Title: Photon Rings and Shadow Size for General Integrable Spacetimes

Speakers: Kiana Salehi

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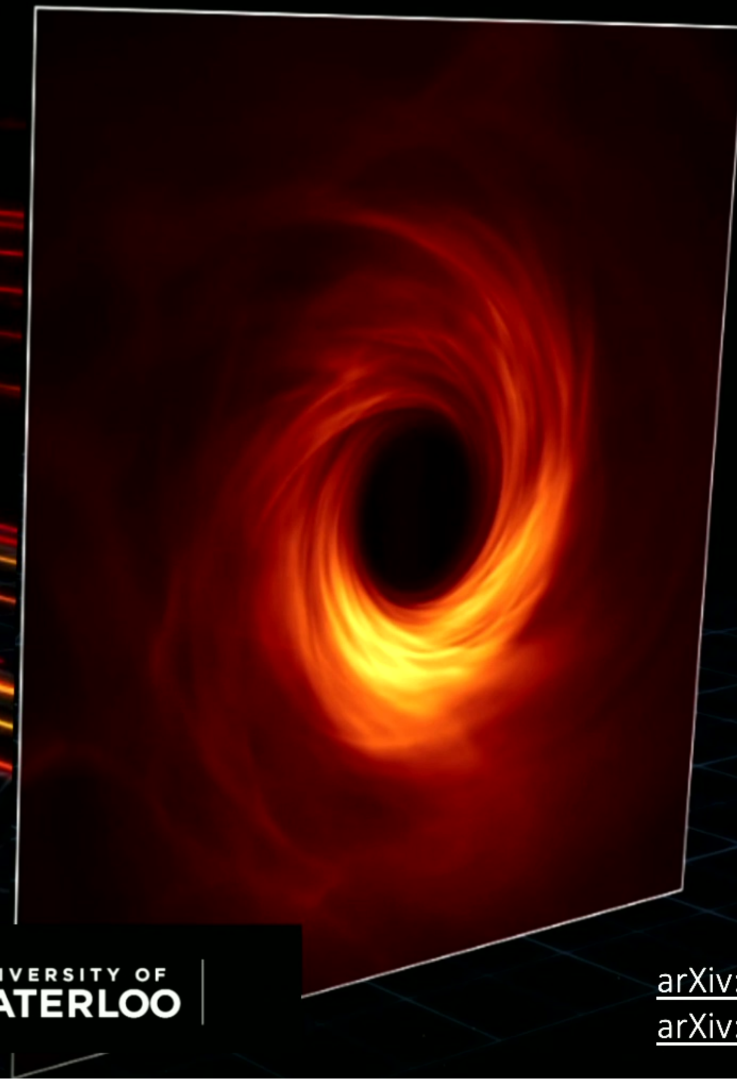
Abstract:

There are now multiple direct probes of the region near black hole horizons, including direct imaging with the Event Horizon Telescope (EHT). As a result, it is now of considerable interest to identify what aspects of the underlying spacetime are constrained by these observations. For this purpose, we present a new formulation of an existing broad class of integrable, axisymmetric, stationary spinning black hole spacetimes, specified by four free radial functions, that makes manifest which functions are responsible for setting the location and morphology of the event horizon and ergosphere. We explore the size of the black hole shadow and high-order photon rings for polar observers, approximately appropriate for the EHT observations of M87*, finding analogous expressions to those for general spherical spacetimes. Of particular interest, we find that these are independent of the properties of the ergosphere, but does directly probe on the free function that defines the event horizon. Based on these, we extend the nonperturbative, nonparametric characterization of the gravitational implications of various near-horizon measurements to spinning spacetimes. Finally, we demonstrate this characterization for a handful of explicit alternative spacetimes.

Photon Rings and Shadow Size for a General Class of Integrable Space Times

Kiana Salehi- Avery Broderick

Perimeter Institute- University of Waterloo



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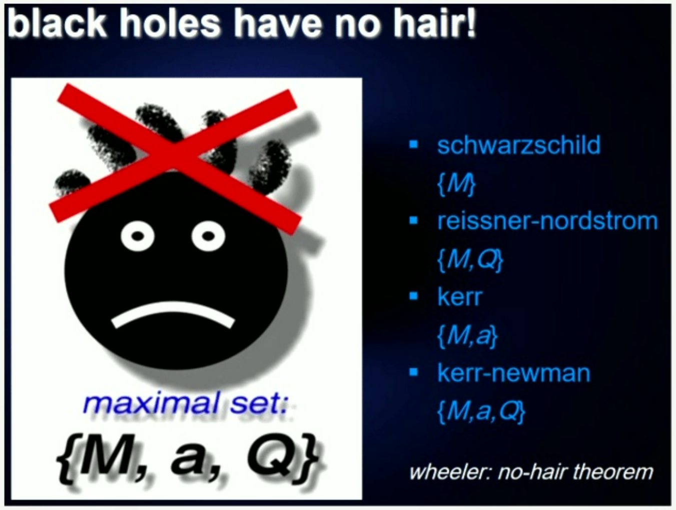
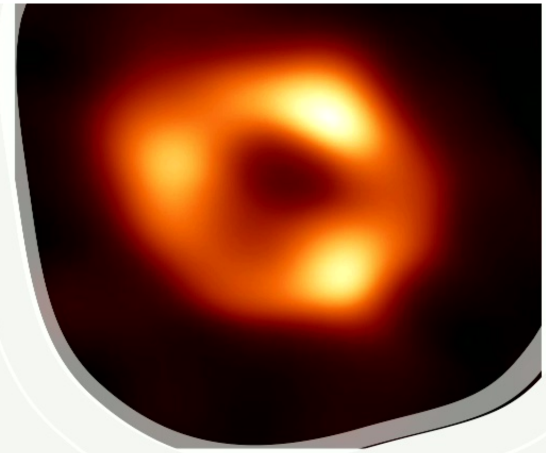
arXiv:2307.15120

arXiv:2311.01495

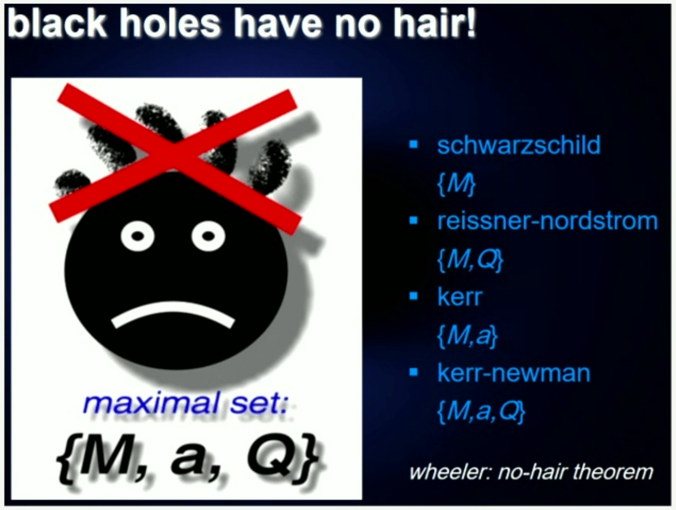
No Hair Theorem



- EHT data for M87 and Sgr A*
- Constraints → the possible deviations.



No Hair Theorem



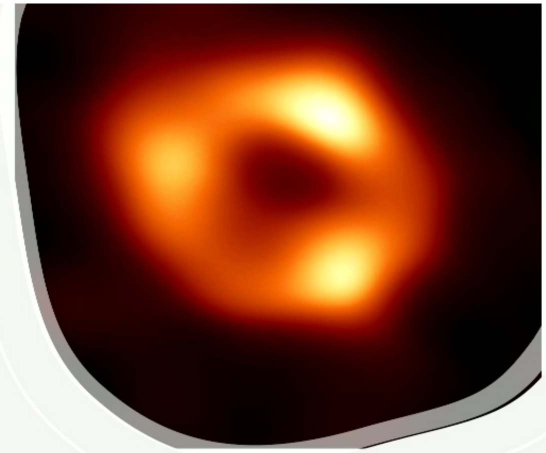
How to quantify these deviations?

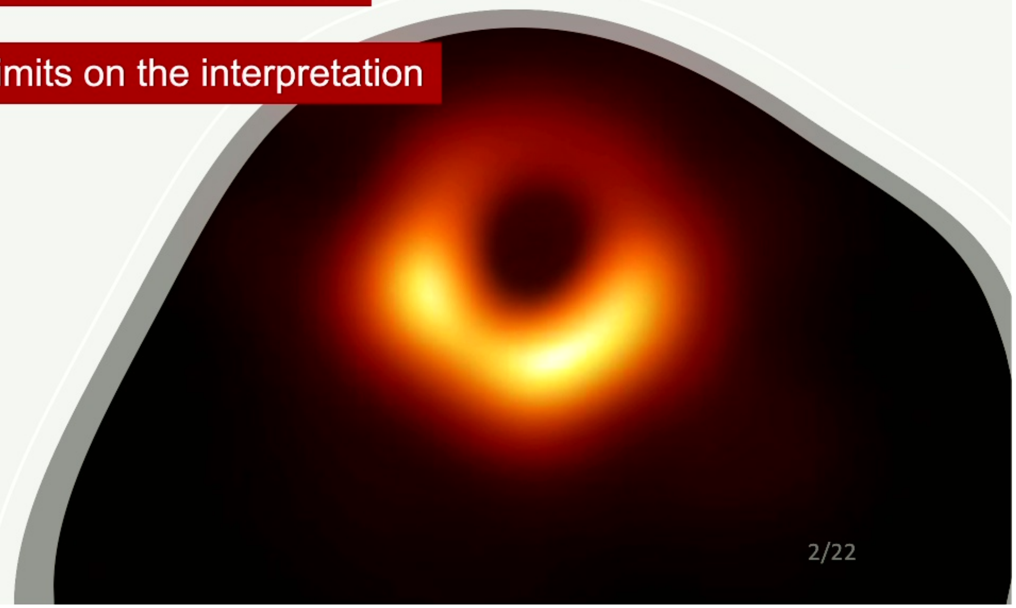
Alternatives?

parametrized

strong underlying assumptions

impose strong limits on the interpretation





- EHT data for M87 and Sgr A*
- Constraints → the possible deviations.

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We need

a Non-perturbative and non-parametric framework to describe/compare near horizon tests

↓

Simple Case

↓

Spherically Symmetric and Static Spacetime

A general spherically symmetric static :

$$ds^2 = -N(r)^2 dt^2 + \frac{B(r)^2}{N(r)^2} dr^2 + r^2 d\Omega$$

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a Non-perturbative and non-parametric framework to describe/compare near horizon tests

Simple Case

Spherically Symmetric and Static Spacetime

Symmetries :

$$\partial_t \rightarrow E = g_{tt} \frac{dt}{d\lambda}$$

$$\partial_\varphi \rightarrow L_z = g_{\varphi\varphi} \frac{d\varphi}{d\lambda}$$

A general spherically symmetric static :

$$ds^2 = -N(r)^2 dt^2 + \frac{B(r)^2}{N(r)^2} dr^2 + r^2 d\Omega$$

Photon Circular Orbit

$$ds^2 = -N(r)^2 dt^2 + \frac{B(r)^2}{N(r)^2} dr^2 + r^2 d\Omega$$

rearrange :

$$\dot{r}^2 = -\frac{g^{tt} + b^2 g^{\varphi\varphi}}{g_{rr}} = 0$$

Where,

$$\dot{r} = \frac{dr}{d\lambda}$$

$$b = \frac{L_z}{E}$$

Taking a derivative

$$\ddot{r} = \frac{1}{2} \frac{N(r_\gamma)^2}{{r_\gamma}^2 B(r_\gamma)^2} \left(\frac{r^2}{N(r)^2}\right)' \Bigg|_{r_\gamma} = 0$$

Solving simultaneously :

$$r_\gamma = \frac{N(r_\gamma)}{N'(r_\gamma)}$$

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

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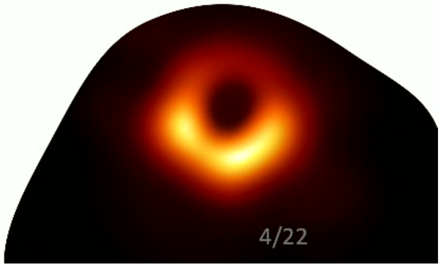
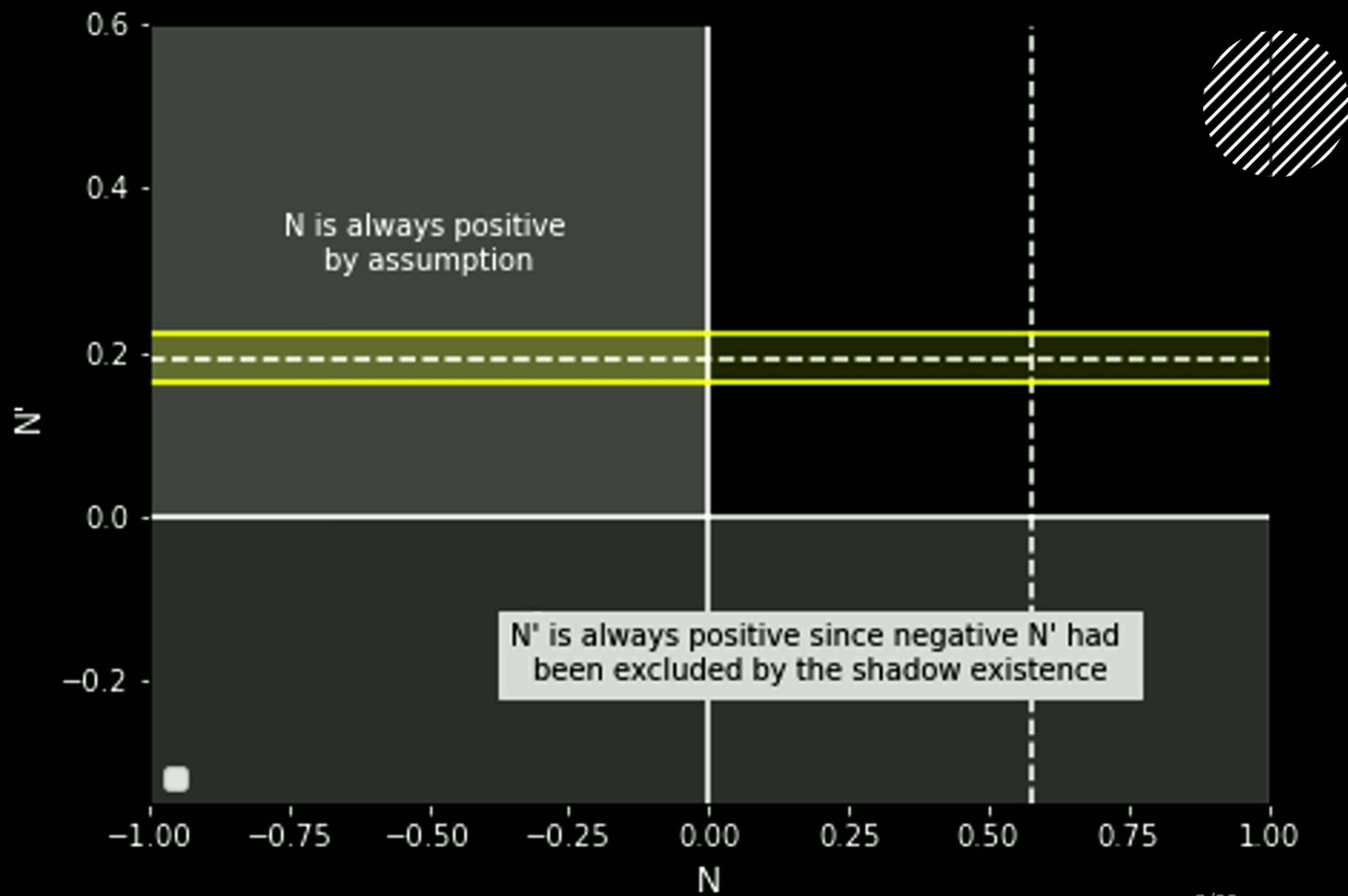


the shadow size is :

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$



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the shadow size is :

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

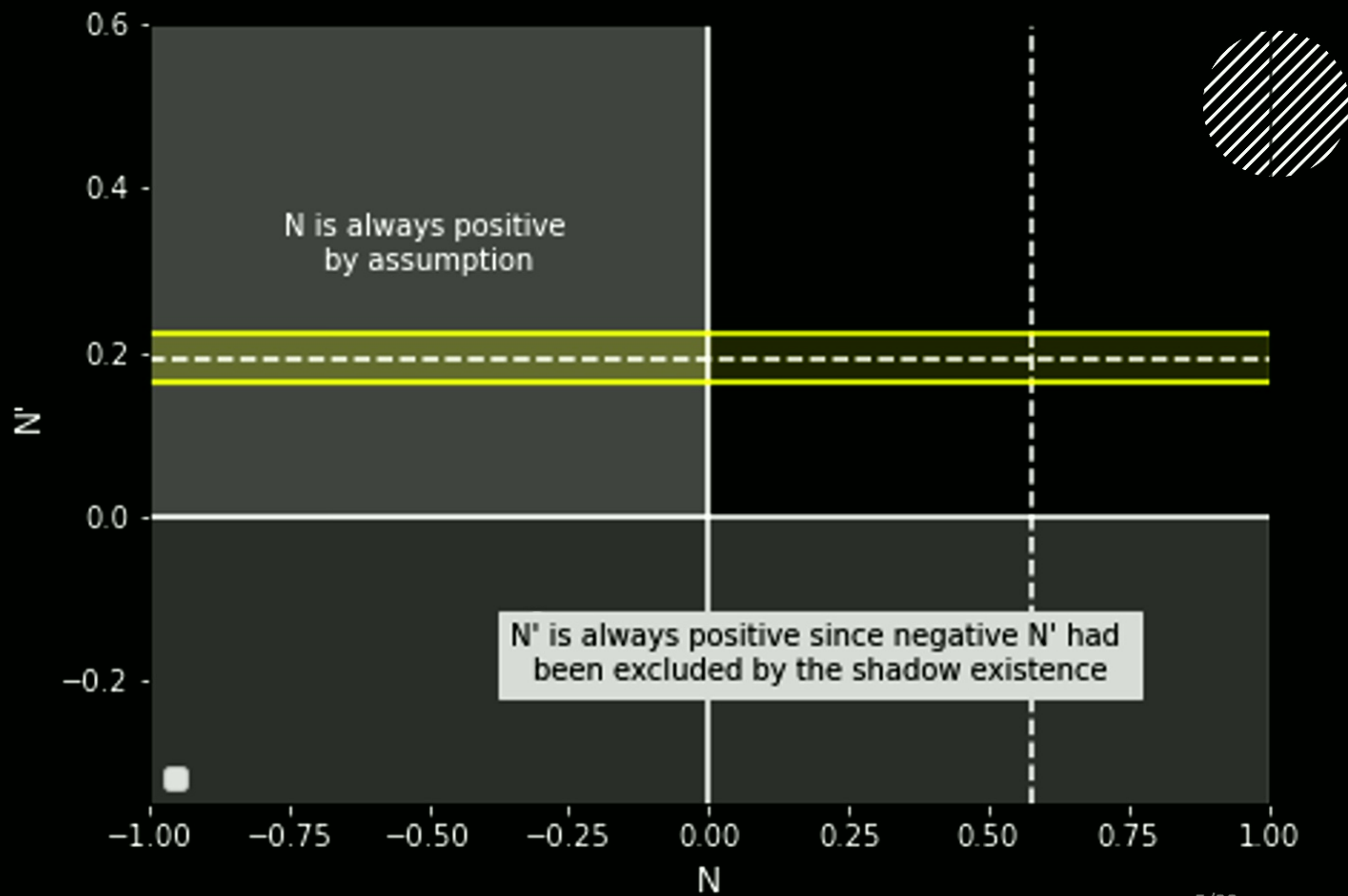
Shadow exist → $N' > 0$

Reminder $g_{tt}(r) = -N^2(r)$

$N > 0$

$N < 0$

Choose $N > 0$



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Multiple Photon Ring

$b = b_\gamma + \delta b$

$\ddot{r} = \ddot{r}|_{r_\gamma} + \ddot{r}'|_{r_\gamma}\,\delta r + \mathcal{O}(\delta r^2)$

$\ddot{r}|_{r_\gamma}=0$

$\delta r = \delta r_0 e^{\omega\tau}$

which

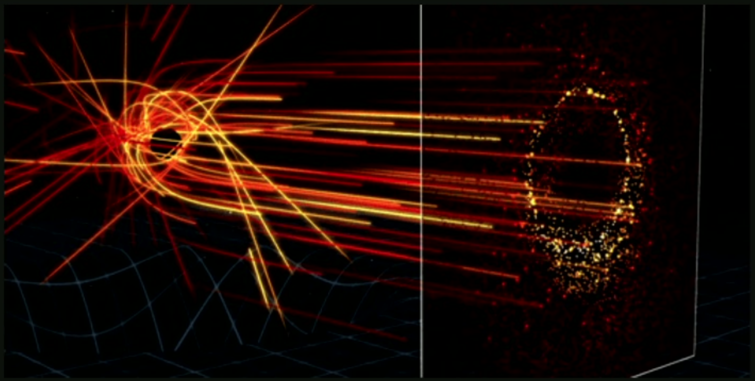
$\omega^2 = \ddot{r}'|_{r_\gamma}$

Lyapunov exponent

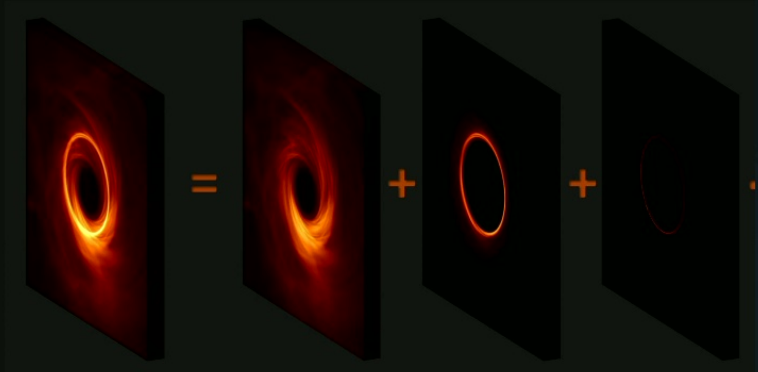
$$\gamma = \pi \frac{N^{3/2}}{N'} \sqrt{\frac{-N''}{B^2}}$$

$$\pi \frac{d\delta r}{d\varphi} = \gamma\, \delta r$$

$$\delta r = \delta r_0 \exp\left(\frac{\gamma}{\pi}\varphi\right)$$



How strongly lensed light creates a photon ring. Credit: Center for Astrophysics, Harvard & Smithsonian



Different photon paths create layers of light. Credit: George Wong (UIUC) and Michael Johnson (CfA)

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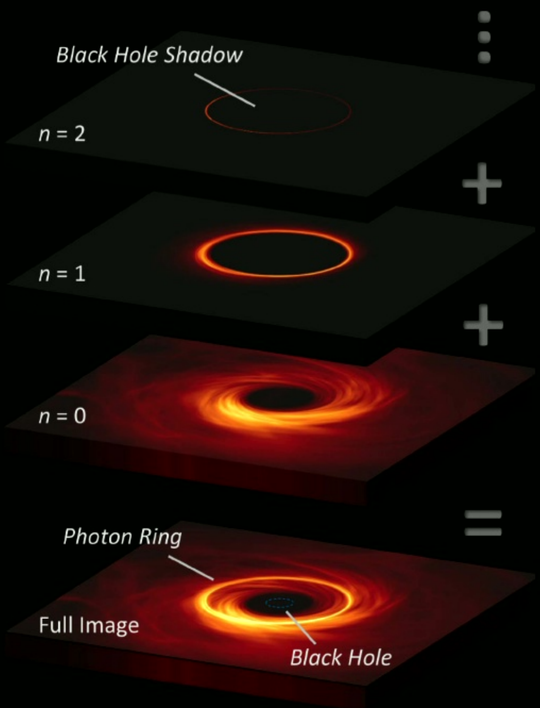
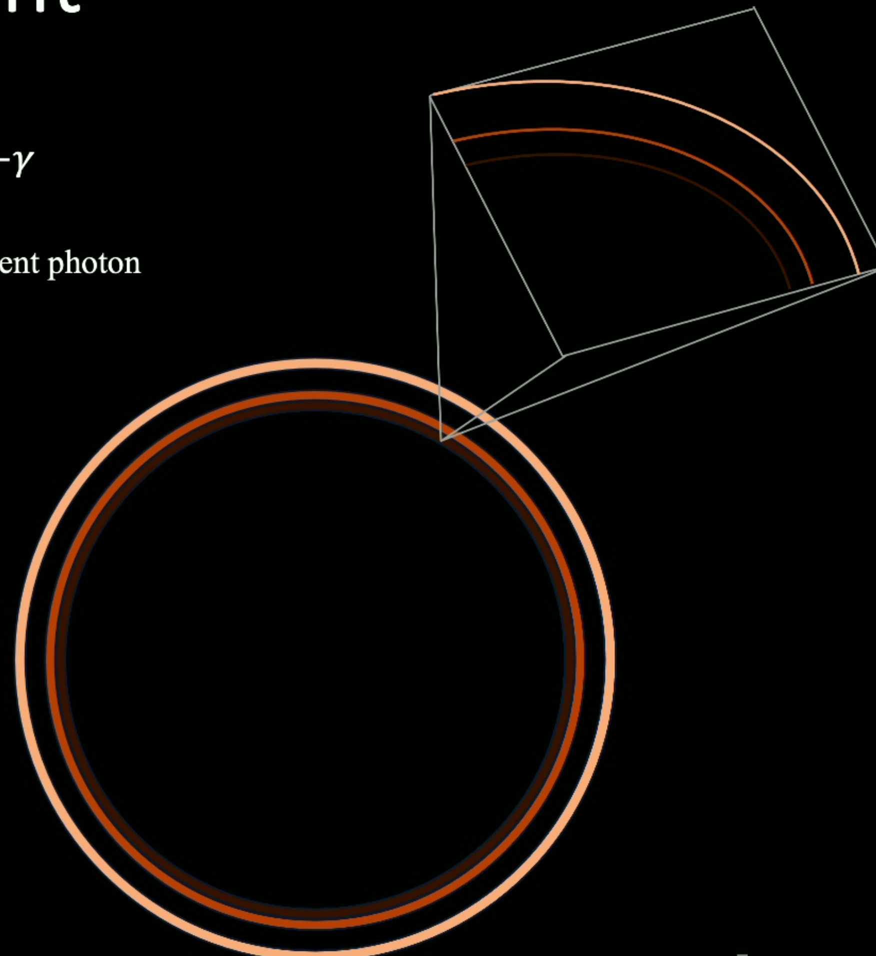
Observing Lyapunov Exponent

$$\frac{R_{m+2}-R_{m+1}}{R_{m+1}-R_m} = e^{-\gamma}$$

R_m: radius of the m/2 subsequent photon





Reference: *Universal Interferometric Signatures of a Black Hole's Photon Ring*
Credit: Michael D. Johnson (CfA), Simulation: George Wong (UIUC)

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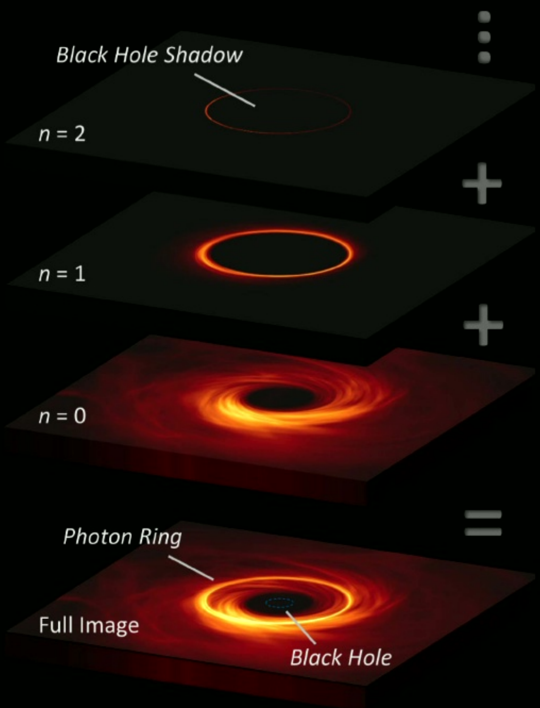

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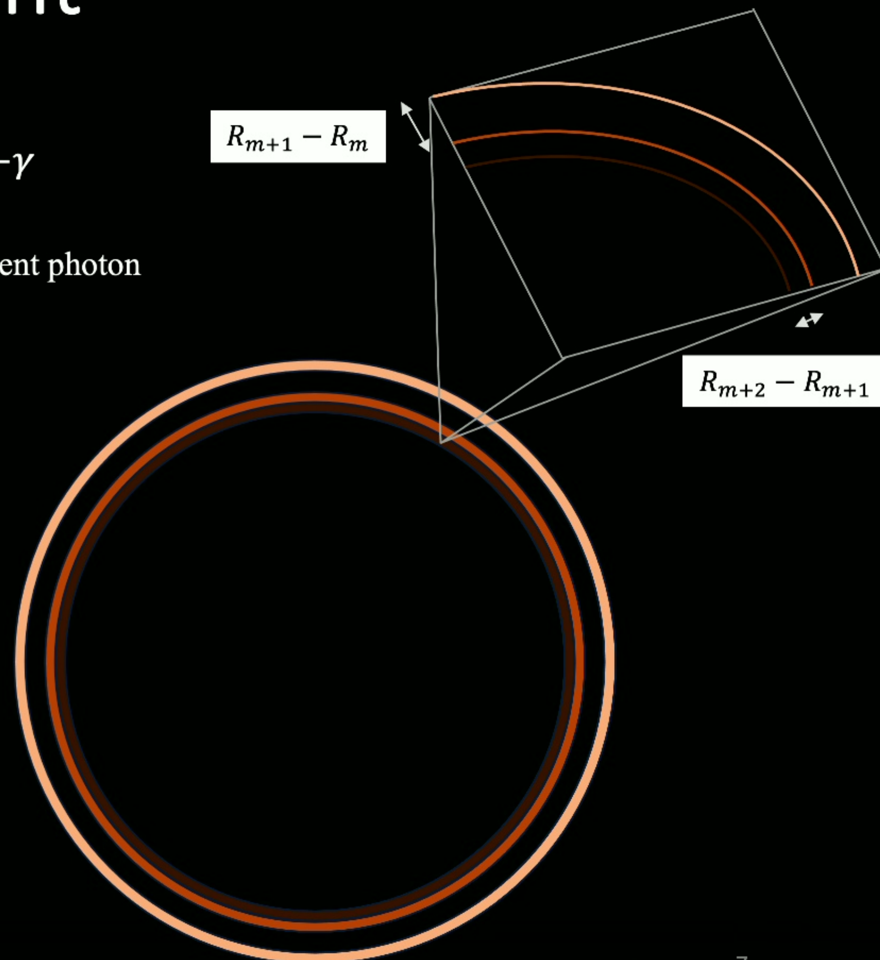

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$N - N'$ diagram

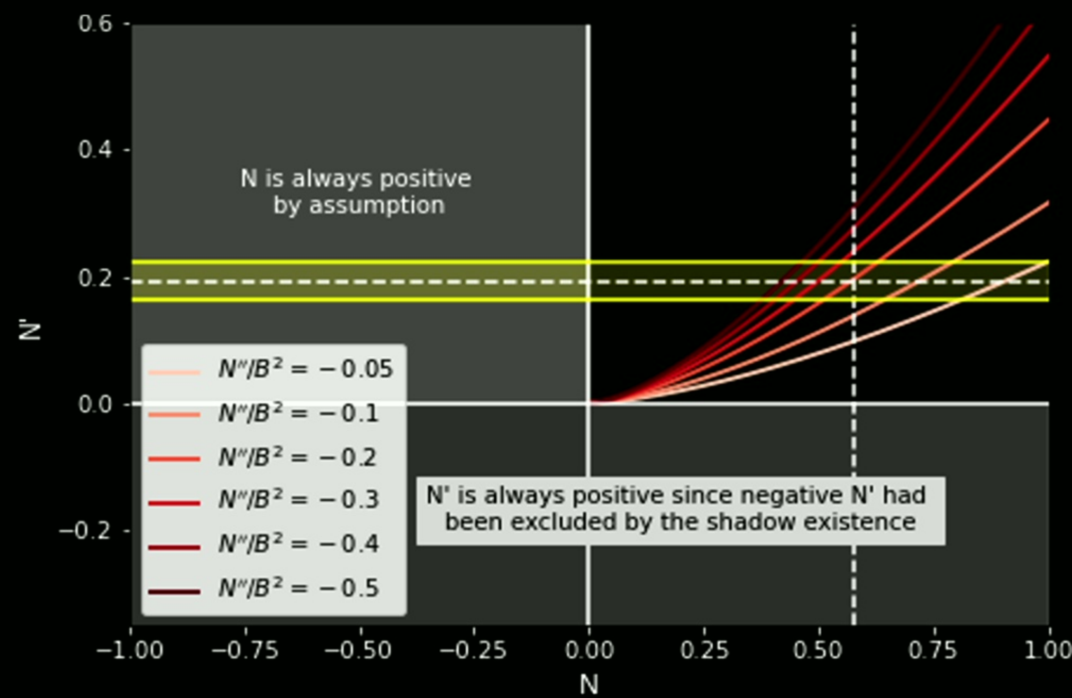
- **Reminder**

Shadow size:

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

Lyapunov exponent:

$$\gamma = \pi \frac{N^{3/2}}{N'} \sqrt{\frac{-N''}{B^2}}$$



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$N - N''/B^2$ diagram

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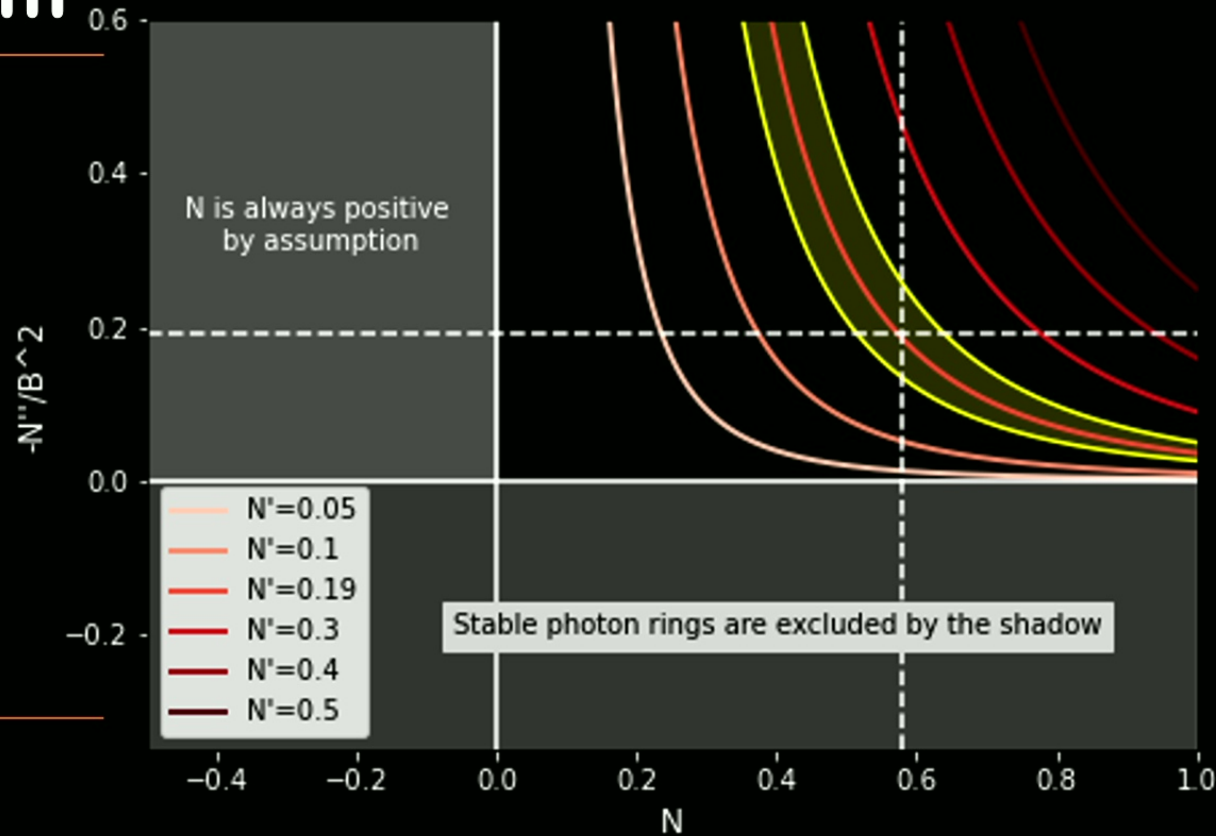

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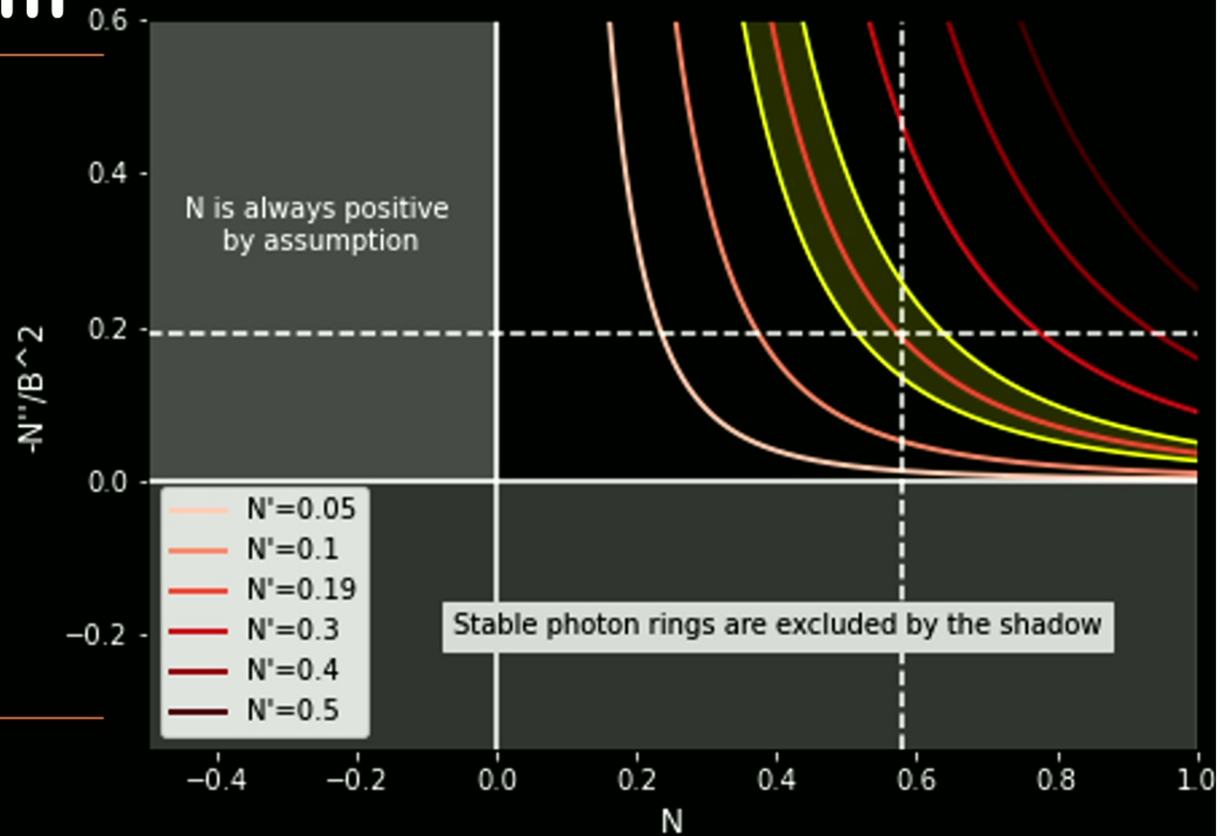

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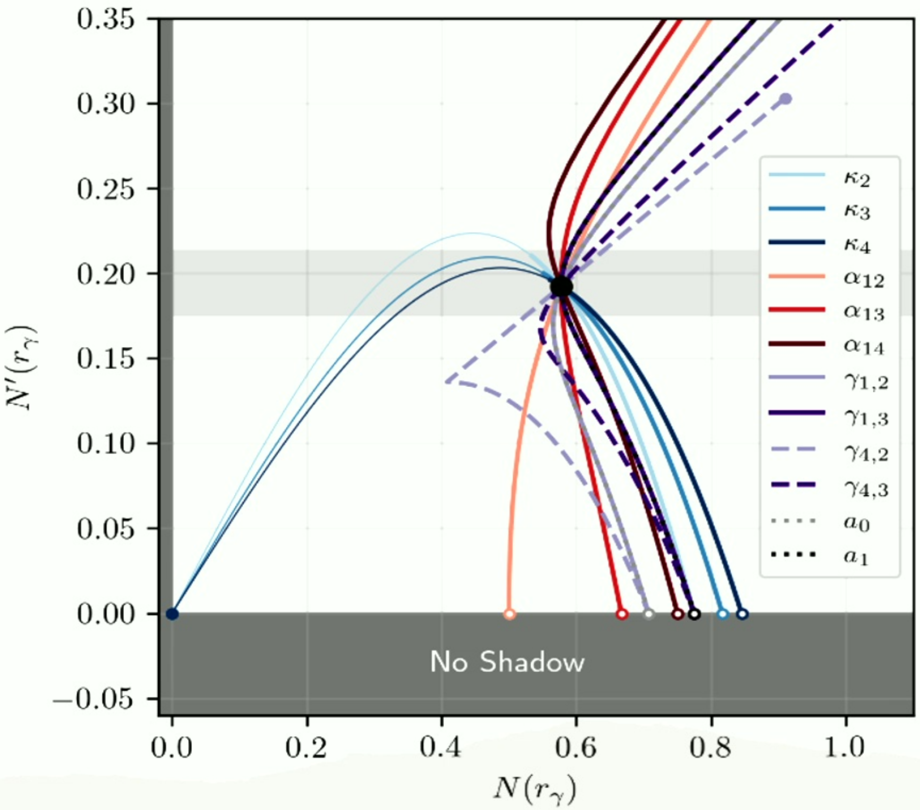
General Spherically Symmetric and Static

Non-perturbative and non-parametric

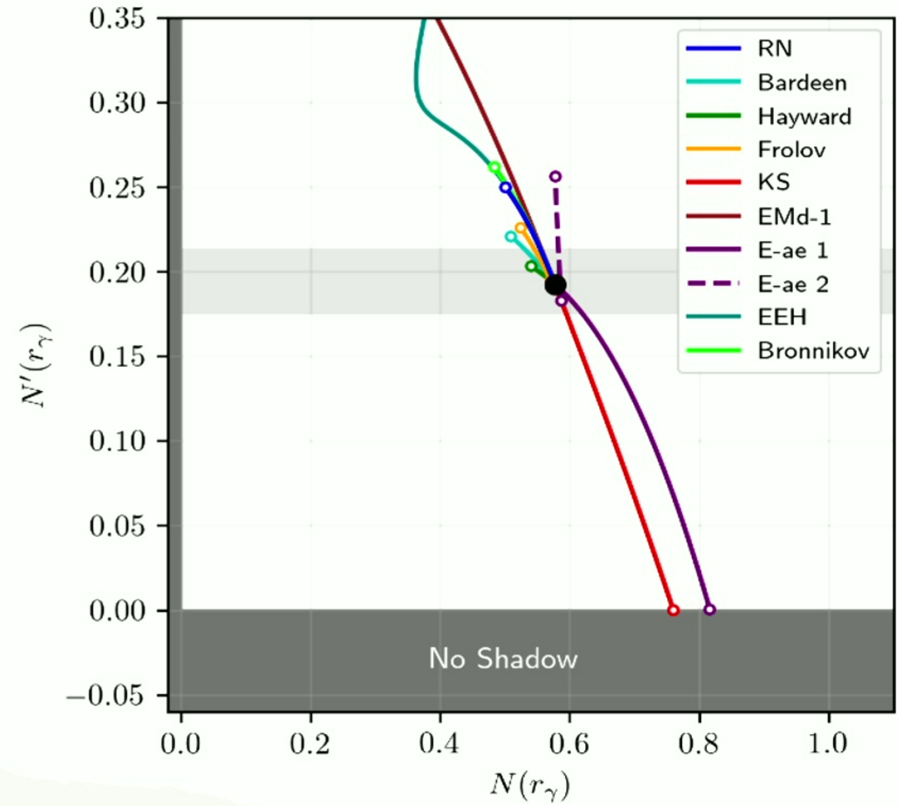
$N - N'$ **diagram** $N - N''/B^2$ **diagram**

different spherically symmetric spacetimes

Metric Expansions



Alternative Spacetimes



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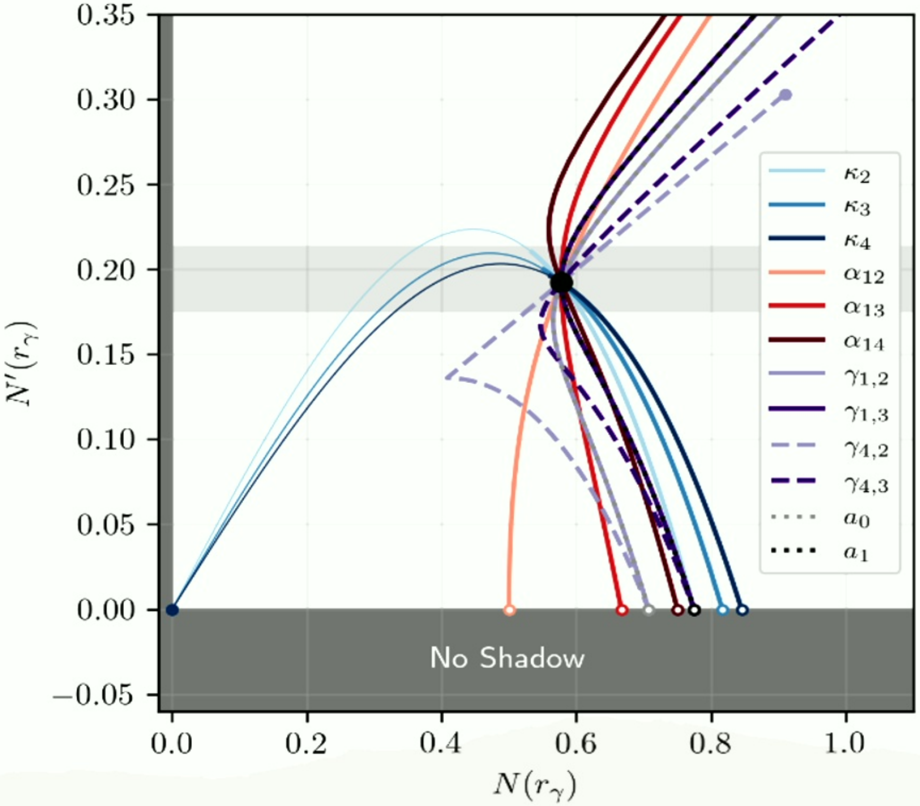
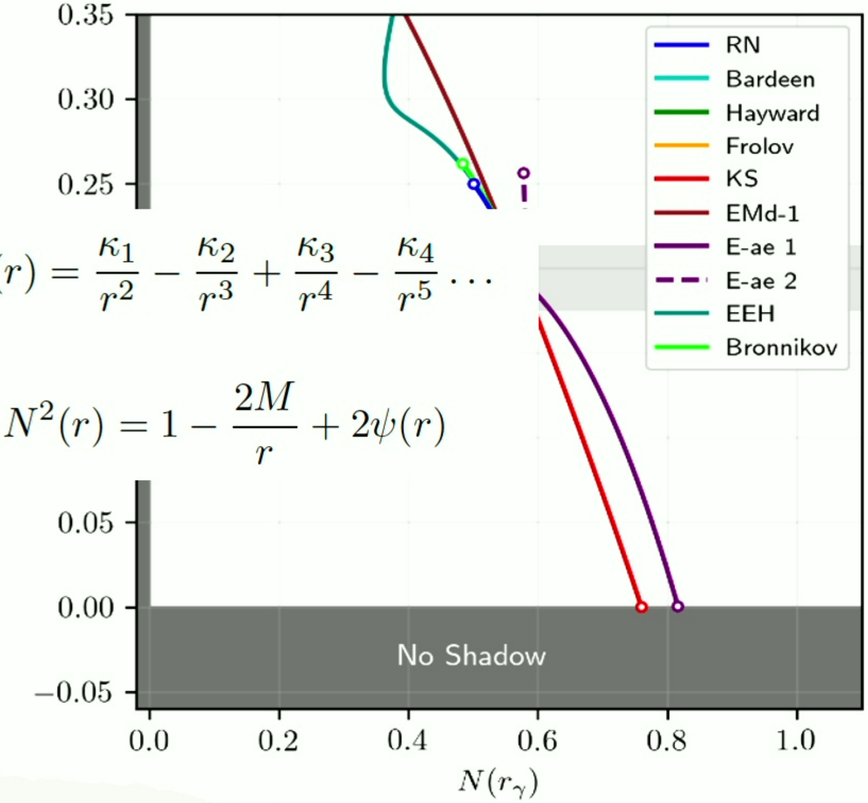
Metric Expansions



Alternative Spacetimes



$$\psi(r) = \frac{\kappa_1}{r^2} - \frac{\kappa_2}{r^3} + \frac{\kappa_3}{r^4} - \frac{\kappa_4}{r^5} \dots$$

$$N^2(r) = 1 - \frac{2M}{r} + 2\psi(r)$$

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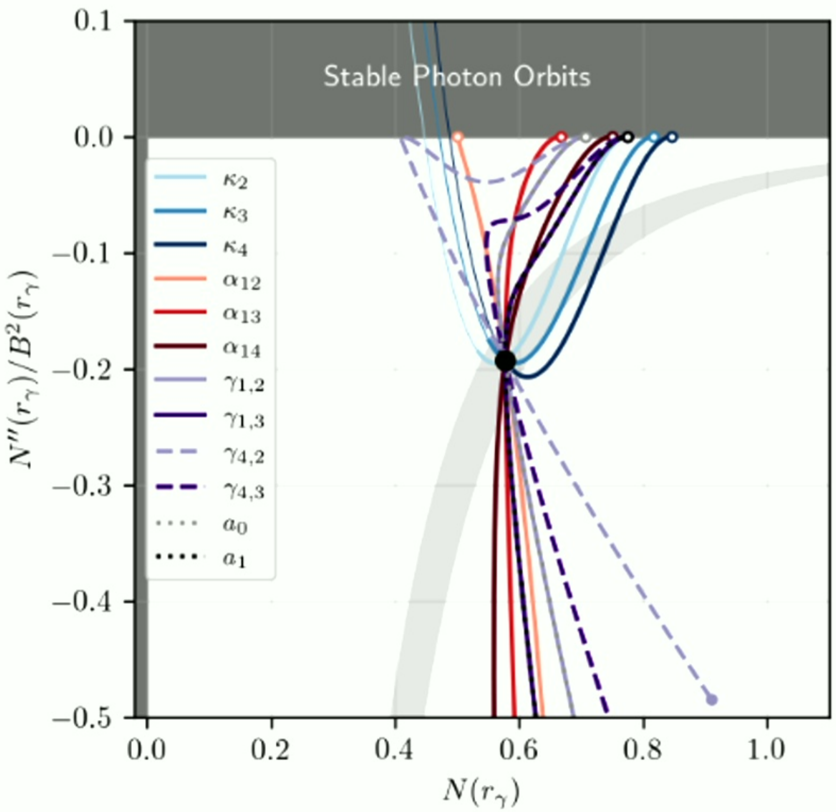
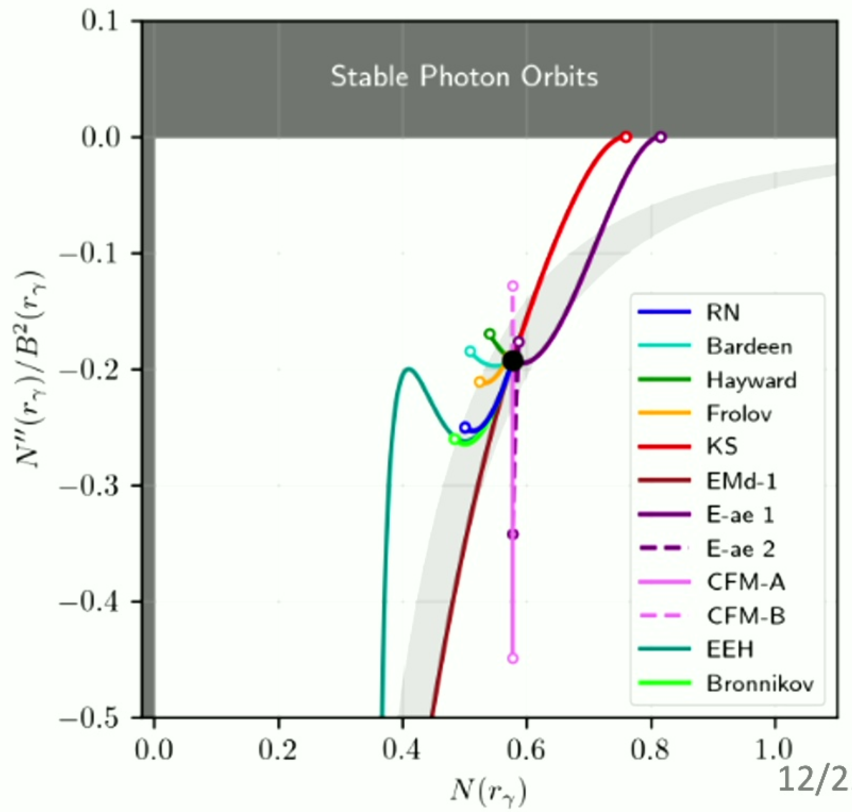
$N - N''/B^2$ diagram





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Summary

arXiv:2307.15120
arXiv:2311.01495

- **a non-perturbative and non-parametric framework to describe/compare near horizon tests.**
- **Shadow size measurements:**

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

- **Relative radii of the subsequent photon rings:**

$$\frac{R_{m+2} - R_{m+1}}{R_{m+1} - R_m} = e^{-\gamma} \qquad , \qquad \bar{\gamma} = -\pi \frac{N^{1.5}}{N'} \sqrt{\frac{-N''}{B^2}}\, 2K\left(\left(\frac{a}{b_\gamma}\right)^2\right)$$

For spherically symmetric and a general class of axisymmetric spacetimes.