Title: Celestial Holography from Euclidean AdS space.

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Abstract: We will explore the connection between Celestial and Euclidean Anti-de Sitter (EAdS) holography in the massive scalar case. Specifically, exploiting the so-called hyperbolic foliation of Minkowski space-time, we will show that each contribution to massive Celestial correlators can be reformulated as a linear combination of contributions to corresponding massive Witten correlators in EAdS. This result will be demonstrated explicitly both for contact diagrams and for the four-point particle exchange diagram, and it extends to all orders in perturbation theory by leveraging the bootstrapping properties of the Celestial CFT (CCFT).Â Within this framework, the Kantorovic-Lebedev transform plays a central role, which will be introduced at the end of the talk. This transform will allow us to make broader considerations regarding non-perturbative properties of a CCFT.Â

Celestial Holography from Euclidean AdS space

The role of the Kantorovic-Lebedv transform in Celestial Holography

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Key Points of the Presentation:

In this presentation we are going to:

- Analyze Celestial holography in the massive scalar case from a bulk perspective;
- Compute Celestial contact diagrams and the one-particle exchange diagram;
- Reformulate Celestial correlators in terms of corresponding EAdS correlators;
- Introduce the Kantorovich-Lebedev transform and discuss its role in Celestial holography.

References:

- Iacobacci, L., Sleight, C. & Taronna, M. From celestial correlators to AdS, and back. J. High Energ. Phys. 2023, 53 (2023);
- Iacobacci, L., Sleight, C. & Taronna, M. Celestial holography revisited. Part II. Correlators and Källén-Lehmann. J. High Energ. Phys. 2024, 33 (2024).

Minkowski's Hyperbolic Foliation

In $(d+2)$-dimensions, $X^\mu \in \mathbb{M}^{d+2}$, $\mu = 0, \ldots, d+1$.
Define $X^\pm = X^0 \pm X^{d+1}$.

- In the region $\mathscr{A}_\pm$:

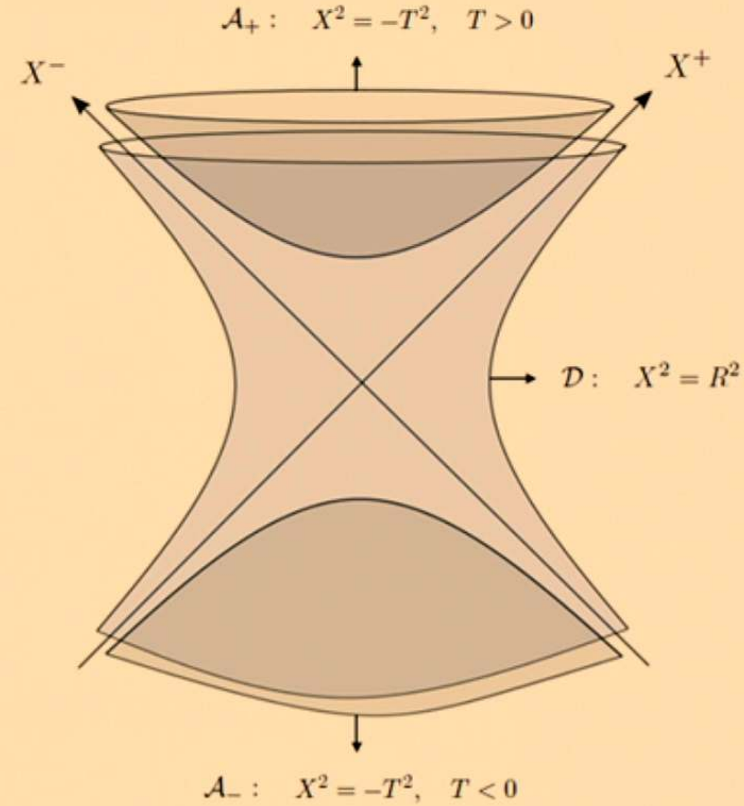
$$X^\pm > 0, \qquad X^2 = -T^2, \quad T \gtrless 0,$$
$$\mathrm{d}s^2_{\mathscr{A}_\pm} = -\mathrm{d}T^2 + T^2 \mathrm{d}s^2_{\mathscr{H}^+_{d+1}}.$$

- In the region $\mathscr{D}$:

$$X^2 = R^2, \quad R > 0,$$
$$\mathrm{d}s^2_{\mathscr{D}} = \mathrm{d}R^2 + R^2 \mathrm{d}s^2_{dS_{d+1}}.$$

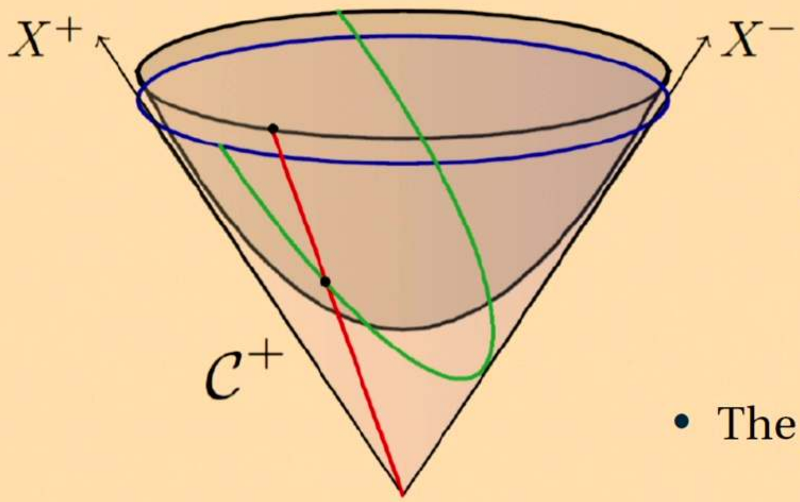
- We can further divide the region $\mathscr{D}$.

$$\mathscr{D}_\pm : \qquad X^+ = X^0 + X^{d+1} \gtrless 0.$$



EAdS Conformal Compactification

- In Poincaré coordinates, a point $\hat{X}$ on the hyperboloid $\mathscr{H}^+_{d+1}$ is parameterized as follows:



$$\hat{X} = \left(\frac{1+y^2+|\vec{\omega}|^2}{2y}, \frac{1-y^2-|\vec{\omega}|^2}{2y}, \frac{\vec{\omega}}{y} \right)^T, \qquad (1)$$

where $\vec{\omega} \in \mathbb{R}^d$ and $y > 0$. The metric is

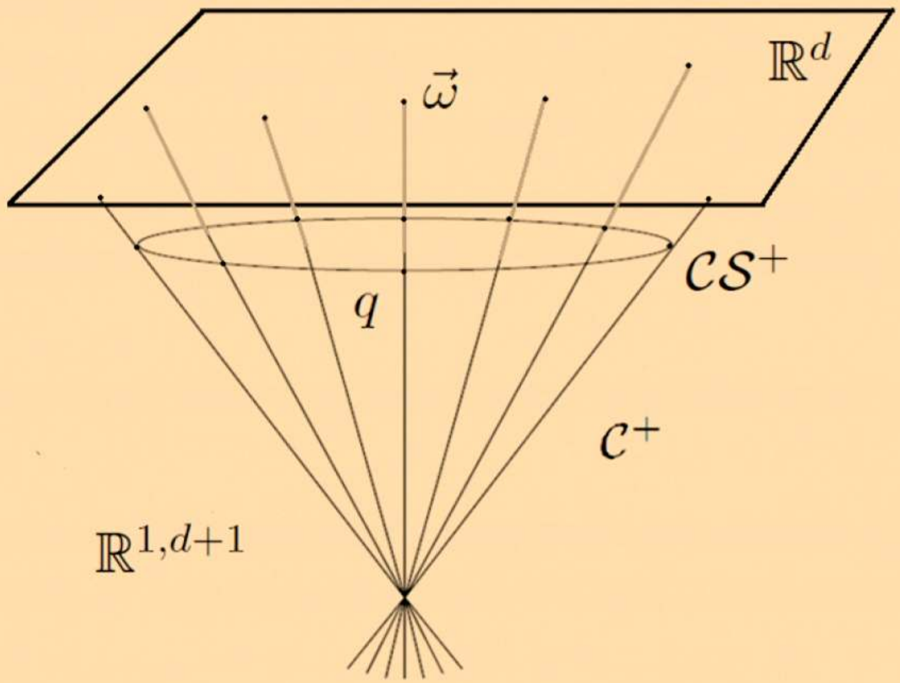
$$\mathrm{d}s^2 = y^{-2}[\mathrm{d}y^2 + \mathrm{d}\vec{\omega} \cdot \mathrm{d}\vec{\omega}]. \qquad (2)$$

- The Poincaré section of $\mathscr{C}^+$ is shown in green:

$$Q_+ := \lim_{y \to 0^+} y\hat{X} = \left(\frac{1+|\vec{\omega}|^2}{2}, \frac{1-|\vec{\omega}|^2}{2}, \vec{\omega} \right)^T, \qquad \mathrm{d}s^2_{\mathbb{E}} := \lim_{y \to 0^+} y^2 \mathrm{d}s^2 = \mathrm{d}\vec{\omega} \cdot \mathrm{d}\vec{\omega}. \quad (3)$$

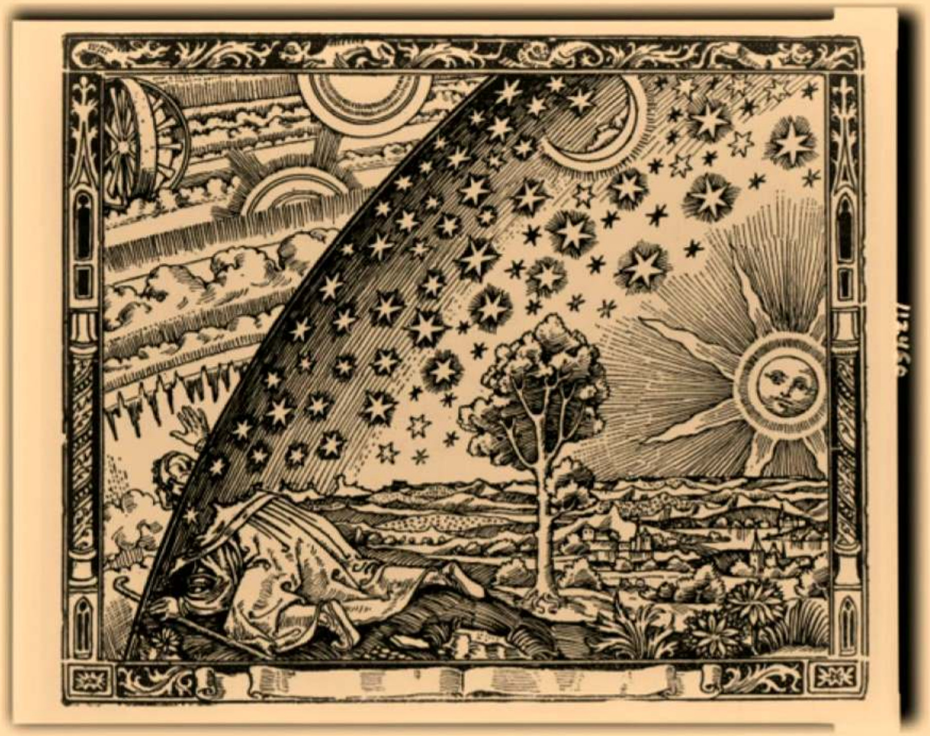
- Points on the the Celestial Sphere (in blue) are in one-to-one correspondence with points on the Poincaré section of $\mathscr{C}^+$ through a projective map (in red).

The Celestial Sphere as the Projective space



- The future (past) Celestial Sphere $\mathscr{CS}^{\pm}$ is the set of future-directed (past-directed) light-rays passing through the origin;
- $\mathscr{CS}^{\pm}$ can be visualized as the sphere that each light-ray intersects in one point at infinity;
- Points $Q_{\pm} \in \mathscr{CS}^{\pm}$ are mapped into $\vec{\omega} \in \mathbb{R}^d$ by a projective map;
- $Q_{\pm}(\vec{\omega})$ transforms as a scalar conformal primary operator of conformal weight 1 under the action of SO(1, d+1).

Celestial Holography: An Introduction



- Celestial Holography establishes a duality between a QFT in asymptotically flat space-time and a CFT defined on $\mathscr{CS}$;
- In this description, external states are manifestly packed into unitary irreducible representations (UIR) of SO(1, d+1);
- SO(1, d+1) acts on $\mathscr{CS}$ as the d-dimensional Euclidean Conformal group $\Longrightarrow$ the boundary CFT is Euclidean;
- We will refer to the boundary theory as a *Celestial CFT* (CCFT);
- In the following, we will focus our attention on the <u>massive scalar case</u>.

Conformal Primary Wavefunction

- A *new description* of QFT is found by employing *conformal primary wavefunctions*;
- *Conformal primary wavefunctions,* $\phi_\Delta(X; Q(\vec{\omega}))$, are solutions of the relativistic equation of motion labeled by a boundary point $Q(\vec{\omega})$ and a conformal weight Δ;
- Under the action of SO(1, d+1), they transform covariantly as Lorentz tensors with respect to X and as conformal primary tensors of conformal weight Δ with respect to $\vec{\omega}$.

References:

[Pasterski, Shao, Strominger 2016], → spin 0, 1, 2 in arbitrary d;
[Law, Zlotnikov 2020] → massive arbitrary integer spin in $4d$;
[Iacobacci, Mück 2020] → Dirac spinors in arbitrary d;
[Narayanana 2020] → Dirac and Rarita-Schwinger fields in $4d$.

Scalar Conformal Primary Basis

- The massive conformal primary wavefunctions in the scalar case are defined as follows ([Pasterski, Shao 2017]):

$$\phi^{\pm}_{\Delta}(X;Q_{\pm}) = \mathcal{N}_{\Delta} \int_{\mathcal{H}^{\pm}_{d+1}} [\mathrm{d}\hat{p}]\, K^{\mathrm{AdS}}_{\Delta}(\hat{p};Q_{\pm})\, \mathrm{e}^{im\hat{p}\cdot X}, \tag{4}$$

where $p = m\hat{p}$, $Q_{\pm} \in \mathcal{CS}^{\pm}$ and $K^{\mathrm{AdS}}_{\Delta}(\hat{p};Q) = \left(-2Q\cdot\hat{p}\right)^{-\Delta}$ is the scalar bulk-to-boundary propagator in $\mathcal{H}^{\pm}_{d+1}$.
- Massive conformal primary wavefunctions form a δ-orthogonal basis for $\Delta \in \frac{d}{2} + i\mathbb{R}^{+}$;
- The massless scalar conformal primary wavefunctions take the form of Mellin transform of plane waves

$$\varphi^{\pm}_{\Delta}(X;Q_{\pm}) = \mathcal{N}_{\Delta} \int_{0}^{+\infty} \mathrm{d}\omega\, \omega^{\Delta-1}\, \mathrm{e}^{i\omega Q_{\pm}\cdot X}; \tag{5}$$

- They form a basis for $\Delta \in \frac{d}{2} + i\mathbb{R}$.

Massive Celestial Amplitudes

- In the scalar case, massive Celestial Amplitudes are related to momentum Amplitudes as follows [Pasterki, Shao, Strominger 2016]:

$$\widetilde{\mathscr{A}}_n(\Delta_i, Q_{\pm,i}) = \left(\prod_{k=1}^{n} \int_{\mathscr{H}^{\pm}_{d+1}} [\mathrm{d}\hat{p}_k]\, K^{\mathrm{AdS}}_{\Delta_k}(\hat{p}_k; Q_{\pm,k}) \right) \mathscr{A}_n(\pm m_k \hat{p}_k) \tag{6}$$

- The authors used this formula to compute the Celestial contact 3-point function for two incoming particles with mass $m_1 = m_2 = m$, and one outgoing particle with mass $m_3 = 2m(1+\epsilon)$.

- They found that the Celestial 3-point contact amplitude is proportional to the 3-point contact diagram on EAdS at lowest order in ϵ, which is $\sqrt{\epsilon}$

Closed Expression of Conformal Primary Wavefunctions

- The integral defining the massive conformal primary wavefunctions

$$\phi^{\pm}_{\Delta}(X;Q_{\pm}) = \mathcal{N}_{\Delta}\int_{\mathcal{H}^{\pm}_{d+1}}[\mathrm{d}\hat{p}]\,K^{\mathrm{AdS}}_{\Delta}(\hat{p};Q_{\pm})\,\mathrm{e}^{im\hat{p}\cdot X}, \tag{7}$$

is divergent $\implies$ We need to insert a regulator!

- We start from the convergent integrals

$$\psi^{\pm}_{\Delta}(X;Q_{\pm}) = \mathcal{N}_{\Delta}\int_{\mathcal{H}^{\pm}_{d+1}}[\mathrm{d}\hat{p}]\,K^{\mathrm{AdS}}_{\Delta}(\hat{p};Q_{\pm})\,\mathrm{e}^{m\hat{p}\cdot X}, \qquad X\in\mathcal{A}_{\pm}. \tag{8}$$

- Setting $X=\tau\hat{X}$, with $\hat{X}\in\mathcal{H}^{+}_{d+1}$ and $\tau \gtrless 0$ in $\mathcal{A}_{\pm}$, we found the result

$$\psi^{\pm}_{\Delta}(\tau,\hat{X};Q_{\pm}) = \mathcal{N}_{\Delta}\,K^{\mathrm{AdS}}_{\Delta}(\pm\hat{X};Q_{\pm})\widetilde{K}_{\Delta-\frac{d}{2}}(\pm m\tau), \quad \widetilde{K}_{\Delta-\frac{d}{2}}(\pm m\tau) = \frac{2\tau^{-d/2}}{\Gamma(\Delta-\frac{d}{2})}K_{\Delta-\frac{d}{2}}(\pm m\tau). \tag{9}$$

in the region $\mathcal{A}_{\pm}$.

Closed Expression of Conformal Primary Wavefunctions

- Rotate continuously τ in the complex plane,

$$\tau \to \mathrm{e}^{i\theta} T \implies m\tau\hat{p}\cdot\hat{X} \to m\mathrm{e}^{i\theta} T\hat{p}\cdot\hat{X}, \qquad \theta \in [0, \pi/2 - \epsilon]. \tag{10}$$

- The integrals

$$\psi^{\pm}_{\Delta}(X; Q_{\pm}) = \mathcal{N}_{\Delta} \int_{\mathcal{H}^{\pm}_{d+1}} [\mathrm{d}\hat{p}]\, K^{\mathrm{AdS}}_{\Delta}(\hat{p}; Q_{\pm})\, \mathrm{e}^{m\tau\hat{p}\cdot\hat{X}}, \qquad X = \tau\hat{X} \in \mathcal{A}_{\pm}. \tag{11}$$

are convergent along all the path. At the final point $\theta = \pi/2 - \epsilon$, $\psi^{\pm}_{\Delta}(X; Q_{\pm})$ coincide with $\phi^{\pm}_{\Delta}(X; Q_{\pm})$ regularized in $\mathcal{A}_{\pm}$.

- In $\mathcal{A}_{\pm}$, we found the closed expressions

$$\phi^{\pm}_{\Delta}(T, \hat{X}; Q_{\pm}) = \mathcal{N}_{\Delta}\, K^{\mathrm{AdS}}_{\Delta}(\pm\hat{X}; Q_{\pm})\, \widetilde{K}_{\Delta-\frac{d}{2}}(\pm mT), \qquad T \gtrless 0,\ \hat{X} \in \mathcal{H}^{+}_{d+1}. \tag{12}$$

Analytical Continuation to the other Regions

- We can pass from one region to another by suitable analytical continuations.

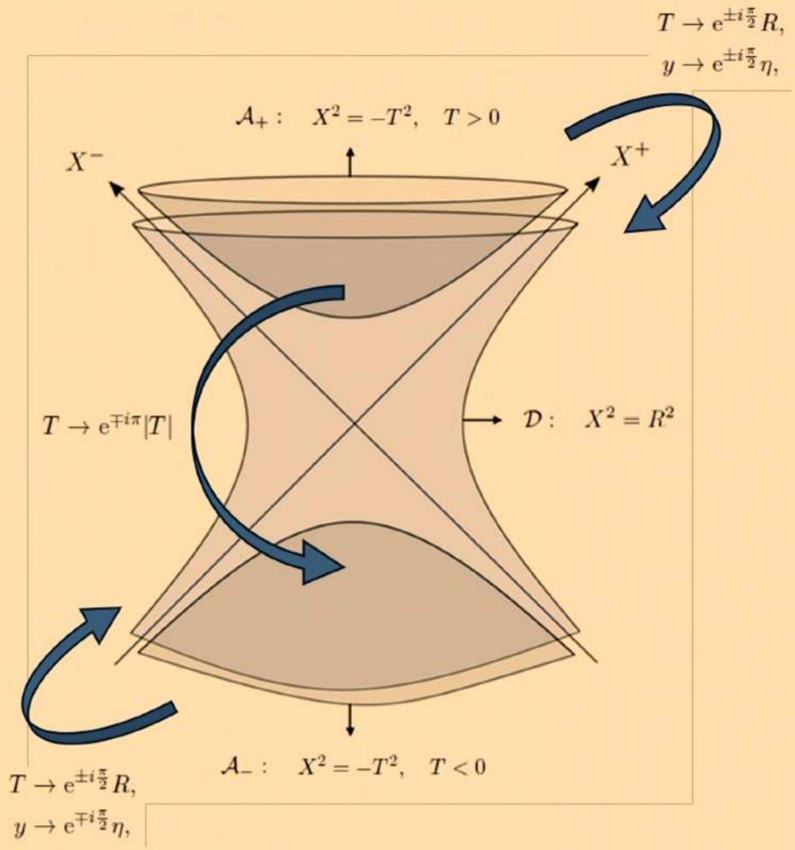
$$\mathscr{A}_+ : \qquad X = +\frac{T}{y}\left(\frac{1+y^2+|\vec{z}|^2}{2}, \frac{1-y^2-|\vec{z}|^2}{2}, \vec{z}\right),$$

$$\mathscr{A}_- : \qquad X = +\frac{T}{y}\left(\frac{1+y^2+|\vec{z}|^2}{2}, \frac{1-y^2-|\vec{z}|^2}{2}, \vec{z}\right),$$

$$\mathscr{D}_+ : \qquad X = +\frac{R}{\eta}\left(\frac{1-\eta^2+|\vec{z}|^2}{2}, \frac{1+\eta^2-|\vec{z}|^2}{2}, \vec{z}\right),$$

$$\mathscr{D}_- : \qquad X = -\frac{R}{\eta}\left(\frac{1-\eta^2+|\vec{z}|^2}{2}, \frac{1+\eta^2-|\vec{z}|^2}{2}, \vec{z}\right).$$

where $T \gtrless 0$ in $\mathscr{A}_\pm$, $R, y, \eta > 0$ and $\vec{z} \in \mathbb{R}^d$.



Closed Expression of Conformal Primary Wavefunctions: Final Result

- Here we show the closed expressions of the conformal primary wavefunctions in the four distinct regions:

$$X \in \mathscr{A}_+ : \qquad \phi^{\pm}_{\Delta}(X;Q_\pm) = \mathcal{N}_\Delta K^{\rm AdS}_{\Delta}(\pm\hat{X}_{\rm AdS};Q_\pm)\tilde{K}_{\Delta-\frac{d}{2}}\left(m\,Te^{\pm\frac{\pi i}{2}}\right), \tag{13}$$

$$X \in \mathscr{A}_- : \qquad \phi^{\pm}_{\Delta}(X;Q_\pm) = \mathcal{N}_\Delta K^{\rm AdS}_{\Delta}(\pm\hat{X}_{\rm AdS};Q_\pm)\tilde{K}_{\Delta-\frac{d}{2}}\left(m|T|e^{\mp\frac{\pi i}{2}}\right), \tag{14}$$

$$X \in \mathscr{D}_+ : \qquad \phi^{\pm}_{\Delta}(X;Q_\pm) = \mathcal{N}_\Delta K^{\rm AdS}_{\Delta}(e^{+\frac{\pi i}{2}}\hat{X}_{\rm dS};Q_\pm)\tilde{K}_{\Delta-\frac{d}{2}}(mR), \tag{15}$$

$$X \in \mathscr{D}_- : \qquad \phi^{\pm}_{\Delta}(X;Q_\pm) = \mathcal{N}_\Delta K^{\rm AdS}_{\Delta}(-e^{-\frac{\pi i}{2}}\hat{X}_{\rm dS};Q_\pm)\tilde{K}_{\Delta-\frac{d}{2}}(mR). \tag{16}$$

- From now on, we will set $Q_\pm = \pm Q$, since $Q_+ = -Q_- \Longleftarrow$ Antipodal map!

$$K^{\rm AdS}_{\Delta}(\hat{X}_{\rm AdS};Q) = \left(-2Q\cdot\hat{X}_{\rm AdS}\right)^{-\Delta}, \qquad \tilde{K}_{\Delta-\frac{d}{2}}(mR) = \frac{2R^{-d/2}}{\Gamma(\Delta-\frac{d}{2})}K_{\Delta-\frac{d}{2}}(mR). \tag{17}$$

Contact Amplitudes

- We want to use the closed expressions of the conformal primary wavefuctions to compute amplitudes:

$$\tilde{\mathscr{A}}^{\mathrm{c}}_{\Delta_1\ldots\Delta_n}(\pm_1 Q_1,\ldots,\pm_n Q_n) = -ig\int \mathrm{d}^{d+2}X\,\phi^{\pm_1}_{\Delta_1}(X,\pm_1 Q_1)\ldots\phi^{\pm_n}_{\Delta_n}(X,\pm_n Q_n)\,, \tag{18}$$

- Split the integral into to the four regions

$$\int \mathrm{d}^{d+2}X = \int_{\mathscr{A}_+}\mathrm{d}^{d+2}X + \int_{\mathscr{A}_-}\mathrm{d}^{d+2}X + \int_{\mathscr{D}_+}\mathrm{d}^{d+2}X + \int_{\mathscr{D}_-}\mathrm{d}^{d+2}X, \tag{19}$$

- Contact Amplitudes divide into

$$-ig\int \mathrm{d}^{d+2}X\prod_{i=1}^{n}\phi^{\pm_i}_{\Delta_i}(X,\pm_i Q_i) = -ig\left(I_{\mathscr{A}_+} + I_{\mathscr{A}_-} + I_{\mathscr{D}_+} + I_{\mathscr{D}_-}\right), \tag{20}$$

- In each region, we can write

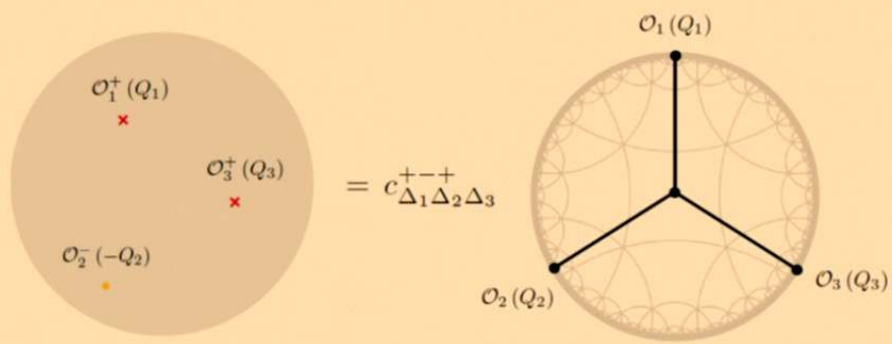
$$\int_{\mathscr{A}_\pm}\mathrm{d}^{d+2}X = \int_{\mathbb{R}^\pm}|T|^{d+1}\mathrm{d}T\int_{\mathscr{H}^+_{d+1}}\mathrm{d}\hat{X}_{\mathrm{AdS}}, \qquad \int_{\mathscr{D}_\pm}\mathrm{d}^{d+2}X = \int_0^\infty R^{d+1}\mathrm{d}R\int_{dS^\pm_{d+1}}\mathrm{d}\hat{X}_{\mathrm{dS}}. \tag{21}$$

Contact Amplitudes: Final Result

- Summing up the various contributions, we find out the proportionality law

$$\tilde{\mathscr{A}}^{\mathrm{c}}_{\Delta_1\ldots\Delta_n}(\pm_1 Q_1,\ldots,\pm_n Q_n) = \underbrace{\left(c^{\pm_1\cdots\pm_n}_{\mathscr{A}_+} + c^{\pm_1\cdots\pm_n}_{\mathscr{A}_-} + c^{\pm_1\cdots\pm_n}_{\mathscr{D}_+} + c^{\pm_1\cdots\pm_n}_{\mathscr{D}_-}\right)}_{c^{\pm_1\cdots\pm_n}_{\Delta_1\ldots\Delta_n}} \times \tilde{\mathscr{A}}^{\mathrm{c}}_{\Delta_1\ldots\Delta_n}(Q_1,\ldots,Q_n)\,. \qquad (22)$$

The n-point Celestial contact amplitude is proportional to the corresponding EAdS contact diagram by a coefficient that depends on the masses and the conformal weights of the fields.



Conformally Coupled Scalar

- Consider the 3-point contact amplitude with two incoming modes and one outgoing mode.
- In the simplest example of conformally coupled scalars, corresponding to $\Delta_i = \frac{d+1}{2}$,

$$c^{++-}_{\frac{d+1}{2}\frac{d+1}{2}\frac{d+1}{2}}|_{\mathscr{A}} = ig\frac{\cos\left(\frac{d\pi}{2}\right)\Gamma\left(\frac{1-d}{2}\right)}{\sqrt{2m_1 m_2 m_3}}(m_3 - m_1 - m_2)^{\frac{d-1}{2}}, \quad (23)$$

$$c^{++-}_{\frac{d+1}{2}\frac{d+1}{2}\frac{d+1}{2}}|_{\mathscr{D}} = ig\frac{\cos\left(\frac{d\pi}{2}\right)\Gamma\left(\frac{1-d}{2}\right)}{\sqrt{2m_1 m_2 m_3}}(m_1 + m_2 + m_3)^{\frac{d-1}{2}}. \quad (24)$$

- Setting $m_1 = m_2 = m$, $m_3 = 2m(1+\epsilon)$ and $d = 2$, the contribution from regions $\mathscr{A}_\pm$ recovers the result given in equation (3.13) of [Pasterski, Shao, Strominger 2016];
- We differ from their result by the contribution from region $\mathscr{D}$, which is regular in ϵ.

Many $i\epsilon$-prescription in Celestial Holography

- The $i\epsilon$-prescription present in the closed-form expressions of the conformal primary functions arises from the procedure we applied to regularize the divergent integrals defining $\phi^{\pm}_{\Delta}$;
- In standard QFT, the $i\epsilon$-prescription is related to the temporal ordering and comes from the Feynman propagator,

$$\Pi_T^{(m)}(X_1, X_2) = -i \int \frac{\mathrm{d}^{d+2} p}{(2\pi)^{d+2}} \frac{\mathrm{e}^{ip\cdot(X_1 - X_2)}}{p^2 + m^2 - i\epsilon}. \tag{25}$$

- In Celestial Holography Revisited, we explored the possibility of introducing an $i\epsilon$-prescription related to temporal ordering, as in standard QFT.

The Celestial Bulk-to-Boundary Propagator

- Recover the conformal primary wavefunctions using the prescription [Sleight, Taronna 2023]:

$$Y = t\hat{Y}, \qquad \phi^{\pm}_{\Delta}(X;Q_\pm) = \lim_{\hat{Y}\to Q_\pm} \int_0^{+\infty} \frac{\mathrm{d}t}{t} t^{\Delta} \underbrace{\int_{\mathcal{H}^{(m)}_{d+1}} [\mathrm{d}p]\, e^{ip\cdot(X-t\hat{Y})}}_{\mathcal{W}(X,t\hat{Y})} \tag{26}$$

- Define the Celestial bulk-to-boundary propagator in a similar way, replacing $\mathcal{W}(X,Y)$ with the Feynman propagator $\Pi^{(m)}_{T}(X,Y)$ ([Sleight, Taronna 2023]):

$$\Pi^{(m)}_{\Delta}\left(X,Q_\pm\right) = \int_0^{\infty} \frac{\mathrm{d}t}{t} t^{\Delta} \lim_{\hat{Y}\to Q_\pm} \Pi^{(m)}_{T}\left(X,t\hat{Y}\right). \tag{27}$$

- The closed-form expression is

$$\Pi^{(m)}_{\Delta}\left(X,Q_\pm\right) = c^{\text{dS-AdS}}_{\Delta}\, \widetilde{K}_{\frac{d}{2}-\Delta}\left(m\sqrt{X^2+i\epsilon}\right) G^{\text{AdS}}_{\Delta}(X_\epsilon, Q_\pm), \qquad G^{\text{AdS}}_{\Delta}(X_\epsilon,Q) = C^{\text{AdS}}_{\Delta} \frac{\left(\sqrt{X^2+i\epsilon}\right)^{\Delta}}{(-2X\cdot Q+i\epsilon)^{\Delta}} \tag{28}$$

Spectral Decomposition of the Feynman Propagator

- The Feynman propagator can be recast as [Iacobacci, Sleight, Taronna 2024]

$$\Pi_T^{(m)}(X_1,X_2) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}\nu}{2\pi}\,\rho_\nu^{(m)}(X_1,X_2)\,\Omega_\nu(\sigma_\epsilon), \tag{29}$$

where

$$\rho_\nu^{(m)}(X_1,X_2) = \frac{1}{4}\Gamma(i\nu)\Gamma(-i\nu)\widetilde{K}_{-i\nu}\left(m\sqrt{X_1^2+i\epsilon}\right)\widetilde{K}_{i\nu}\left(m\sqrt{X_2^2+i\epsilon}\right) \tag{30}$$

is its spectral density and

$$\Omega_\nu(\sigma_\epsilon) = \Omega_\nu(0)\,{}_2F_1\left(\Delta,\Delta^*;\frac{d+1}{2};\sigma_\epsilon\right), \quad \Delta = \frac{d}{2}+i\nu,\ \nu\in\mathbb{R}. \tag{31}$$

- The function $\Omega_\nu(\sigma_\epsilon)$ has the same functional expression of Harmonic function in EAdS, but the argument is different:

$$\sigma_\epsilon = \frac{1+\hat{z}}{2}, \qquad \hat{z} = \frac{\hat{X}_1\cdot\hat{X}_2 - i\epsilon}{\sqrt{\hat{X}_1^2+i\epsilon}\sqrt{\hat{X}_1^2+i\epsilon}} \Longrightarrow \boxed{\Omega_\nu(\sigma_\epsilon)\text{ is rather a Propagator!}}$$

Celestial Correlators

- Celestial correlators are defined as [Iacobacci, Sleight, Taronna 2024]:

$$\left\langle \mathcal{O}_{\Delta_1}(Q_1)\ldots\mathcal{O}_{\Delta_n}(Q_n)\right\rangle = \prod_i \lim_{\hat{X}_i\to Q_i}\int_0^\infty \frac{\mathrm{d}t_i}{t_i}\,t_i^{\Delta_i}\left\langle \phi_1(t_1\hat{X}_1)\ldots\phi_n(t_n\hat{X}_n)\right\rangle. \tag{32}$$

- The Celestial contact diagram decomposes into four contributions

$$-ig\int \mathrm{d}^{d+2}X\prod_{i=1}^{n}\Pi_{\Delta_i}^{(m_i)}\left(X,Q_i\right) = -ig\left(I_{\mathscr{A}_+}+I_{\mathscr{A}_-}+I_{\mathscr{D}_+}+I_{\mathscr{D}_-}\right), \tag{33}$$

where we defined

$$I_\bullet = \int_\bullet \mathrm{d}^{d+2}X\prod_{i=1}^{n}\Pi_{\Delta_i}^{(m_i)}\left(X,Q_i\right), \tag{34}$$

with $\bullet = \mathscr{A}_+, \mathscr{A}_-, \mathscr{D}_+, \mathscr{D}_-$.

Celestial Contact Diagram

- The Celestial contact diagram is therefore the sum of contributions from regions $\mathscr{A}_+$ and $\mathscr{D}_+$,

$$\mathscr{A}_c^{(n)} = -ig\sin\left(\frac{\pi}{2}\left(-d+\sum_{i=1}^{n}\Delta_i\right)\right)\tilde{R}_{\Delta_1\dots\Delta_n}(m_1,\dots,m_n)\times {}^{(\mathrm{AdS})}\tilde{\mathscr{A}}^{c}_{\Delta_1\dots\Delta_n}(Q_1,\dots,Q_n) \quad (35)$$

where

$$\tilde{R}_{\Delta_1\dots\Delta_n}(m_1,\dots,m_n) = \int_0^\infty \mathrm{d}R\, R^{d+1}\prod_{i=1}^{n}\tilde{K}_{\frac{d}{2}-\Delta_i}(m_i R) \quad (36)$$

encodes the dependence on the radial direction.

- Within this formalism

$$\boxed{I_{\mathscr{A}_-} + I_{\mathscr{D}_-} = 0} \quad (37)$$

Particle Exchange Diagram

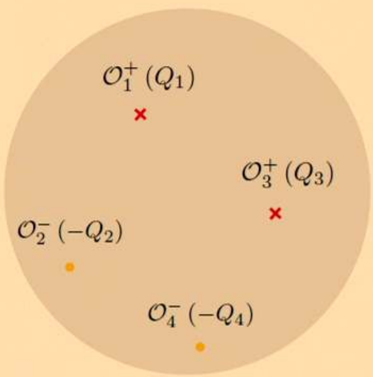
- The celestial exchange diagram is the sum of the contributions from the regions $\mathscr{A}_+$ and $\mathscr{D}_+$, giving

$$\mathscr{E}^{m_1,m_2|m|m_3,m_4}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}(Q_1,Q_2,Q_3,Q_4) = g^2 \int_{-\infty}^{\infty} \frac{\mathrm{d}\nu}{2\pi} \frac{c^{\text{flat-AdS}}_{\Delta_1\Delta_2\frac{d}{2}+i\nu}(m_1,m_2,m)\, c^{\text{flat-AdS}}_{\frac{d}{2}+i\nu\Delta_3\Delta_4}(m,m_3,m_4)}{c^{\text{flat-AdS}}_{\frac{d}{2}+i\nu}} \times \mathscr{A}^{\text{AdS}}_{\Delta_1,\Delta_2|\frac{d}{2}+i\nu|\Delta_3,\Delta_4}\left(Q_1,Q_2,Q_3,Q_4\right).$$

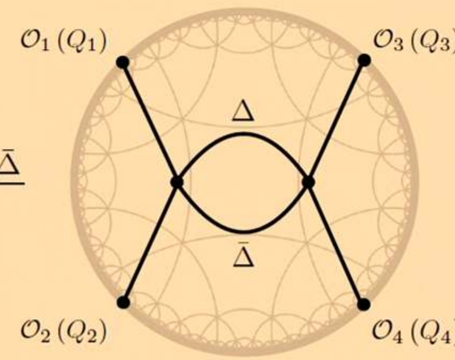
where $\mathscr{A}^{\text{AdS}}_{\Delta_1,\Delta_2|\frac{d}{2}+i\nu|\Delta_3,\Delta_4}\left(Q_1,Q_2,Q_3,Q_4\right)$ is the four-point exchange of a particle with scaling dimension $\Delta = \frac{d}{2} + i\nu$ in EAdS.

Celestial Correlators from EAdS Witten Diagram

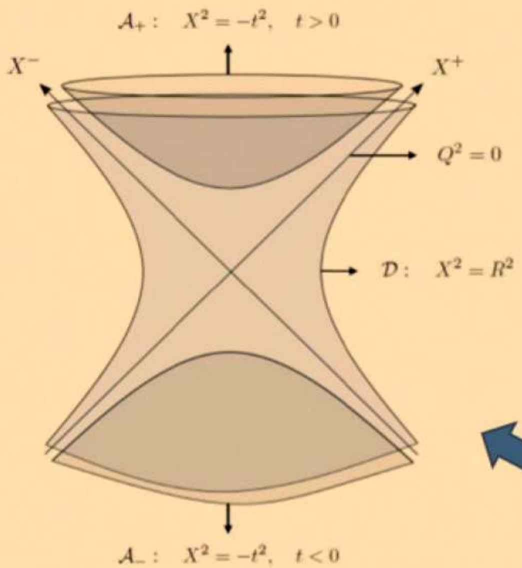
Each contribution to massive Celestial correlators can be recast in terms of a linear combination of contributions of corresponding massive Witten correlators in EAdS.

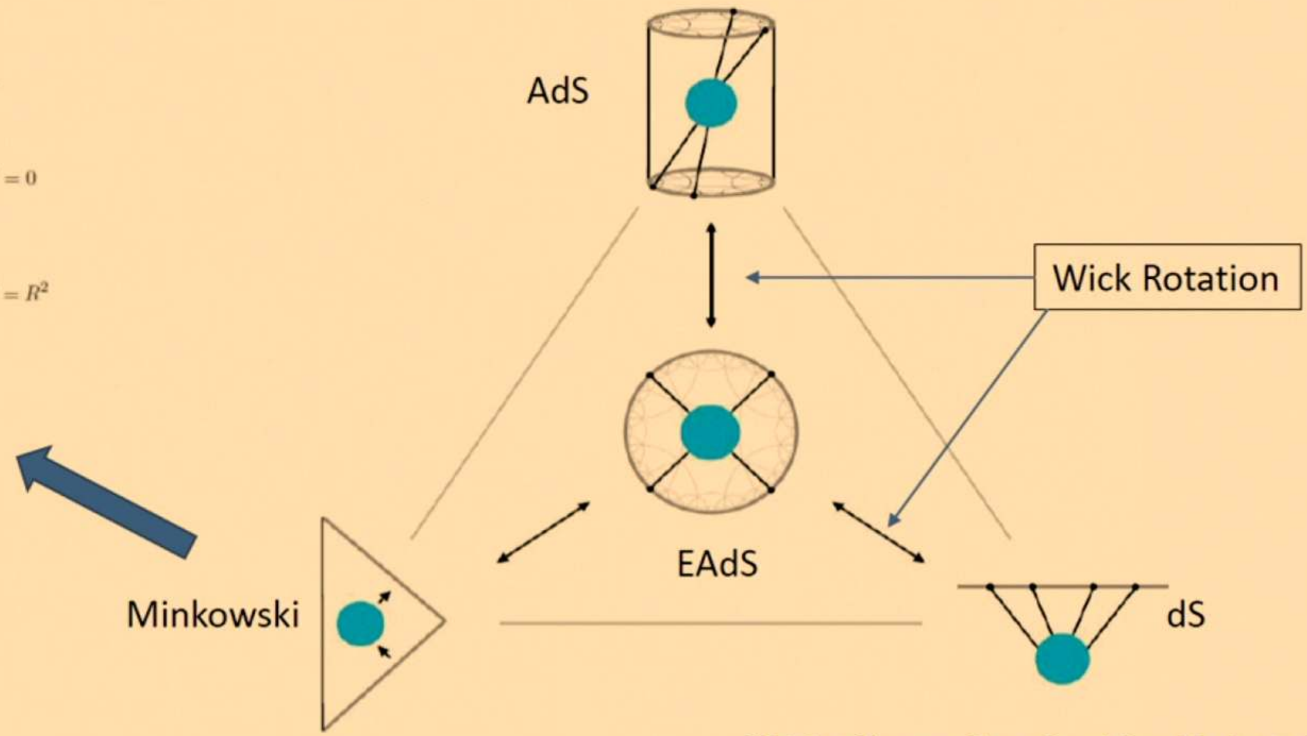


$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i}\frac{d\bar{\Delta}}{2\pi i} \sum_{\pm\hat{\pm}} \frac{c^{+-\pm\hat{\pm}}_{\Delta_1\Delta_2\Delta\bar{\Delta}}\, c^{+-\mp\hat{\mp}}_{\Delta_3\Delta_4\Delta\bar{\Delta}}}{c_{\Delta} c_{\bar{\Delta}}}$$



The Holographic Triangle





[Sleight & Taronna, *From dS to AdS and back*, 2021]

The Kantorovic-Lebedev Transform

- In the context of Celestial holography, an orthogonal basis to expand elements of $L^2(\mathbb{R}^+, dRR^{d-1})$ is given by Bessel-K functions with pure imaginary order:

$$\tilde{K}_{i\alpha}(R) = \langle R|\tilde{K}_{i\alpha}\rangle = \frac{2R^{-d/2}}{\Gamma(i\alpha)} K_{i\alpha}(mR)\,, \tag{38}$$

which possess the properties of completeness and orthogonality:

$$\langle \tilde{K}_{i\alpha}|\tilde{K}_{i\beta}\rangle = 2\pi\delta(\beta-\alpha) + \frac{2\pi\Gamma(i\alpha)\delta(\alpha+\beta)}{\Gamma(-i\alpha)}\,, \tag{39}$$

$$\frac{1}{2}\int_{-\infty}^{+\infty} \frac{\mathrm{d}\alpha}{2\pi} \langle R_1|\tilde{K}_{i\alpha}\rangle\langle \tilde{K}_{i\alpha}|R_2\rangle = R_1^{-d+1}\delta(R_1-R_2)\,. \tag{40}$$

The Kantorovic-Lebedev Transform (2)

- In the hyperbolic slicing of Minkowski, a field $\phi(X)$ can be decomposed into fields that live on the hyperbolic slices. In the region $\mathscr{D}$:

$$\phi(X) = \frac{1}{2}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{\mathrm{d}\Delta}{2\pi i}\,\hat{\phi}_{\mu(\Delta)}(\hat{X})\,\tilde{K}_{\Delta-\frac{d}{2}}(mR)\,, \tag{41}$$

where

$$\tilde{K}_{\Delta-\frac{d}{2}}(mR) = \frac{2R^{-d/2}}{\Gamma(\Delta-\frac{d}{2})}K_{\Delta-\frac{d}{2}}(mR), \tag{42}$$

- It is important to note that

$$(\Box - m^2)\phi(X) = 0 \iff (\nabla^2_{\mathrm{dS}} - \underbrace{\Delta(d-\Delta)}_{\mu(\Delta)})\hat{\phi}_{\mu(\Delta)}(\hat{X}) = 0. \tag{43}$$

Conformal Primary Basis Formalism Recovered

- We can analytically extend the KL kernel inside the light-cone,

$$\phi(X) = \frac{1}{2}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i}\hat{\phi}_{\mu(\Delta)}(\hat{X})\tilde{K}_{\Delta-\frac{d}{2}}(\pm imT), \qquad \tilde{K}_{\Delta-\frac{d}{2}}(\pm imT) = \frac{2(\pm iT)^{-d/2}}{\Gamma(\Delta-\frac{d}{2})}K_{\Delta-\frac{d}{2}}(\pm imT), \tag{44}$$

- Insider the light-cone, the conformal primary basis decomposition follows using the AdS/CFT correspondence:

$$\hat{\phi}_{\mu(\Delta)}(\hat{X}) = \int d^d\omega\, K_{\Delta}^{\text{AdS}}(\hat{X};Q(\vec{\omega}))\mathcal{O}_{\Delta^*}(\vec{\omega}), \qquad K_{\Delta}^{\text{AdS}}(\hat{X};\vec{\omega}) = \left(-2\hat{X}\cdot Q(\vec{\omega})\right)^{-\Delta}. \tag{45}$$

The Flat/CCFT correspondence results from the sequential combination of the KL transform and the AdS/CFT correspondence.

Conclusions

- Each contribution to massive Celestial correlators can be recast in terms of a linear combination of contributions of corresponding massive Witten correlators in EAdS;
- Different $i\epsilon$-prescriptions give rise to different results in Celestial Holography;
- The Flat/CCFT correspondence results from the sequential combination of the KL transform and the AdS/CFT correspondence;
- EAdS emerges as the foundational theory from which overarching considerations and properties concerning holography in general can be derived.

Thank you for listening !

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