Title: Everything that can be learned about a causal structure with latent variables by observational and interventional probing schemes

Speakers: Marina Maciel Ansanelli

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URL: https://pirsa.org/24090191

Abstract:

What types of differences among causal structures with latent variables are impossible to distinguish by statistical data obtained by probing each visible variable? If the probing scheme is simply passive observation, then it is well-known that many different causal structures can realize the same joint probability distributions. Even for the simplest case of two visible variables, for instance, one cannot distinguish between causal influence of one variable on the other and the two variables sharing a latent common cause. However, it is possible to distinguish between these two causal structures if we have recourse to more powerful probing schemes, such as the possibility of intervening on one of the variables and observing the other. Herein, we address the question of which causal structures remain indistinguishable even given the most informative types of probing schemes on the visible variables. We find that two causal structures remain indistinguishable if and only if they are both associated with the same mDAG structure (as defined by Evans (2016)). We also consider the question of when one causal structure dominates another in the sense that it can realize all of the joint probability distributions that can be realized by the other using a given probing scheme. (Equivalence of causal structures is the special case of mutual dominance.) Finally, we investigate to what extent one can weaken the probing schemes implemented on the visible variables and still have the same discrimination power as a maximally informative probing scheme.

Everything that can be learned about a causal structure with latent variables by observations and interventions

Marina Maciel Ansanelli

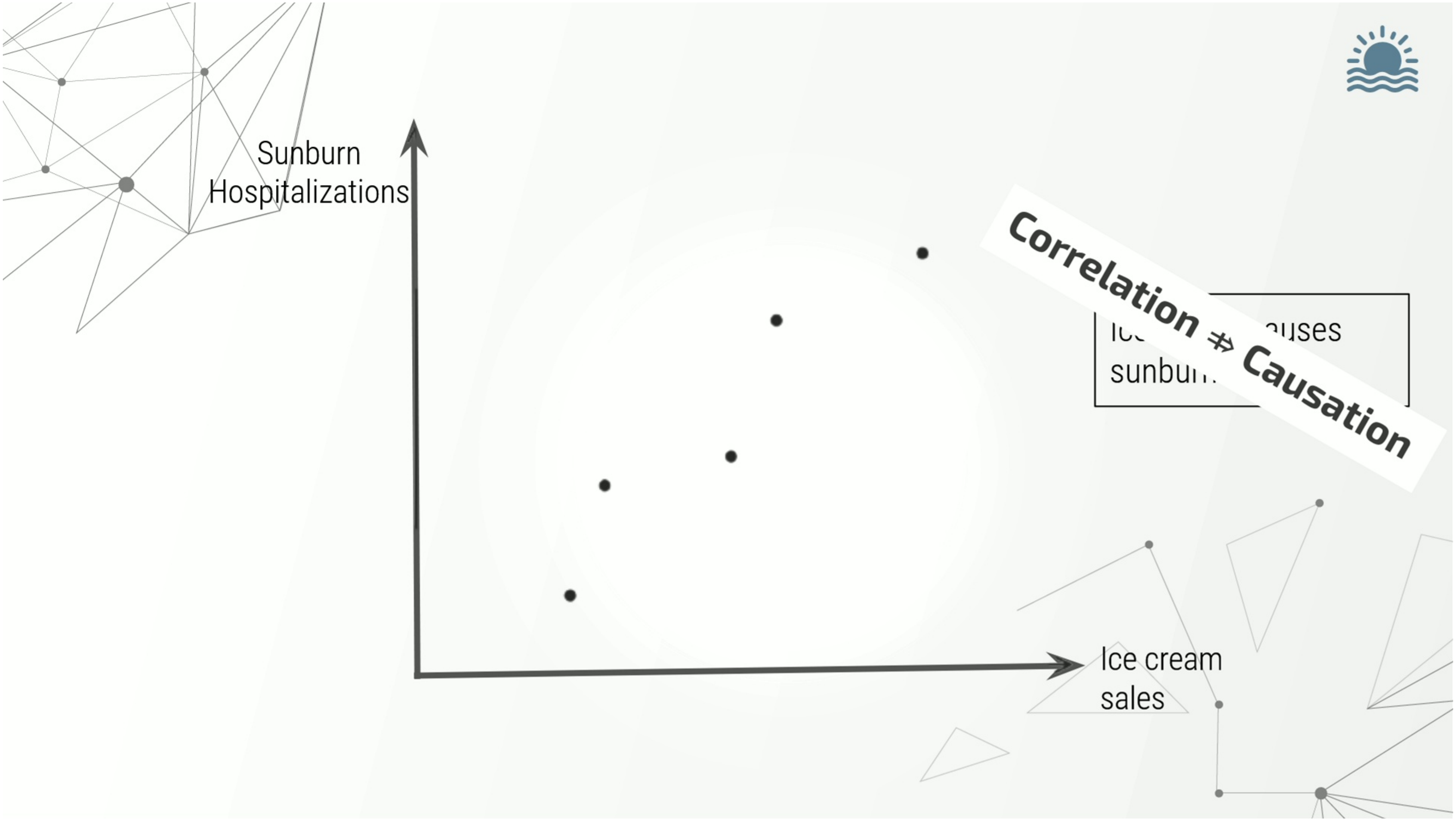
Joint work with Elie Wolfe and Robert Spekkens

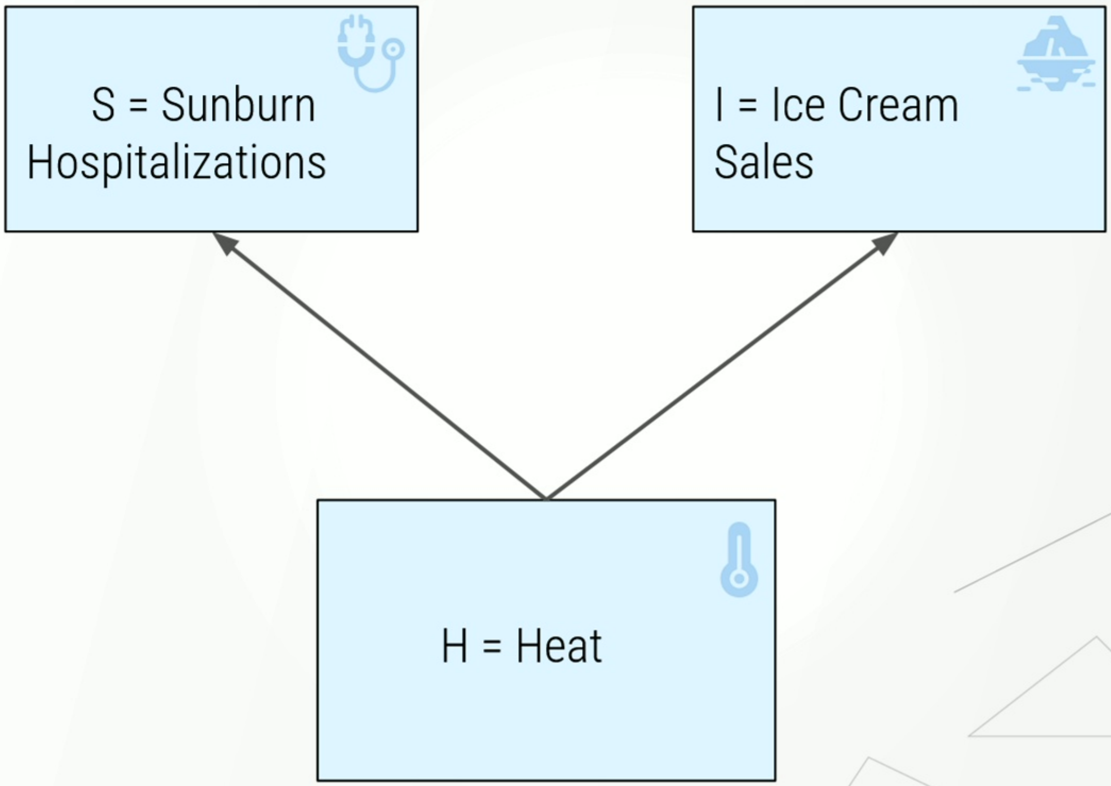
arXiv: 2407.01686

PERIMETER INSTITUTE

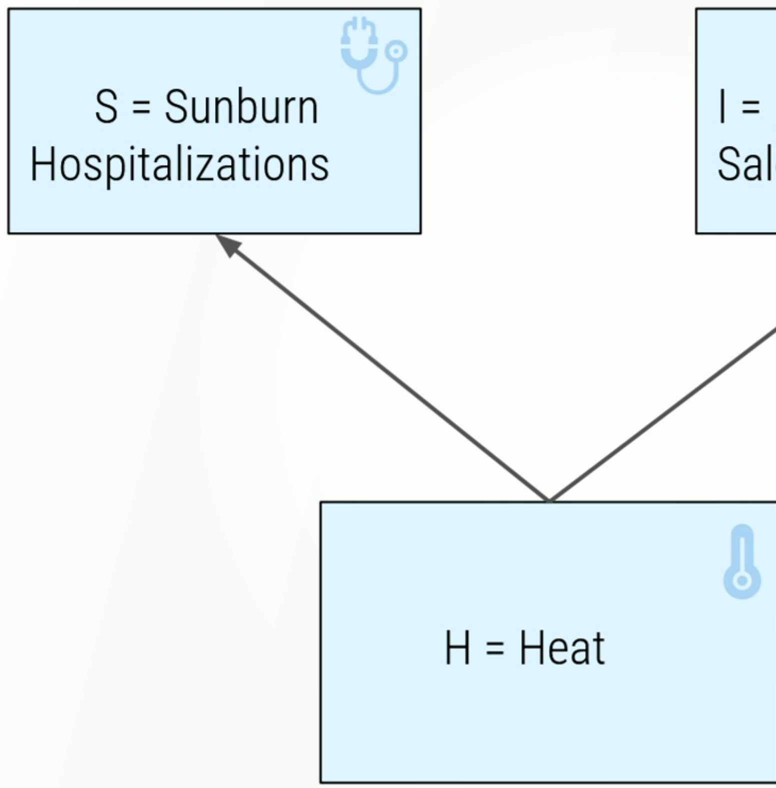
UNIVERSITY OF WATERLOO

Motivation: Causal Inference in Statistics and Physics


Sunburn
Hospitalizations
Ice cream
sales
Correlation ⇏ Causation



H is a common cause

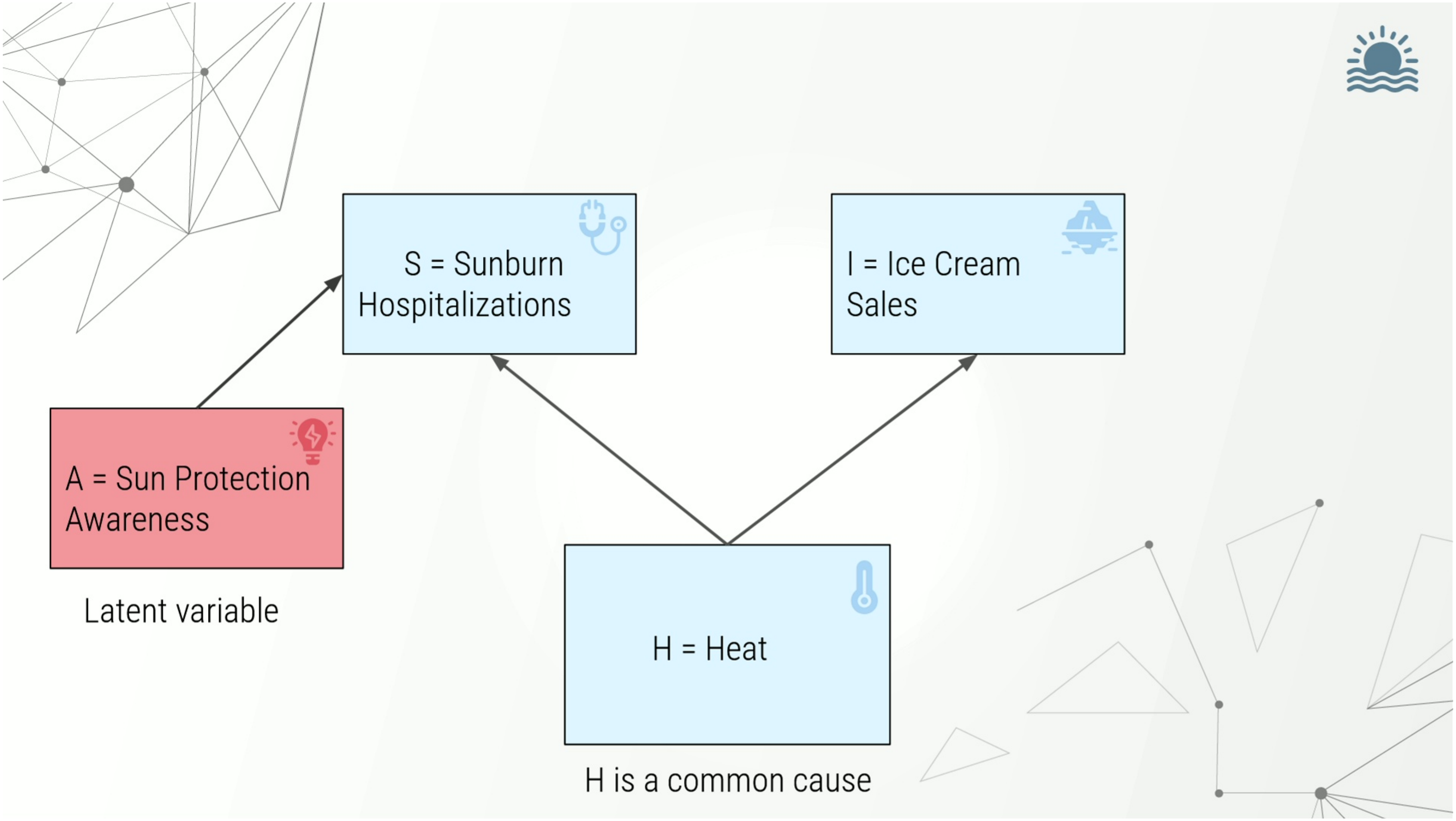


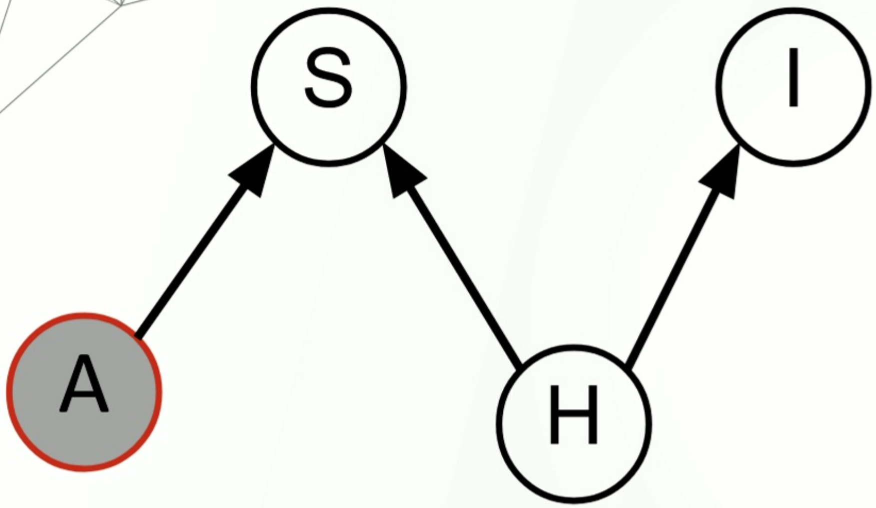
H is a common cause

When we can measure H:

$$P(IS|H) = P(I|H)P(S|H)$$

Restriction on the probability distributions that are compatible with this causal structure


S = Sunburn Hospitalizations
I = Ice Cream Sales
A = Sun Protection Awareness
Latent variable
H = Heat
H is a common cause



DAG = Directed Acyclic Graph

Applications to Classical Data Analysis

JOURNAL ARTICLE

Adolescent Paranoia: Prevalence, Structure, and Causal Mechanisms

Jessica C Bird, Robin Evans, Felicity Waite, Bao S Loe, Daniel Freeman

Schizophrenia Bulletin, Volume 45, Issue 5, September 2019, Pages 1134–1142, htt

doi.org/10.1093/schbul/sby180

Published: 10 December 2018

ELSEVIER

Journal of Econometrics

Volume 220, Issue 1, January 2021, Pages 23-62

Causal impact of masks, policies, behavior on early covid-19 pandemic in the U.S.

Victor Chernozhukov [a], Hiroyuki Kasahara [b], Paul Schrimpf [b]

Article | Open Access | Published: 11 August 2020

Improving the accuracy of medical diagnosis with causal machine learning

Jonathan G. Richens, Ciarán M. Lee & Saurabh Johri

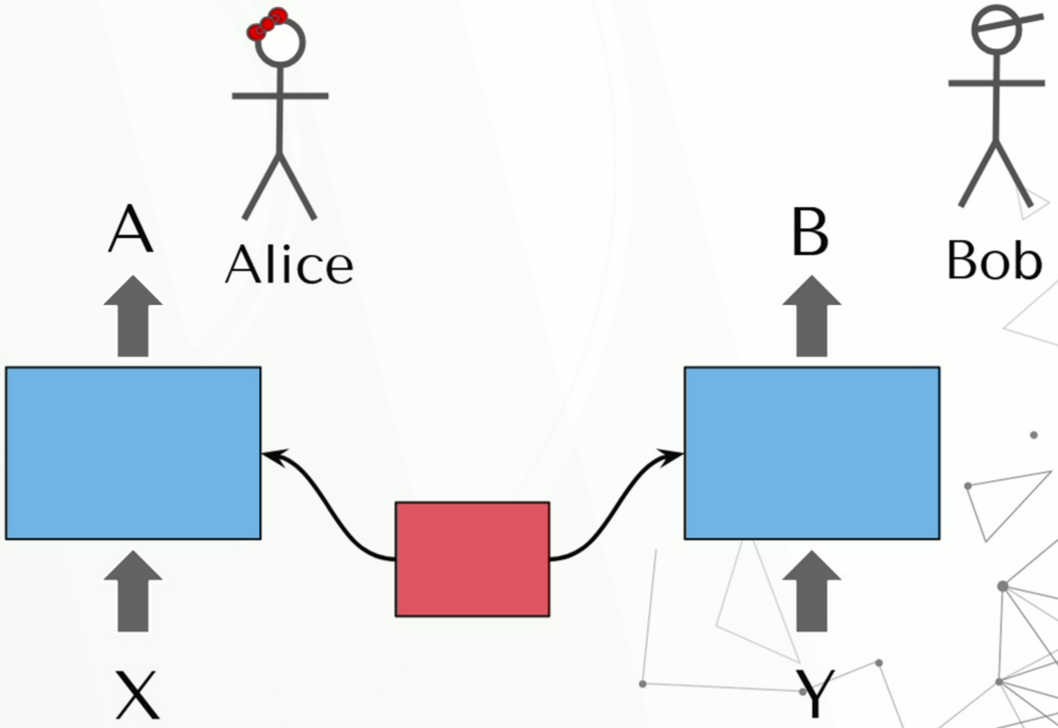
Nature Communications **11**, Article number: 3923 (2020) | Cite this article

Applications to Quantum Foundations

Bell-CHSH Scenario

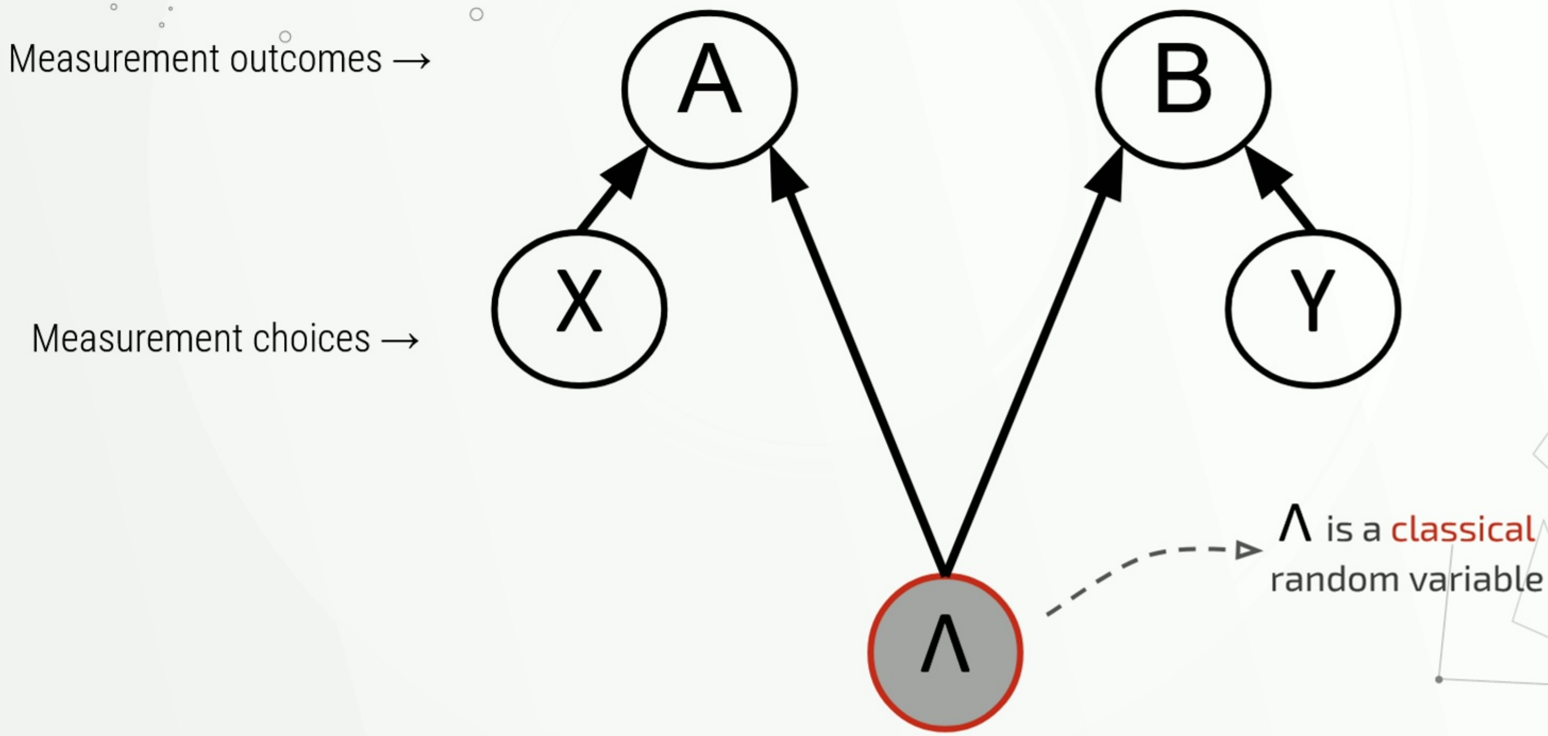
Measurement Outcomes →

Measurement Choices→



Applications to Quantum Foundations

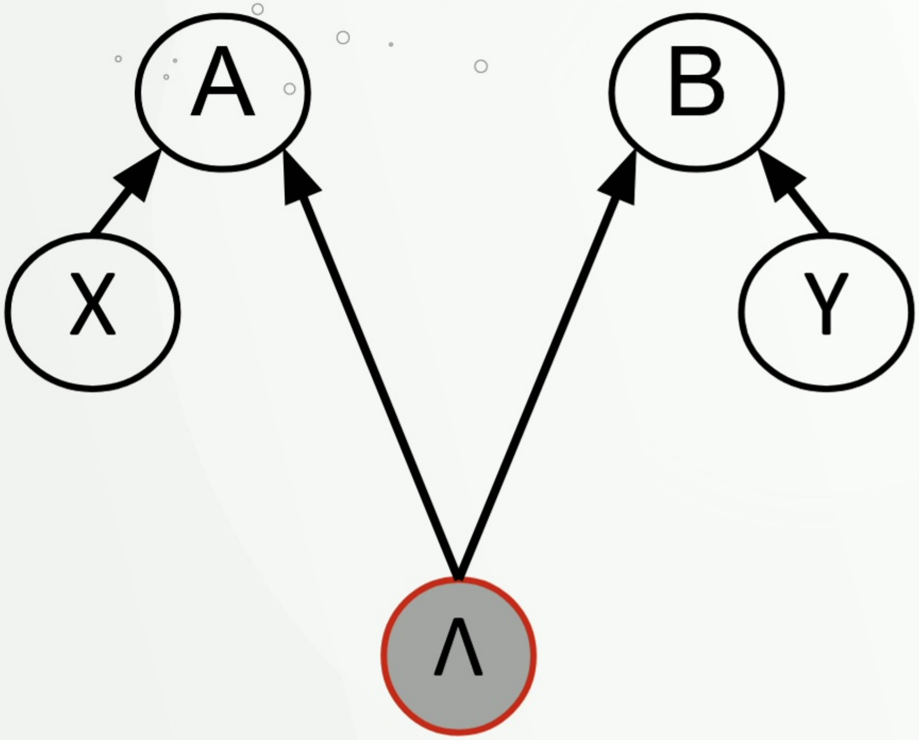
C.J. Wood and R.W. Spekkens: arxiv 1208.4119 (2015)



Bell DAG

Applications to Quantum Foundations

C.J. Wood and R.W. Spekkens: arxiv 1208.4119 (2015)



Bell DAG

P(AB|XY) classically compatible with this DAG need to obey:

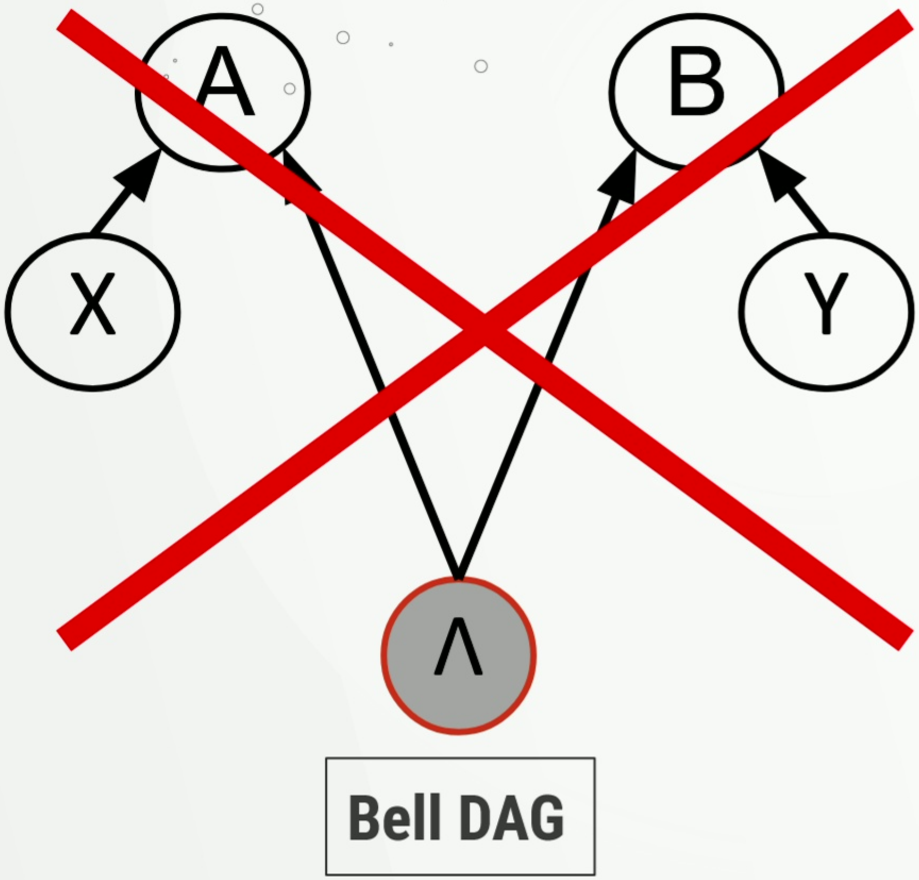
$$P(A|XY) = P(A|X)$$

$$P(B|XY) = P(B|Y)$$

$+$ Bell inequalities

Applications to Quantum Foundations

C.J. Wood and R.W. Spekkens: arxiv 1208.4119 (2015)



P(AB|XY) classically compatible with this DAG need to obey:

$$P(A|XY) = P(A|X)$$

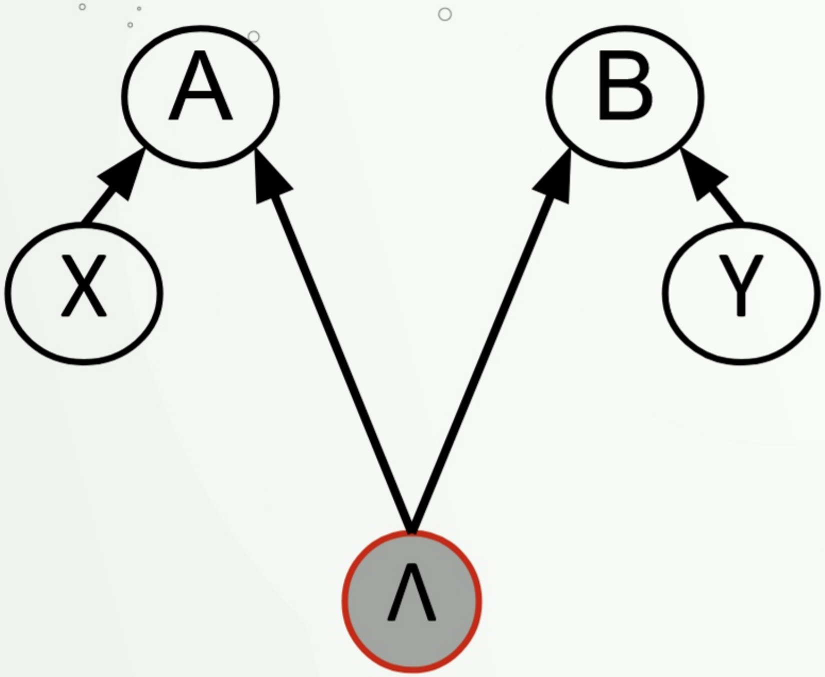
$$P(B|XY) = P(B|Y)$$

+ Bell inequalities

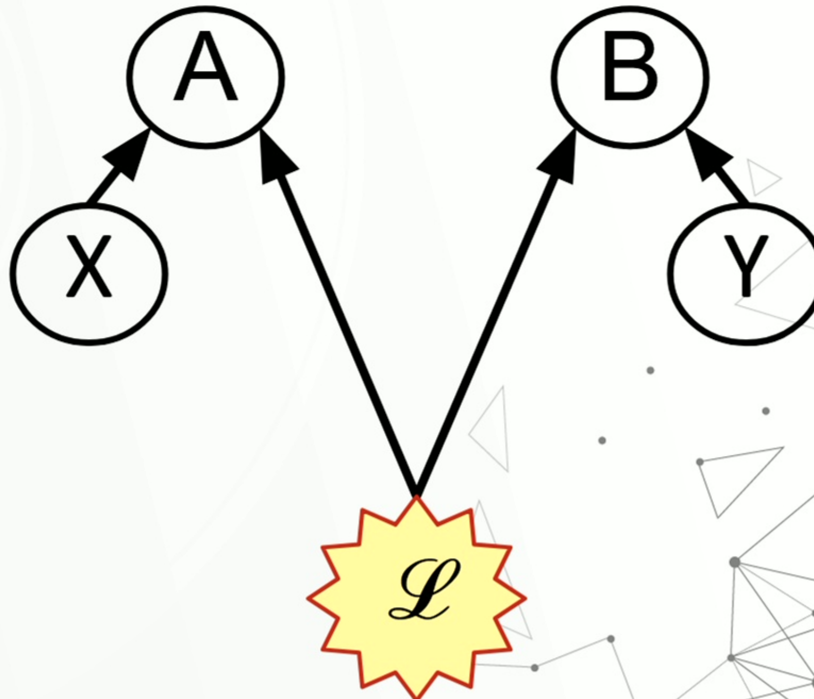
QM and experiments violate this inequality

Applications to Quantum Foundations

C.J. Wood and R.W. Spekkens: arxiv 1208.4119 (2015)



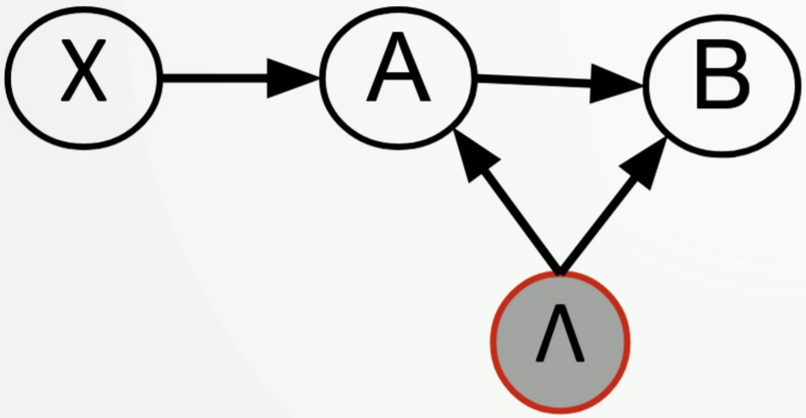
Λ is a classical random variable



$\mathscr{L}$ is a quantum system

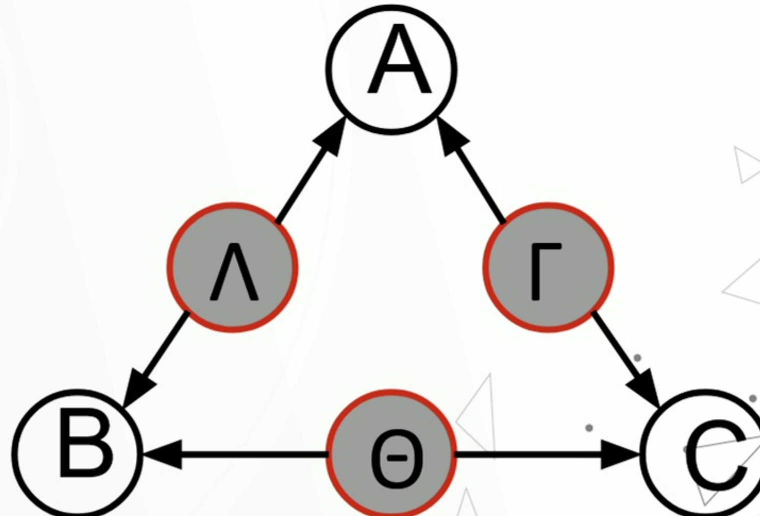
Applications to Quantum Foundations

Other Causal Structures that have Quantum-Classical Gaps



Instrumental Scenario

T. Van Himbeeck et.al.: arxiv 1804.04119 (2019)

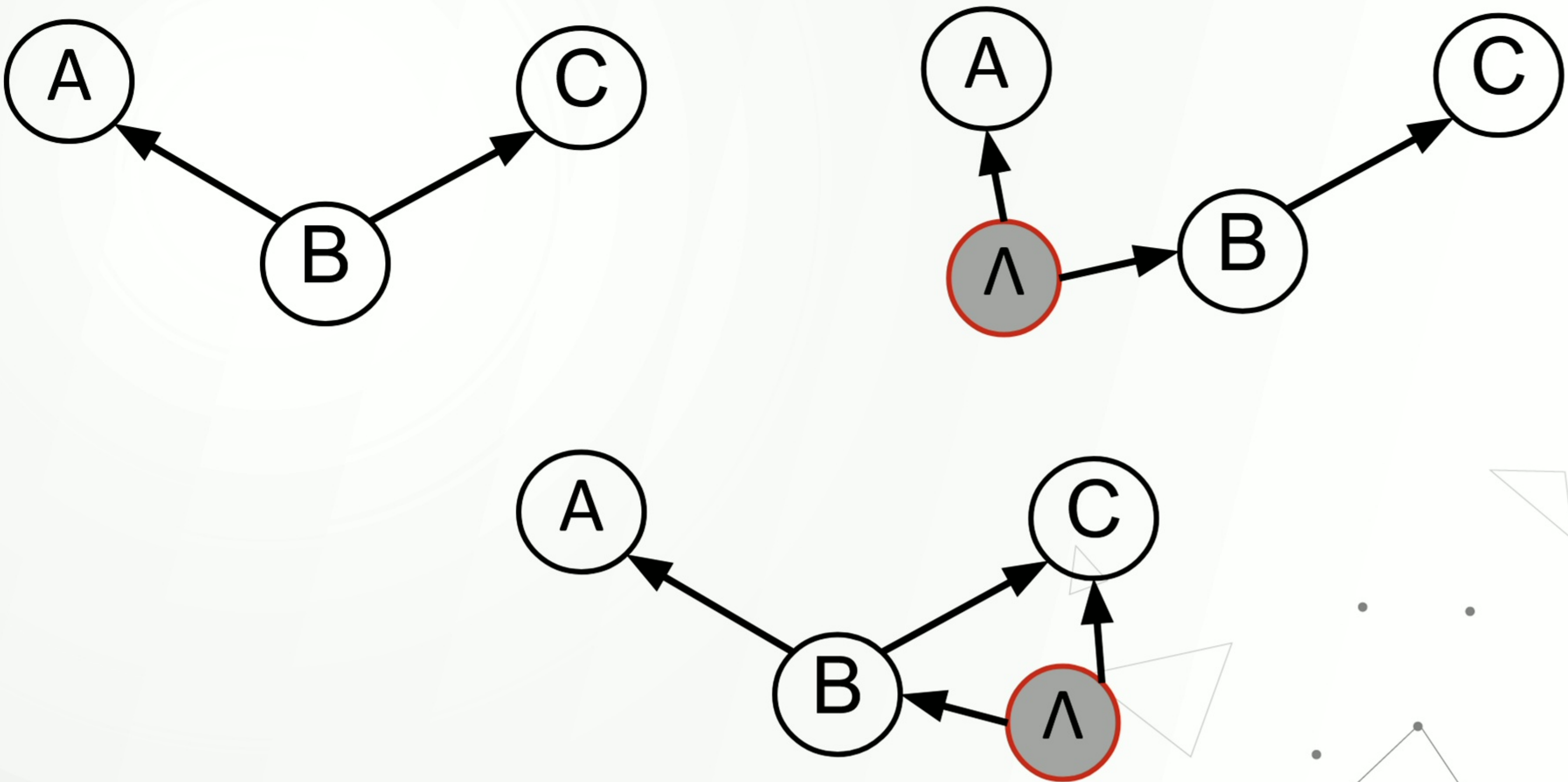


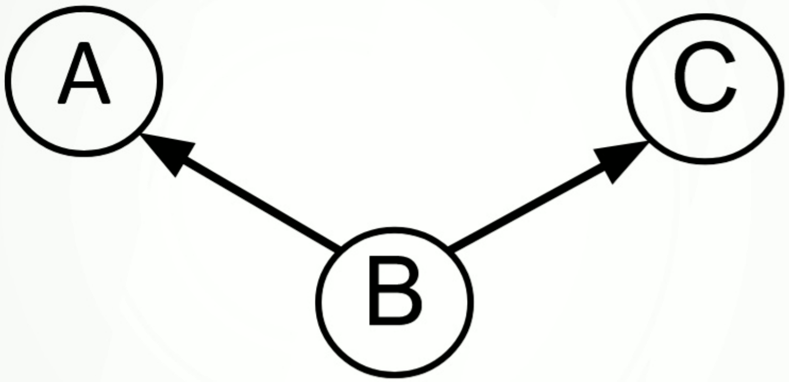
Triangle Scenario

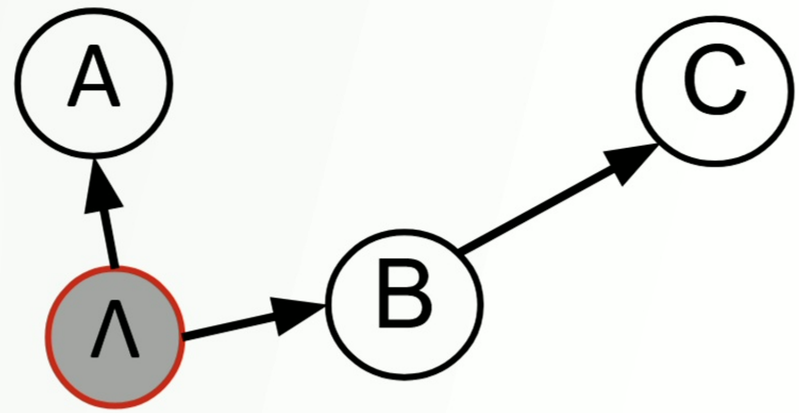
E. Wolfe et.al. : arxiv 1909.10519 (2021)

Distinguishing causal structures from statistical data

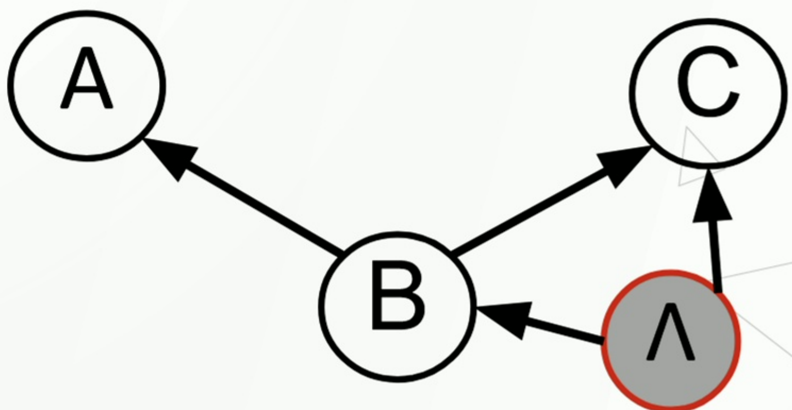
Indistinguishability of Causal Structures under passive observations


A
C
B
A
C
Λ
B
A
C
B
Λ

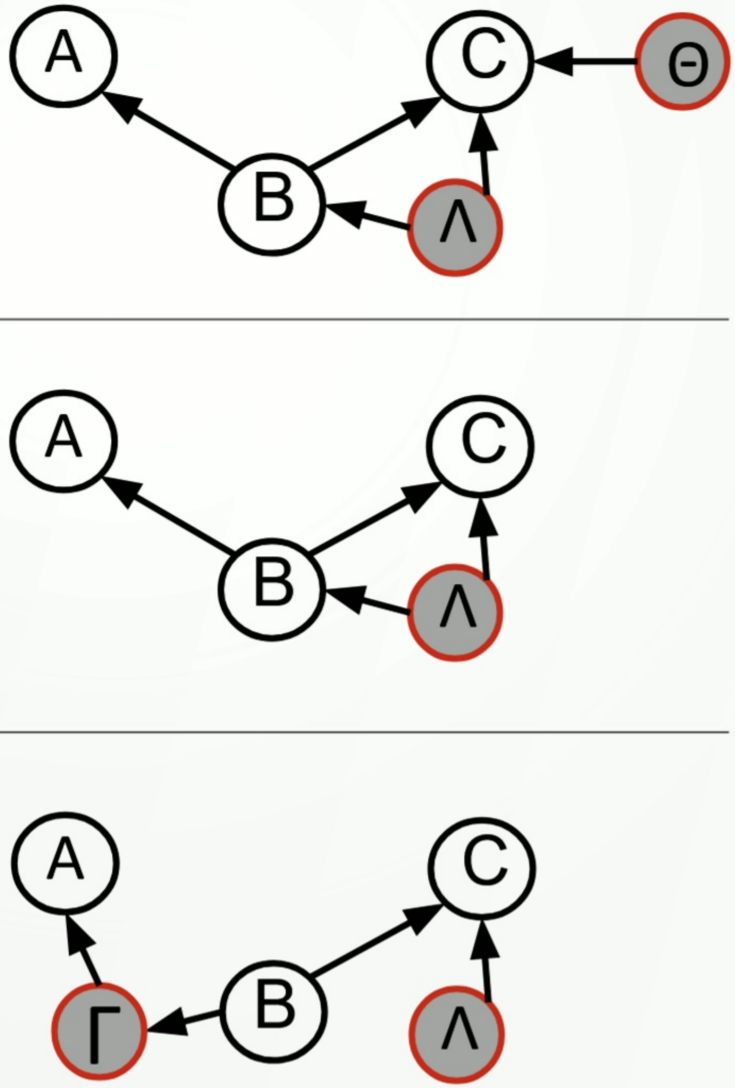




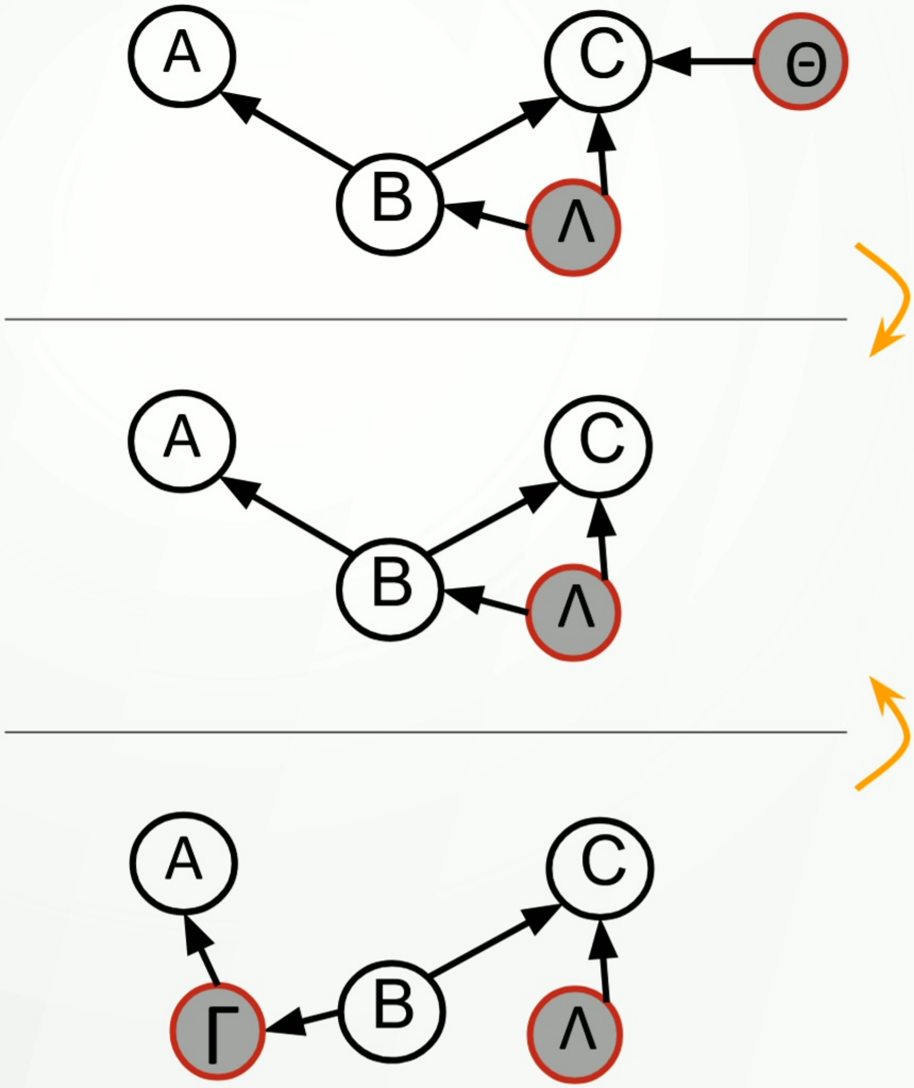
Observationally
equivalent



$$P(AC|B) = P(A|B)P(C|B)$$



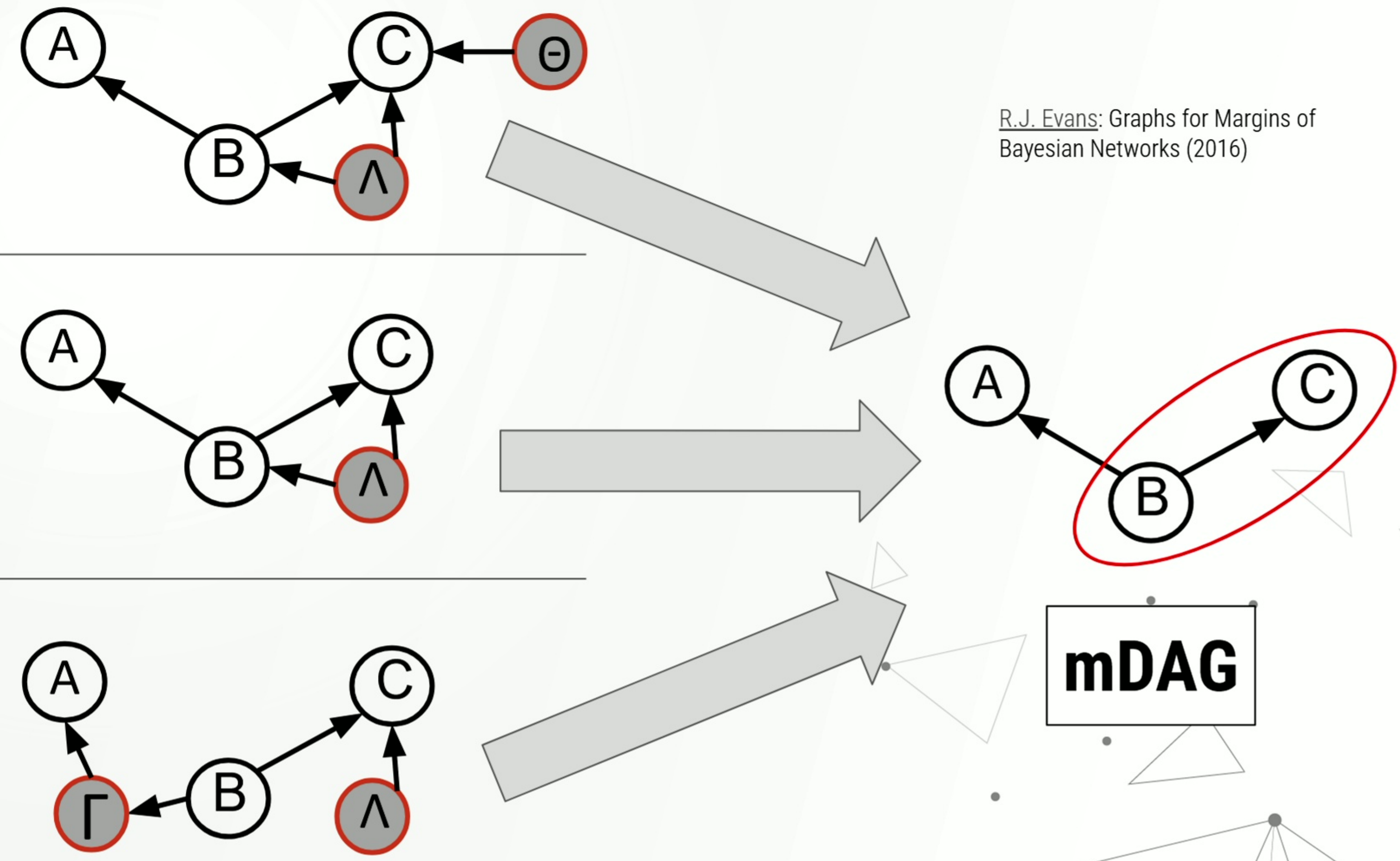
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)



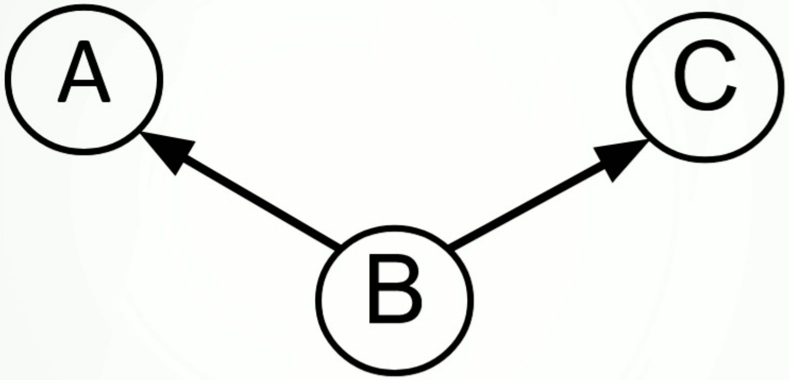
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

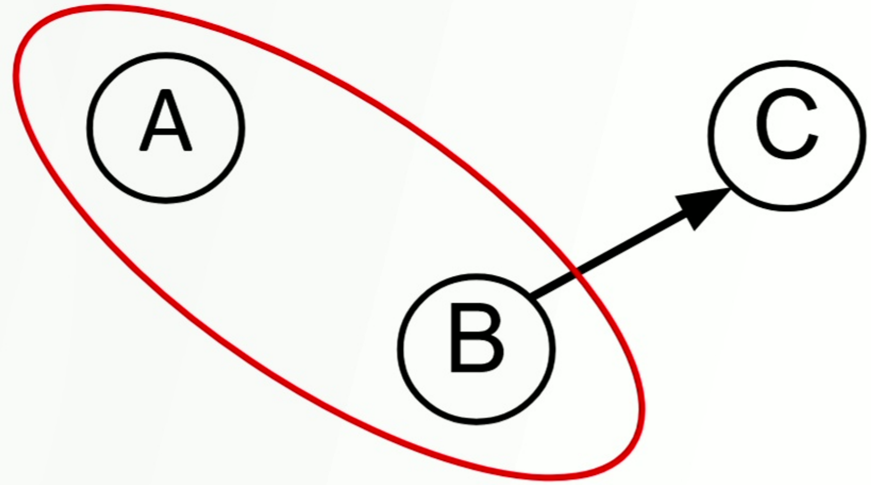
Remove redundant latent nodes

Make latent nodes parentless

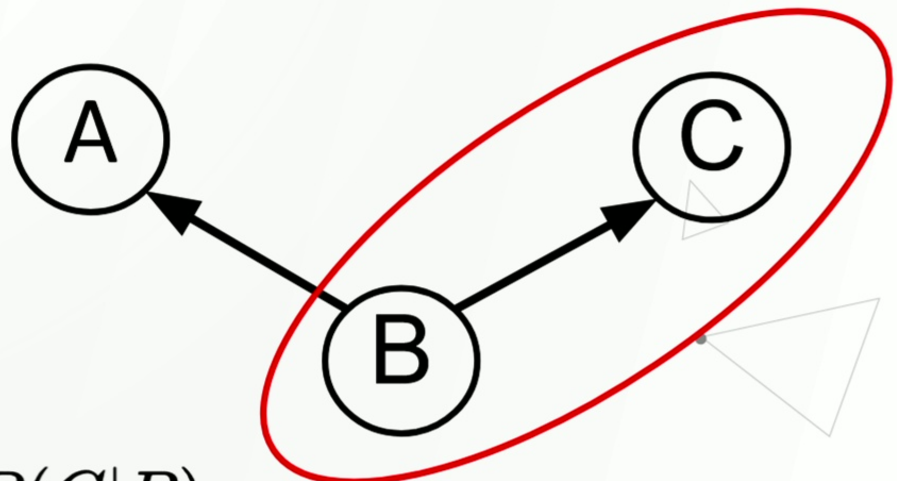


R.J. Evans: Graphs for Margins of Bayesian Networks (2016)





Different mDAGs can be observationally equivalent!

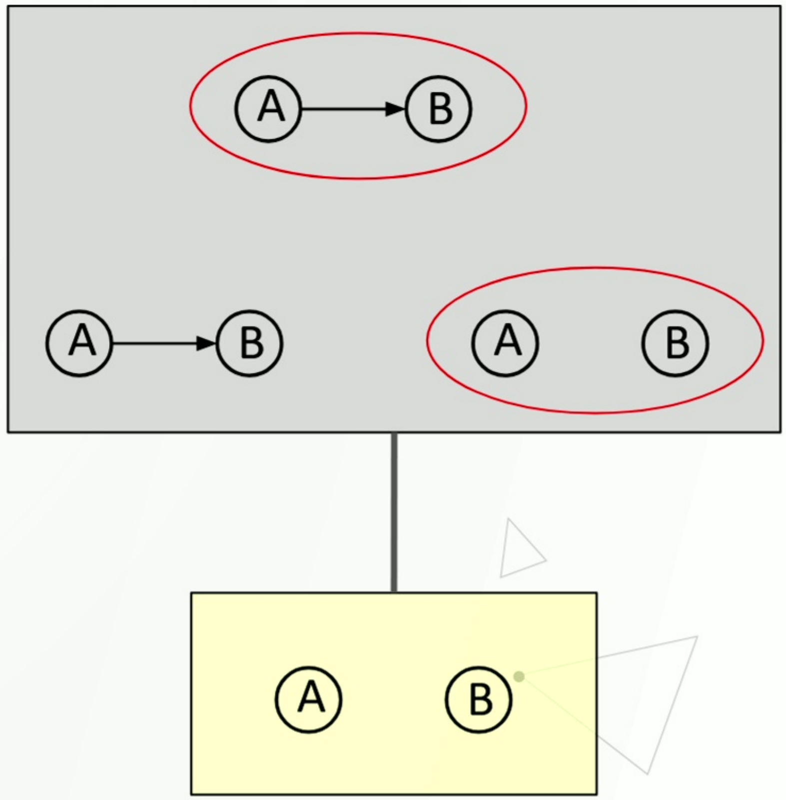


$$P(AC|B) = P(A|B)P(C|B)$$

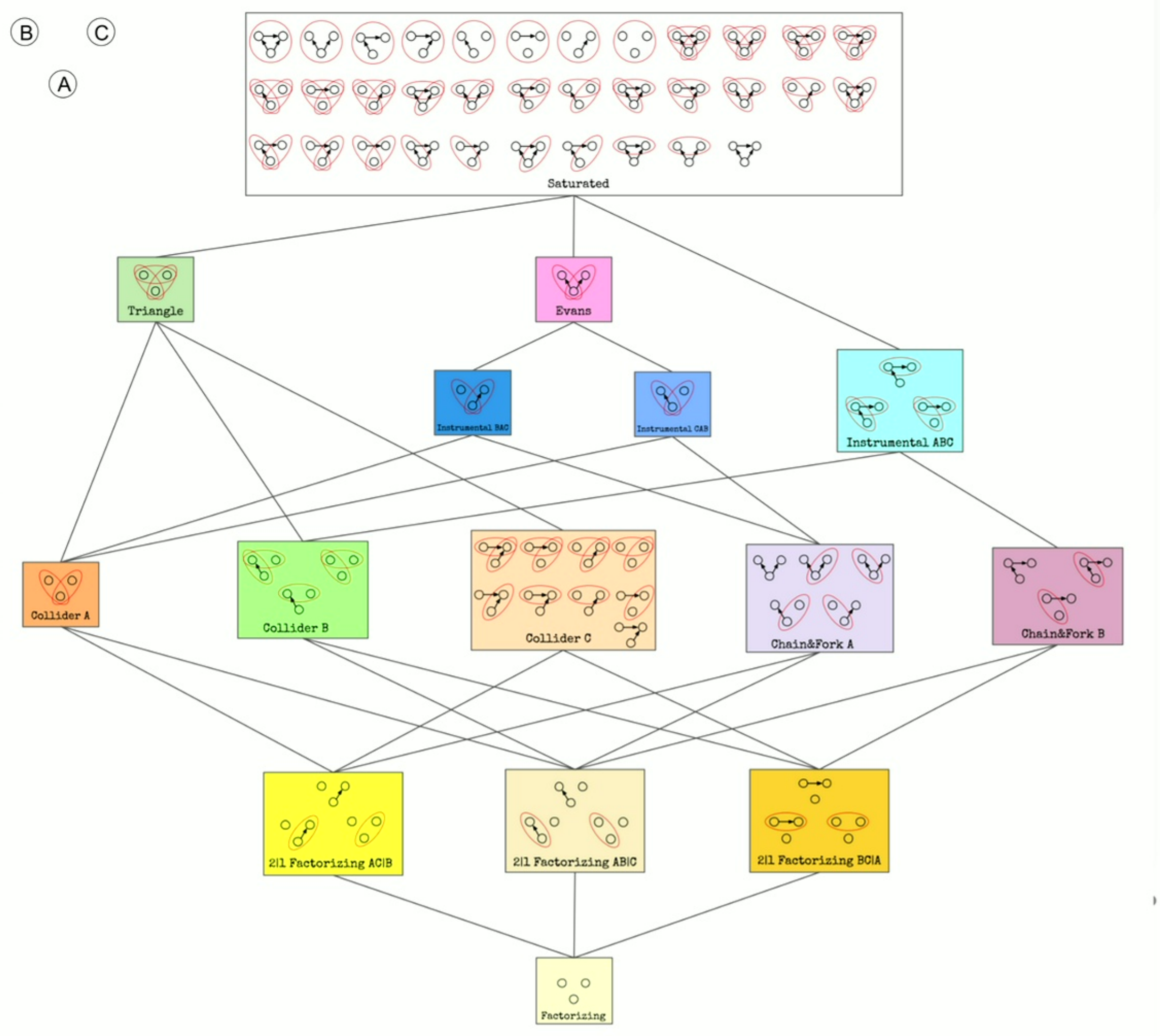
2 visible nodes

No constraints

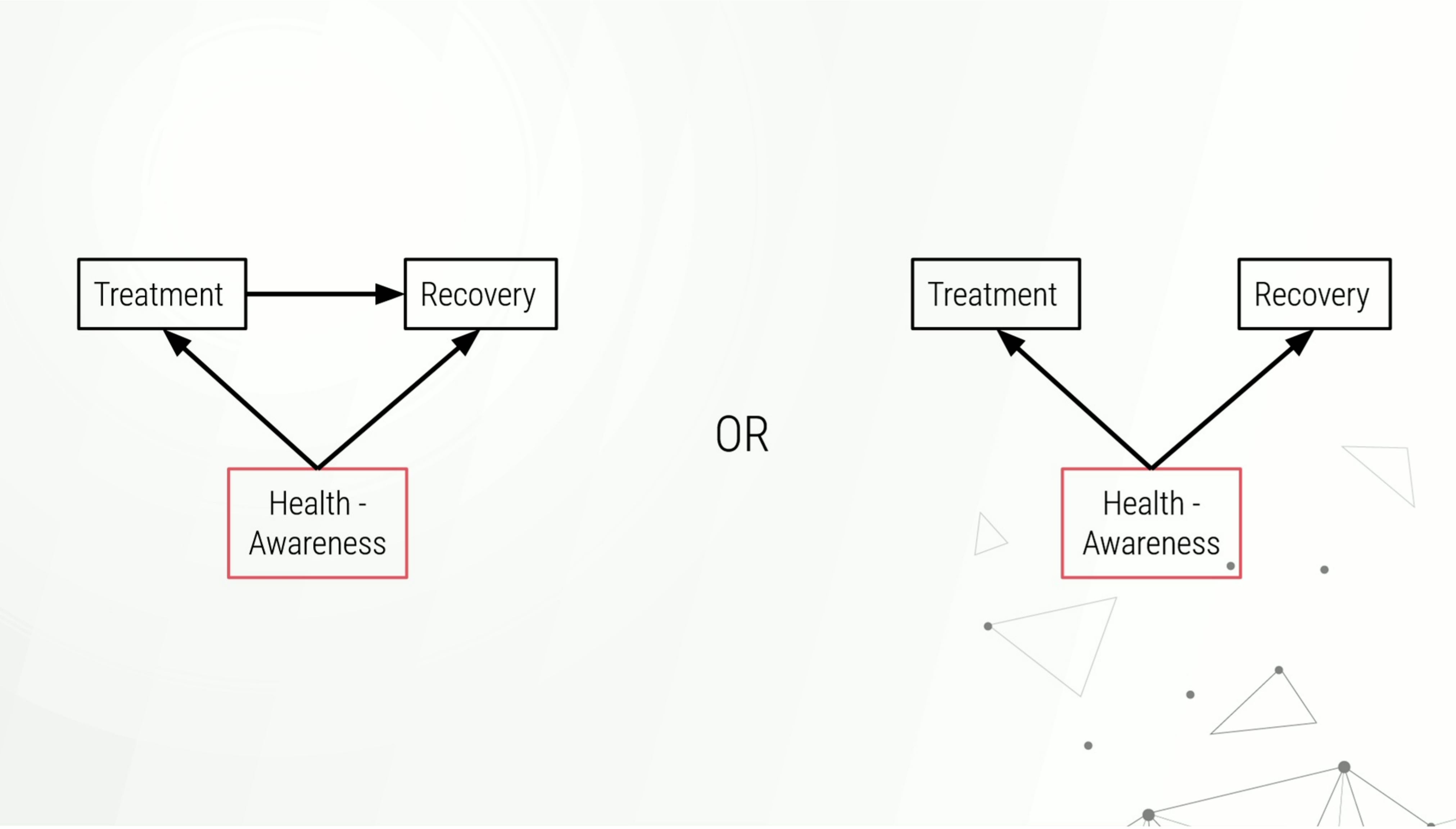
$$P(AB) = P(A)P(B)$$



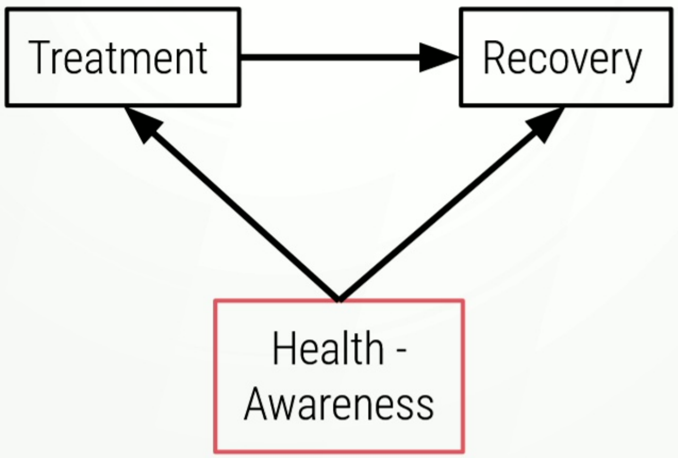
3 visible nodes



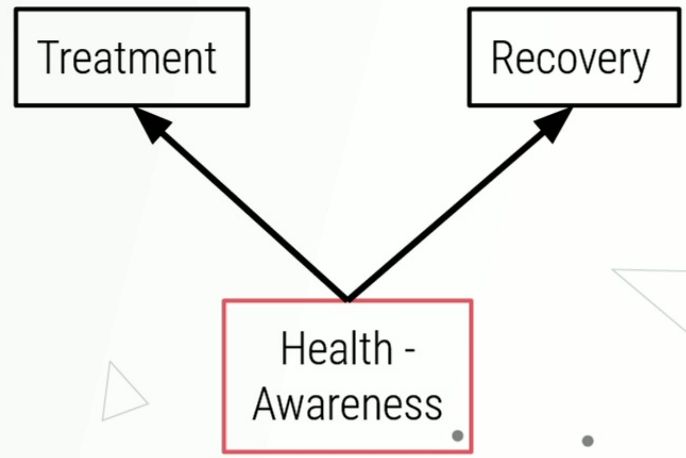
Does the treatment cause the recovery?


Treatment
Recovery
Health -
Awareness
OR
Treatment
Recovery
Health -
Awareness

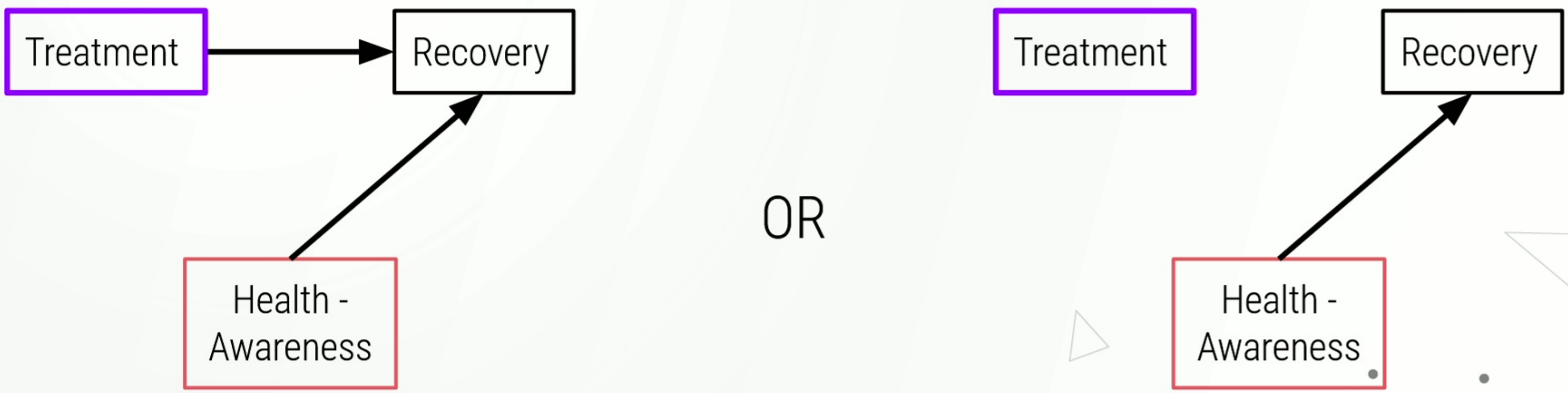
Observationally
equivalent



OR



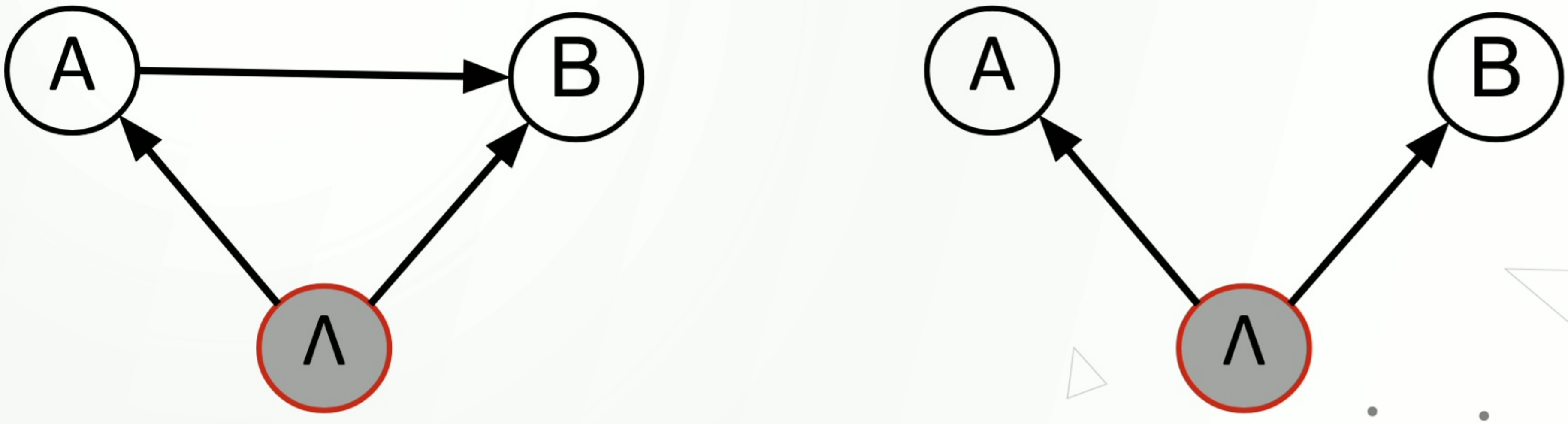
Passive observation only: Indistinguishable



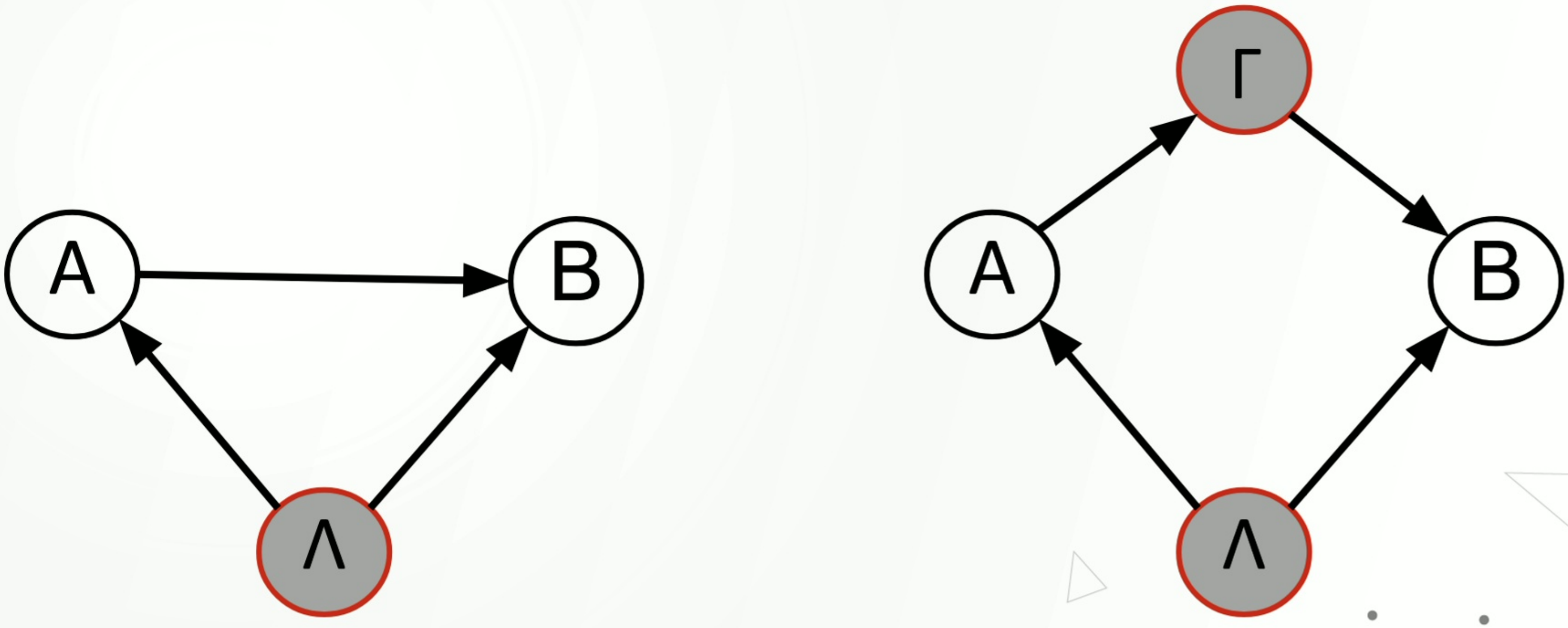
Passive observation only: Indistinguishable
Intervene on Treatment: Distinguishable

When is it impossible to distinguish two causal structures *even when there is access to interventions*?

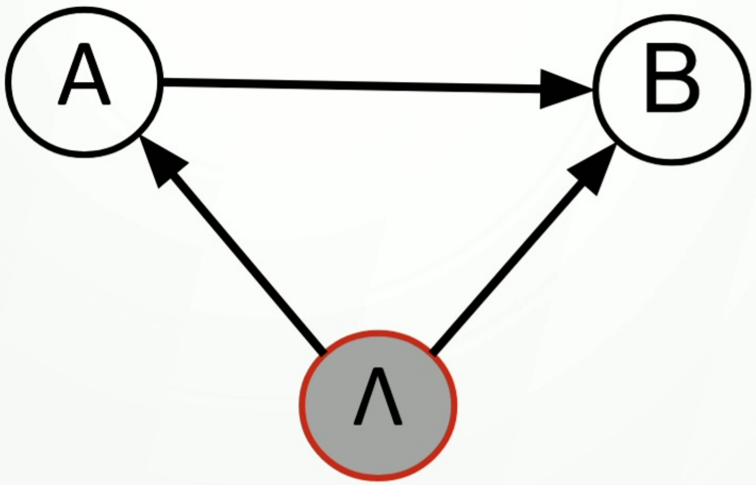
Observational and Interventional Equivalence

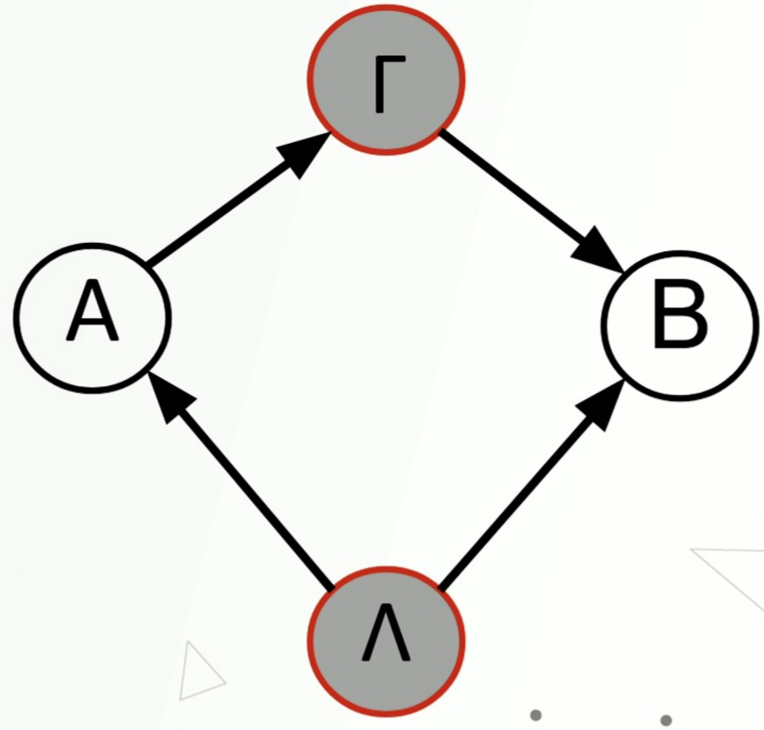


Observationally Equivalent ✅



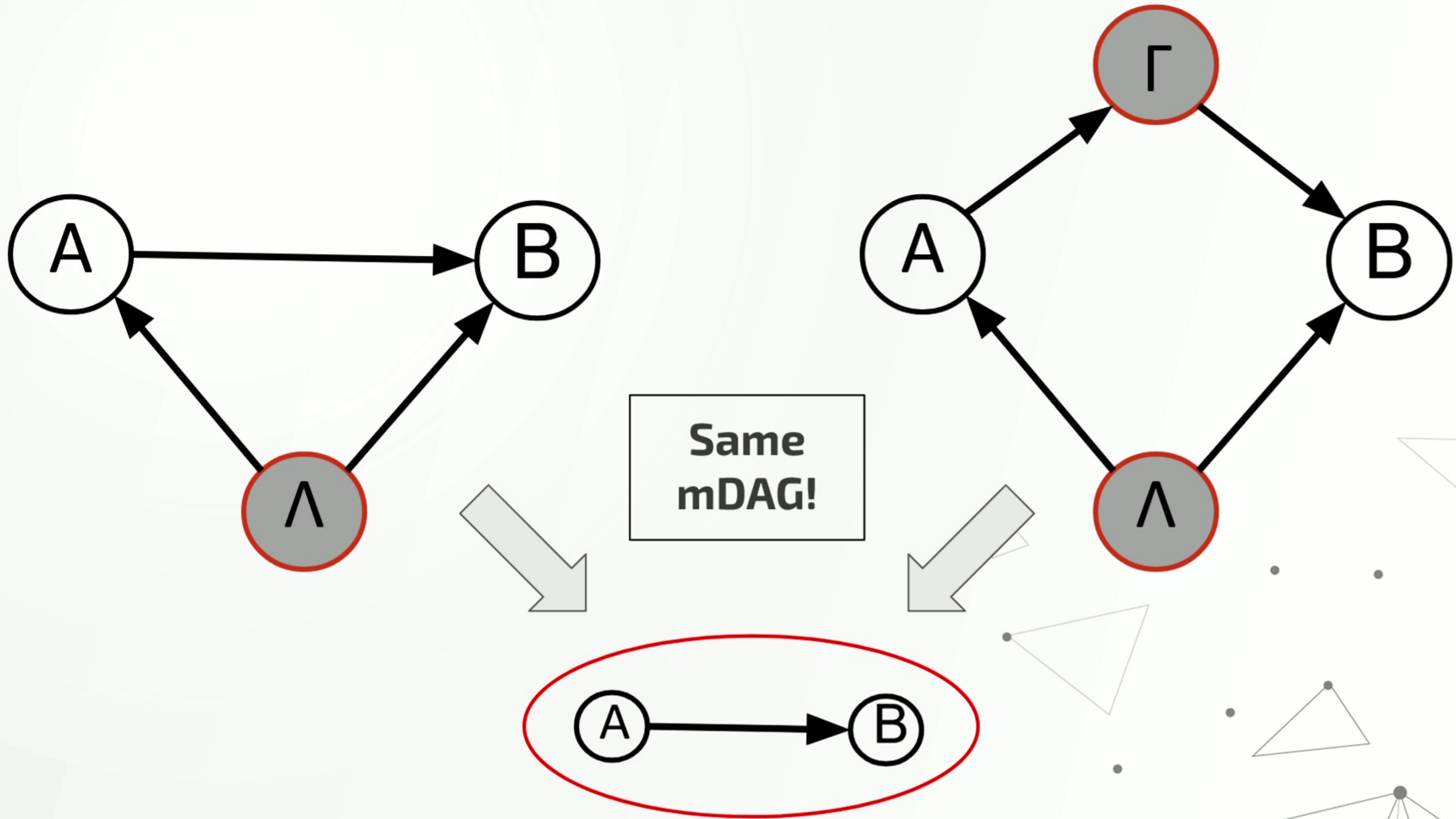
Observationally Equivalent ✅





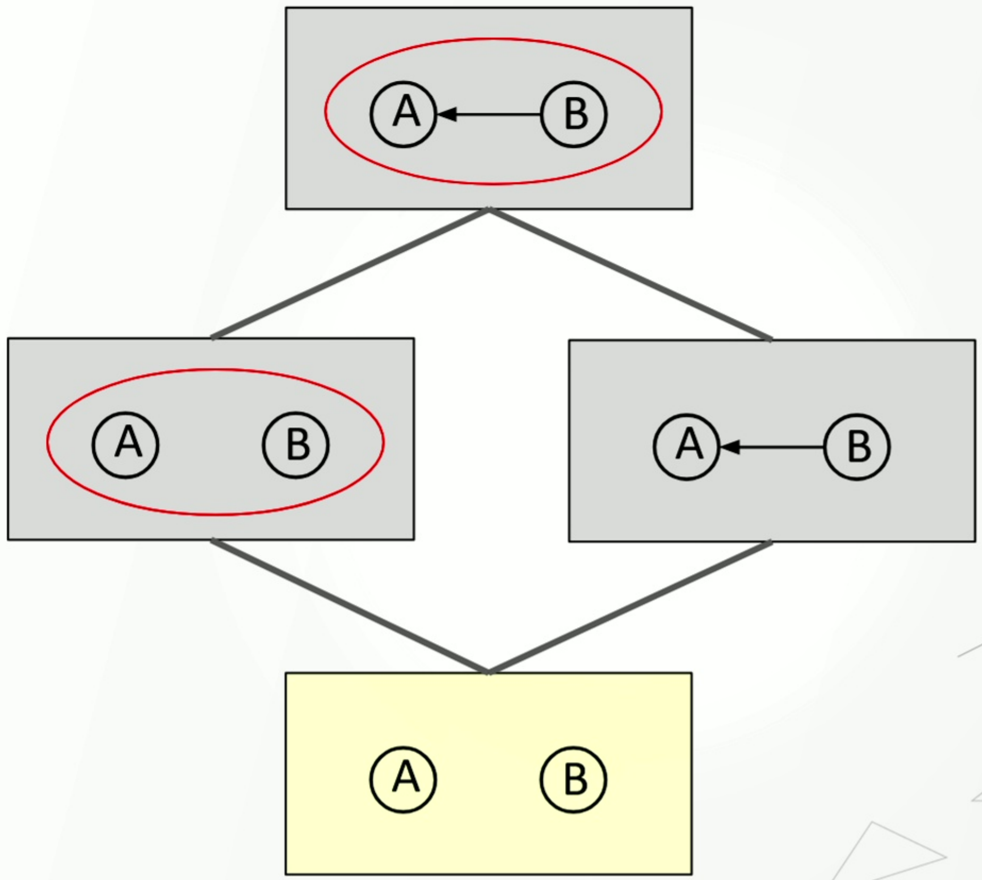
Observationally Equivalent ✅

Interventionally Equivalent ✅


A
B
Λ
Γ
A
B
Λ
Same
mDAG!
A
B

Interventionally Equivalent ⟺ Correspond to the same mDAG

2 visible nodes



2 visible nodes