Title: Coherent Control and Readout of a 25-level Trapped 137Ba+ Ion Qudit

Speakers: Pei Jiang Low

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Subject: Quantum Information

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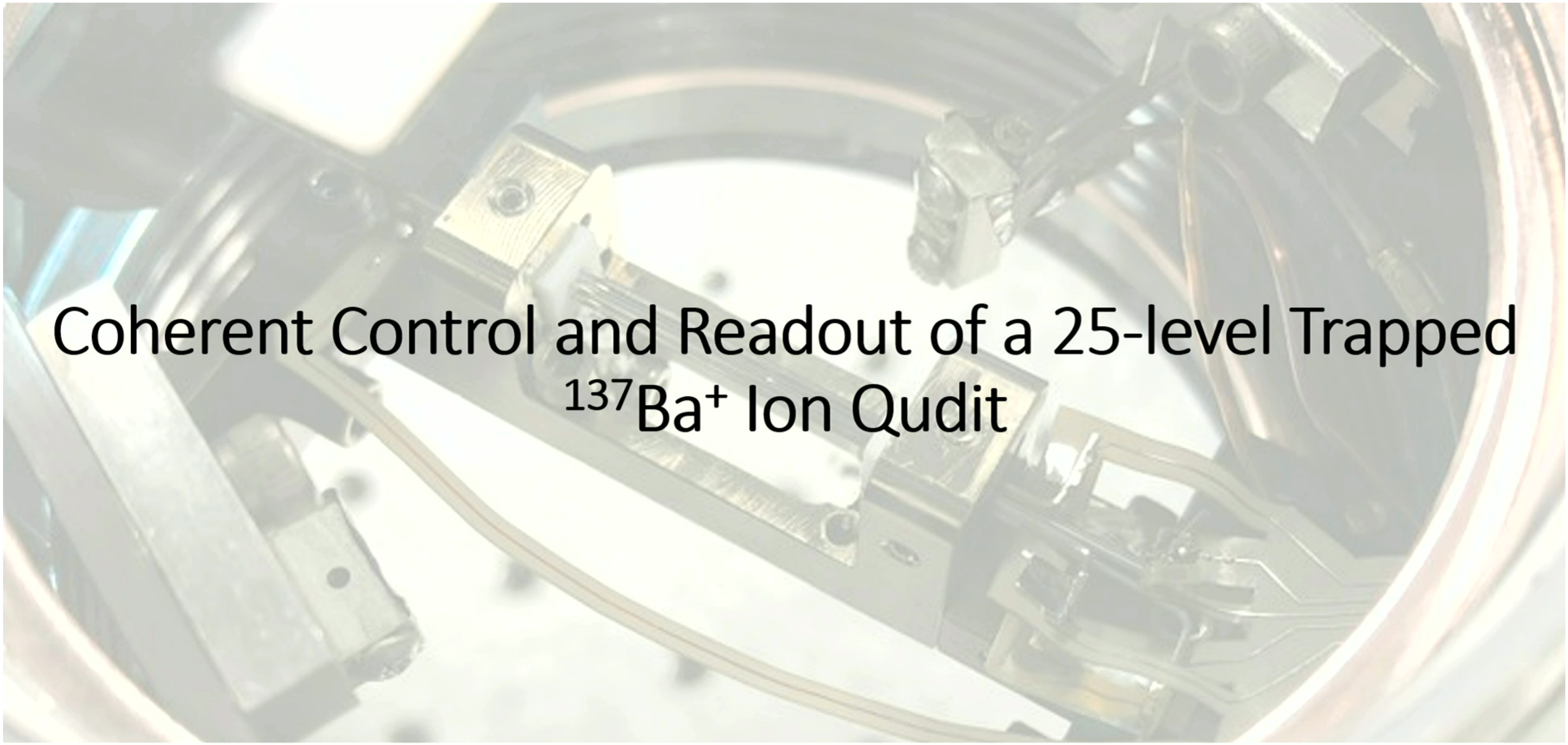
Abstract:

Scaling up the Hilbert space of a quantum computer is essential to demonstrate quantum advantage over classical computing. Other than packing more quantum information carrier in a quantum computing system, one avenue for increasing the quantum computing Hilbert space is by encoding more logical states per quantum information carrier (making it a qudit), rather than a traditional 2-state qubit encoding. Trapped ions typically exhibit multiple (meta)stable electronic energy states that are suitable for logical state encodings and are a suitable platform for qudit applications. In this talk, I present our work on realizing a 25-level trapped 137Ba+ ion qudit, which is the maximal encoding allowed by the single-shot readout protocol that we employ. We show that the harnessing of these additional logical states per ion can be done by just adding a few more lasers to the control system, without any trapping architecture modification compared to a qubit system. I will also present the experimental challenges and limitations, fundamental and technical, relevant to realizing a 25-level trapped 137Ba+ ion qudit.



Coherent Control and Readout of a 25-level Trapped $^{137}\text{Ba}^{+}$ Ion Qudit

Pei Jiang Low

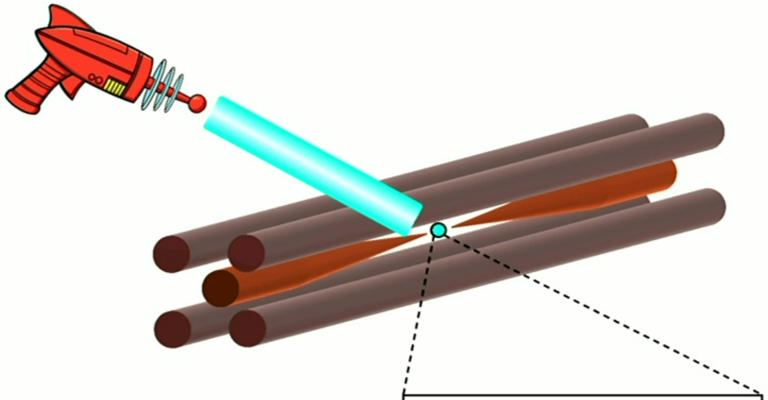
UNIVERSITY OF WATERLOO

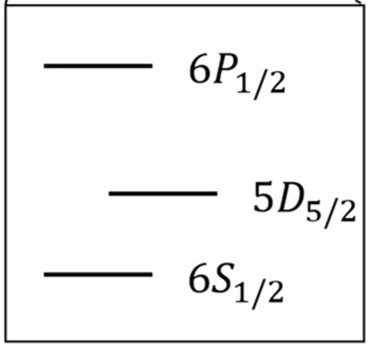
IQC Institute for Quantum Computing

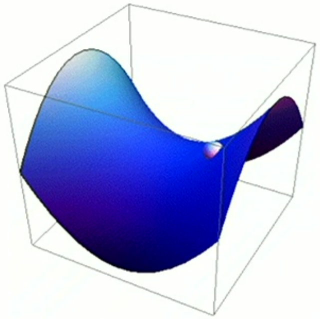
30th September 2024

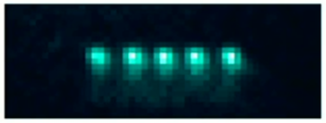
Brief Summary of Trapped Ion Quantum Computing

- Ion(s) trapped in vacuum with electric field.
- Electronic energy levels of the ion used to encode computational states.
- Lasers used to manipulate the electronic states.
- Chain of ion move with shared phonon modes due to Coulomb interactions, used for entanglement between ions.





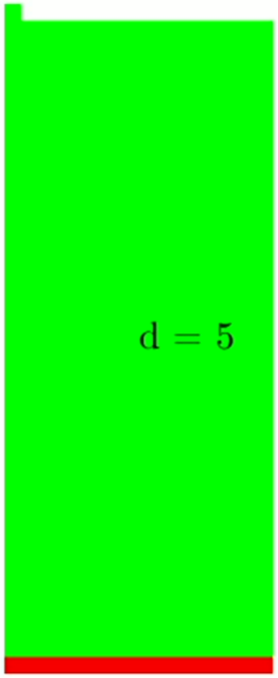




Why Qudits?

Qudits: 2 or more encoding per information carrier.

- Quantum advantage over classical computing requires sufficiently large computational space.
- Qudit scales up Hilbert space with the same number of physical information carrier units: 2^N (qubits) vs d^N (qudits).



2D representation of the size of Hilbert space

Red – Hilbert space spanned by qubits

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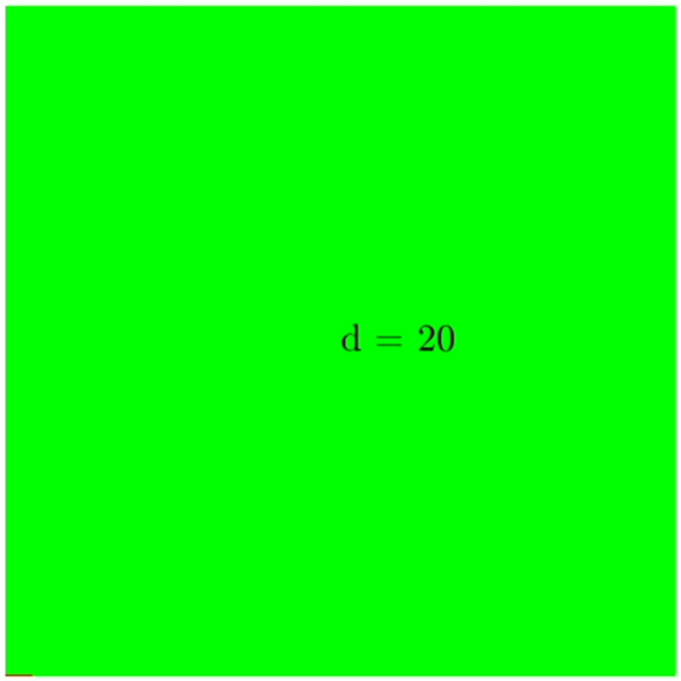

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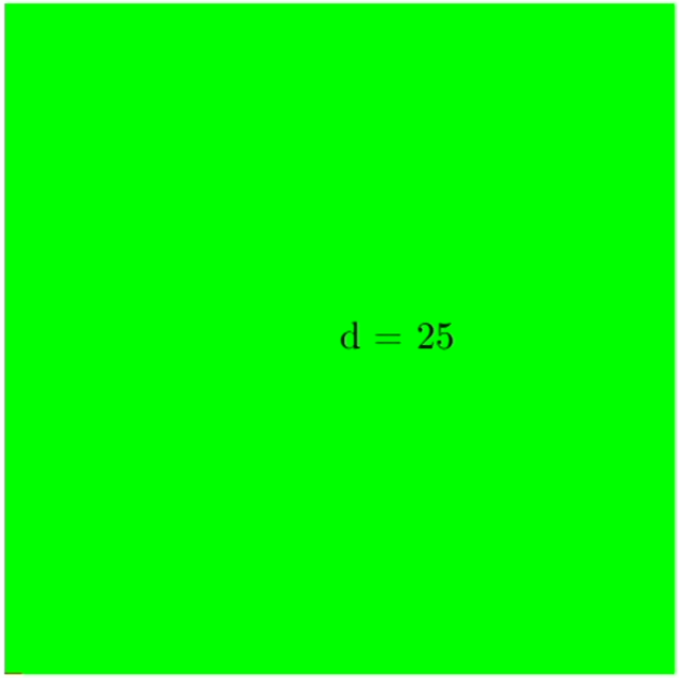
- Qudit scales up Hilbert space with the same number of physical information carrier units: 2^N (qubits) vs d^N (qudits).

- Larger Hilbert space can be used for simplifying qubit operations.[1,2]
- Higher spin systems' quantum simulation more direct with qudits. E.g. Spin 1 system using qutrits.[3,4]
- More relaxed quantum error correction threshold.[5,6,7]



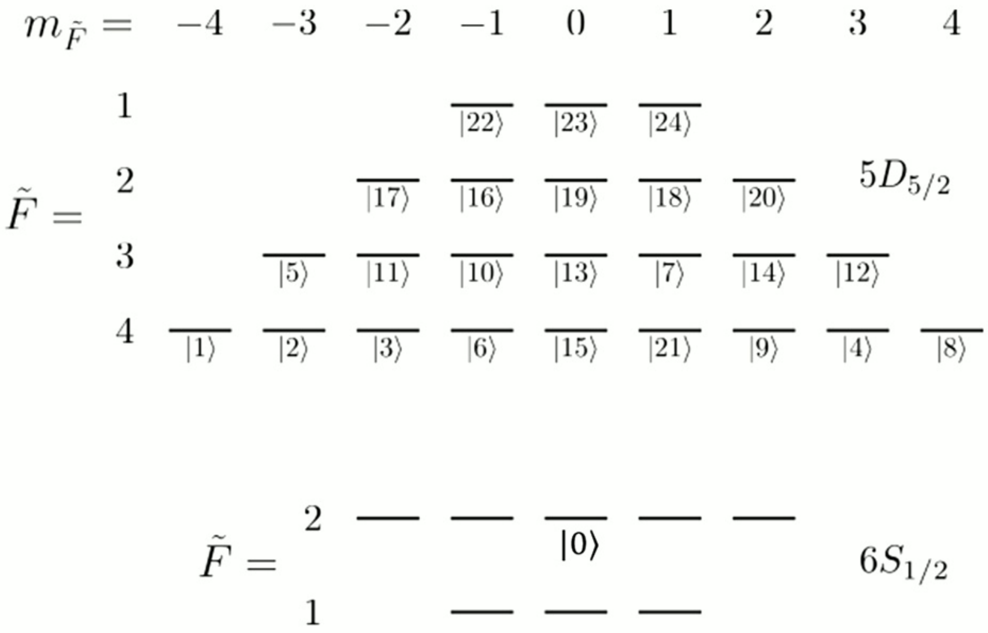
2D representation of the size of Hilbert space

Red – Hilbert space spanned by qubits

[1] Ralph, T. C., Resch, K. J. & Gilchrist, A. Phys. Rev. A 75, 022313, DOI: 10.1103/307 PhysRevA.75.022313 (2007).
[2] Lanyon, B. P. et al. Nat. Phys. 5, 134–140, DOI: 305 10.1038/nphys1150 (2008).
[3] Senko, C. et al. Phys. Rev. X 5, 021026, DOI: 301 10.1103/PhysRevX.5.021026 (2015).
[4] Andrade, B. et al. Quantum Sci. Technol. 7, 034001, DOI: 10.1088/2058-9565/ac5f5b (2022).
[5] Campbell, E. T., Anwar, H. & Browne, D. E. Phys. Rev. X 2, 041021, DOI: 10.1103/PhysRevX.2.041021 (2012).
[6] Andrist, R. S., Wootton, J. R. & Katzgraber, H. G. Phys. Rev. A 91, 042331, DOI: 10.1103/PhysRevA.91.042331 (2015).
[7] Watson, F. H. E., Anwar, H. & Browne, D. E. Phys. Rev. A 92, 299 032309, DOI: 10.1103/PhysRevA.92.032309 (2015).

Trapped Ions as Qudits

- Common to find more than 2 stable energy states.
- Binary restriction usually for magnetic field insensitivity.
- Magnetic field noise can be overcome[1], encode more!
- Main question: Qudits advantageous in theory, realistically advantageous? Practical challenges vs theoretical gain.
- Universal single qudit operations + one ion-ion entangling gate form universal gate set.[2,3,4]
  - Any 2 encoded states in an ion connected via a series of coherent Givens rotations → ability to perform universal single qudit operations.
- Entangling gate can be the same as qubit entangling gate. Assessing single qudit gates challenges = assessing universal qudit gate challenges.



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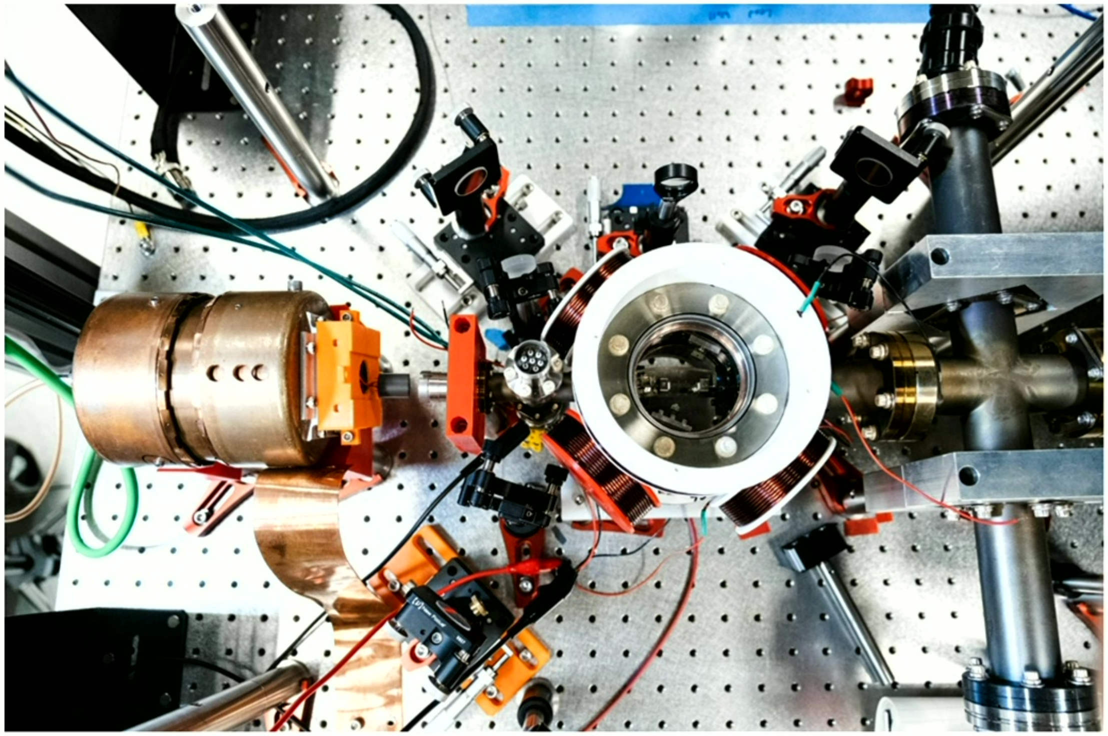
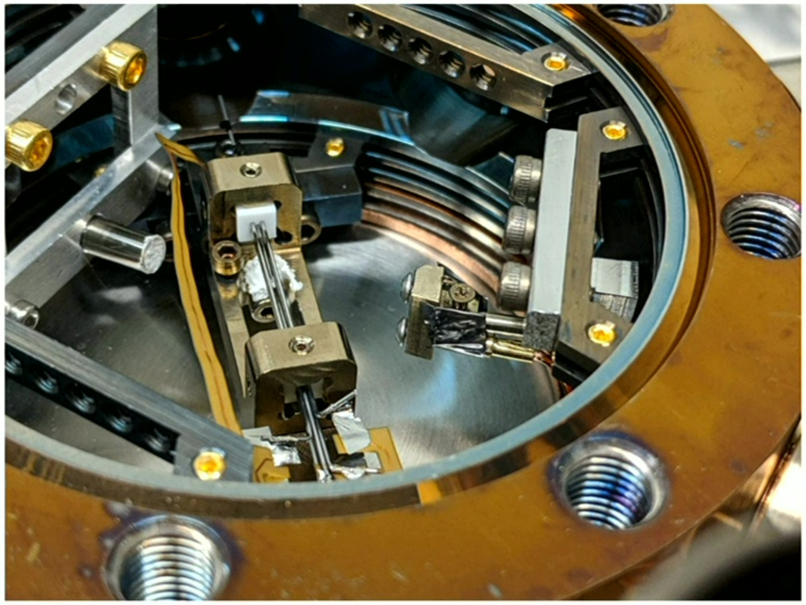
[1] Ruster, T. et al. Appl. Phys. B 122, 254, DOI: 10.1007/s00340-016-6527-4 (2016).
[2] Schirmer, S. G., Greentree, A. D., Ramakrishna, V. & Rabitz, H. J. Phys. A: Math. Gen. 35, 8315, DOI: 10.1088/0305-4470/35/39/313 (2002).
[2] Ivanov, P. A., Kyoseva, E. 332 S. & Vitanov, N. V. Phys. Rev. A 74, 022323, DOI: 10.1103/PhysRevA.74.022323 (2006).
[4] Ringbauer, M. et al. Nat. Phys. 18, 1053–1057, DOI: 10.1038/278 s41567-022-01658-0 (2022).

Our Ion Trap: Aquamarine



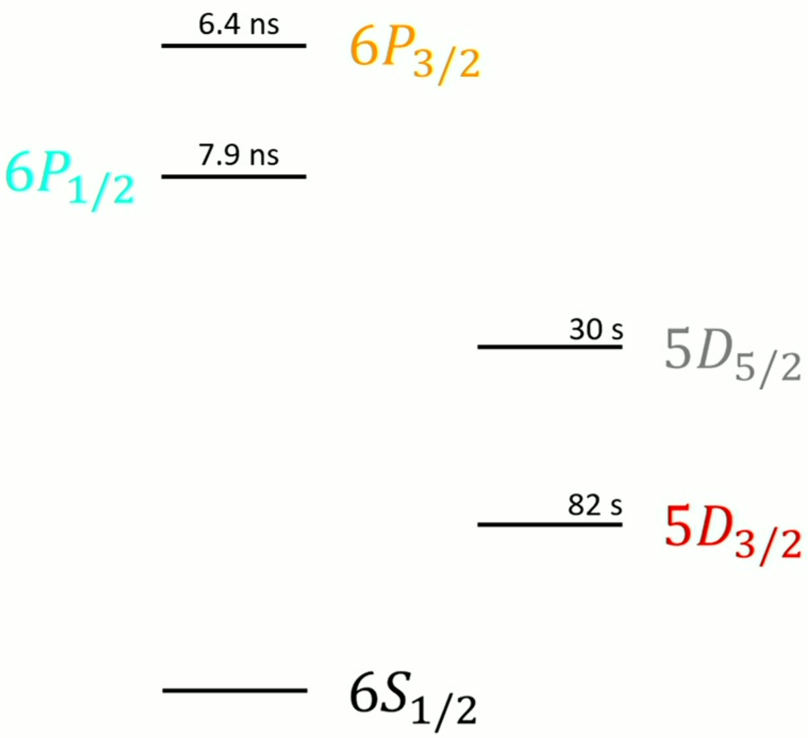


Barium+ Ion's Energy Levels

- $6S_{1/2}$: stable ground state.

Fluorescence group

- $6P_{1/2}$: drive with 493 nm laser for fluorescence.
- $5D_{3/2}$: driven with 650 nm laser to repump to $6P_{1/2}$.



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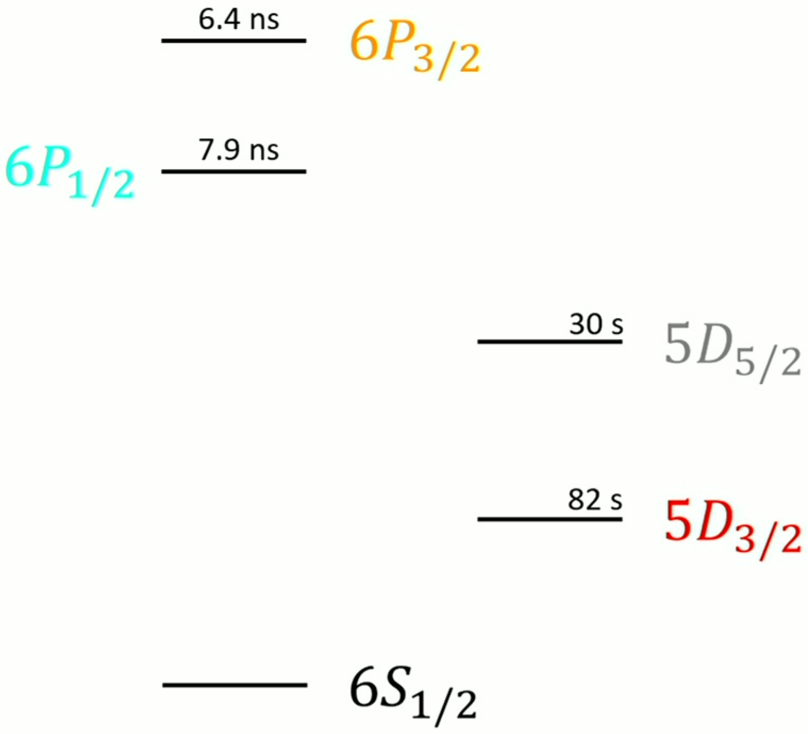
Metastable state control group

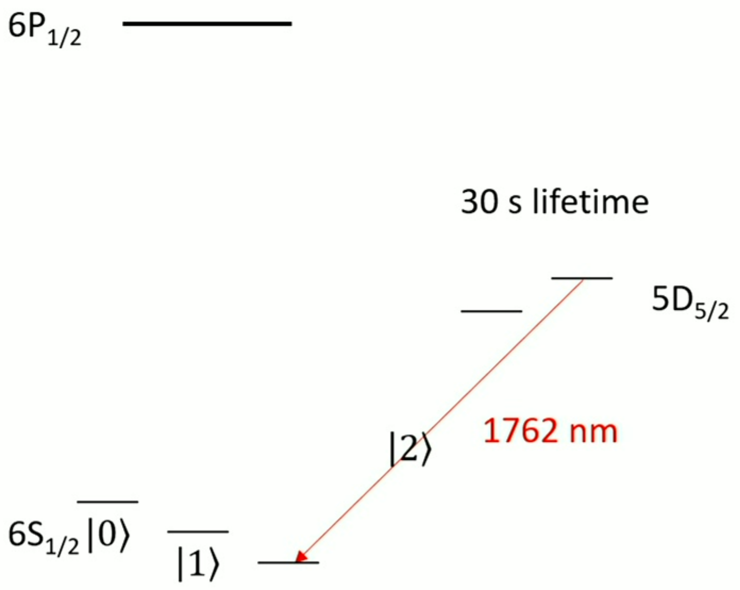
- $5D_{5/2}$: metastable* state for QC**. Drive with 1762 nm laser.
- $6P_{3/2}$: drive with 614 nm to reset $5D_{5/2}$ experiments.

*Long decay time **QC: quantum computing



State Measurement

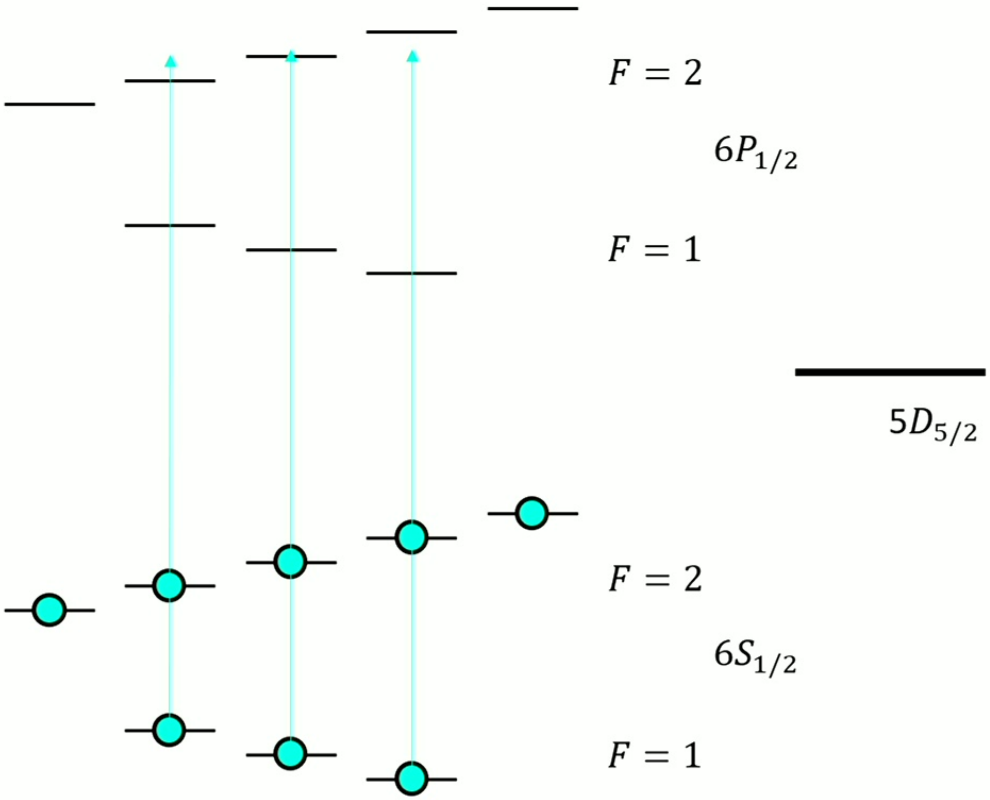
1. Shelve states in $S_{1/2}$ to corresponding metastable states in $D_{5/2}$ except for $|0\rangle$.
2. Fluoresce all remaining states in $S_{1/2}$.
3. Bring back state $|1\rangle$ to $S_{1/2}$ level.
4. Repeat steps 2 and 3 for different states.



State Preparation

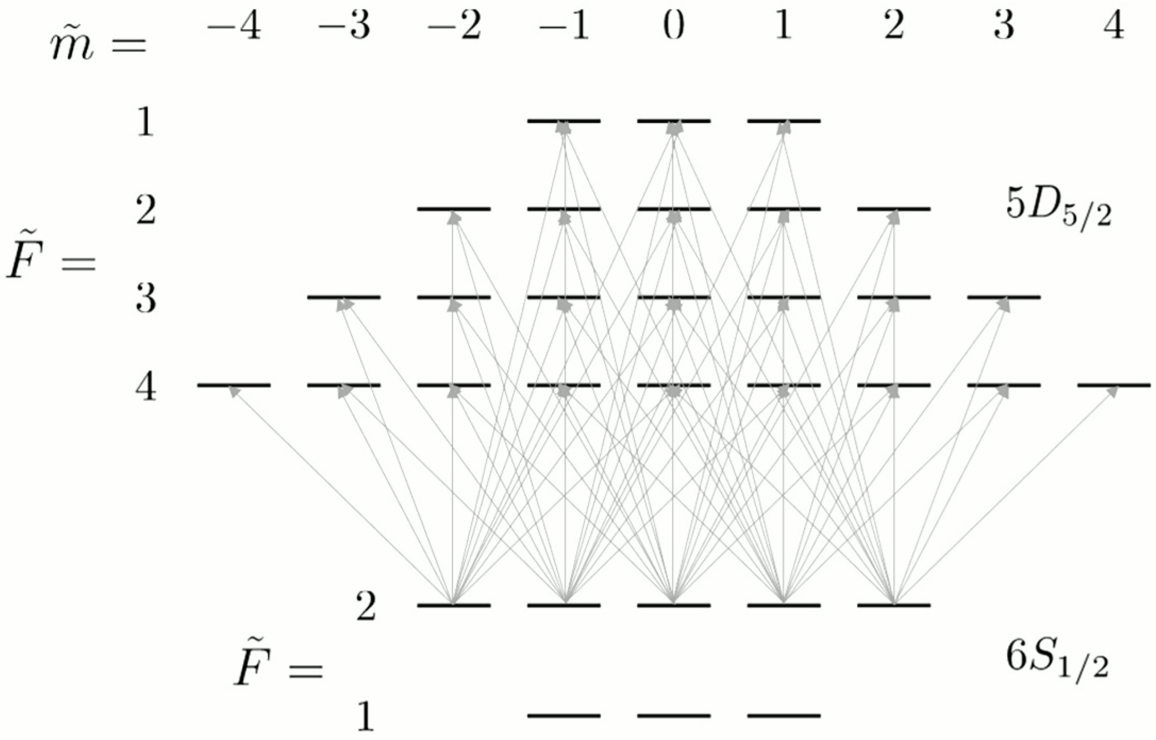
Initializing in $6S_{1/2}$ state*:

1. Turn on 493 nm light with only frequency for pumping out $6S_{1/2}$, $F = 1$ state populations.
2. Turn on 1762 nm light with frequencies to drive out population in $6S_{1/2}$, $F = 2$ states that we do not want to initialize. Each frequency turned on for a π-pulse time.
3. Turn on 614 nm light to drive out state populations in $5D_{5/2}$ and allow the state population to decay back to $6S_{1/2}$.
4. Repeat steps 1-3 until state population saturates at the desired state to be initialized.



*An, F. A. et al. Phys. Rev. Lett. 129, 130501, DOI: 10.1103/PhysRevLett.129.130501 (2022).

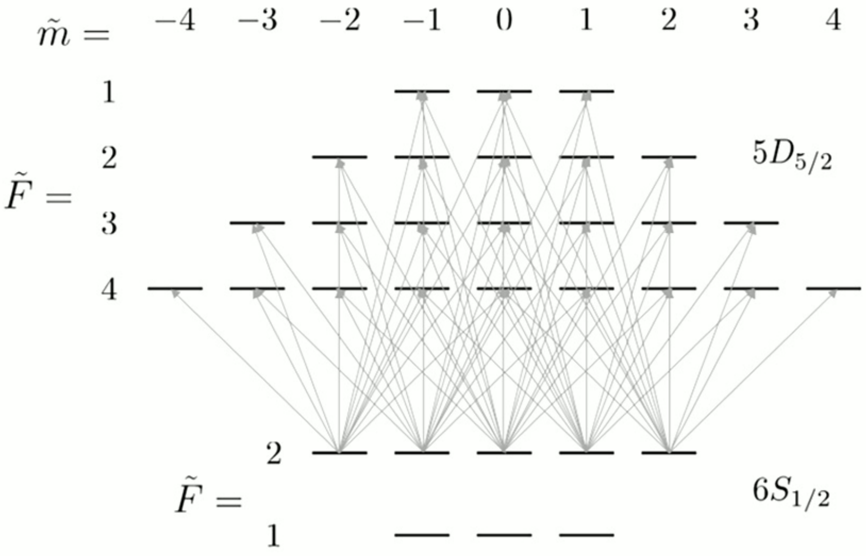
High Dimensional Qudit Encoding with ^{137}Ba^{+}



- ^{137}Ba^{+} $6S_{1/2}$ and $5D_{5/2}$ have 32 non-degenerate states in non-zero magnetic field!
- At most 25-level qudit can be encoded.
- So many transitions! Which ones are useful?

Choosing Useful Coherent Transitions in ^{137}Ba^{+}

- Calculate transition strengths, pick the ones strong enough for practical use.



Choosing Useful Coherent Transitions in ^{137}Ba^{+}

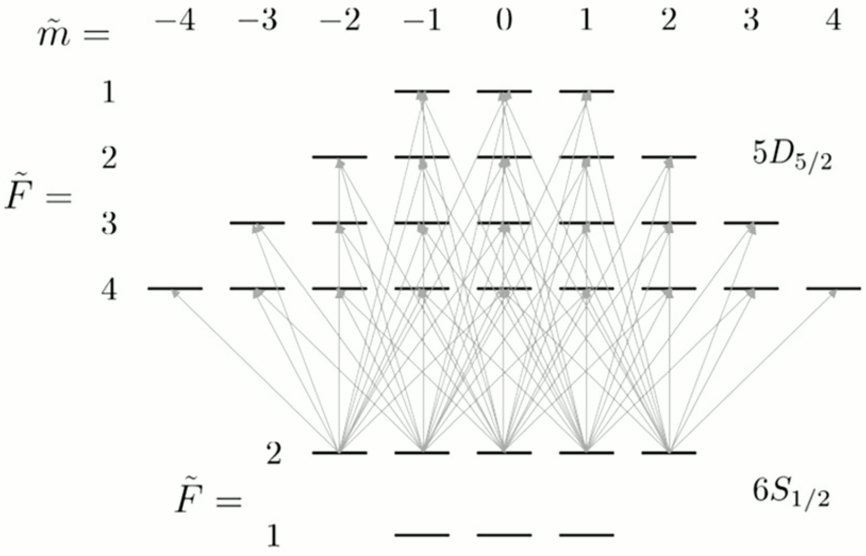
- Calculate transition strengths, pick the ones strong enough for practical use.

$\langle b|\hat{Q}|a\rangle$

$\hat{Q}$ is electric quadrupole operator

$\langle D_{5/2}, F, m_F|$?

$|S_{1/2}, F, m_F\rangle$?



Choosing Useful Coherent Transitions in $^{137}\mathrm{Ba}^+$

- Calculate transition strengths, pick the ones strong enough for practical use.
- $5\mathrm{D}_{5/2}$ level has close F=3 and F=4 hyperfine splitting.
- Small B assumption quickly falls apart.

$$\langle b|\hat{Q}|a\rangle$$

$\hat{Q}$ is electric quadrupole operator

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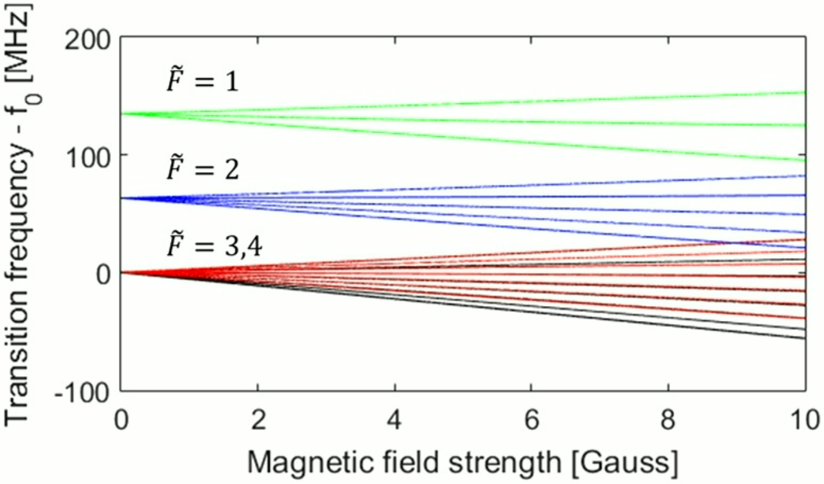


f$_0$ is the energy level of $\tilde{F} = 4$ at zero magnetic field.

Choosing Useful Coherent Transitions in ^{137}Ba$^+$

- Calculate transition strengths, pick the ones strong enough for practical use.
- 5D$_{5/2}$ level has close F=3 and F=4 hyperfine splitting.
- Small B assumption quickly falls apart.
- Energy eigenstates are not the $|F, m_F\rangle$ states.
- Obtain energy eigenstates from solving the Hamiltonian.

$$\hat{H}_R = \frac{2\pi A_D}{\hbar}\vec{I}\cdot\vec{J} + hB_Q\frac{\frac{3}{\hbar^4}\left(\vec{I}\cdot\vec{J}\right)^2 + \frac{3}{2\hbar^2}\vec{I}\cdot\vec{J} - I\left(I+1\right)J\left(J+1\right)}{2I\left(2I-1\right)J\left(2J-1\right)} + \frac{B_e\mu_B}{\hbar}\left(g_J\hat{J}_z + g_I\hat{I}_z\right)$$

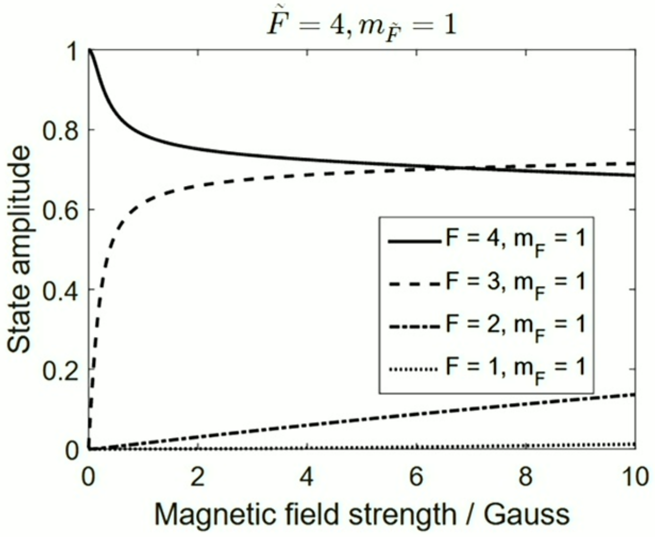
A_D is the dipole hyperfine constant
B_Q is the quadrupole hyperfine constant
B_e is the magnetic field strength

$$\langle b|\hat{Q}|a\rangle$$

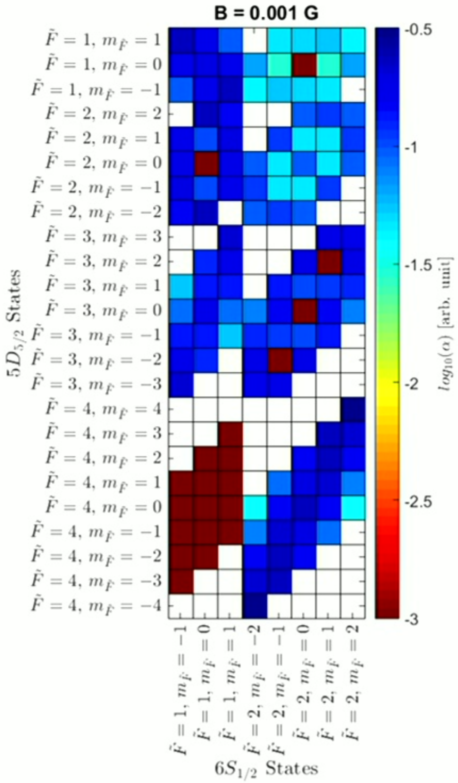
$\hat{Q}$ is electric quadrupole operator

$\langle D_{5/2}, F, m_F|$?

$|S_{1/2}, F, m_F\rangle$?



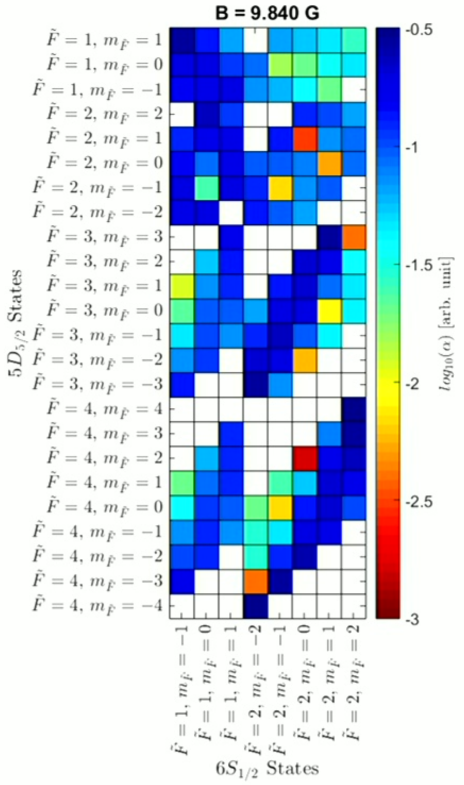
Evolution of $6S_{1/2}$ to $5D_{5/2}$ Transition Strengths



α is the relative transition strength

Evolution of $6S_{1/2}$ to $5D_{5/2}$ Transition Strengths

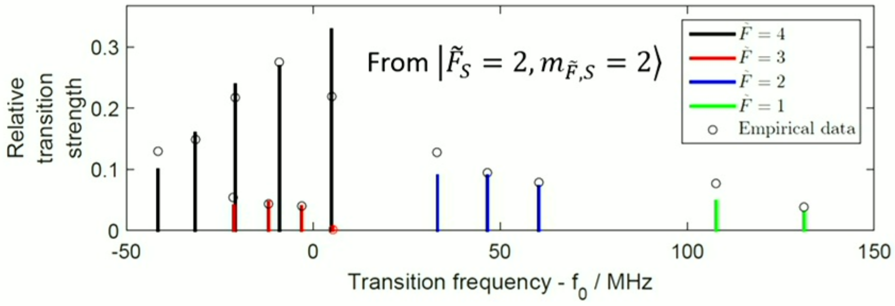
- Increasing magnetic field quickly changes the transition strengths significantly.

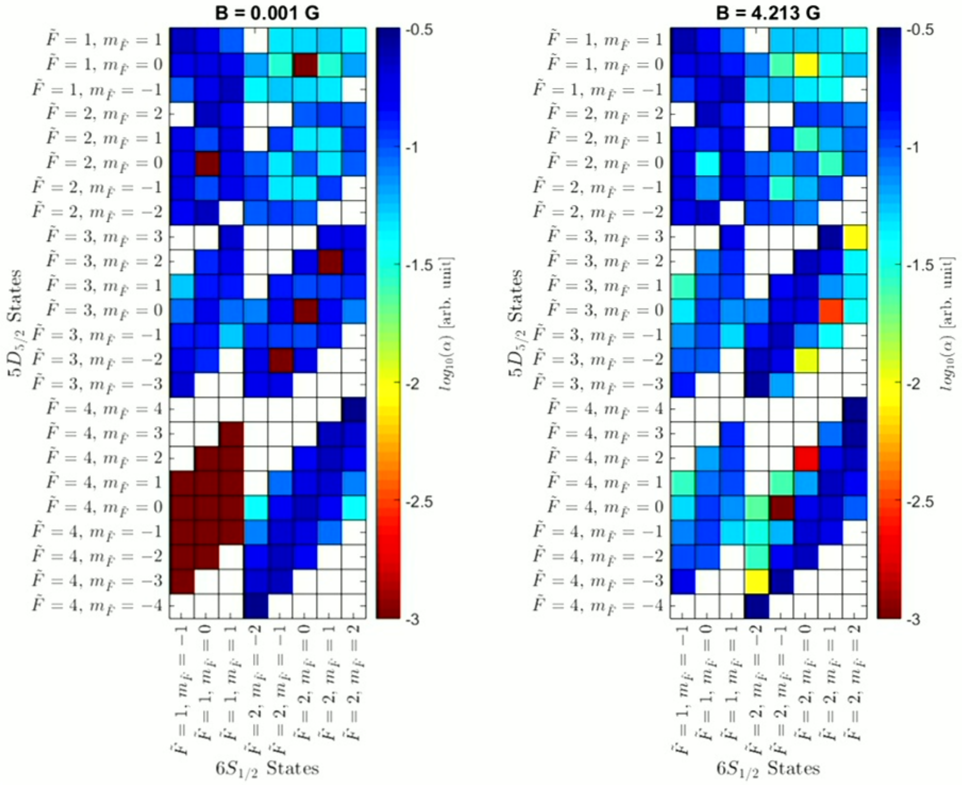


α is the relative transition strength

Evolution of $6S_{1/2}$ to $5D_{5/2}$ Transition Strengths

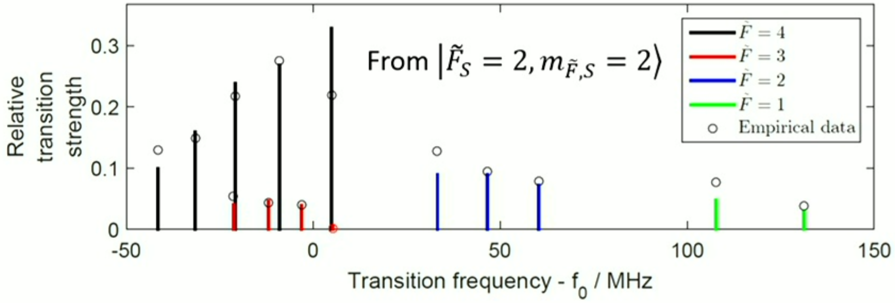
- Increasing magnetic field quickly changes the transition strengths significantly.
- Allowed hyperfine transitions become practically forbidden and vice versa.
- Empirical data agree well with theoretical calculations.

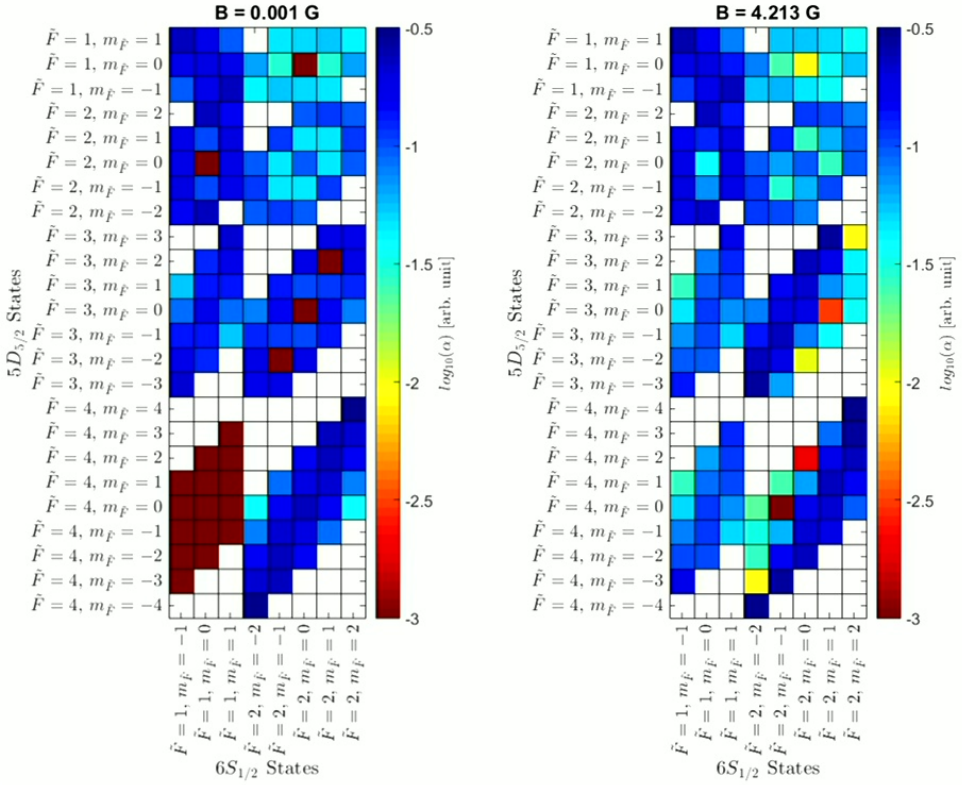




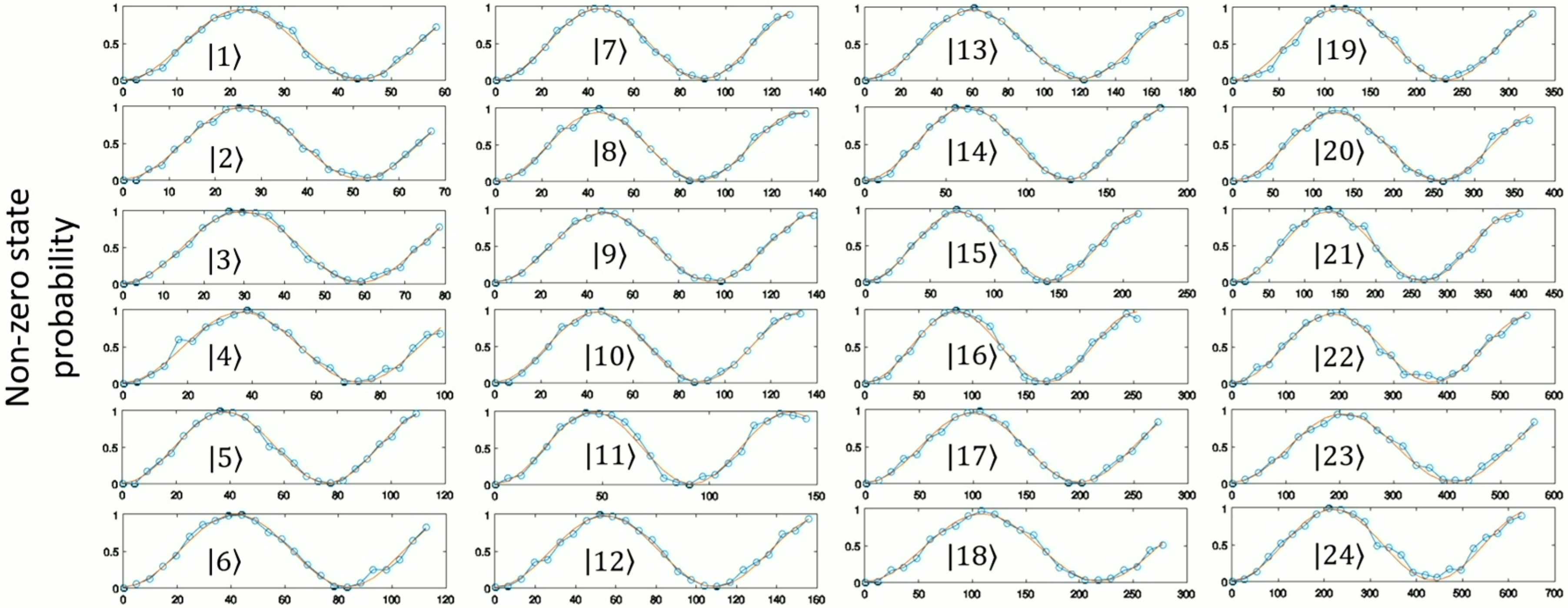
Evolution of $6S_{1/2}$ to $5D_{5/2}$ Transition Strengths

- Increasing magnetic field quickly changes the transition strengths significantly.
- Allowed hyperfine transitions become practically forbidden and vice versa.
- Empirical data agree well with theoretical calculations.
- Deviations verified to be from inaccurate assumption of pure linear polarization of 1762 nm laser. Data fits theory perfectly once laser geometric factor is calibrated for (shown later).





Coherent Rabi Cycling of Qudit States



Pulse time [μs]

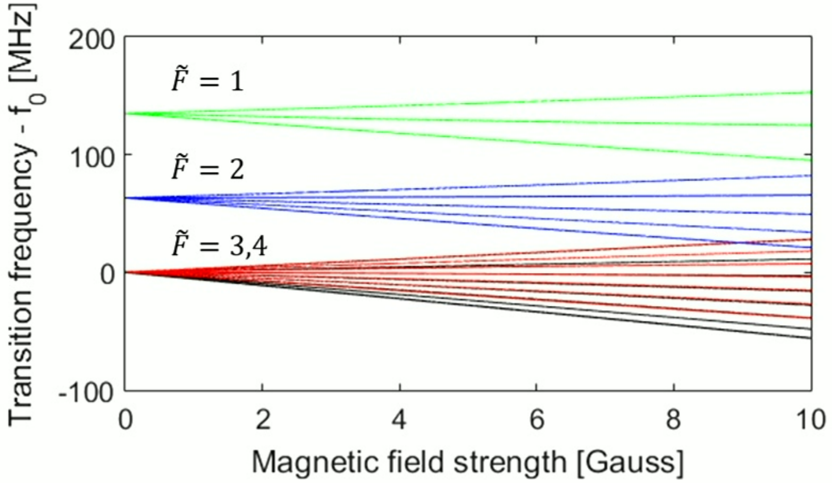
Rabi cycling of state population of $|0\rangle$ (a state in $S_{1/2}$) and other non-zero states.

Transitions not overcrowded for Rabi frequencies in the order 10 kHz at ~4 G magnetic field.
Sufficient control to perform universal single qudit gates.

Laser Parameters Calibrations for Qudits

- Qubits: 1 transition – 1 laser frequency, 1 pulse time calibrations.
- Qudits: many transitions – many calibrations?

Laser Parameters Calibrations for Qudits



- Qubits: 1 transition – 1 laser frequency, 1 pulse time calibrations.
- Qudits: many transitions – 2 frequencies, 5 pulse times calibrations.
- Frequency calibrations:
  - Measure 2 frequencies, f_{offset} and f_B, can compute the difference $\Delta f = f_B - f_{offset}$ as a proxy for the magnetic field strength.
  - f_{offset} also acts as a calibration measurement for drifts of frequency locking devices (e.g. Fabry–Pérot cavity).
  - Relying on theory alone still gives frequency deviations of order 1 kHz.
  - Do a polynomial fit with historical data, linear fit is sufficient for our magnetic field drifts.

$$f_n = a_{n,1}\Delta f + f_1 + a_{n,2}$$

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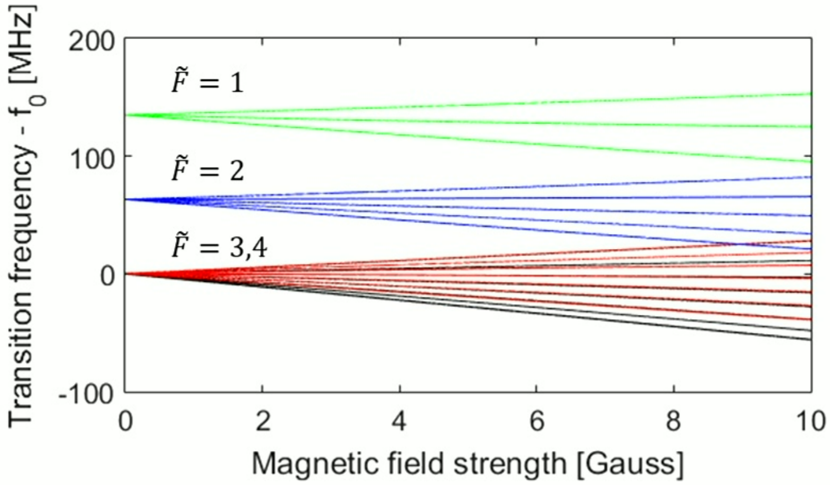
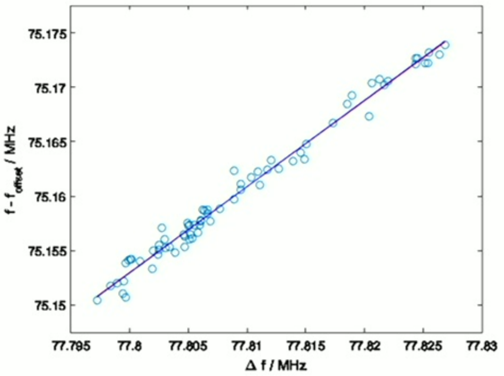




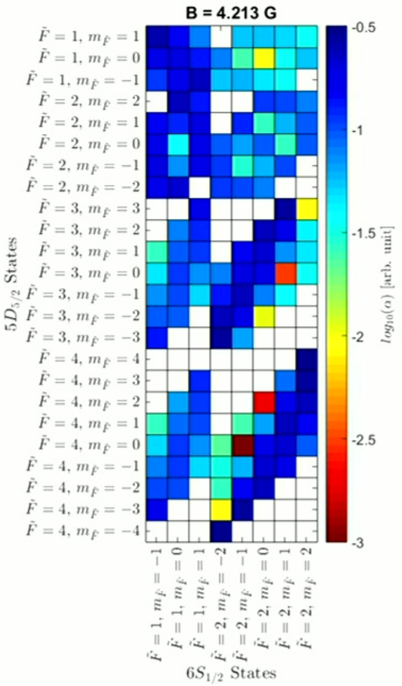
Red line: Free parameter fit for both gradient, $a_{n,1}$, and offset, $a_{n,2}$.

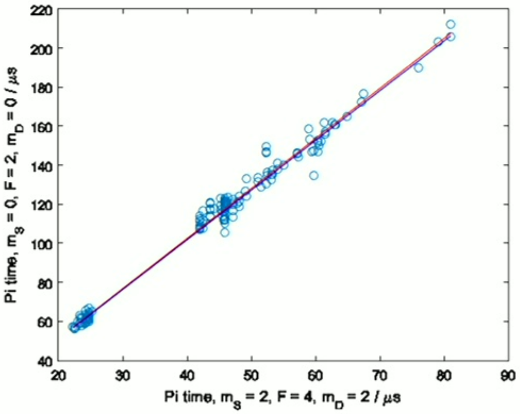
Blue line: Free parameter fit for offset only, gradient from theoretical B field sensitivity.

B field sensitivity from theory is accurate enough for practical use!

Laser Parameters Calibrations for Qudits

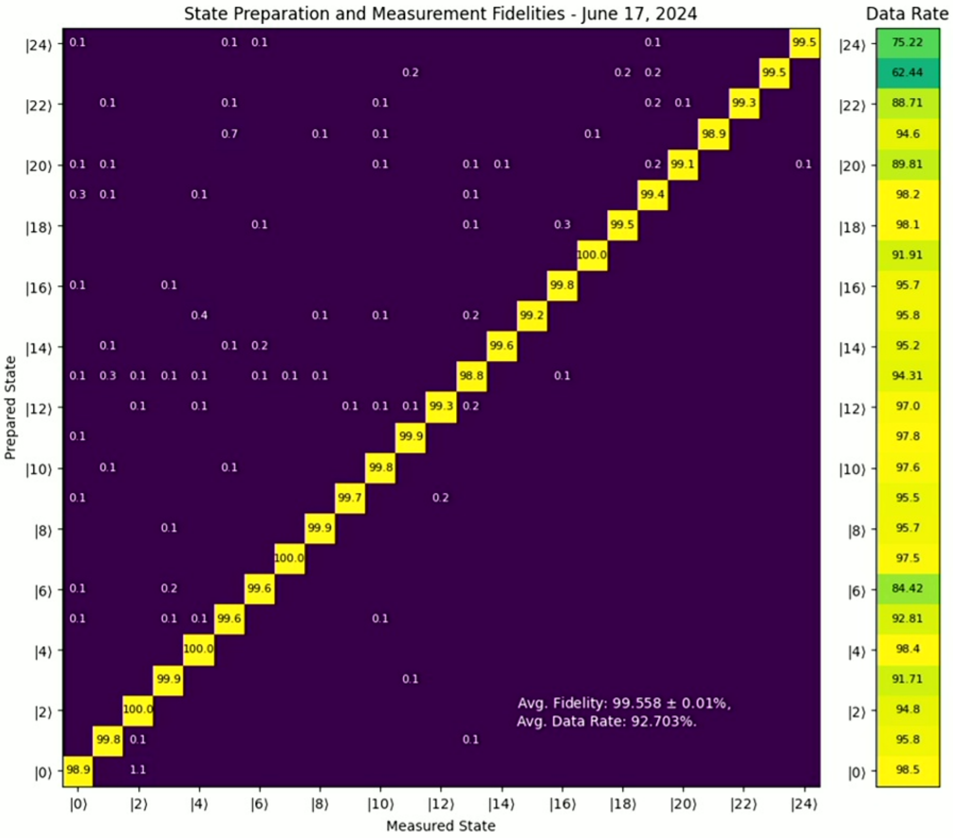
- Qubits: 1 transition – 1 laser frequency, 1 pulse time calibrations.
- Qudits: many transitions – 2 frequencies, 5 pulse times calibrations.
- Pulse time calibrations:
  - Measure 5 reference π-pulse times $\tau_{\Delta m,0}$, each corresponding to a $-2 \leq \Delta m \leq 2$ transition. Having a reference pulse time for each Δm makes it robust to laser polarization drifts.
  - Scale the pulse times of other transitions with a scaling parameter b_n, either determined from historical data or calculated from theory
    $$\tau_n = b_n \tau_{\Delta m,0}$$
  - Theory calculations are practically sufficient!





Red line: Free parameter fit for both gradient, b_n.
Blue line: Gradient calculated from theory.

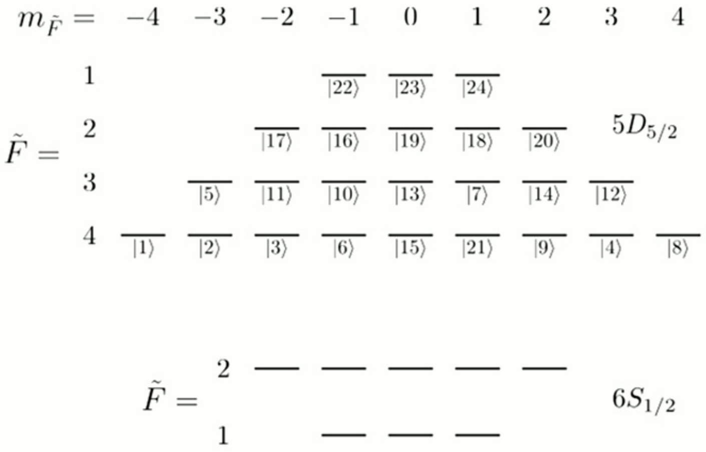
25-Level Qudit SPAM Results



State Preparation and Measurement Fidelities - June 17, 2024

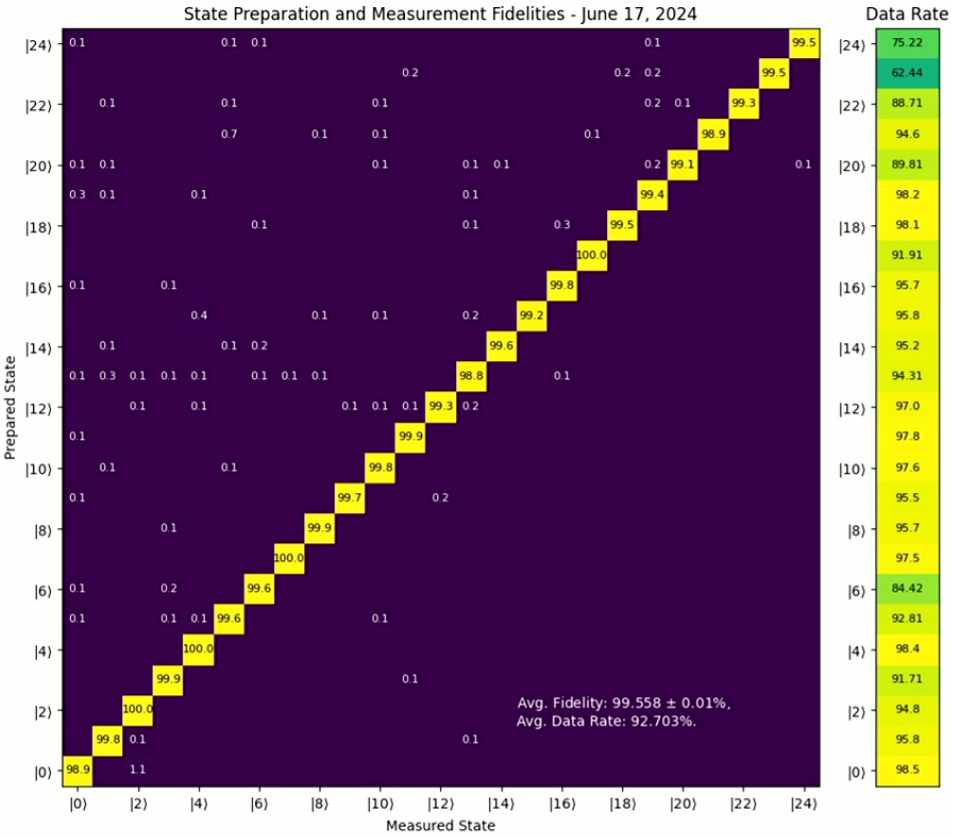
	|0⟩		|2⟩		|4⟩		|6⟩		|8⟩		|10⟩		|12⟩		|14⟩		|16⟩		|18⟩		|20⟩		|22⟩		|24⟩	Data Rate
|24⟩	0.1					0.1	0.1													0.1					99.5	75.22
												0.2							0.2	0.2				99.5		62.44
|22⟩		0.1				0.1					0.1									0.2	0.1		99.3			88.71
						0.7			0.1		0.1							0.1				98.9				94.6
|20⟩	0.1	0.1									0.1			0.1	0.1					0.2	99.1				0.1	89.81
	0.3	0.1			0.1									0.1						99.4						98.2
|18⟩							0.1							0.1			0.3		99.5							98.1
																		100.0								91.91
|16⟩	0.1			0.1													99.8									95.7
					0.4				0.1		0.1			0.2		99.2										95.8
|14⟩		0.1				0.1	0.2								99.6											95.2
	0.1	0.3	0.1	0.1	0.1		0.1	0.1	0.1					98.8			0.1									94.31
|12⟩			0.1		0.1					0.1	0.1	0.1	99.3	0.2												97.0
	0.1											99.9														97.8
|10⟩		0.1				0.1					99.8															97.6
	0.1									99.7			0.2													95.5
|8⟩				0.1					99.9																	95.7
								100.0																		97.5
|6⟩	0.1			0.2			99.6																			84.42
	0.1			0.1	0.1	99.6					0.1															92.81
|4⟩					100.0																					98.4
				99.9								0.1														91.71
|2⟩			100.0																							94.8
		99.8	0.1											0.1												95.8
|0⟩	98.9		1.1																							98.5

Avg. Fidelity: 99.558 ± 0.01%,
Avg. Data Rate: 92.703%.

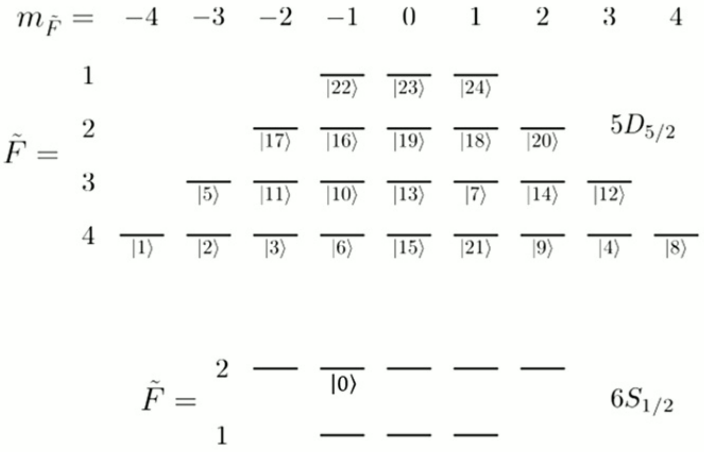


*Ana S Sotirova. High-fidelity state preparation and measurement protocol using multiple qubit encodings. Presentation at DAMOP 2024

25-Level Qudit SPAM Results



- Heralded state preparation (thanks to Oxford's group*) – repeat experiment if first detect bright measurement for $|n \neq 0\rangle$.
- Average 25-level SPAM fidelity of $99.56 \pm 0.1\%$. For reference, 4 qubits (16 states) at 99.9% error ~99.6% error.



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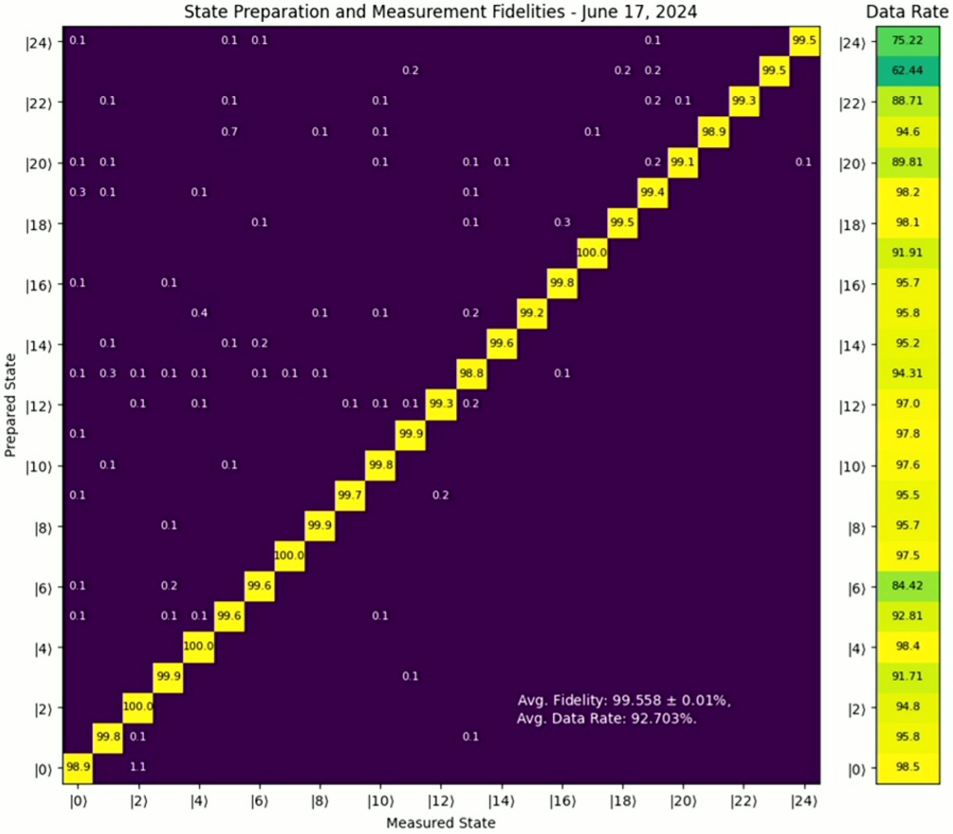


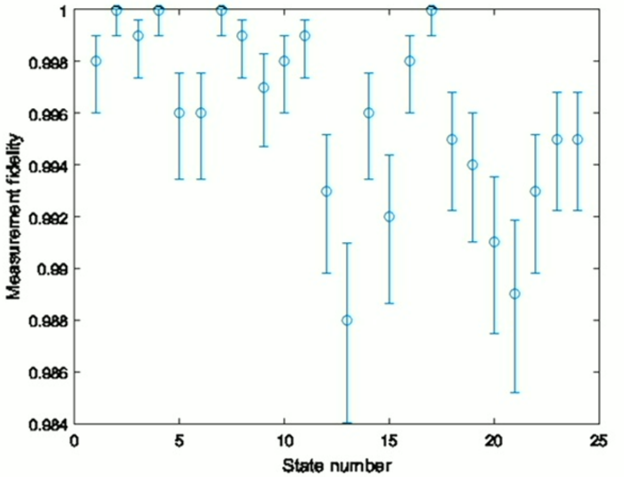
State Preparation and Measurement Fidelities - June 17, 2024

Prepared State	|0⟩		|2⟩		|4⟩		|6⟩		|8⟩		|10⟩		|12⟩		|14⟩		|16⟩		|18⟩		|20⟩		|22⟩		|24⟩	Data Rate
|24⟩	0.1					0.1	0.1													0.1					99.5	75.22
												0.2							0.2	0.2				99.5		62.44
|22⟩		0.1				0.1					0.1									0.2	0.1		99.3			88.71
						0.7			0.1		0.1							0.1				98.9				94.6
|20⟩	0.1	0.1									0.1			0.1	0.1					0.2	99.1				0.1	89.81
	0.3	0.1			0.1									0.1						99.4						98.2
|18⟩							0.1							0.1			0.3		99.5							98.1
																		100.0								91.91
|16⟩	0.1			0.1													99.8									95.7
					0.4				0.1		0.1			0.2		99.2										95.8
|14⟩		0.1				0.1	0.2								99.6											95.2
	0.1	0.3	0.1	0.1	0.1		0.1	0.1	0.1					98.8			0.1									94.31
|12⟩			0.1		0.1					0.1	0.1	0.1	99.3	0.2												97.0
	0.1											99.9														97.8
|10⟩		0.1				0.1					99.8															97.6
	0.1									99.7			0.2													95.5
|8⟩				0.1					99.9																	95.7
								100.0																		97.5
|6⟩	0.1			0.2			99.6																			84.42
	0.1			0.1	0.1	99.6					0.1															92.81
|4⟩					100.0																					98.4
				99.9								0.1														91.71
|2⟩			100.0																							94.8
		99.8	0.1											0.1												95.8
|0⟩	98.9		1.1																							98.5

Measured State

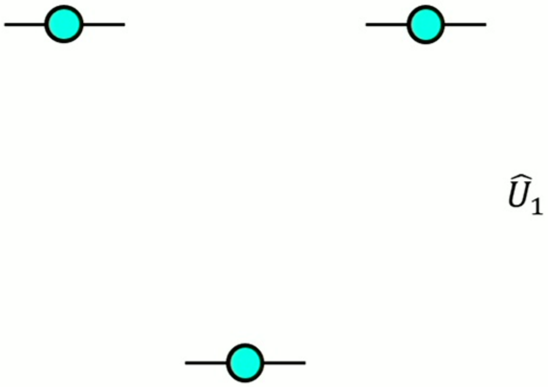
Avg. Fidelity: 99.558 ± 0.01%,
Avg. Data Rate: 92.703%.

- Heralded state preparation (thanks to Oxford's group*) – repeat experiment if first detect bright measurement for $|n \neq 0\rangle$.
- Average 25-level SPAM fidelity of $99.56 \pm 0.1\%$. For reference, 4 qubits (16 states) at 99.9% error $\sim$99.6% error.
- Errors mainly from spontaneous $5D_{5/2}$ decay. Predicted average error of 0.2%.



Qudit Coherence

- Demonstrate coherence among all the encoded states with a generalized qudit Ramsey experiment.
- Prepare a state with equal superposition with $\hat{U}_1$.
- "Bring back" state $|n\rangle$ to state $|0\rangle$ with a θ-phase shifted (relative to $\hat{U}_1$) 2-level rotation in series during $\hat{U}_2$.



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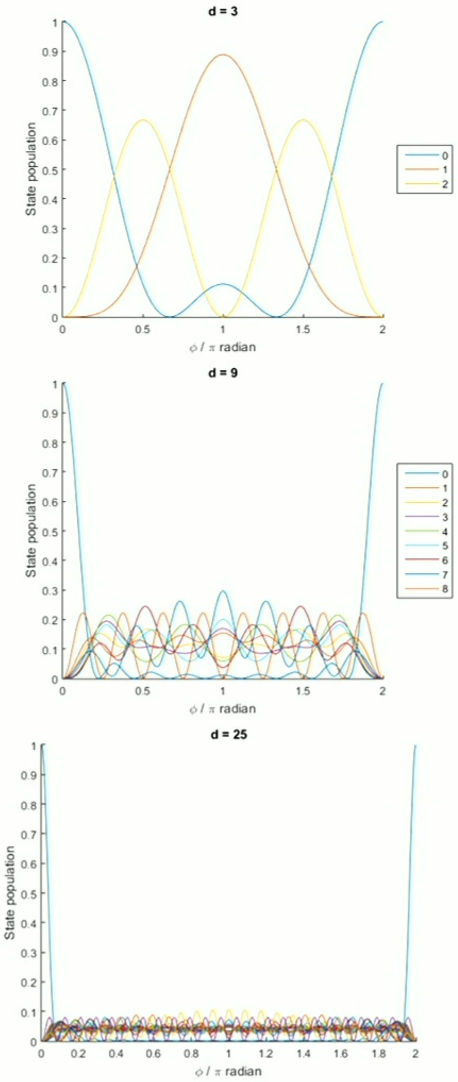
- Set $\theta = n\phi + \pi$, the $|0\rangle$ state population with respect to ϕ is:

$$|\langle 0|\psi\rangle|^2 = \frac{1}{d} + \frac{2}{d^2}\sum_{m=1}^{d-1}(d-m)\cos(m\phi)$$

- $|\langle 0|\psi\rangle|^2 = 1$ when ϕ is zero or integer multiples of 2π.



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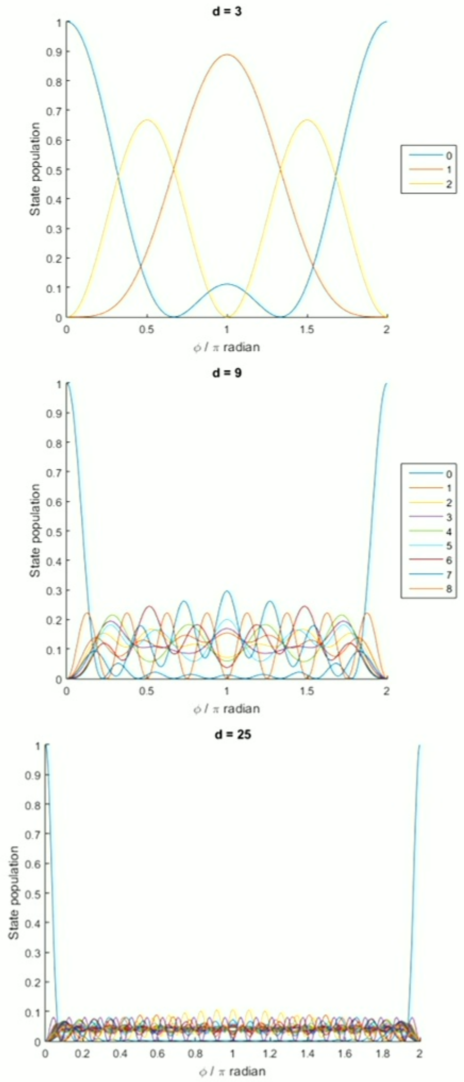
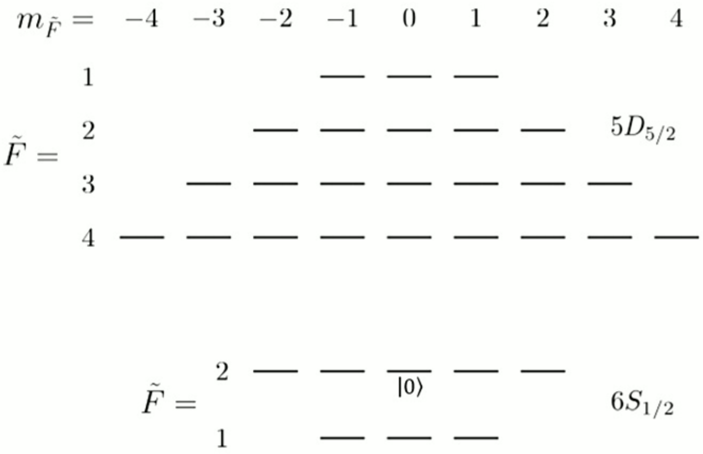
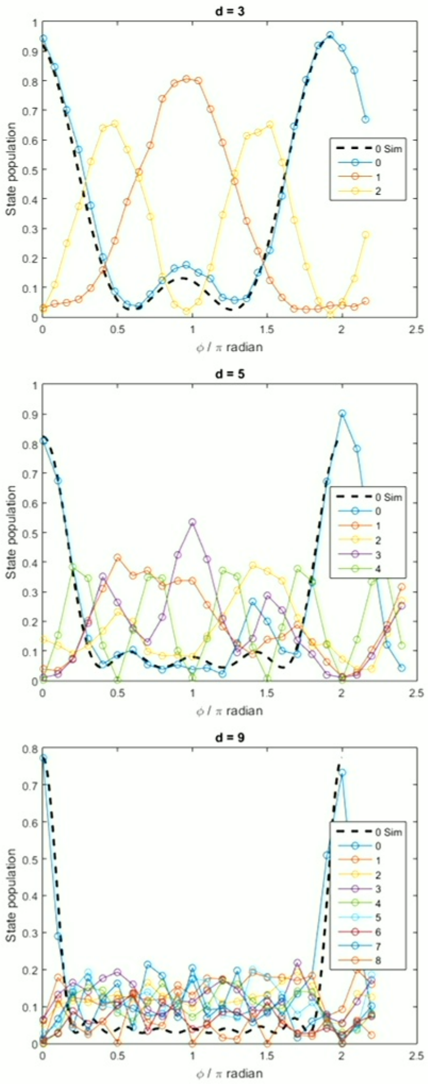
- $|\langle 0|\psi\rangle|^2 = 0$ when ϕ is integer multiples of $\frac{2\pi}{d}$.
- Cosine terms average out to zero if ϕ is not coherently controlled in experiments, $|\langle 0|\psi\rangle|^2$ converge to $\frac{1}{d}$.



Qudit Coherence

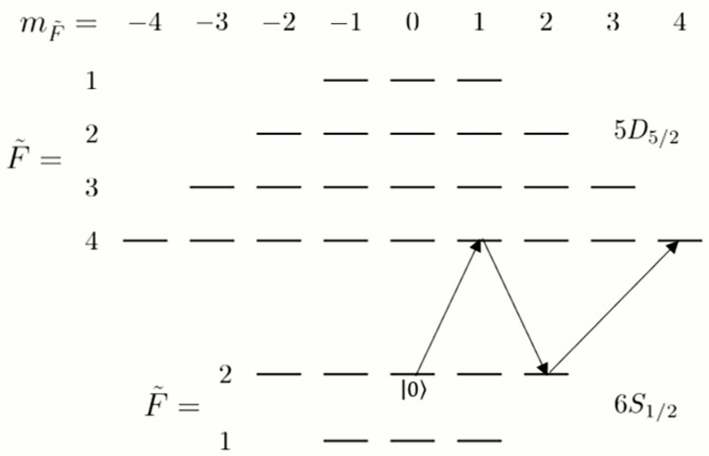
- $|0\rangle$ encoded in the $\mathrm{F} = 2, m = 0$ ground state.
- States reachable from $|0\rangle$ with only one 1762 nm transition allows encoding up to 9 states with our calibrated data.
- Successful demonstration of qudit coherence up to $d = 9$.

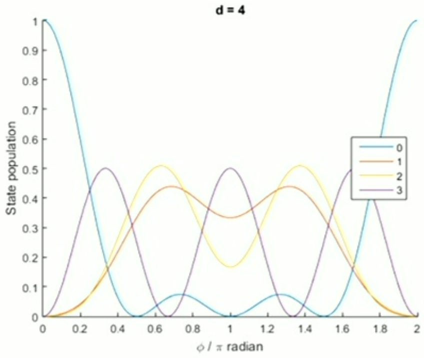


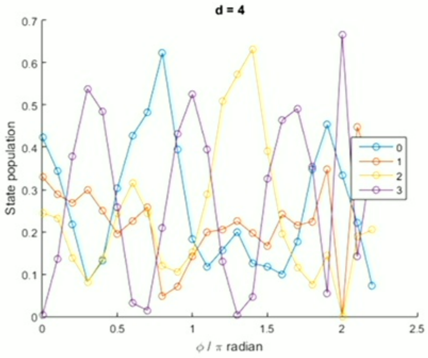


Qudit Coherence

- Need to “bus” the state to reach up to 25 levels.
- Tried 4-level qudit Ramsey phase scan with bussing, unexpected results obtained.







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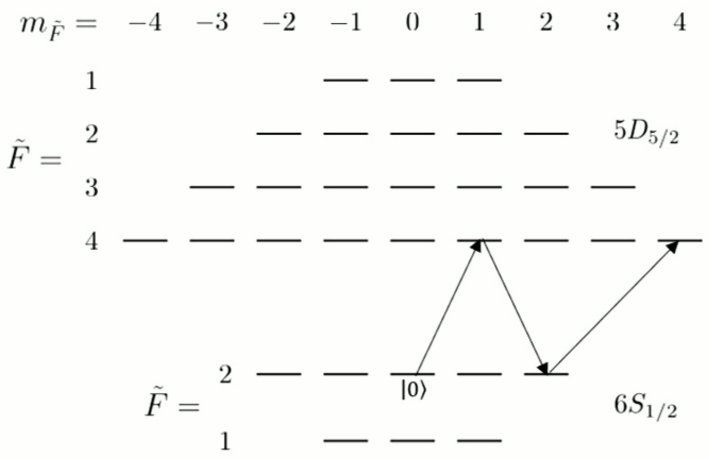


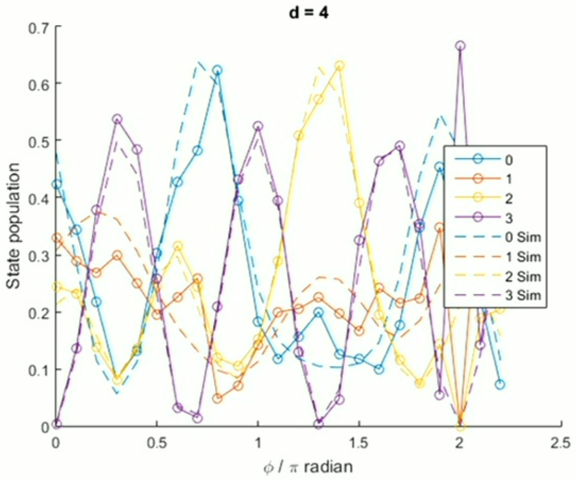
- Need to “bus” the state to reach up to 25 levels.
- Tried 4-level qudit Ramsey phase scan with bussing, unexpected results obtained.
- Allowing frequency detunings in the simulations explains the discrepancy.

$$|\langle 0|\psi\rangle|^2 = \frac{1}{d} + \frac{2}{d^2}\sum_{m=1}^{d-1}\cos(m\phi + \Delta_m t) + \frac{2}{d^2}\sum_{m=1}^{d-1}\sum_{m=1}^{d-1}\cos\big((m-n)\phi + (\Delta_m - \Delta_n)t\big)$$

- Two states with distance D (D 2-level transitions) compounds D number of frequency detunings:

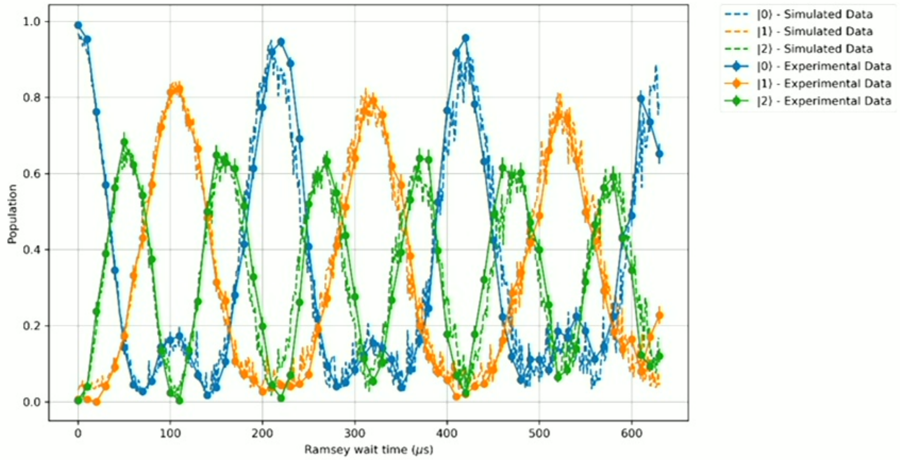
$$\Delta_{net} = \Delta_1 + \Delta_2 + \cdots + \Delta_D$$
$$\mathrm{std}(\Delta_{net}) = \sqrt{Var(\Delta_1) + Var(\Delta_2) + \cdots + Var(\Delta_D)}$$

- Qudit encoding with distances D needs calibration parameters that are overall $\sqrt{D}$ more accurate.



Qudit Coherence

- Can add wait time between $\hat{U}_1$ and $\hat{U}_2$ to study decoherence time, vary ϕ with time.
- Investigation ongoing.



Summary

- Main question: Qudits advantageous in theory, realistically advantageous? Practical challenges vs theoretical gain.

- Solved challenges:
  - Unique to $^{137}\mathrm{Ba}^+$ or $^{137}\mathrm{Ba}^+$-like ions – non-trivially different energy level structures.
  - Calibration time for parameters do not scale with qudit dimension.
  - Transition frequencies are not too crowded for order 10 kHz Rabi frequencies.
  - Demonstrated SPAM with error approaching the spontaneous metastable state decay limit, 25-level SPAM at ~99.6% fidelity.

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