Title: Open Quantum On Lie Group: An Effective Field Theory Approach

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Abstract:

In this work, we propose a systematic method to obtain the effective field theory of the quantum dissipative systems which nonlinearly realize symmetries. We focus on the high temperature or Brownian limit, in which the effective action of the dissipative dynamics is localized in time. We first introduce a microscopic model at the linear response level, which shows how the dissipative dynamics on Lie group emerges effectively through the reduced dynamics of a system interacting with a thermal bath. The model gives a systematic method to give the Langevin equation which is covariant with respect to the symmetries of the system. In addition, the model shows a systematic way to go beyond the Gaussian white noise and the interaction between the noise and dissipation. Then, using the dynamical KMS symmetry, without any reference to the microscopic structure of the bath, we obtain the most general effective action of the nonlinearly realized dissipative dynamics at high temperature. The universal dissipative coefficients are larger than the case of the linear response approximation. Then, we focus on the case of Ohmic friction where the corresponding dissipative coefficients are more restricted; we suggest an alternative model, the bulk model, to describe any Ohmic dissipative system at high temperature. The Bulk model provides a geometrical picture for the noise in the case of Ohmic friction.

Open Quantum Dynamics On Lie Group: An Effective Field Theory Approach

PI Graduate Students Seminar

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All Physical Systems Are Open

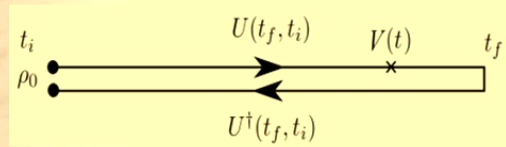
- It’s not always necessary nor possible to include the microscopic details of
- Example: Brownian motion

$$m\frac{d^2x(t)}{dt^2} = -\gamma\frac{dx(t)}{dt} + \eta(t)$$

$$\langle \eta(t)\eta(t') \rangle = 2D\delta(t-t')$$

Double Time-Path Integral:

$$\rho(t) = U(t,t_i)\rho_0 U^\dagger(t,t_i)\ .$$



$$S[q_+,q_-] = \int dt\left(\frac{1}{2}\dot{q}_+^2 - \frac{1}{2}V(q_+)\right) - \int dt\left(\frac{1}{2}\dot{q}_-^2 - \frac{1}{2}V(q_-)\right)$$

$$\bar{q} = \frac{1}{2}(q_+ + q_-) \qquad \hat{q} = \frac{1}{2}(q_+ - q_-)$$

$$S[\bar{q},\hat{q}] = -\int_{-\infty}^{+\infty} \mathrm{d}t\left[2\hat{q}\left(\ddot{\bar{q}} + V'(\bar{q})\right) + \mathcal{O}\left((\hat{q})^3\right)\right]$$

$$\ddot{\bar{q}} + V'(\bar{q}) = 0$$

Dynamics On A Group Manifold

- The group manifold:

$$[G_i, G_j] = C^k{}_{ij} G_k$$

$$\Omega = g^{-1} dg = \Omega^i_j(q) dq^j G_i$$

$$\Omega = g^{-1} dg = \Omega^i_j(q) dq^j G_i$$

- The left invariant dynamics on the group manifold

$$g(q) \mapsto g(\tilde{q}) = g_L \, g(q)$$

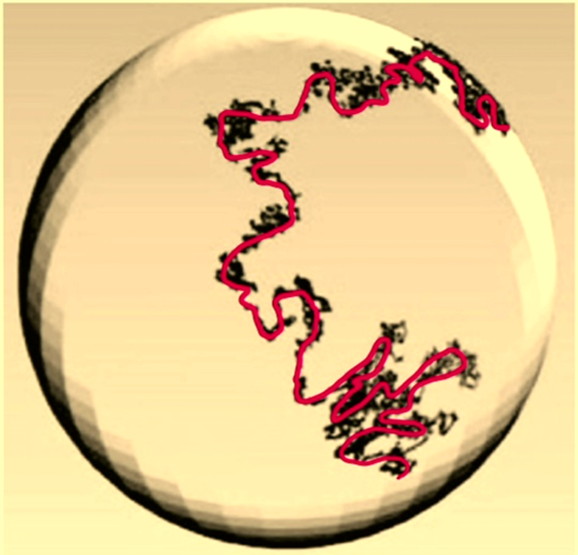
$$D_t q^i = \Omega^i_j(q) \dot{q}^j$$

$$S_{\mathrm{body}} = \int dt \, \frac{1}{2} J_{ij} D_t q^i D_t q^j$$

$$J_{ij} \partial_t D_t q^j + J_{jl} C_{ikl} D_t q^j D_t q^k = 0$$

Group Covariant Classical & Quantum Fields

$$\mathrm{e}^{q_\pm G} = \mathrm{e}^{\bar{q}G}\mathrm{e}^{\pm\hat{q}G} \qquad\qquad q_+ \ominus q_- = 2\hat{q}$$

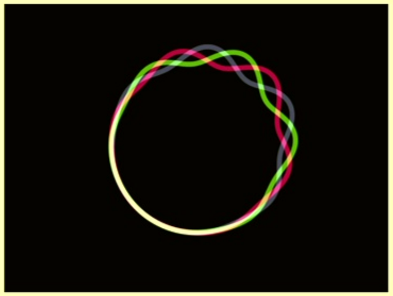


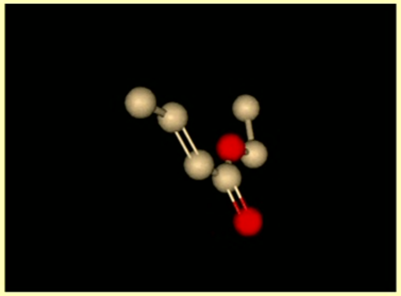
- The leading order Shwinger-Keldysh action of a free body

$$S_{\text{body}} = \int dt\,\frac{1}{2}J_{ij}(D_tq^i_+D_tq^j_+ - D_tq^i_-D_tq^j_-) \simeq -2\int dt\,\hat{q}^i\big(J_{ij}\partial_tD_t\bar{q}^j + J_{jl}{C^l}_{ik}D_t\bar{q}^jD_t\bar{q}^k\big)$$

$$J_{ij}\,\partial_tD_t\bar{q}^j + J_{jl}\,C_{ikl}\,D_t\bar{q}^j\,D_t\bar{q}^k = 0$$

Interaction with a bath and its spectral decomposition.





$$V(\theta,\chi) = \sum_{j=-\infty}^{\infty} \mathrm{e}^{ij\theta}\, \mathcal{O}^j(\chi)$$

$$V(\alpha,\beta,\gamma;\chi) = \sum_{j,m,m'} U^j_{mm'}(\alpha,\beta,\gamma) O^j_{m'm}(\chi)$$

$$V(q,\chi) = \sum_{raa'} U^r_{aa'}(q) \mathcal{O}^{\dagger r}_{a'a}(\chi)$$

Integrating Out The Bath & The Influence Functional

$$e^{i\mathcal{I}[q_+,q_-]} = \langle \tilde{\mathcal{T}}\big(e^{ig\int_0^t dt'\hat{V}(t')}\big)\mathcal{T}\big(e^{-ig\int_0^t dt'\hat{V}(t')}\big)\rangle_{\rho_\chi}$$

The free Schwinger-Keldysh action functional is modified by the influence functional

$$S_{\text{eff}} = \int dt\,\frac{1}{2}J_{ij}(D_t q_+^i D_t q_+^j - D_t q_-^i D_t q_-^j) + \mathcal{I}[q_+,q_-]$$

The Influence Functional At The Linear Response Approximation & With Time Reversal Symmetry

- Two-point functions of the bath projected on the body's degrees of freedom: it carries group indices.

$$\mathcal{I}[q_+, q_-] = \frac{i}{2}\int dt_1 dt_2 \Big\{ U^r_{a'b'}(q_{+2} \ominus q_{+1})\mathcal{G}^r_{a'b'}(t_1 - t_2) + U^r_{a'b'}(q_{-2} \ominus q_{-1})\tilde{\mathcal{G}}^r_{a'b'}(t_1 - t_2)$$
$$- U^r_{a'b'}(q_{+2} \ominus q_{-1})\mathcal{K}^r_{a'b'}(t_1 - t_2) - U^r_{a'b'}(q_{-2} \ominus q_{+1})\tilde{\mathcal{K}}^r_{a'b'}(t_1 - t_2) \Big\}$$

$$\sum_a U^{\dagger r}_{a'a}(q_1) U^r_{ab'}(q_2) = U^r_{a'b'}(q_2 \ominus q_1)$$

The Green's functions carry the group indices. In close analogy: plane wave expansion in terms of IR of SO(3)

$$e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^\ell \sqrt{\frac{4\pi}{2\ell+1}} j_\ell(kr)\, U^\ell_{m0}(\alpha, \beta, \gamma)\, Y_{\ell m}(\theta, \phi)$$

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Locality In Time?

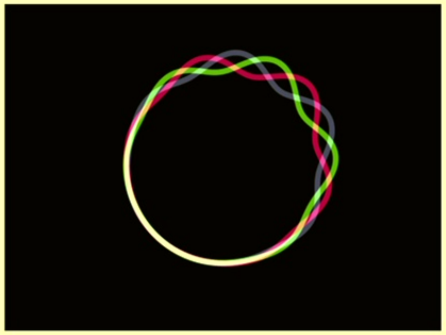
- At high Temperature, and if the response in the bath dies quickly.

$$\mathcal{K}^r_{ab}(t) = \frac{2\pi}{\beta}\varrho^r_{0,ab}\,\delta(t) + \pi i\varrho^r_{0,ab}\,\delta'(t) + \dots$$

$$\tilde{\mathcal{K}}^r_{ab}(t) = \frac{2\pi}{\beta}\varrho^r_{0,ab}\,\delta(t) - \pi i\varrho^r_{0,ab}\,\delta'(t) + \dots$$

$$\mathcal{G}^r_{ab}(t) = \frac{2\pi}{\beta}\varrho^r_{0,ab}\,\delta(t) + \pi i\varrho^r_{0,ab}\,\delta'(t) - 2\pi i\varrho^r_{0,ab}\,\theta(-t)\delta'(t) + \dots$$

$$\tilde{\mathcal{G}}^r_{ab}(t) = \frac{2\pi}{\beta}\varrho^r_{0,ab}\,\delta(t) + \pi i\varrho^r_{0,ab}\,\delta'(t) - 2\pi i\varrho^r_{0,ab}\,\theta(t)\delta'(t) + \dots$$



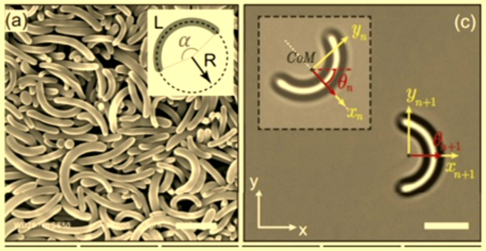
Implication: Coupling of Stochastic Modes

PHYSICAL REVIEW E **107**, L042602 (2023)

Letter | Editors' Suggestion

Effect of curvature on the diffusion of colloidal bananas

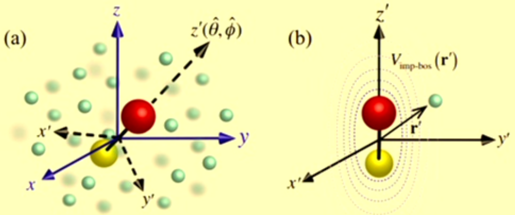
Justin-Aurel Ulbrich,[1,2] Carla Fernández-Rico,[1,2,*] Brian Rost,[3] Jacopo Vialetto,[2] Lucio Isa,[2] Jeffrey S. Urbach,[3,†] and Roel P. A. Dullens[1,4,‡]



PRL **114**, 203001 (2015) PHYSICAL REVIEW LETTERS week ending 22 MAY 2015

Rotation of Quantum Impurities in the Presence of a Many-Body Environment

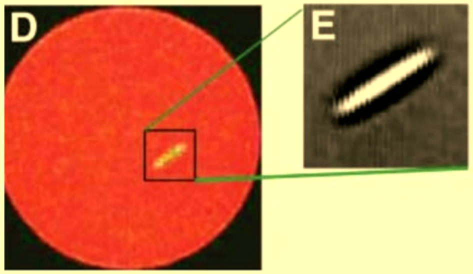
Richard Schmidt[1,2,*] and Mikhail Lemeshko[3,†]



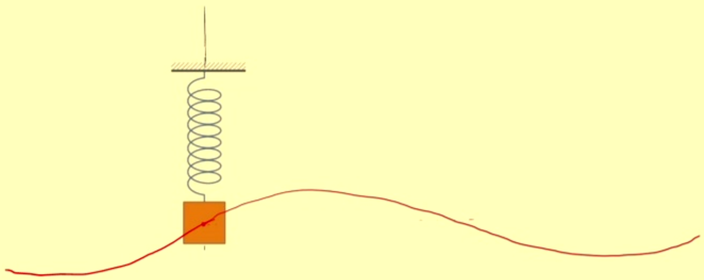
Brownian Motion of an Ellipsoid

Y. Han,[1] A. M. Alsayed,[1] M. Nobili,[2] J. Zhang,[1] T. C. Lubensky,[1*] A. G. Yodh[1]

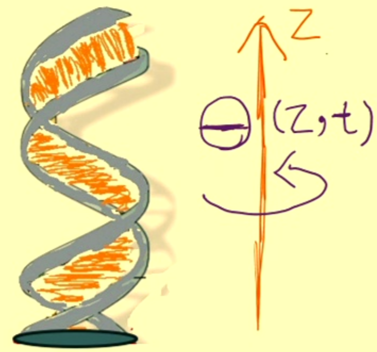
We studied the Brownian motion of isolated ellipsoidal particles in water confined to two dimensions and elucidated the effects of coupling between rotational and translational motion.

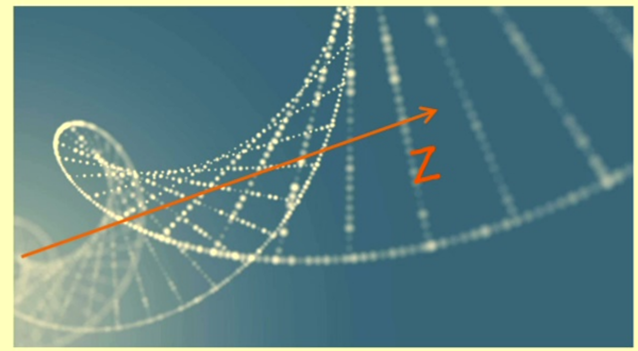


An Alternative For The Strictly Ohmic Dissipation: Non-Abelian Calderia-Leggett Model

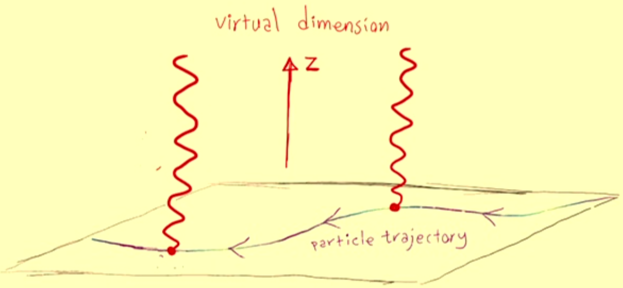


$$\theta(t) = \Theta(z=0,t)$$





$$x(t) = \mathcal{X}(z=0,t) \qquad y(t) = \mathcal{Y}(z=0,t)$$



The Ohmic condition from DKMS

$$f_{ij} = o_{ijk} = E_{ijkl} = d_{ij} = m_{ijk} = 0$$

PHYSICAL REVIEW E **109**, L052103 (2024)

A.B, Y Radkovski, S Sibiryakon


Thank you