Title: Partitions in quantum theory

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Series: Quantum Foundations

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Abstract:

The standard perspective on subsystems in quantum theory is a bottom-up, compositional one: one starts with individual "small" systems, viewed as primary, and composes them together to form larger systems. The top-down, decompositional perspective goes the other way, starting with a "large" system and asking what it means to partition it into smaller parts. In this talk, I will 1/ argue that the adoption of the top-down perspective is the key to progress in several current areas of foundational research; and 2/ present an integrated mathematical framework for partitions into three or more subsystems, using sub-C* algebras. Concerning the first item, I will explain how the top-down perspective becomes crucial whenever the way in which a quantum system is partitioned into smaller subsystems is not unique, but might depend on the physical situation at hand. I will display how that precise feature lies at the heart of a flurry of current hot foundational topics, such as quantum causal models, Wigner's friend scenarios, superselection rules, quantum reference frames, and debates over the implementability of the quantum switch. Concerning the second item, I will argue that partitions in (finite-dimensional) quantum theory can be naturally pinned down using sub-C* algebras. Building on simple illustrative examples, I will discuss the often-overlooked existence ofÂ non-factorÂ C*-algebras, and how it leads to numerous subtleties -- in particular a generic failure of local tomography. I will introduce a sound framework for quantum partitions that overcomes these challenges; it is the first top-down framework that allows to consider three or more subsystems. Finally, as a display of this framework's technical power, I will briefly present how its application to quantum causal modelling unlocked the proof that all 1D quantum cellular automata admit causal decompositions.
(This is joint work with Octave Mestoudjian and Pablo Arrighi. This talk is complementary to my Causalworlds 2024 presentation, which will focus

on the issue of causal decompositions.)

Partitions in quantum theory

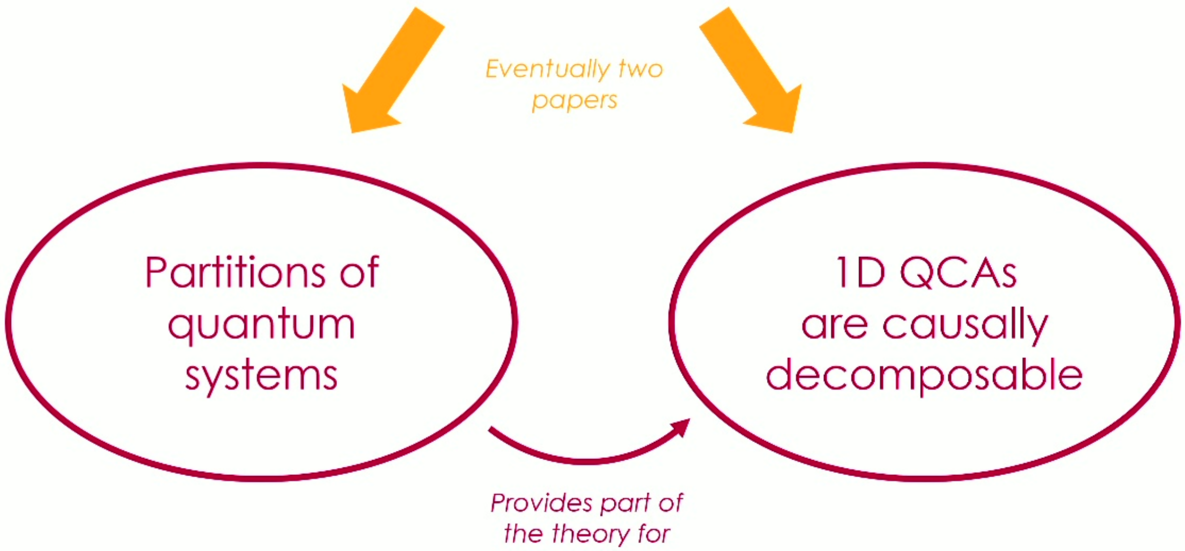
(why you should care about them, and how to)

Augustin Vanrietvelde
Perimeter Institute
September 12th, 2024

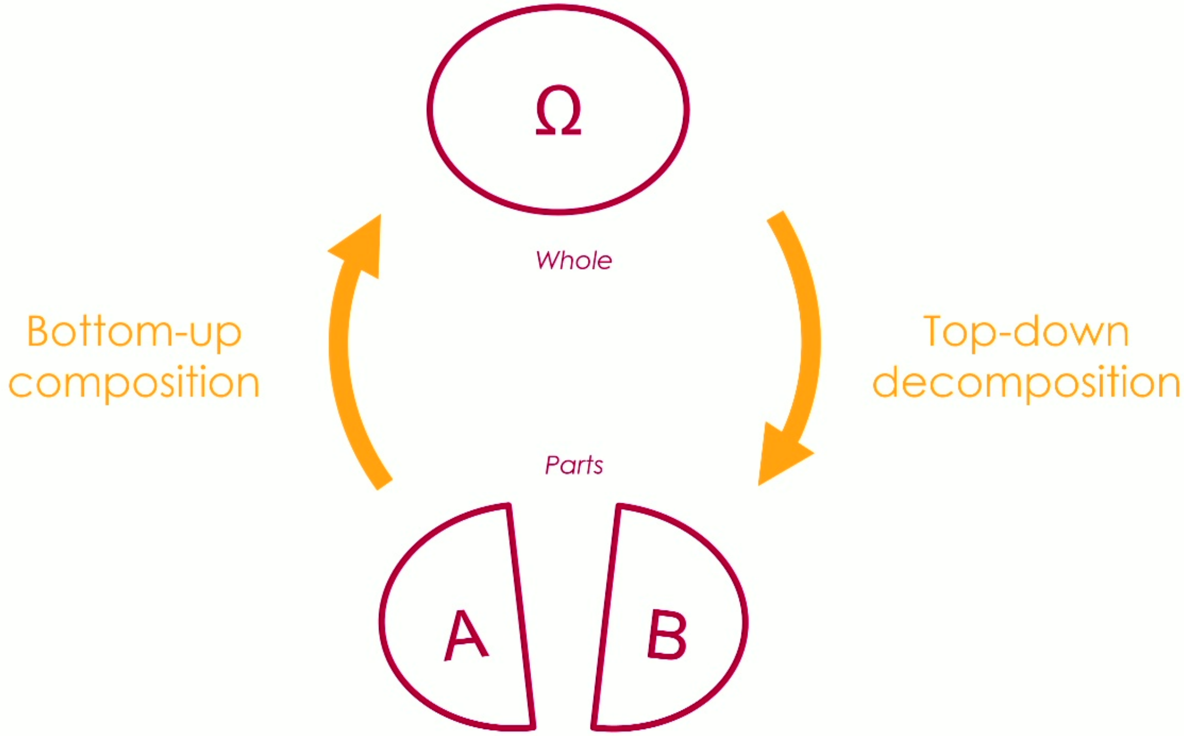
Pablo Arrighi

+ Octave Mestoudjian

Ongoing work



A bit of mereology



1. Why you should think top-down

2. How to do it

3. An application: causal decompositions

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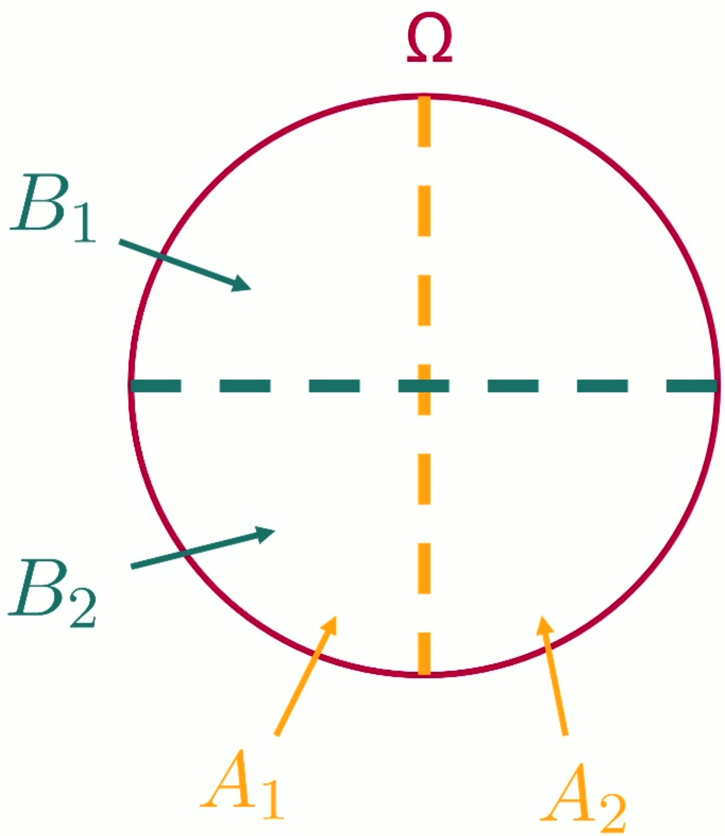
Comparing partitions



Several different ways of partitioning the same system

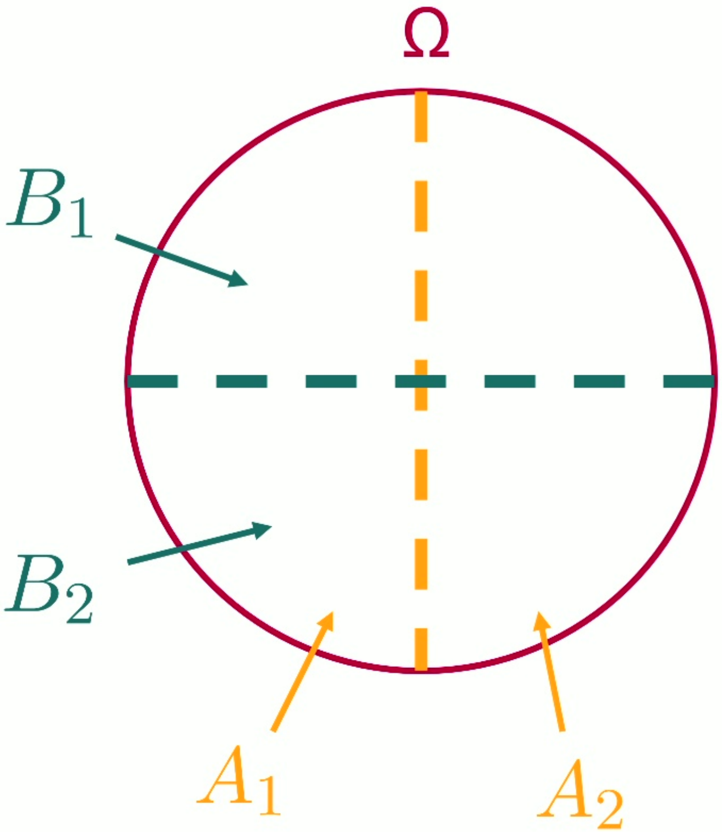
How do these interplay?

Inclusions?

Common fine-graining?

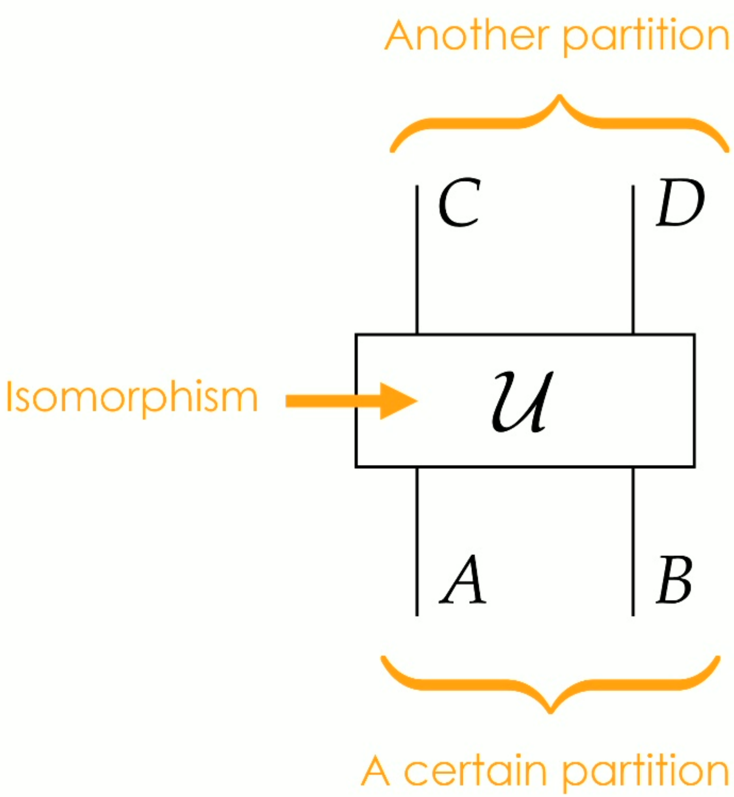
Symmetries?

Comparing partitions



Studying the interplay of partitions requires the top-down perspective

Example 1: quantum causal structure

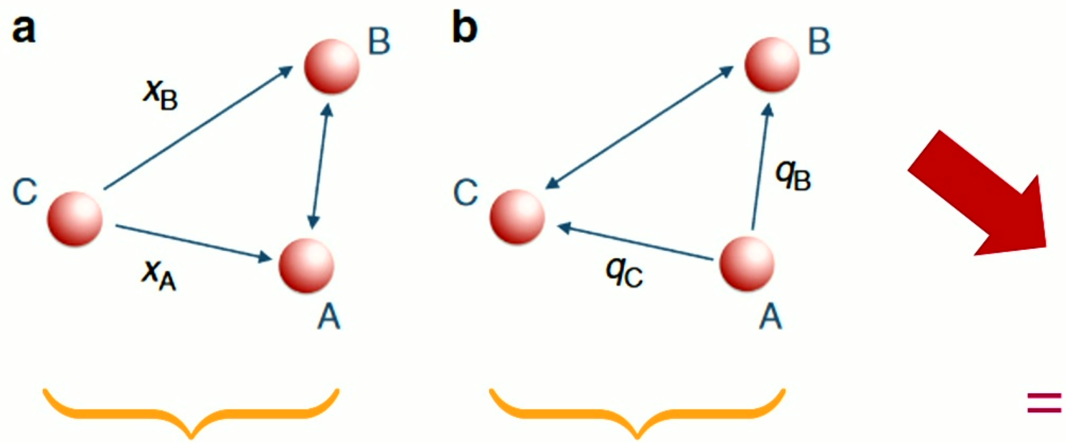


"Passive picture": U translates between two partitions of the same system

How these relate

= U's causal structure

Example 2: quantum reference frames



Partition into
C, A | C, B | C

Partition into
A, B | A, C | A

How these relate

= QRF transformations

Giacomini, Castro-Ruiz, Brukner, "Quantum mechanics and the covariance of physical laws in quantum reference frames", 2017 (1712.07207)

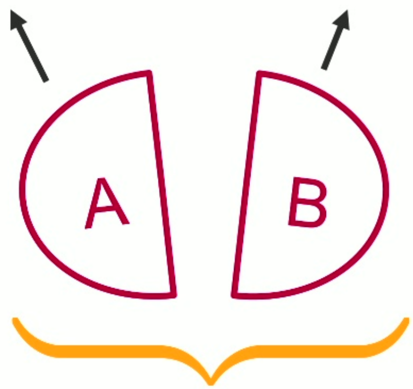
Castro-Ruiz & Oreshkov, "Relative subsystems and quantum reference frame transformations", 2021 (2110.13199)

Example 3: superselection rules

“I can’t modify / measure superpositions of the **total** charge of a quantity Q”

(fermionic parity, energy, momentum…)

Can’t change Q_A Can’t change Q_B

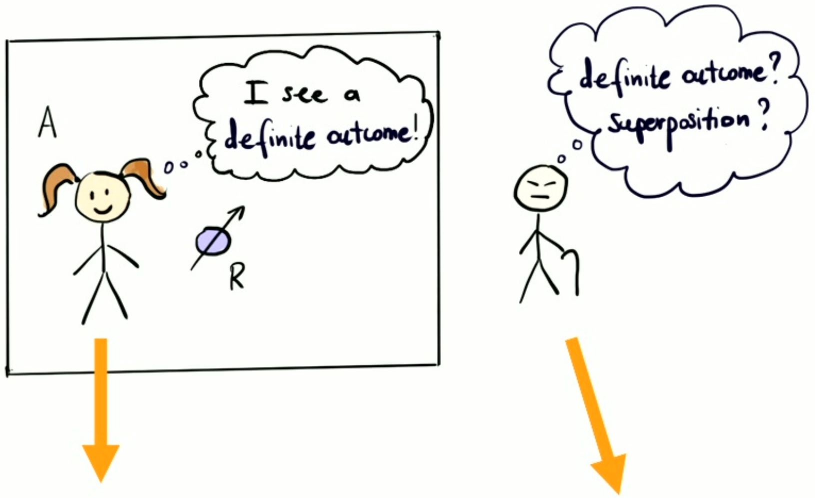


Global operations can modify local charges!
(as long as they preserve the global one)

→ Non-standard subsystems

Example 4: Wigner's friend scenarios

(drawing from Fraser et al., 2009.00321)



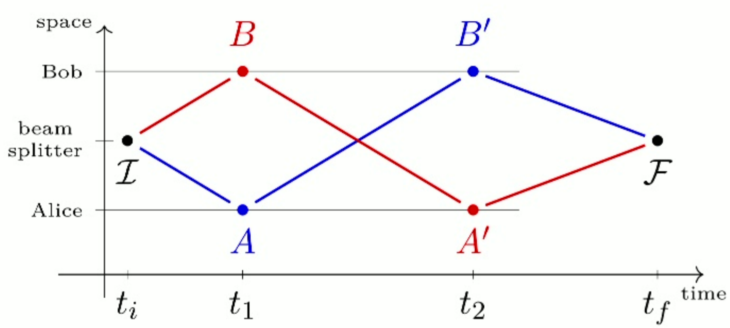
Splits the world into A and R

Splits the world into AR and W

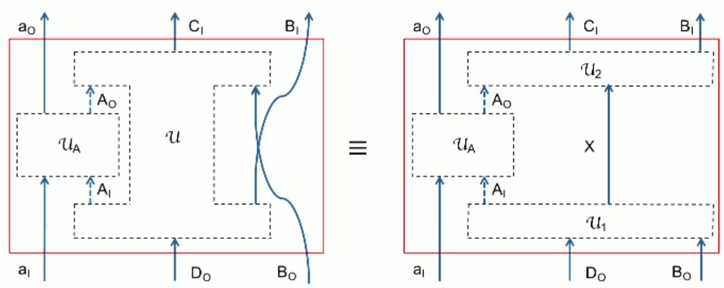
Maybe paradoxes can be understood as partitions incompatibilities?

Example 5: the Switch controversy

(Paunkovic & Vojinovic, 1905.09682)



(Oreshkov, 1801.07594)



"Events are spacetime points!"

"Events are time-delocalised subsystems!"

Different ways to partition the world into loci of intervention

Boils down to a theory of subsystems not only in space, but also in time!

Why you should think top down

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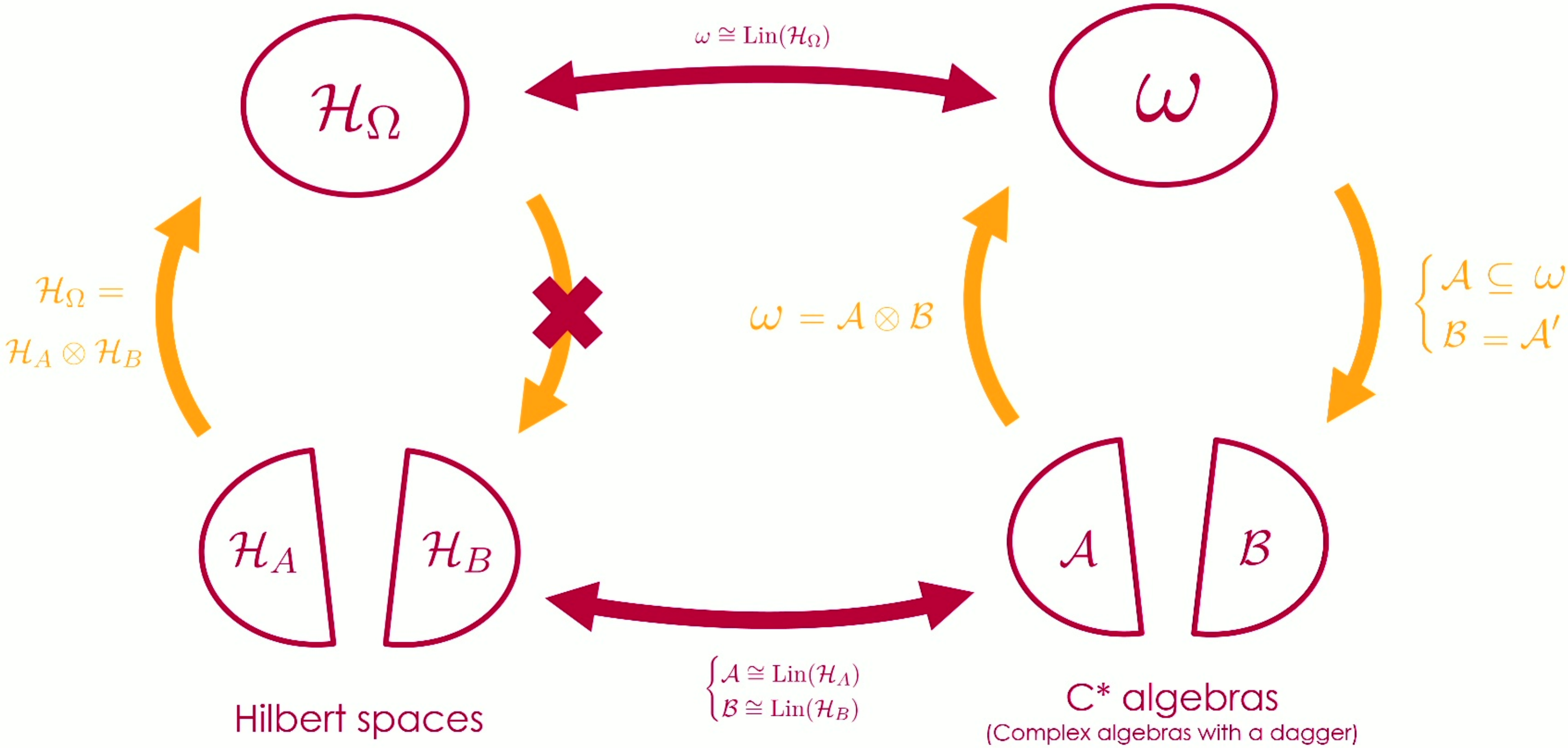
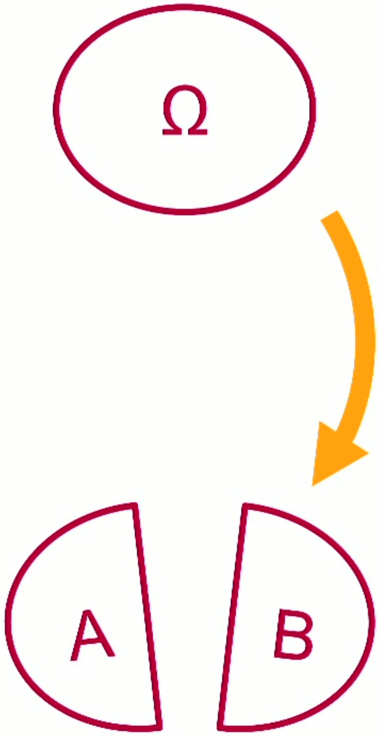
How to do it

A bit of quantum mereology



Subsystems as sub-C* algebras

- A **subsystem** = a sub-C* algebra
- A **bipartition** = two sub-C* algebras that are each other's commutant



Definition 2.2 (Bipartitions of factors). *Let ω be a factor. We say that a pair of two sub-C^* algebras $(\mathcal{A}_1, \mathcal{A}_2)$ of it forms a bipartition if*

$$\mathcal{A}_1 = \mathcal{A}_2' \,. \tag{7}$$

(see e.g. algebraic QFT, quantum error-correction, operational approaches…)

Chiribella, "Agents, subsystems, and the conservation of information", 2018 (1804.01943)

What does a C* algebra look like?

A C* algebra can be:

- A **factor** → isomorphic to an algebra of matrices
- Not a factor → isomorphic to an algebra of **block-diagonal** matrices

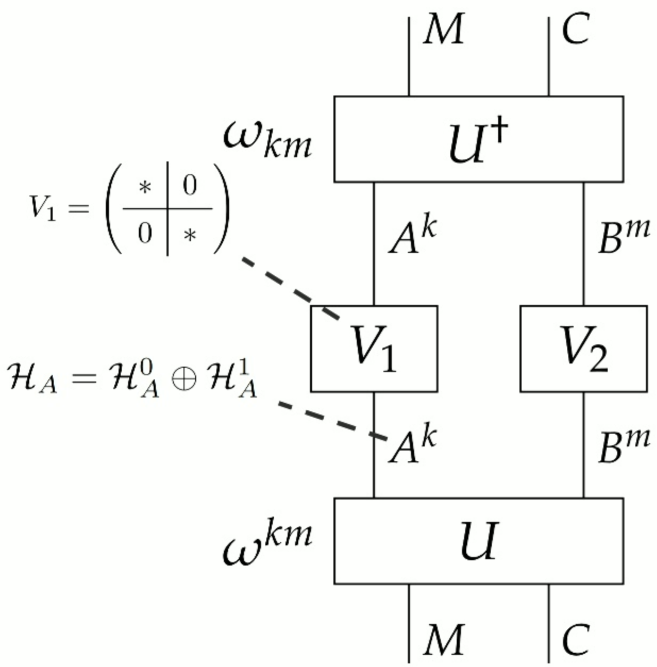
$$\left(\begin{array}{c|c} * & 0 \\ \hline 0 & * \end{array}\right)$$

A C-algebra of block-diagonal matrices*

The **centre** $\mathcal{Z}(\omega)$ of a C* algebra is the set of elements that commute with all of it.

- Non-factor C* algebras are those with a non-trivial centre
- The centre is spanned by the projectors onto the algebra's blocks

An example of non-factor subsystems



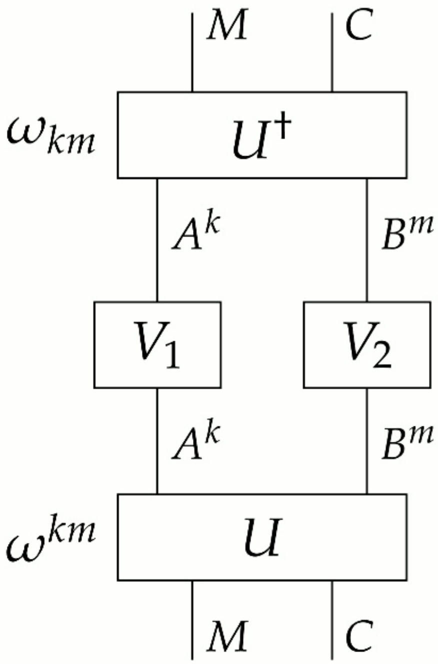
Superposition of trajectories

If we believe that A&B are acting on subsystems, then these have to correspond to <u>non-factor</u> sub-C* algebras.

(Can be linked with partial classicality / superselection rules, but with subtleties!!)

AV, "Routed quantum circuits: an extended framework for coherent control and indefinite causal order", 2022 (PhD thesis)

A disturbing feature: Failure Of Local Tomography (FOLT)



Mathematical level

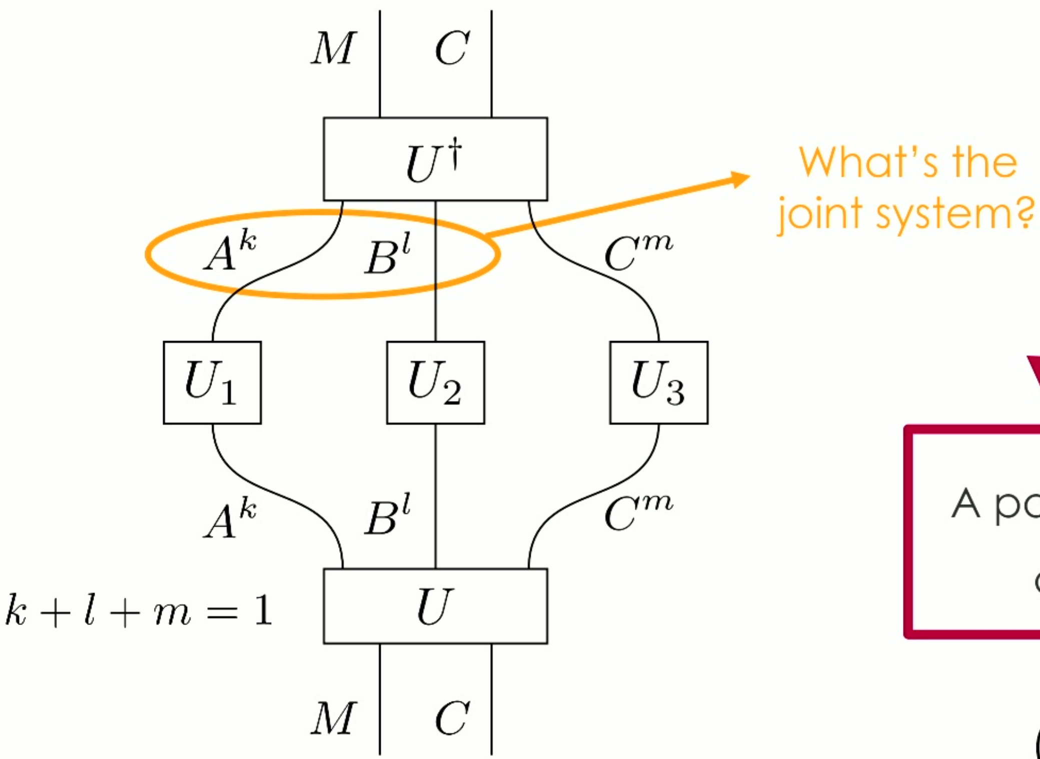
The algebraic span of $\mathcal{A}$ and $\mathcal{B}$ is not equal to the whole algebra.

⇕

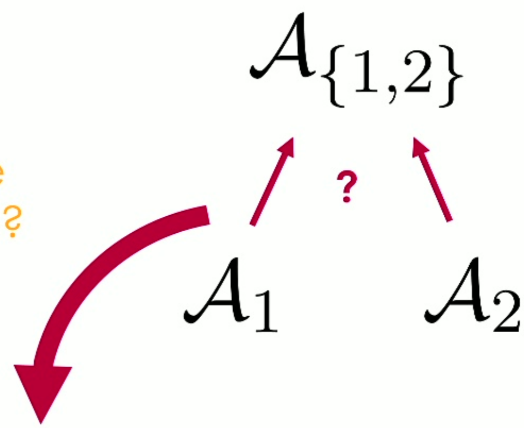
Operational level

The relative phase between paths is inaccessible to A, B, and even to their correlations, while it is accessible to a global agent.

Challenge 1: composite systems for ≥ 3 parts



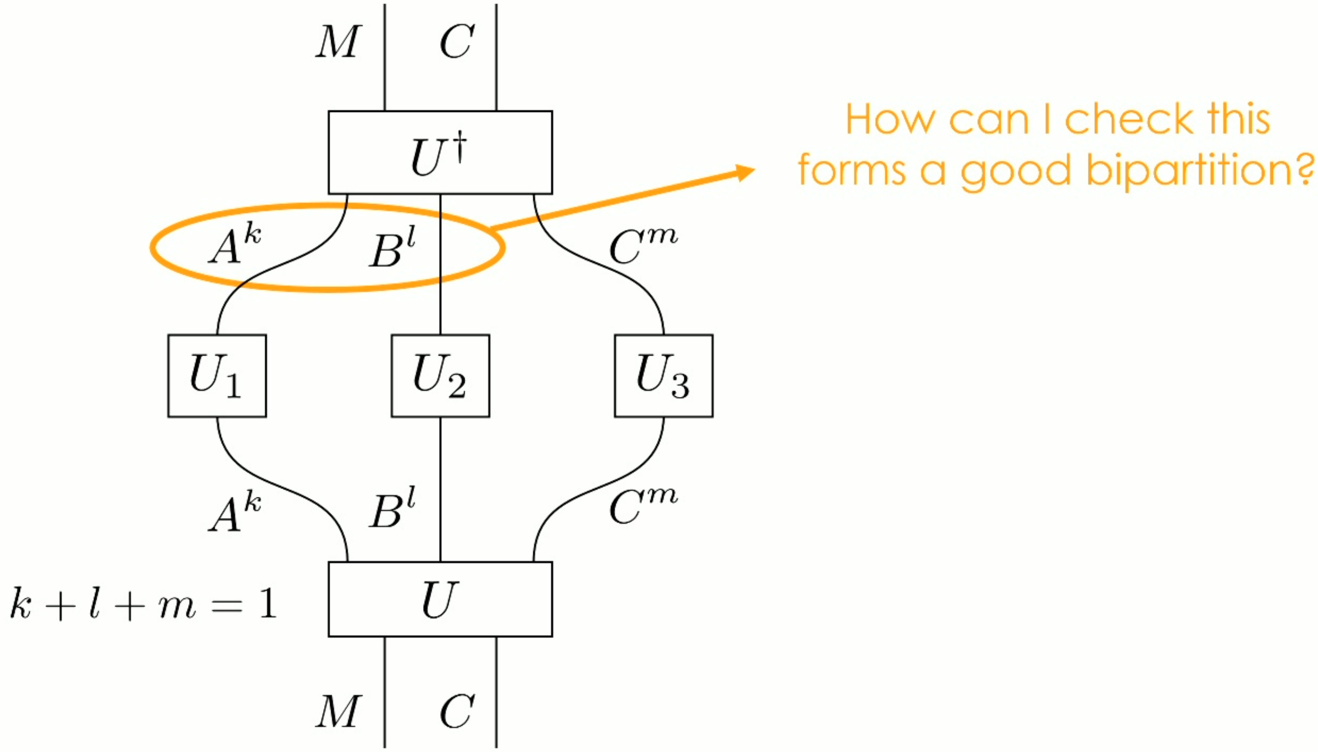
Superposition of 3 trajectories



A partition has to be defined by assigning an algebra to every **set** of indices

$$(\mathcal{A}_x)_{x\in X} \Rightarrow (\mathcal{A}_S)_{S\subseteq X}$$

Challenge 2: partitioning non-factor algebras $\left(\omega \cong \bigoplus_{k \in K} \mathrm{Lin}\left(\mathcal{H}_{\Omega}^{k}\right)\right)$



Superposition of 3 trajectories

Challenge 2: partitioning non-factor algebras $\left(\omega \cong \bigoplus_{k\in K} \mathrm{Lin}\left(\mathcal{H}_\Omega^k\right)\right)$

Standard definition

$$\mathcal{A}_1 = \mathcal{A}_2'$$

Standard def is not suitable due to centre issues

Better definition

$$\begin{cases} \forall k \in K,\ \pi^k \mathcal{A}_1' = \pi^k \mathcal{A}_2\,; \\ \mathcal{Z}(\omega) \subseteq \mathcal{Z}(\mathcal{A}_1) \vee \mathcal{Z}(\mathcal{A}_2) \end{cases}$$

"I have a standard partition within each block of ω"

"The parts allow to recover the whole centre of ω"

Defining multipartitions (at last)

→ Just rely on bipartitions!

Definition

Definition 4.2 (Partitions). *Let $\mathcal{W}$ be a C^* algebra. A* <u>*partition*</u> *of it, labelled by the finite set X, is a mapping*

$$\begin{aligned} \mathcal{A}:\ \mathcal{P}(X) &\to \mathrm{Sub}\,(\mathcal{W}) \\ S &\mapsto \mathcal{A}_S\,, \end{aligned} \tag{42}$$

satisfying the following conditions:

$$\mathcal{A}_X = \mathcal{W}\,; \tag{43}$$

$$\mathcal{A}_\emptyset = \{\lambda \mathbb{1} \,|\, \lambda \in \mathbb{C}\}\,; \tag{44}$$

$$\forall S, T \subseteq X,\ \text{disjoint},\ (\mathcal{A}_S, \mathcal{A}_T)\ \text{forms a bipartition of}\ \mathcal{A}_{S \sqcup T}\,. \tag{45}$$

Back to Hilbert spaces?

Partitioned C* algebra

Concrete representation?

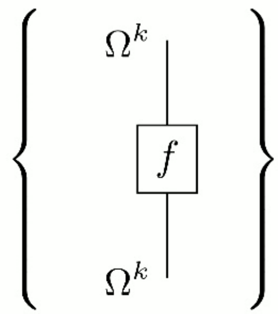
Partitioned Hilbert space

→ Requires **routed circuits** to deal with the non-factorness

Maps are supplemented by *routes:*
Boolean matrices encoding constraints on whether they can relate some input and output sectors

e.g. to represent a non-factor algebra

$\omega \cong$



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Example: representing the superposition of 3 trajectories

→ Represent as operators on

$$\mathcal{H}_\Omega := \bigoplus_{k_1,k_2,k_3 \text{ such that } \sigma_{\vec{k}}=1} \mathcal{H}_{A_1^{k_1}} \otimes \mathcal{H}_{A_2^{k_2}} \otimes \mathcal{H}_{A_3^{k_3}}$$

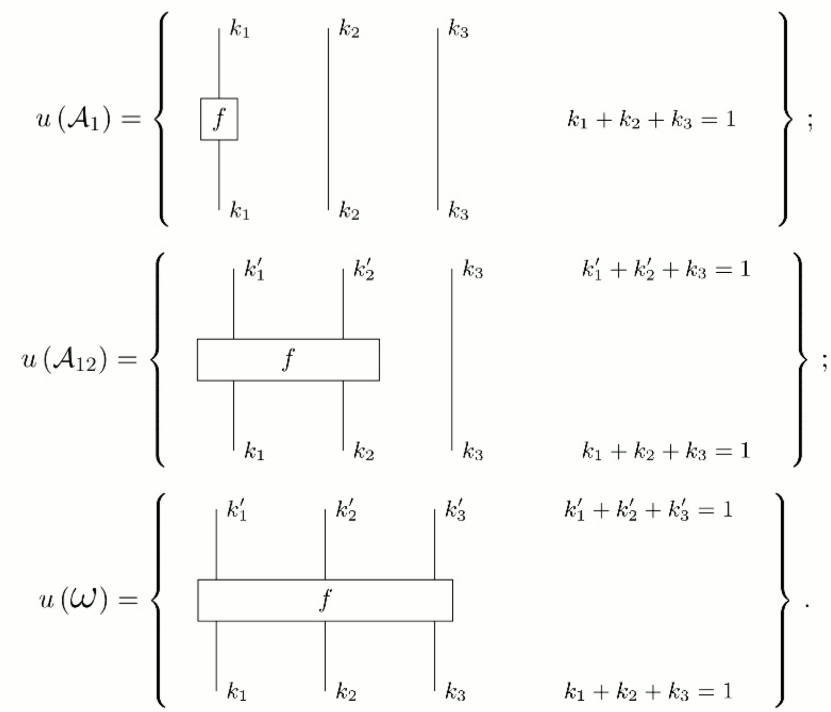
Sectorial correlations

$$\forall k_1, k_2, k_3 \in \{0,1\}, \quad \sigma_{\vec{k}} := \begin{cases} 1 \text{ if exactly one of the } k_n\text{'s is equal to } 1, \\ 0 \text{ otherwise,} \end{cases}$$

… but **embed it** to get a nice tensor product:

$$\mathcal{H}_\Omega \subseteq \mathcal{H}_\Omega^{\text{ext}} := \underbrace{\left(\bigoplus_{k_1 \in \{0,1\}} \mathcal{H}_{A_1^{k_1}} \right)}_{=: \mathcal{H}_{A_1}} \otimes \underbrace{\left(\bigoplus_{k_2 \in \{0,1\}} \mathcal{H}_{A_2^{k_2}} \right)}_{=: \mathcal{H}_{A_2}} \otimes \underbrace{\left(\bigoplus_{k_3 \in \{0,1\}} \mathcal{H}_{A_3^{k_3}} \right)}_{=: \mathcal{H}_{A_3}}.$$

→ This yields a representation in which the algebras are localised:



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Is there always a representation?

For any partition, one can find a representation using routed circuits, in which all the **individual systems' algebras** are localised

i.e. $\mathcal{A}_1$, $\mathcal{A}_2$, etc.

… But it is sometimes impossible to find a representation in which the **composite systems' algebras** are also all localised!

i.e. those like $\mathcal{A}_{\{1,2\}}$

→ A typical example is the case of ≥ 3 fermionic modes

Friis, "Reasonable fermionic quantum information theories require relativity", 2015 (1502.04476)

Why you should think top down

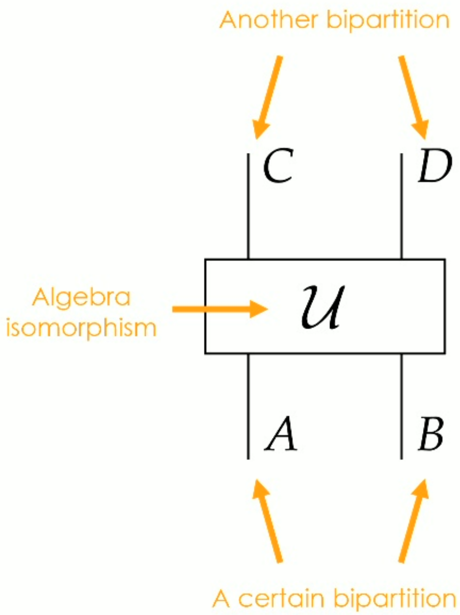
↓

How to do it

↓

An application: causal decompositions

A better view of quantum causal structure

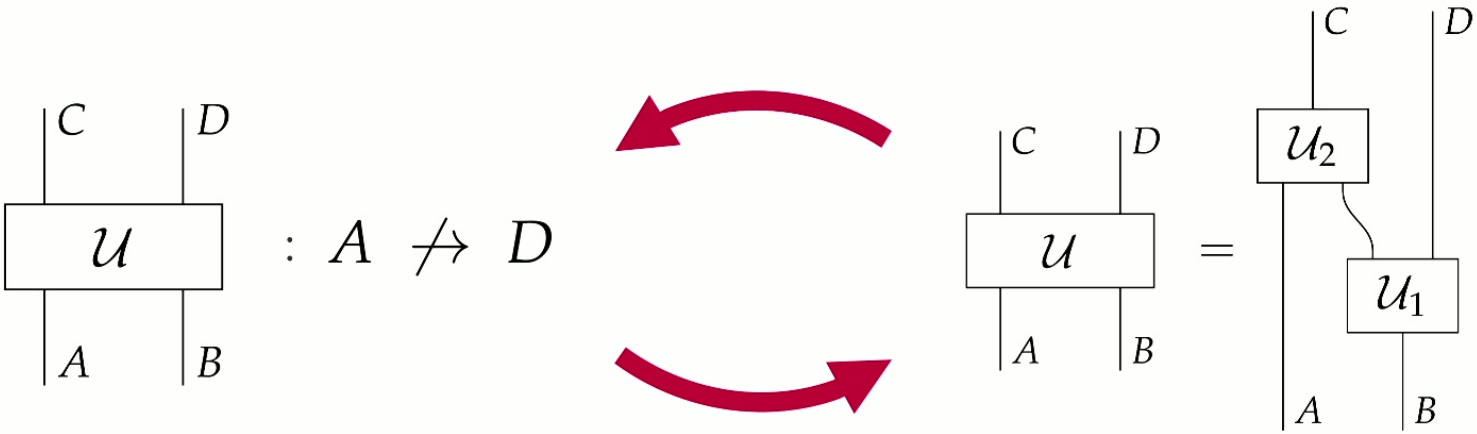


"Passive picture": U specifies the relation between two bipartitions of the same global algebra

Re-expressing causality

$$A \not\to D \iff [\mathcal{A}, \mathcal{D}] = 0$$

Causal decompositions: one-way causation

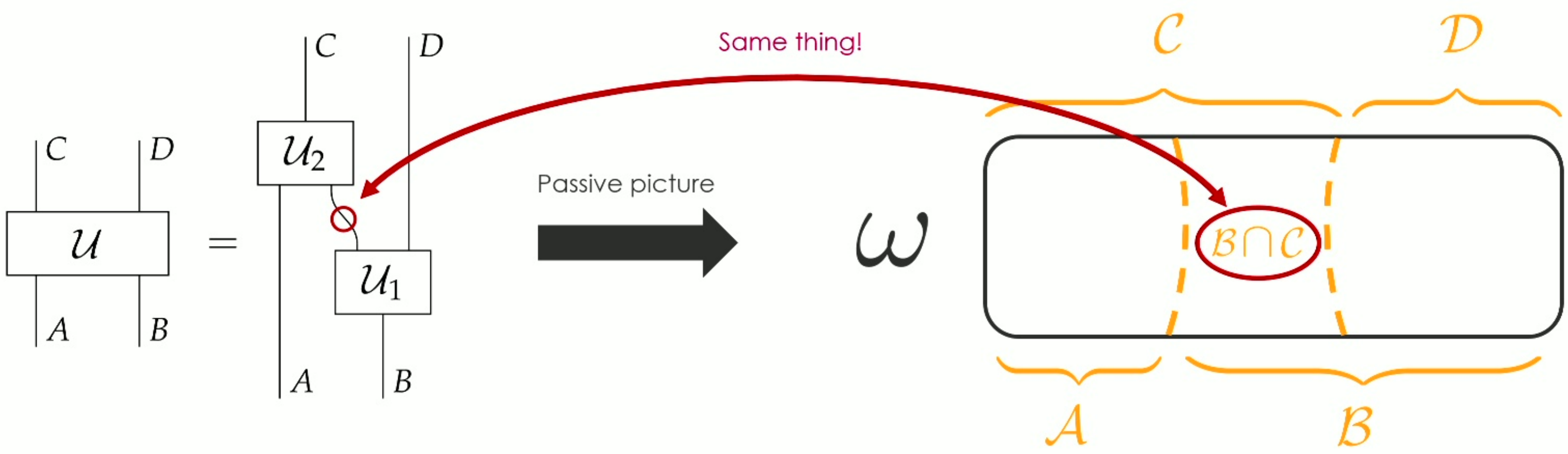


Causal structure
(operational / phenomenological)

Compositional structure
(mathematical / structural)

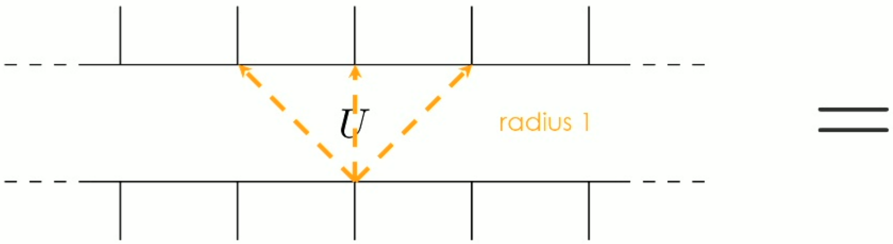
Eggeling, Schlingemann and Werner, "Semicausal operations are semilocalizable", 2001 (quant-ph/0104027)

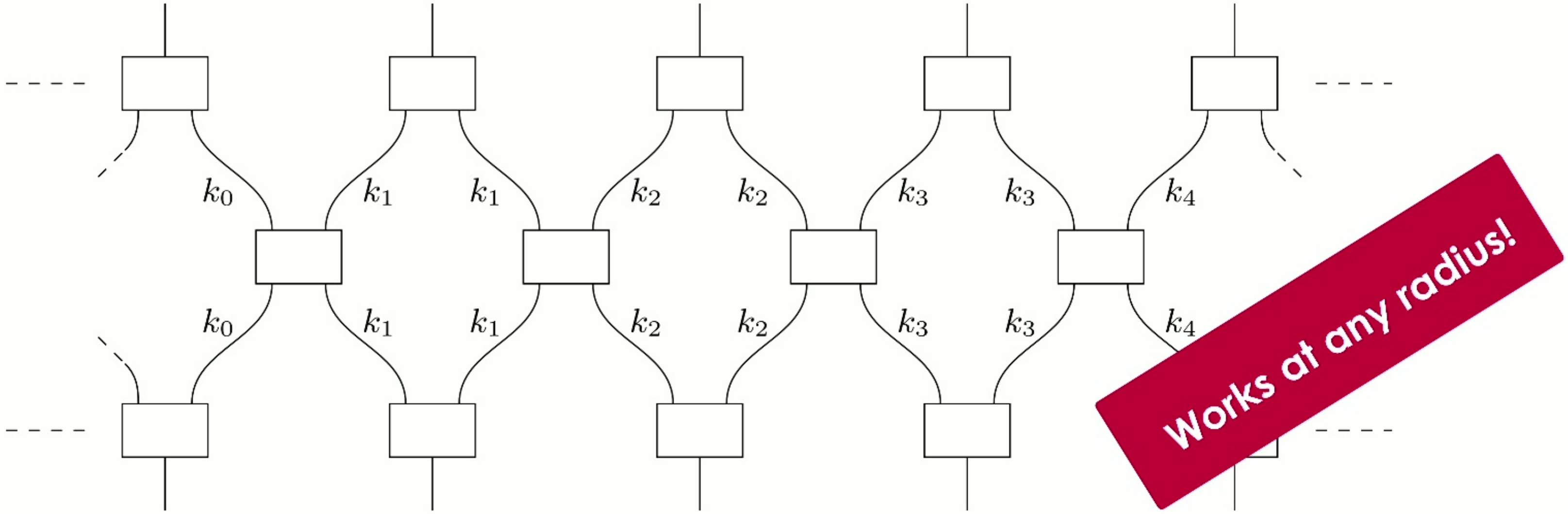
Causal decompositions as repartitioning



Causal decompositions are a problem of **fine-graining partitions**

Our result: all 1D QCAs are causally decomposable





Some lessons from this talk

Are there several possible partitions of your system? Then you should work top-down

Partitions yield a surprisingly rich mathematical structure

Quantum causal structure is about the interplay of different partitions

We would be wise to pay more attention to **non-factor C* algebras**

Prospects

Applying it

- Quantum causal modelling
- Quantum reference frames
- Superselection rules
- Wigner's friend scenarios
- Time-delocalised subsystems
- … and more?


1. Why you should think top-down

2. How to do it

3. An application: causal decompositions

Questions?

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