

Title: Quantizing Null Hypersurfaces

Speakers: Luca Ciambelli

Series: Quantum Gravity

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Abstract: In this talk, we first present a detailed analysis of the classical geometry of generic null hypersurfaces. We then reformulate the Einstein equations as conservation laws for the intrinsic geometric data on these hypersurfaces. Following this, we derive the symplectic structure and the corresponding Poisson bracket. Upon quantizing this phase space, we propose that the projected Einstein tensor obeys the operator product expansion of the stress tensor in a conformal field theory along null time. This hypothesis is supported by explicit computations in simplified scenarios, such as the absence of radiation and within the framework of perturbative gravity.

Notably, we discover a non-vanishing central charge, which counts the local geometric degrees of freedom and diverges in the classical limit. We suggest that this central charge is a fundamental principle underlying the emergence of time in quantum gravity. If time permits, we will conclude by introducing a mesoscopic model of quantum gravity on null hypersurfaces, based on the concept of the "embadon," an operator that creates localized bits of area on cuts.

Quantizing Null Hypersurface  
ciambelli.luca@gmail.com

Quantizing Null Hyp  
ciambelli.luca@gmail.com



ULTRA-LOCALITY



hypersurface

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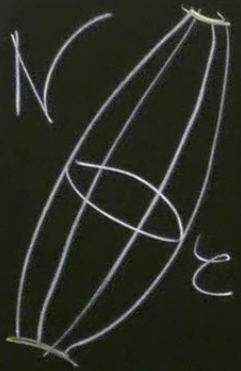
CONFORMALITY

Einstein  $\rightarrow$

1) Raychaudhuri

2) Damour

$$G \rightarrow DT=0$$



# Collision Structure

$$\theta_a^b k_b = 0$$

$$l^a \neq 0, \quad l^a q_{ab} = 0 \quad (l, q)$$

Ruling  $ka l^a = 1$   $q_a^b = \epsilon_a^b - ka l^b$

$$\theta_{ab} = \frac{1}{2} \mathcal{L}_l q_{ab} \quad \theta_a^b = \frac{1}{2} \theta q_a^b + \sigma_a^b \quad \theta_a^b q_{bc} = \theta_{ac}$$



$$\theta_{ab} = \frac{1}{2} \alpha_l \theta_{lab} \quad \theta_a^b = \frac{1}{2} \theta \theta_a^b + \sigma_a^b \quad \theta_a^b \theta_{bc} = \theta_{ac}$$

$$D_a E_N = -\omega_a E_N$$

$$E_N = k \wedge E_\varphi$$

$$\omega_a = k k_a + \pi_a$$

$$\pi_a l^a = 0$$

$$T_a^b \equiv$$

$$D_{ab} = \frac{1}{2} \alpha_l \theta_{ab} \quad \theta_a^b = \frac{1}{2} \theta \theta_{ab} + \sigma_a^b \quad \theta_a \theta_{bc} = \theta_{ac}$$

$$D_a E_N = -\omega_a E_N \quad \boxed{\mu = k + \frac{\theta}{2}}$$

$$E_N = k \wedge E_\varphi$$

$$\omega_a = k k_a + \pi_a$$

$$\pi_a l^a = 0$$

$$l^a D_b T_a^b = C = (L + \theta) \theta - \mu \theta + \sigma_a^b \sigma_b^a$$

$$q_c^a D_b T_a^b = P_c = (L + \theta) \pi_a + \dots$$

$$T_a^b =$$

Null Br

$$\boxed{D_b T_a^b}$$



$$\partial_a \eta_{bc} = \partial_{ac} \eta_b$$

$$T_a^b \equiv D_a l^b - \delta_a^b D_c l^c$$

Noll Brown-York

$$D_b T_a^b \stackrel{!}{=} T_{ab} l^b$$

$$T_{ab} = D_a l^c \eta_{bc}$$

$$T_{\mu\nu} = 2D_\mu n_\nu$$

$$+\frac{\theta}{2}$$

$$+\pi a$$

a



$$\Theta^{\text{can}} = \frac{1}{8\pi G} \left( \frac{1}{2} \pi^{ab} \delta q_{ab} - \pi_a \delta l^a \right)$$

$$T_a^b = \pi_a^b + \pi_{al}^b$$

$$\Theta^{\text{can}} = \int T^{\mu\nu} \delta g_{\mu\nu}$$

$\pi_a^b k_b = 0$

e<sup>a</sup>)

$$\Omega^{\text{con}} = \delta\theta^{\text{con}}$$

$$Q_{\xi} = \underbrace{\int_{\mathcal{V}^b} D_a T_b^a}_{\mathcal{R}} + \underbrace{\int_{\mathcal{S}^b} \epsilon_b \xi^a T_a^b}_{\mathcal{E}}$$



$$\Theta^{\text{can}} = \frac{1}{8\pi G} \left( \frac{1}{2} \pi^{ab} \delta q_{ab} - \pi_a \delta l^a \right)$$

$$T_a^b = \pi_a^b + \pi_{al}^b$$

$$\Theta^{\text{can}} = \int T^{\mu\nu} \delta g_{\mu\nu} \quad \pi_a^b k_b = 0$$

$$\xi^a = f l^a + y^b q_b^a$$

$$\delta_\xi l = [f l, l]$$

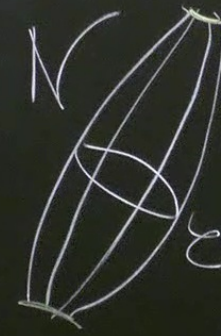


$$\Omega^{\text{con}} = \delta \theta^{\text{con}}$$

$$\int \left( \int_{\Sigma} D_a T_b^a + \int_{\Sigma} \epsilon_b \xi^a T_{ac}^b \right)$$

$$\int_{\Sigma} \lambda = 0$$

$$\lambda = \ell(\xi)$$



Ruling

$\Theta_{ab} =$

$$\int_{\Sigma} D_b$$

$$\delta \omega^{\alpha\beta} = \delta \theta^{\alpha\beta}$$

$$\int \left( \xi^b \mathcal{D}_a T_b^a + \left( \epsilon_b \xi^a T_{ab} + \frac{1}{2} \nabla_\alpha \xi^\alpha \right) \right)$$

$$\int_{\Sigma_{\lambda}} \delta_{\xi, \lambda} \mathcal{L} = 0$$

$$\lambda = \mathcal{L}(\mathcal{F})$$

$$\lambda = \partial \nu$$

$$g_{\mu\nu} \mapsto \mathcal{F} g_{\mu\nu}$$

$$g_{\mu\nu} \mapsto \omega g_{\mu\nu}$$

$$g_{\mu\nu} \rightarrow g_{\mu\nu}$$



Ruling

$\partial_{ab} =$

$\partial_{\xi}^a \mathcal{D}_b$



$$\sigma_{ab} = \frac{1}{2} (\sigma_a^c \sigma_b^d - \sigma_a^d \sigma_b^c) \quad \sigma_a = \frac{1}{2} (\sigma_a^b \sigma_b^c + \sigma_a^c \sigma_c^b) \quad \sigma_{ab} = \sigma_{ba}$$

$$Q_f = \frac{1}{8\pi G} \int_V f \epsilon + \int_{\partial V} (\Omega \partial_\nu f - f \partial_\nu \Omega)$$

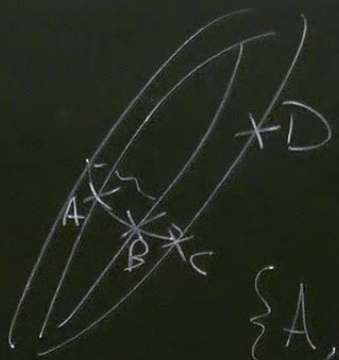
$$q_{ab} = \Omega \bar{q}_{ab} \quad \int_{\partial V} \Omega = A$$

$$q_{ab}, \bar{q}_{ab}, \mu \rightarrow (\Omega, \mu), (\sigma^{ab}, \bar{q}_{ab})$$



$$U_a^b|_{bc} = Uac$$

$$\partial_\nu \Omega \Big| \Omega^{\text{can}} = \int_V \left( \delta(\rho \sigma^{ab}) \wedge \delta q_{ab} - \delta \Omega \wedge \delta \mu \right)$$



$$\begin{aligned} \{A, B\} &= 0 \\ \{C, D\} &\neq 0 \end{aligned}$$

$$x_1 = (v_1, z_1, \bar{z}_1)$$

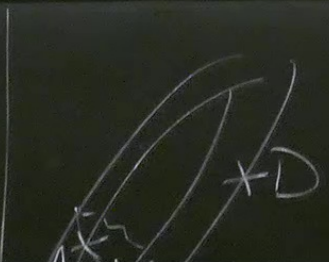
$$\{A_s^1, B_s^2\} = F(v_1, v_2) C_{is} \begin{bmatrix} \delta(z_1 - z_2) \\ \delta(\bar{z}_1 - \bar{z}_2) \end{bmatrix}$$

$$(B_i, A_j) = (\Omega, \mu, \sigma, \bar{q}, \pi, l)$$

$\delta \pi \alpha \lambda \delta \Omega^a$



$$\{C_f, C_g\} = -8\pi G C_{[f, g]}, \quad C_f = \int_{\mathbb{R}} f C \Rightarrow \{C_f, \cdot\} = \delta_f$$



$$(B, A) = (\Omega, \mu, \sigma, \bar{q}, \dots)$$

$8\pi G \Lambda \delta$

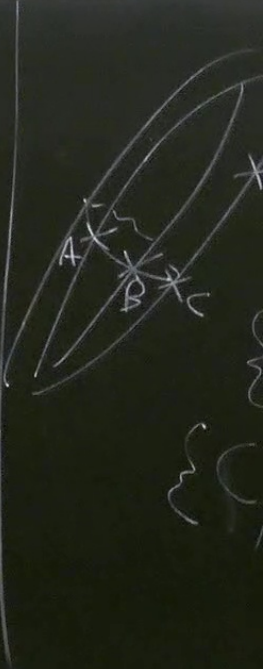
$(\Lambda, D^2)$



$$\text{"} \hat{C} \text{"} \rightarrow \frac{\hat{C}}{4G\frac{c}{\hbar}}$$

$$\langle \phi_1 \phi_2 \rangle = \delta_{12}$$

$$\hat{C}_1 \hat{C}_2 \sim \left( \frac{cN}{z(v_{12}-i\epsilon)^4} + \frac{2\hat{C}_2}{(v_{12}-i\epsilon)^2} + \frac{2v_2\hat{C}_2}{v_{12}-i\epsilon} \right) \delta(z_{12})$$



$$FC \Rightarrow \{L_f, \cdot\} = \delta_f$$

στραλδρα

$$(B, A)_i = (\Omega, \mu, \sigma, \bar{q}, \pi, l)$$

$$\{A_\alpha^1, B_\beta^2\} = F(v_1, v_2) C_{\alpha\beta} \begin{matrix} \delta(z_1 - z_2) \\ \delta(\bar{z}_1 - \bar{z}_2) \end{matrix}$$

$$i = (v_1, z_1, \bar{z}_1)$$

$$\langle \phi_1 | \phi_2 \rangle = \frac{1}{X_{12}^{2\Delta_1 \Delta_2}}$$

$$Q_{ab} = \frac{1}{2} \alpha_{\ell} Q_{ab}$$

$$Q_f = \frac{-1}{8\pi G} FC$$

$$q_{ab} = \Omega \bar{q}_{ab}$$

$$q_{ab}, \bar{q}_{ab}, \mu \rightarrow (\Omega, \mu)$$



$$\sigma_{ab} = \frac{1}{2} \epsilon_{abcd} \sigma^c \sigma^d \quad \sigma_a = \frac{1}{2} (\sigma^b \sigma^c + \sigma^c \sigma^b) \quad \sigma_a \sigma_b = \sigma_b \sigma_a$$

$$C = \partial_\nu^2 \Omega - \mu \partial_\nu \Omega \rightarrow \frac{\hat{C}}{4G\hbar}$$

$$\mathcal{E} = \sqrt{8} \sigma_{ab} \sigma_b$$

$$x_1 = v_1, z_1, \bar{z}_1 \mid \{ \hat{\Omega}_1, \hat{\mu}_2 \} = 8\pi G \delta(v_1 - v_2) \rightarrow [\hat{\Omega}_1, \hat{\mu}_2] = -i\hbar 8\pi G \delta(v_1 - v_2)$$

$$\{x, \bar{x}\} \quad \boxed{\hat{\Omega}_1} \hat{\mu}_2 \sim \frac{-4\hbar G \delta}{v_{12} - i\epsilon}$$

$$\boxed{C_{tot} = 2 + \dots}$$

$$\hat{C}_1 \hat{C}_2 \sim \left( \frac{C_{tot} \delta}{2(v_{12} - i\epsilon)^4} + \frac{2\hat{C} + 2v\hat{C}}{2(v_{12} - i\epsilon)^4} \right) \delta$$

