Title: The g-function and defect changing operators from wavefunction overlap on a fuzzy sphere

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Collection/Series: Perimeter Institute Graduate Students' Conference 2024

Date: September 13, 2024 - 10:00 AM

URL: https://pirsa.org/24090190

Abstract:

This talk will be based on SciPost Phys. 17, 021 (2024). Defects are common in physical systems with boundaries, impurities or extensive measurements. The interaction between bulk and defect can lead to rich physical phenomena. Defects in gapless phases of matter with conformal symmetry usually flow to a defect conformal field theory (dCFT). Understanding the universal properties of dCFTs is a challenging task. In this talk, we propose a computational strategy applicable to a line defect in arbitrary dimensions. Our main assumption is that the defect has a UV description in terms of a local modification of the Hamiltonian so that we can compute the overlap between low-energy eigenstates of a system with or without the defect insertion. We argue that these overlaps contain a wealth of conformal data, including the g-function, which is an RG monotonic quantity that distinguishes different dCFTs, the scaling dimensions of defect creation operators Δ^{+0}_α and changing operators Δ^{+-}_α that live on the intersection of different types of line defects, and various OPE coefficients. We apply this method to the fuzzy sphere regularization of 3D CFTs and study the magnetic line defect of the 3D Ising CFT. Using exact diagonalization and DMRG, we report the non-perturbative results $g=0.602(2),\Delta^{+0}_0=0.108(5)$ and $\Delta^{+-}_0=0.84(5)$ for the first time. We also obtain other OPE coefficients and scaling dimensions. Our results have significant physical implications. For example, they constrain the possible occurrence of spontaneous symmetry breaking at line defects of the 3D Ising CFT. Our method can be potentially applied to various other dCFTs, such as plane defects and Wilson lines in gauge theories.

Conformal defects and boundaries of 3d Ising CFT on fuzzy sphere

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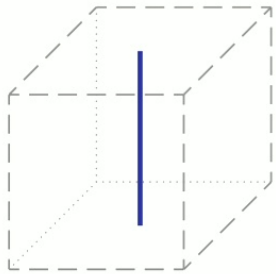
13 September, 2024
Graduate student conference
Perimeter Institute

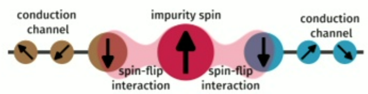
SciPost Phys. ***17****, 021 (2024) / arXiv:2401.00039*
arXiv:2407.15914

Defects and boundaries

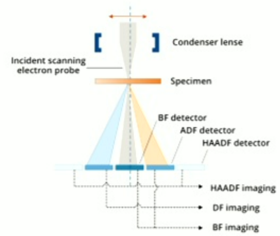
Defect
Spacetime dim-1



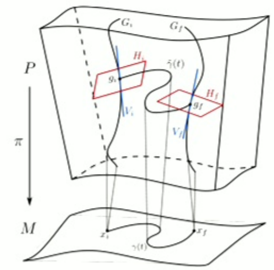
Impurity
(*e.g.*, Kondo problem)



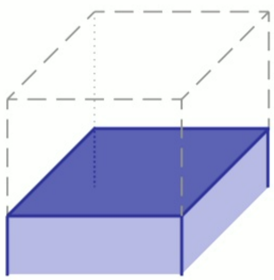
Detectors in experiments



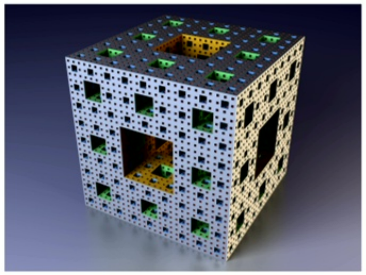
Line operators
(*e.g.*, Wilson line)



Boundary
Spacetime codim-1



Finite Sample

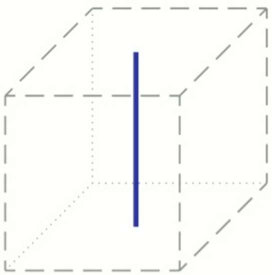


Bulk information
(*e.g.*, AdS/CFT, TO)

Defect and boundary CFTs

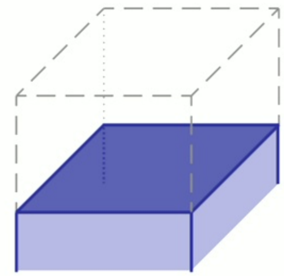
Bulk CFT $SO(d+1,1)$ $\begin{cases} \text{translation } P^{\mu} & \text{rotation } L^{\mu\nu} \\ \text{dilatation } D & \text{SCT } K^{\mu} \end{cases}$

(e.g., $d = 3$ Ising CFT)



$$S = S_{\text{CFT}} + \lambda \int \mathrm{d}x_0 \, \sigma(x_0)$$

$\sigma = 0,$	ordinary
$\sigma = \text{const.},$	normal
$\partial_z \sigma = 0,$	special



Line defect CFT
$SO(2,1) \times SO(d-1)_{\text{L}}$

Boundary CFT
$SO(d,1)$

- Defect primary operators $\hat{\phi}$;
- Bulk-to-defect 1-pt and 2-pt corrlators.

$$\langle \phi(x) \rangle = \frac{a_{\phi}}{x_{\perp}^{\Delta_{\phi}}}, \quad \langle \phi(x) \hat{\phi}(0) \rangle = \frac{b_{\phi\hat{\phi}}}{x_{\perp}^{\Delta_{\phi} - \Delta_{\hat{\phi}}} x^{2\Delta_{\hat{\phi}}}}$$

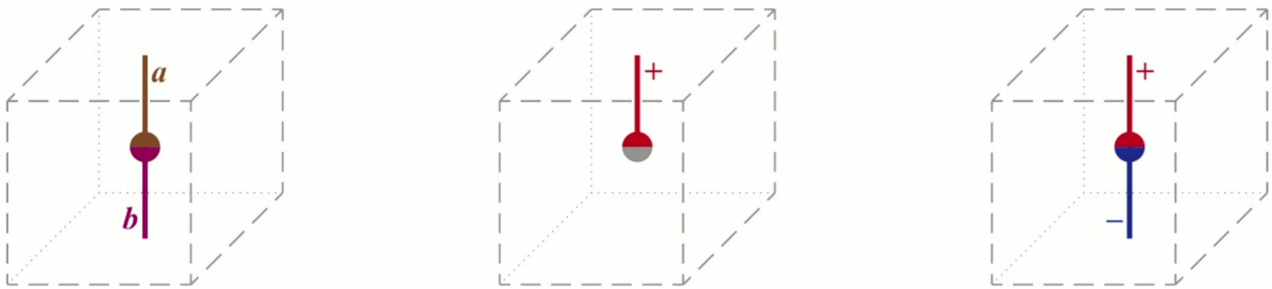
[1]F. Gliozzi *et al.*, JHEP **05** (2015) 036, [2]A. Krishnan, SciPost Phys. **15**, 090 (2023)

g-function and defect changing operators

g-function $\quad g = \frac{Z_{\text{defect CFT}}}{Z_{\text{bulk CFT}}}$

- g-theorem : RG-monotonic quantity.
- Fingerprint of the dCFTs.
- Equivalence in bCFT : boundary central charge c_{bd}.

Defect creation operator **Defect changing operator**



- Fate of symmetry-breaking on defect

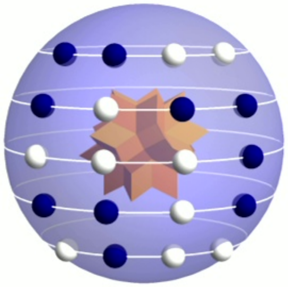
$$\begin{cases} \text{stable,} & \text{if } \Delta_{+-} > 1, g < 1/2 \\ \text{unstable,} & \text{if } \Delta_{+-} < 1, g > 1/2 \end{cases}$$

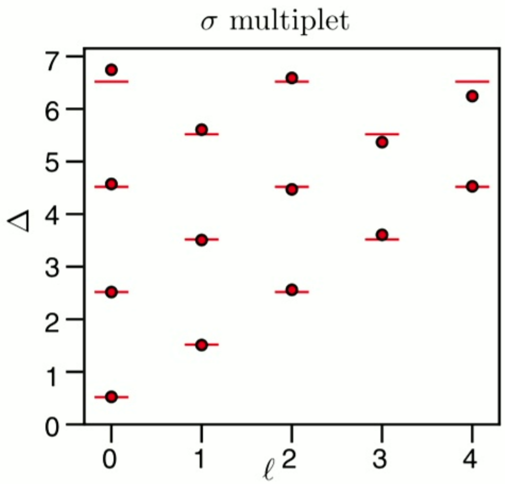
[1] I. Affleck *et al.*, PRL **67**, 161 (1991)

Fuzzy sphere

Electrons moving on a sphere
with a magnetic monopole at centre

- Wide range of spectrum
- Verification of conformal symmetry
- High accuracy on small system size





	Bootstrap	16 spins	Error
σ	0.518	0.524	1.2%
ϵ	1.413	1.414	0.07%
ϵ'	3.830	3.838	0.2%
$\sigma_{\mu\nu}$	4.180	4.214	0.8%
	…		
ϵ^-	NA	10.01	
σ^-	NA	11.19	

[1]W. Zhu *et al.*, PRX **13** 021009 (2023)

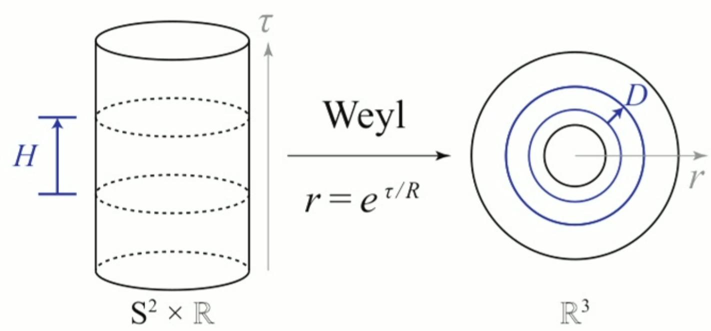
Sphere geometry

'Sphere' — Quantum Hamiltonian defined on S^d

Weyl transformation

$$(\hat{\mathbf{n}}, \tau) \in S^d \times \mathbb{R} \ \mapsto\ \mathbf{r} \in \mathbb{R}^{d+1}$$

$$r = e^{\tau/R}, \quad \mathbf{r} = r\hat{\mathbf{n}}$$



State-Operator correspondance

CFT operators

$\updownarrow$

Eigenstates of the quantum Hamiltonian

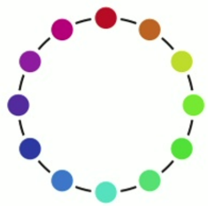
- Corresponding angular momenta, quantum numbers…
- Energy $\leftrightarrow$ scaling dimension

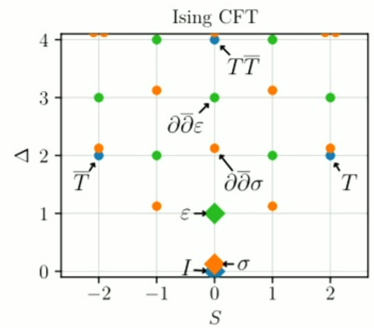
$$E_\Phi - E_0 = \text{constant} \times \Delta_\Phi$$

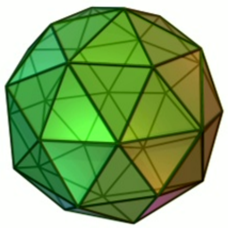
[1] J. L. Cardy, J. Phys. A **17**, L385 (1984)

Sphere geometry

- $(1+1)$d — Lattice chain with PBC easily realise S^1 geometry







- $(2+1)$d — No lattice preserves full SO(3) rotation symmetry

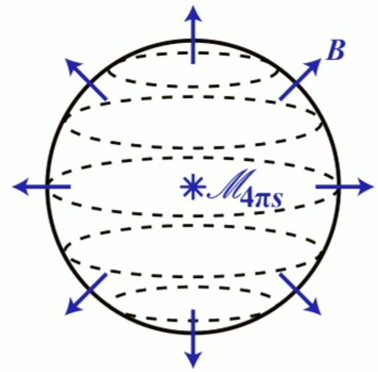
'Fuzzy' — Use spherical Landau levels instead of lattice as single-particle state

[1]J. L. Cardy, J. Phys. A **17**, L385 (1984) [2]Y. Zou *et al.*, PRL **121**, 230402 (2018)

Spherical Landau level

Electrons moving on a sphere with a magnetic monopole at centre

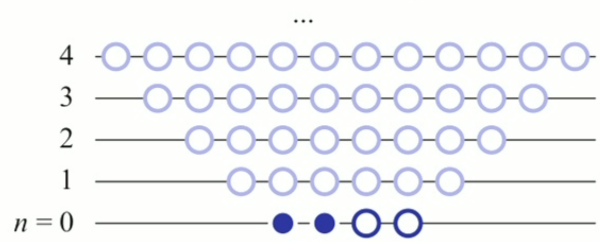
$$H_0 = \frac{(\partial^\mu + iA^\mu)^2}{2MR^2}$$



Single particle eigenstates — Spherical Landau level

$$\psi_{n,m}(\vec{r}) = Y^{(s)}_{lm}(\theta, \phi)$$

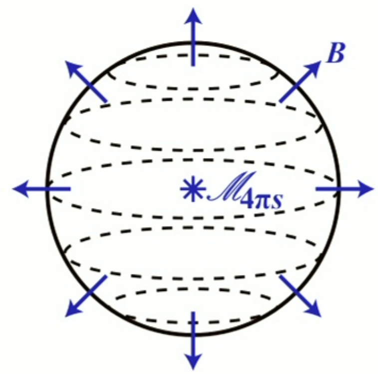
$n =$	$0,$	$1,$	$2,$	$\dots$
$l =$	$s,$	$s+1,$	$s+2,$	$\dots$
Degeneracy	$2s+1,$	$2s+3,$	$2s+5,$	$\dots$



- Set $H_0 \gg H_{\text{int}}$, partially the lowest LL — fluctuation within the lowest LL
- Preserve full SO(3) symmetry — LLL carries spin-s representation
- Fuzziness = Non-commutativity $\Rightarrow$ UV regulator

[1]T. T. Wu & C. N. Yang, Nucl. Phys. B **107**, 365 (1976)

Many-body Hamiltonian on fuzzy sphere



- Project onto the lowest Landau level

$$\hat{\psi}_\sigma(\vec{r}) = \sum_m Y^{(s)}_{sm}(\theta,\phi)\hat{c}_{m\sigma} \quad (m = -s, \dots, s)$$

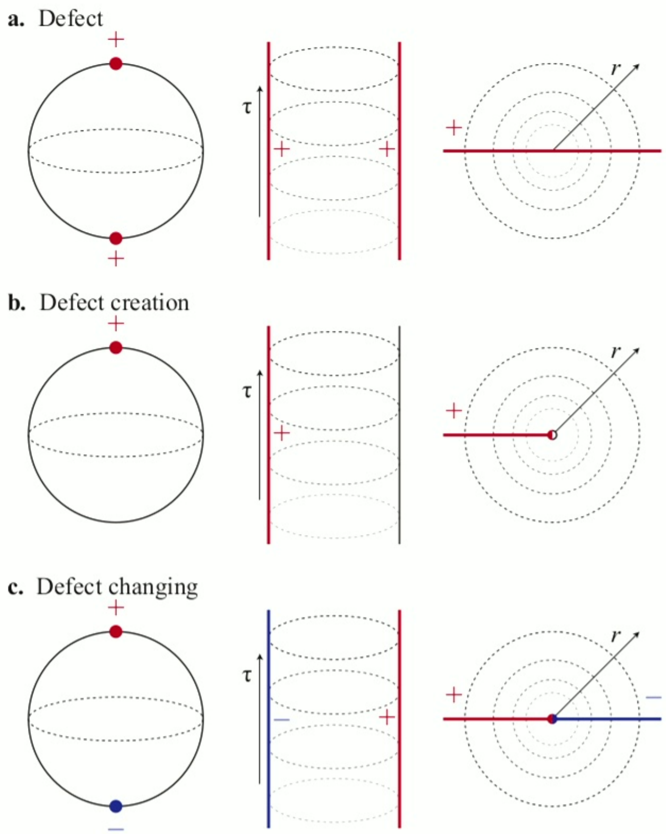
  - $\sim$ fermion chain with $(2s+1)$ sites ;
  - Thermodynamic limit $s \to \infty$.

- Density operator $\hat{n}^i(\vec{r}) = \hat{\boldsymbol{\psi}}^\dagger(\vec{r})\boldsymbol{\sigma}^i\hat{\boldsymbol{\psi}}(\vec{r}), \quad \hat{n}^i_{lm} = \sum_{m_1} a_{lmm_1}\hat{\mathbf{c}}^\dagger_{m+m_1}\sigma^i\hat{\mathbf{c}}_{m_2}$

- Ising CFT $H_{\text{int}} = -\int d^2r_1\, d^2r_2\, U(\vec{r}_{12})n^z(\vec{r}_1)n^z(\vec{r}_2) - h\int d^2r\, n^x(\vec{r})$

[1]W. Zhu *et al.*, PRX **13** 021009 (2023)

Defect on fuzzy sphere



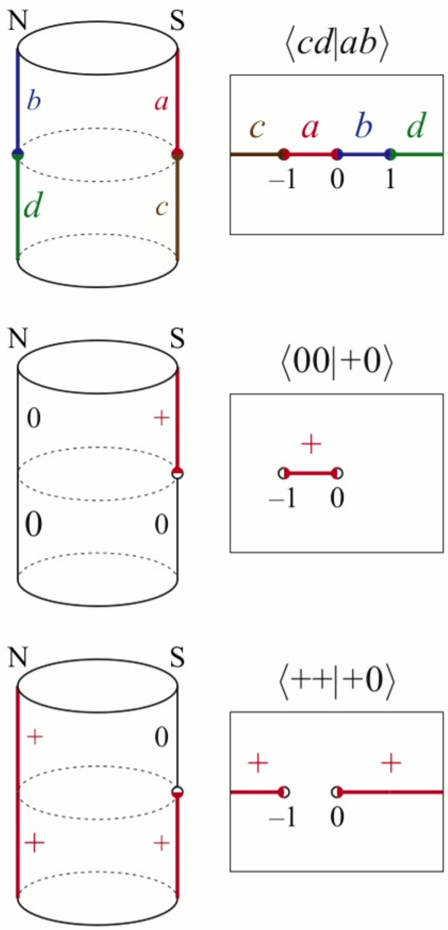
$$H^{ab}_{\text{defect}} = H_{\text{bulk}} - h(an^z(\text{N}) + bn^z(\text{S}))$$

$$\Delta_{+0} = \text{bulk const.} \times \left(E_{+0} - \tfrac{1}{2}(E_{++} + E_{00})\right) = 0.108(5)$$

$$\Delta_{+-} = \text{bulk const.} \times (E_{+-} - E_{++}) = 0.84(5)$$

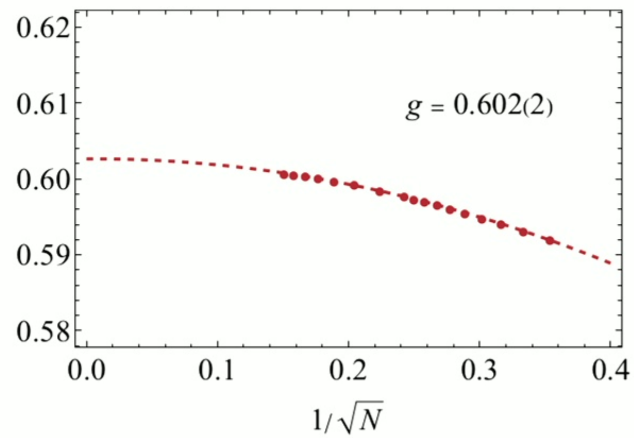
[1]L. Hu *et al.*, NC **15**, 3659 (2024)

Gift from overlap

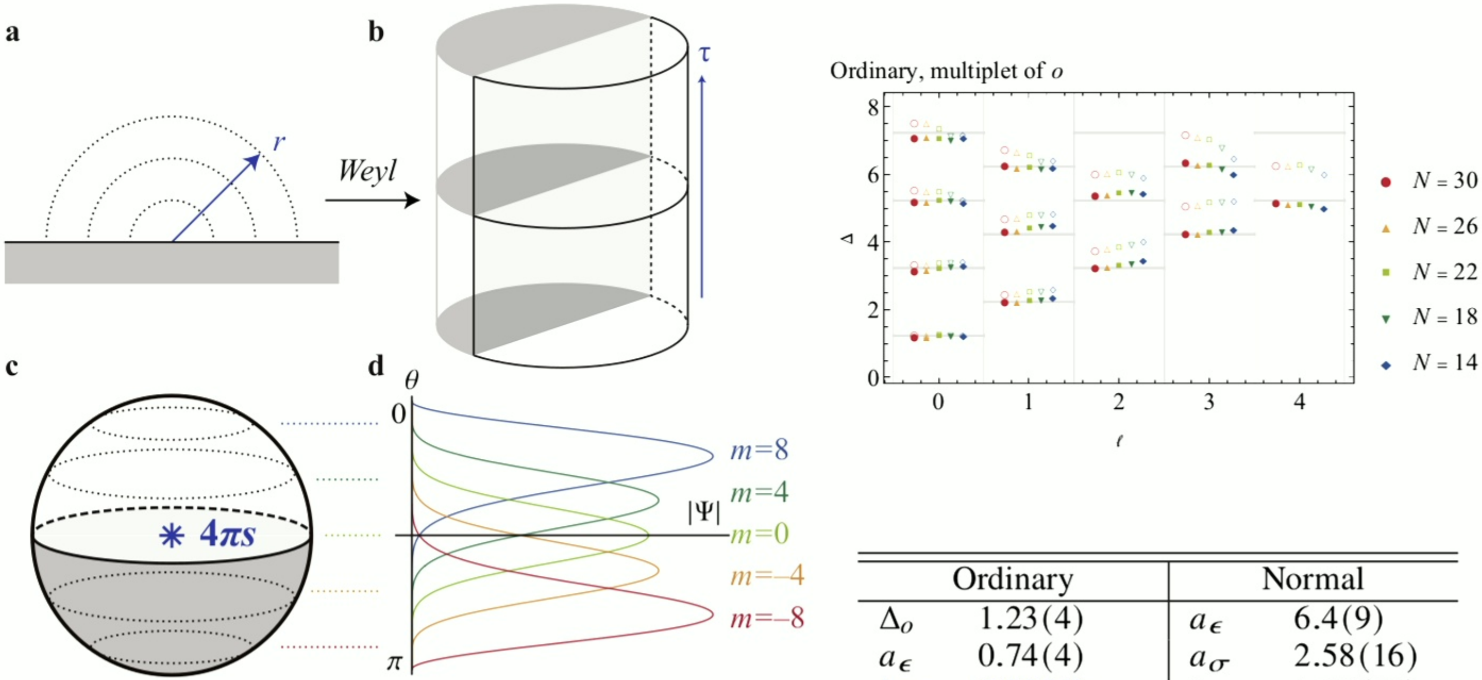


$$\langle cd|ab\rangle = \frac{\langle \phi^{ca}(-1)\phi^{ab}(0)\phi^{bd}(1)\phi^{dc}(\infty)\rangle}{\sqrt{Z_{ab}Z_{cd}}}$$

$$\frac{\langle 00|+0\rangle}{\langle ++|+0\rangle} = \sqrt{\frac{Z_{++}}{Z_{00}}} = \sqrt{g}$$



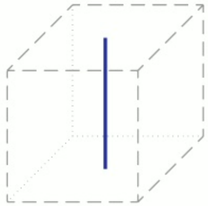
Quantity	Result	ϕ^{00}_α	$C^{0+0}_{00\alpha}$	ϕ^{++}_α	$C^{+0+}_{00\alpha}$	$C^{+-+}_{00\alpha}$
g	0.602(2)	σ	0.869(19)	ϕ^{++}_1	0.22(3)	1.25(12)
Δ^{+0}_0	0.108(5)	ϵ	0.3334(9)	ϕ^{++}_2	0.053(19)	1.01(26)
Δ^{+-}_0	0.84(5)	$T^{\mu\nu}$	0.044(28)	ϕ^{++}_3	/	/
C^{+0-}_{000}	0.77(5)	ϵ'	0.003(7)	ϕ^{++}_4	0.007(3)	0.009(5)

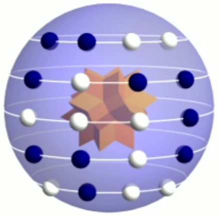


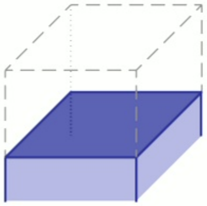
Ordinary		Normal	
Δ_o	1.23(4)	a_ϵ	6.4(9)
a_ϵ	0.74(4)	a_σ	2.58(16)
$b_{\epsilon\mathrm{D}}$	**0.92(4)**	$b_{\epsilon\mathrm{D}}$	1.74(22)
$b_{\sigma o}$	0.87(2)	$b_{\sigma\mathrm{D}}$	0.254(17)
C_D	0.0089(2)	C_D	0.176(2)
c_{bd}	**−0.0159(5)**	c_{bd}	**−1.44(6)**

$$H^i_{\mathrm{bd}} = H_{\mathrm{bulk}} - h\sum_{m<0} \mathbf{c}^\dagger_m \sigma^i \mathbf{c}_m, \quad \begin{cases} i = x, & \text{ordinary} \\ i = z, & \text{normal} \end{cases}$$

Summary







Ising magnetic line defect CFT

- Wealth of data by taking overlap
- g-function $g = 0.602(2)$
- Defect creation/changing operators
- Immpossibility of SSB on defect

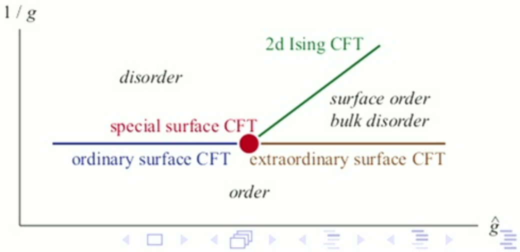
Ising normal & ordinary boundary CFT

- Easy realisation on fuzzy sphere
- Operator spectum
- Bulk-to-boundary OPEs
- Boundary central charges c_{bd}

More defects ?

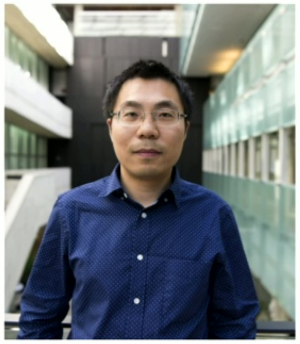
- Wilson line,
- Monodromy,
- …

More boundaries ?



Collaborators

Yin-Chen He | **Davide Gaiotto** | **Yijian Zou**



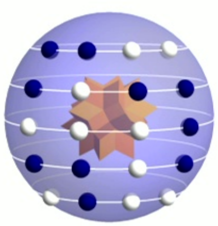
The g-function and defect changing operators from wavefunction overlap on a fuzzy sphere
Zheng Zhou, Davide Gaiotto, Yin-Chen He, and Yijian Zou
SciPost Phys. **17**, 021 (2024) / arXiv : 2401.00039

Studying the 3d Ising surface CFTs on the fuzzy sphere
Zheng Zhou, Yijian Zou
arXiv : 2407.15914

Ongoing quest of the fuzzy sphere

- 2210.13482 — Ising CFT
- 2303.08844 — Ising OPE
- ■ 2306.16435 — SO(5) DQCP
- 2308.01903 — Defect CFT
- 2310.19880 — QMC
- 2312.04047 — O(3) WF
- ■ 2401.00039 — Defect g-function
- 2401.17362 — Bulk F-function
- 2406.10186 — Cusp
- ■ 2407.15914 — Boundary CFT
- 2407.15948 — Boundary CFT
- 2409.02998 — Conformal generators

- ■ *Coming soon* — CS-matter theories
- ■ *Coming soon* — O(4) DQCP
- ■ *Coming soon* — Conformal perturbation
- ■ *Coming soon* — Conformal window of SU(2) QCD
- ■ *Coming soon* — 3d Potts
- *Coming soon* — Lee-Yang singularity
- *Coming soon* — O(2) WF
- ■ More gauge theories ? QED, scalar QED...
- ■ Wilson line defects ?
- ■ Gaussian theory and Lagrangian construction ?
- Hidden structure of 3d CFTs ?
- ■ — *(personally involved)*



FuzzifiED — Toolkit to close the gap for fuzzy sphere numerics

https://www.fuzzified.world