what is the entanglement between subregions A and B?

complementary 



arbitrary

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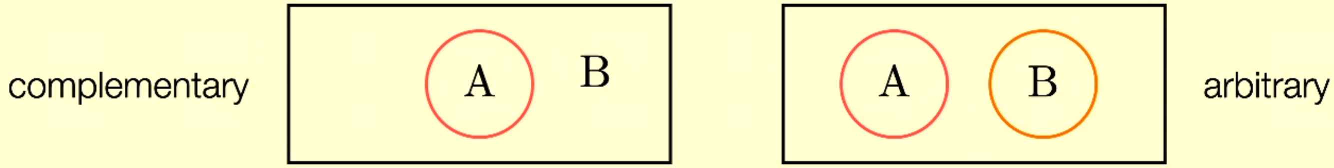


Difficulties with entanglement in the continuum

- Calculating entanglement measures between (arbitrary) subregions:

IN LATTICES: Easy, in principle.

IN CONTINUUM: Notoriously full of conceptual difficulties, ambiguities, divergences.

- Challenges of this sort have historically often been addressed by an operational approach, i.e. focusing on *measurable* quantities. For example:

  - In some spacetimes, notion of “particle” becomes ill-defined
    - ➡ what is the probability of a detector measuring a field excitation?
  - entanglement appears UV-divergent
    - ➡ “harvest” entanglement + perform QI processing tasks with it

An operational approach to entanglement structure

- **Particle detector model / local probe:** system which couples to a field in some finite spacetime region, from which we gain information about the field
- Detector models are inherently regularized by way of their finite spatial and temporal coupling and finite energy gap — you can't measure infinity!

Can detector models reliably quantify entanglement between subregions, or discern properties of a field's entanglement structure?

Goals

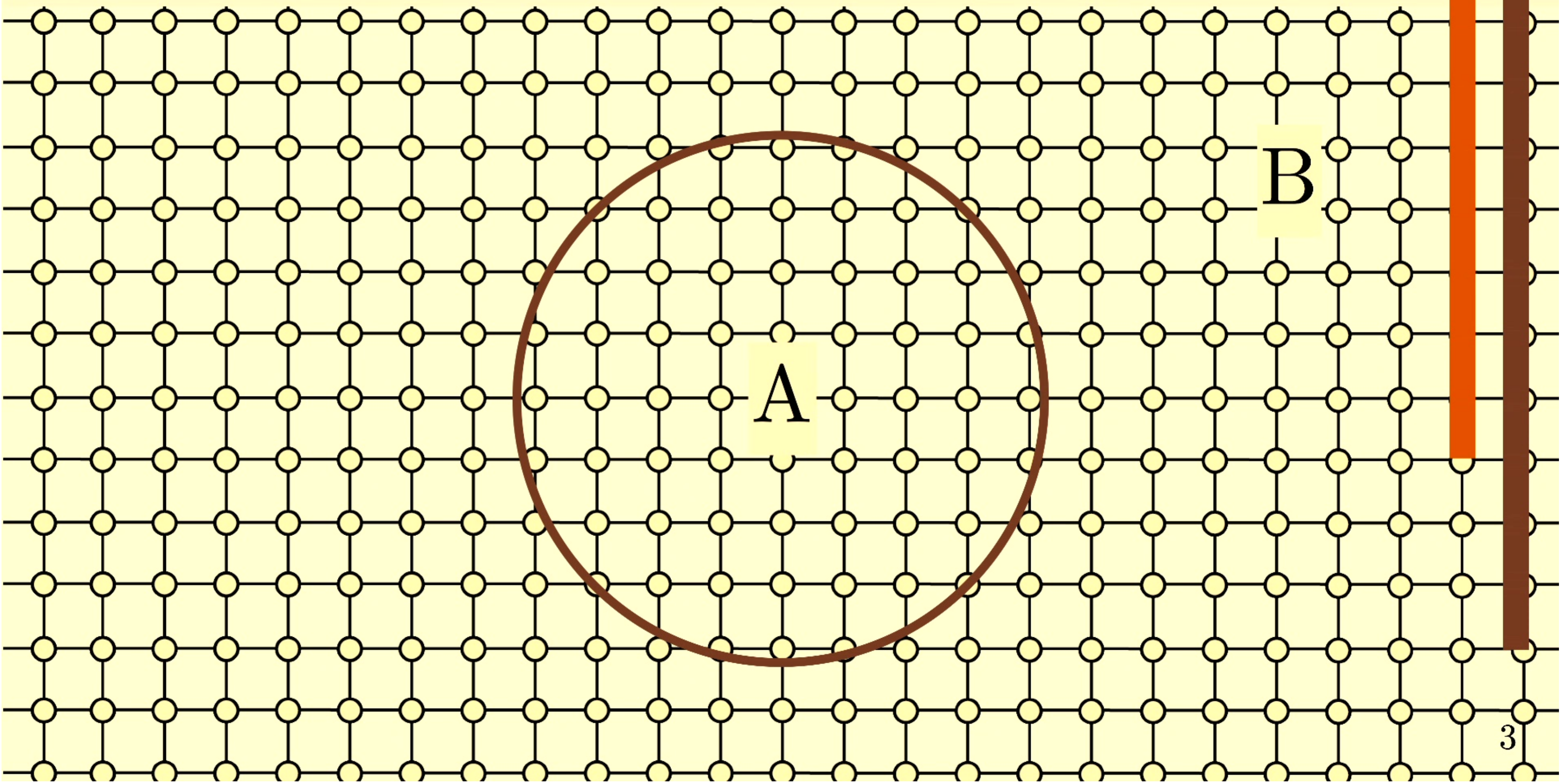
Now-goal:

Use numerical simulations in a lattice QFT to show that local probes can faithfully measure entanglement structure, and what kind of coupling is needed to do so.

Later-goal:

Apply this knowledge to probe fields in the continuum to learn about systems out of reach by numerical simulations, dynamical systems, entanglement transfer in evaporating black holes, etc.

Simulating subregions of a lattice QFT



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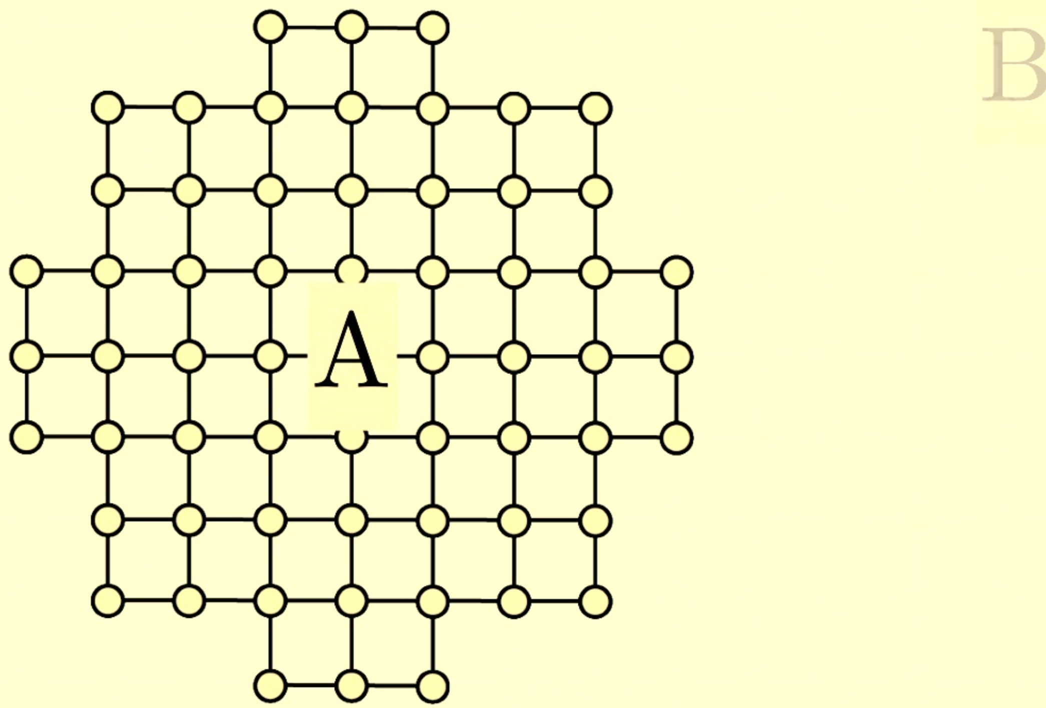

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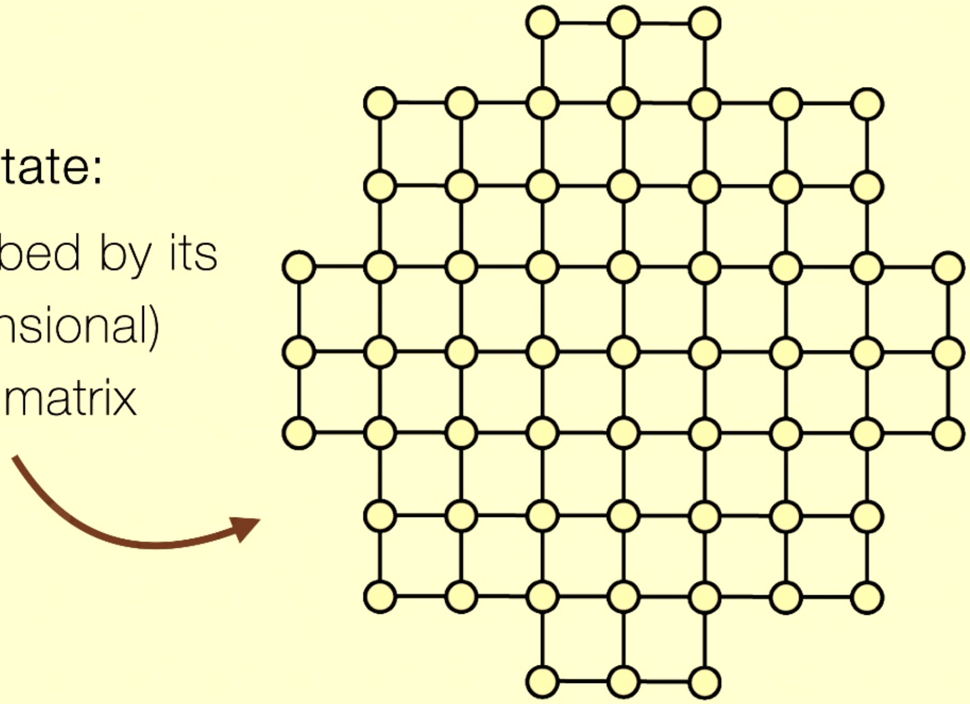
Simulating subregions of a lattice QFT

Gaussian state:

Fully described by its (finite-dimensional) covariance matrix



The covariance matrix in the continuum limit

- Covariance matrix: matrix of two-point correlation functions

$$\Sigma = \begin{pmatrix} \hat{G}_{1,1} & \hat{G}_{1,2} & \cdots & 0 & 0 & \cdots \\ \hat{G}_{2,1} & \hat{G}_{2,2} & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & \hat{H}_{1,1} & \hat{H}_{1,2} & \cdots \\ 0 & 0 & \cdots & \hat{H}_{2,1} & \hat{H}_{2,2} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{pmatrix}$$

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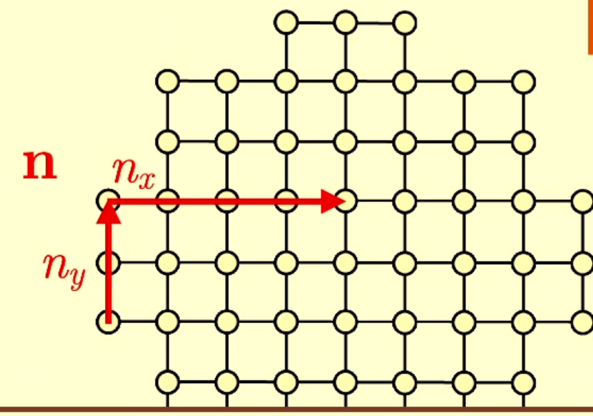
- With *unit lattice spacing*, in the thermodynamic limit (infinite oscillators) [1]:

$$\hat{G}(\mathbf{n}) = \frac{1}{\sqrt{\pi}} \int_0^\infty \mathrm{d}x e^{-(m^2+2D)x^2} \prod_i I_{n_i}(2x^2)$$

integral representation of modified Bessel function of the first kind

$$\hat{H}(\mathbf{n}) = (m^2 + 2D) G(\mathbf{n}) - \sum_{\{\mathbf{n}'\}} G(\mathbf{n}')$$



[1] N. Klco and M. J. Savage, "Entanglement Spheres and a UV-IR Connection in Effective Field Theories," Phys. Rev. Lett., vol. 127, no. 21, 211602, Nov. 2021.

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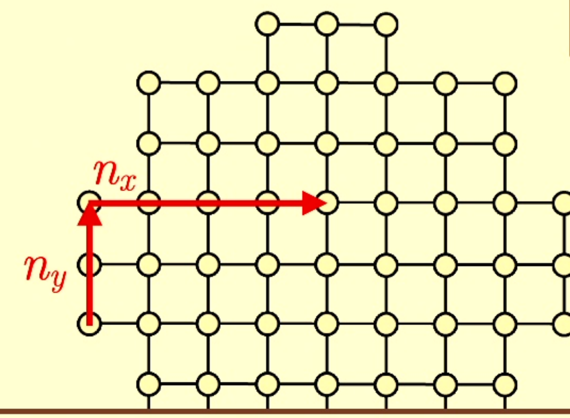
- Continuum limit when $\epsilon = 1$: subsystem size $\rightarrow \infty$
  - Same effect as taking $\epsilon \rightarrow 1$ with fixed system size!



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Mode decomposition of entanglement structure

- **The probe:** a single-mode harmonic oscillator performing some sort of a "swap" interaction with the field at a fixed time
- Entanglement detector can capture in this interaction ≤ entanglement of most entangled mode of subregion

  ➡ prompts *mode-based* decomposition of field entanglement

"Subregion mode" defined by some linear combination of the lattice degrees of freedom in the subregion:

$$\hat{Q} = \sum_i g_i \, \hat{q}_i, \quad \hat{P} = \sum_i f_i \, \hat{p}_i$$

- [2] proves that mode of a subregion that is most entangled with the complement of that subregion is the most mixed normal mode (eigenvector of the reduced covariance matrix)
- Much interest recently in this mode-based decomposition of entanglement structure, e.g. [3-5]

[2] Bruno de S. L. Torres, KW, José Polo-Gómez, Eduardo Martín-Martínez (2023). "Entanglement structure of quantum fields through local probes," *JHEP*.
[3] Natalie Klco, D. H. Beck, Martin J. Savage, (2023). "Entanglement structures in quantum field theories: Negativity cores and bound entanglement in the vacuum." *Phys. Rev. A*.
[4] Boyu Gao, Natalie Klco (2024). "Partial-transpose-guided entanglement classes and minimum noise filtering in many-body Gaussian quantum systems." *Phys. Rev. A*.
[5] Ivan Agullo, Béatrice Bonga, Patricia Ribes-Metidieri, Dimitrios Kranas, Sergi Nadal-Gisbert (2023). "How ubiquitous is entanglement in quantum field theory?" *Phys. Rev. D*.

The energy of entanglement extraction

- Field will become excited post-swap: we must input energy to break the entanglement
- To calculate the energy of a mode: “swap” it out of the field (putting a pure state in its place), and take the difference in energy of the field before and after

$$\hat{H} = \frac{1}{2}\sum_{i}^{N}\omega\left(\hat{q}_i^2 + \hat{p}_i^2\right) - \frac{\alpha}{2}\sum_{\langle i,j\rangle}\hat{q}_i\hat{q}_j \quad \Rightarrow \quad E = \langle\hat{H}\rangle = \frac{1}{2}\sum_{i}^{N}\omega\left(\langle\hat{q}_i^2\rangle + \langle\hat{p}_i^2\rangle\right) - \frac{\alpha}{2}\sum_{\langle i,j\rangle}\langle\hat{q}_i\hat{q}_j\rangle$$

elements of the covariance matrix!

$$E_{\text{mode}} = E_{\text{post-swap}} - E_{\text{ground}}$$

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- Breaking entanglement excites field *outside* the support of the mode: must sum contributions from a large enough* outer region

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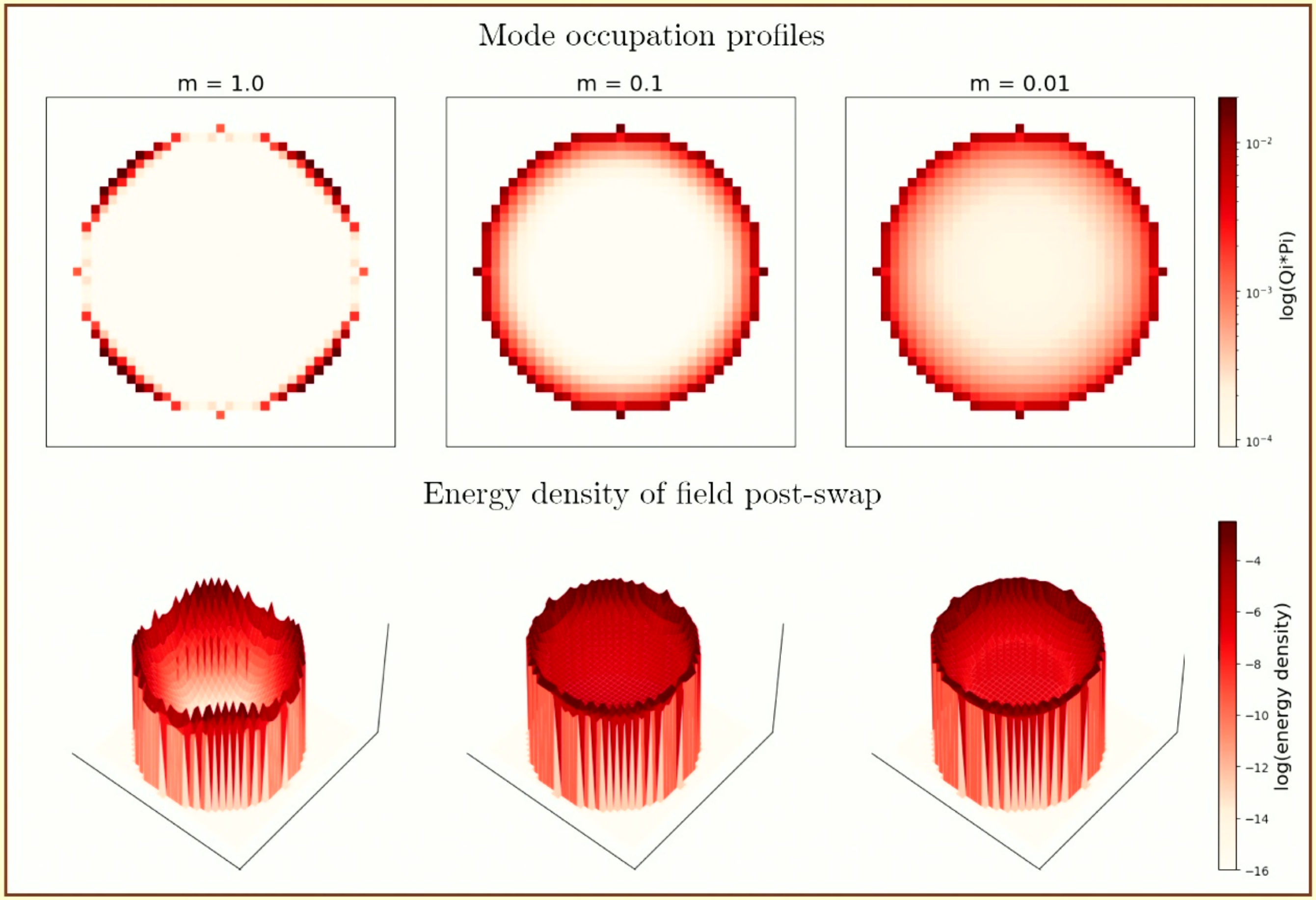
Field energy density post-swap



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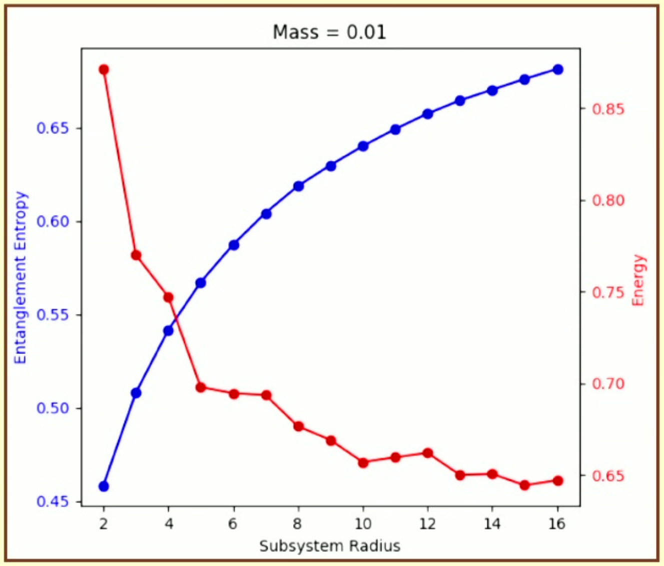
"Continuum limit": High or low energy modes?

- High or low energy? Look at scaling of energy as $r \to \infty$
  - Divergent: High energy
  - Convergent: Low/intermediate energy



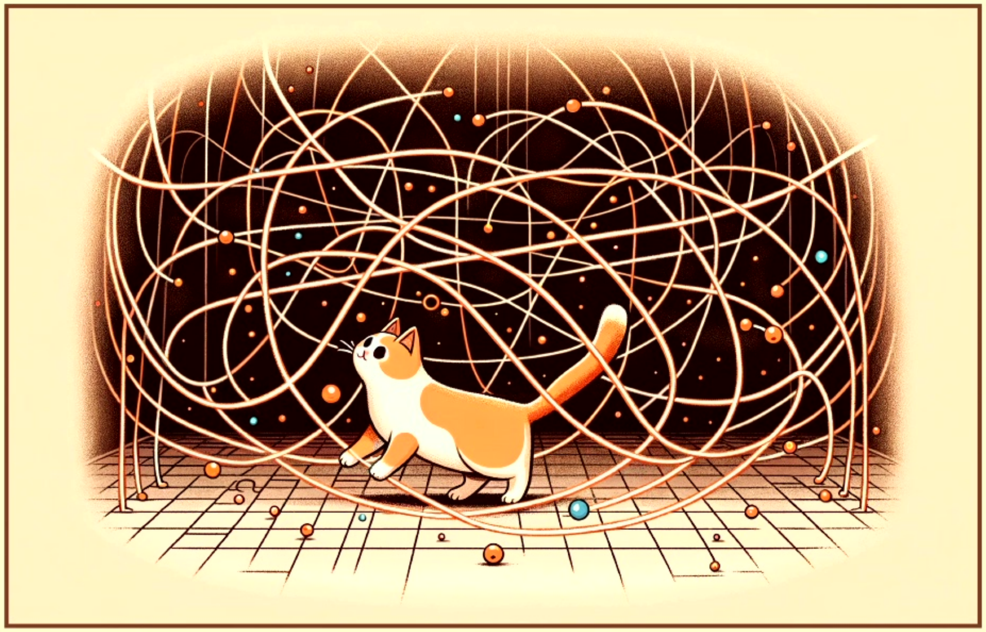
- Not quite enough data yet to tell
  - "Brute force" numerics has proved infeasible; complexity of (likely optimal) algorithms is $\mathcal{O}(r^{3d})$ where d = # of spatial dimensions
- Much larger subsystems achievable with further analytical work [6]
- Entanglement entropy is far easier with large r, and we recover known results

[6] Ivan Agullo, Béatrice Bonga, Eduardo Martín-Martínez, Sergi Nadal-Gisbert, T. Rick Perche, José Polo-Gómez, Patricia Ribes-Metidieri, Bruno de S. L. Torres, "The multimode nature of spacetime entanglement in QFT". *Work in preparation.*

Conclusions

- The framework presented here allows us to study the optimized mode-based entanglement extraction from quantum fields and its energy cost
- Key components:
  1. Direct calculation of subregion covariance matrices
  2. Mode-based entanglement decomposition motivated by detector models
  3. Calculation of entanglement energy through a swap interaction

Thanks for listening!



DALL-E: Cat playing in a quantum field (2024)