

**Title:** Squeezing primordial non-Gaussianity out of the matter bispectrum (and trispectrum) with consistency relations

**Speakers:** Sam Goldstein

**Collection/Series:** Cosmology and Gravitation

**Subject:** Cosmology

**Date:** September 24, 2024 - 11:00 AM

**URL:** <https://pirsa.org/24090153>

**Abstract:**

In this seminar, I will discuss recent progress towards developing robust methods to constrain PNG in the non-linear regime based on the LSS consistency relations — non-perturbative statements about the structure of LSS correlation functions derived from symmetries of the LSS equations of motion. Specifically, I will present non-perturbative models for the squeezed matter bispectrum and collapsed matter trispectrum in the presence of local PNG, as well as in the presence of a more general “Cosmological Collider” signal sourced by inflationary massive particle exchange. Using N-body simulations with modified initial conditions, I will demonstrate that these models yield unbiased constraints on the amplitude of PNG deep into the non-linear regime ( $k \sim 2 \text{ h/Mpc}$  at  $z=0$ ). Finally, I will discuss how these non-perturbative methods can provide insight into the scale-dependent bias signature associated with the Cosmological Collider scenario.

**Squeezing primordial non  
Gaussianity out of the  
matter bispectrum  
(and trispectrum)  
with consistency  
relations**



*Oliver Philcox*



*J. Colin Hill*



*Lam Hui*



*Angelo Esposito*



*Roman Scoccimarro*



*Max Abitbol*

2209.06228

*SG, Esposito, Philcox, Hui, Hill,  
Scoccimarro, Abitbol*

2310.12959

*SG, Philcox, Hill, Esposito, Hui*

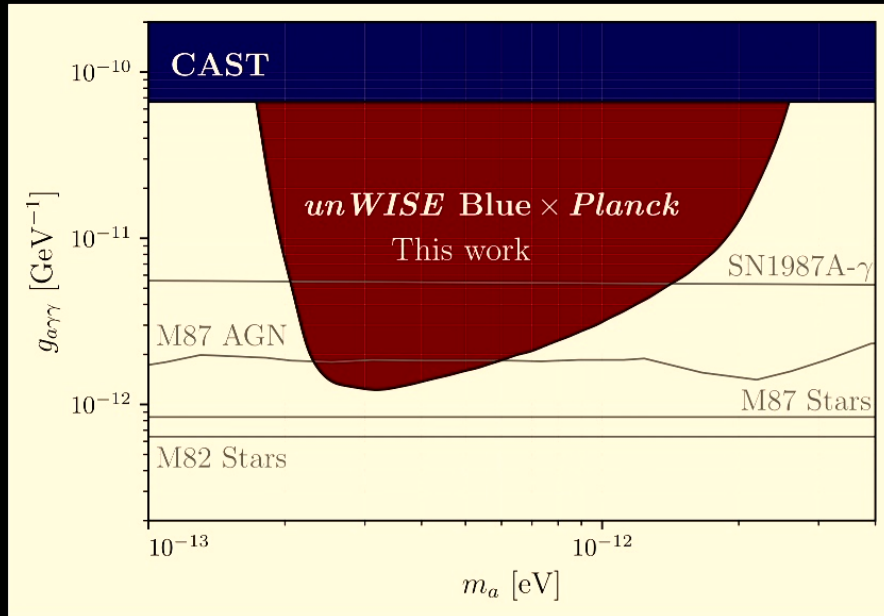
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*SG, Philcox, Hill, Hui*

**Sam Goldstein**

 COLUMBIA UNIVERSITY

## Axion bounds from CMBxLSS

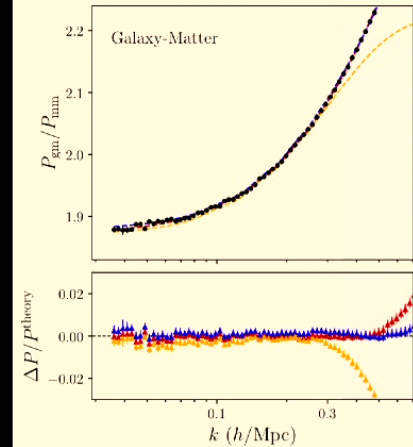


- **SG**, McCarthy, Mondino, Hill, Huang, Johnson [2409.10514](#), submitted

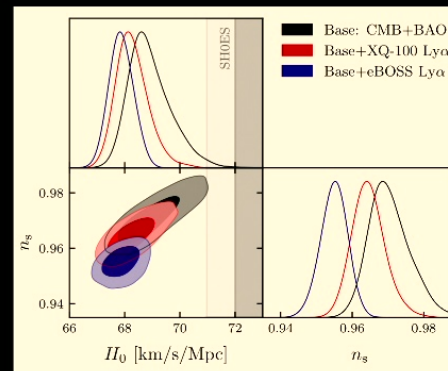
*What I won't talk about...*

## Galaxy-Halo Connection

- Comparing simulations with analytic models for LSS
- **Galaxy bias models: SG**, Pandey, Slosar, Blazek, and Jain, [2111.00501](#)
- **Splashback radius: 2111.06499** and [2105.05914](#)



*Lya forest disfavors EDE as a resolution to Hubble tension!*



- **SG**, Hill, Irsić, and Sherwin, [2303.00746](#)
- **Phys Rev Lett.** **131**, 201001
- Editors' Suggestion
- Featured in Physics

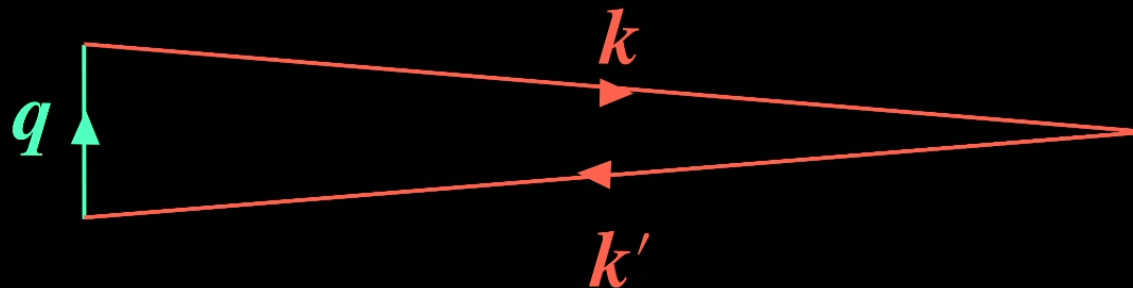
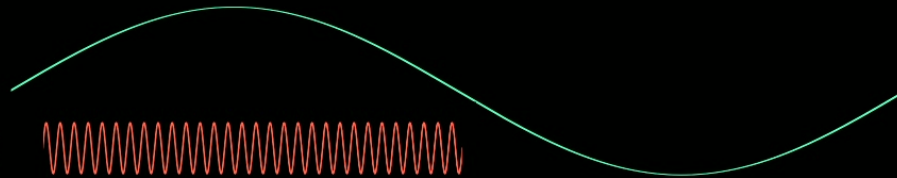
**Early Dark Energy and Ly $\alpha$  Forest**

# Note Notation

Correlations between **long** and **short** wavelength cosmological perturbations are highly constrained by symmetries

Soft/long wavelength mode:

Hard/short wavelength mode:



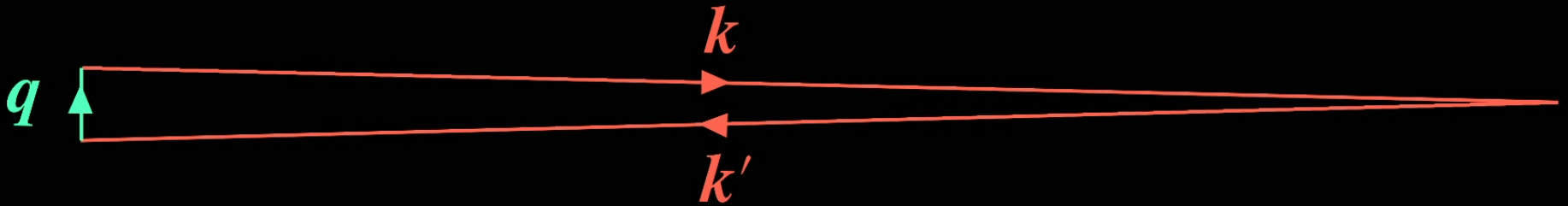
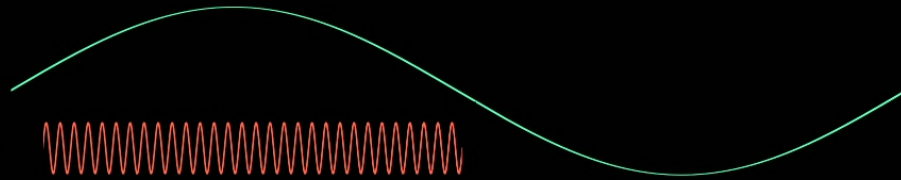
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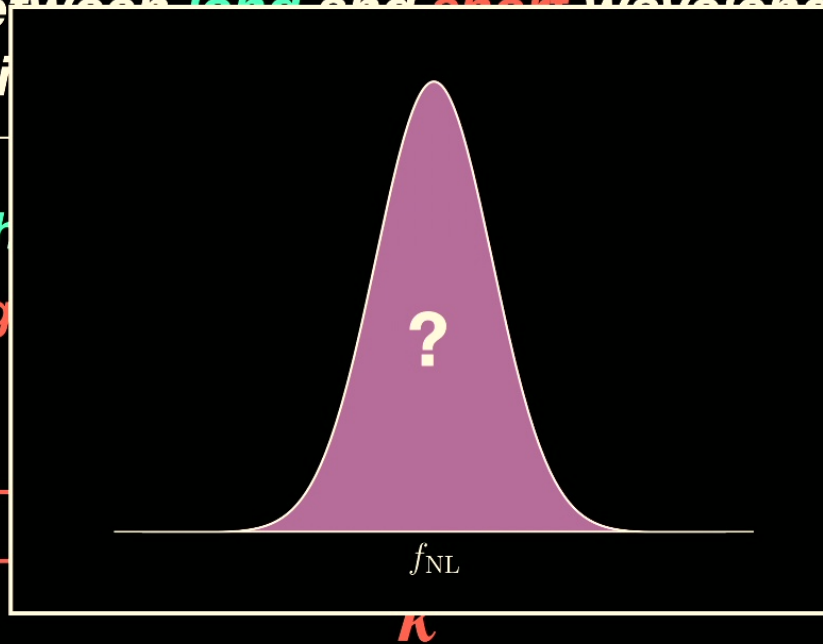
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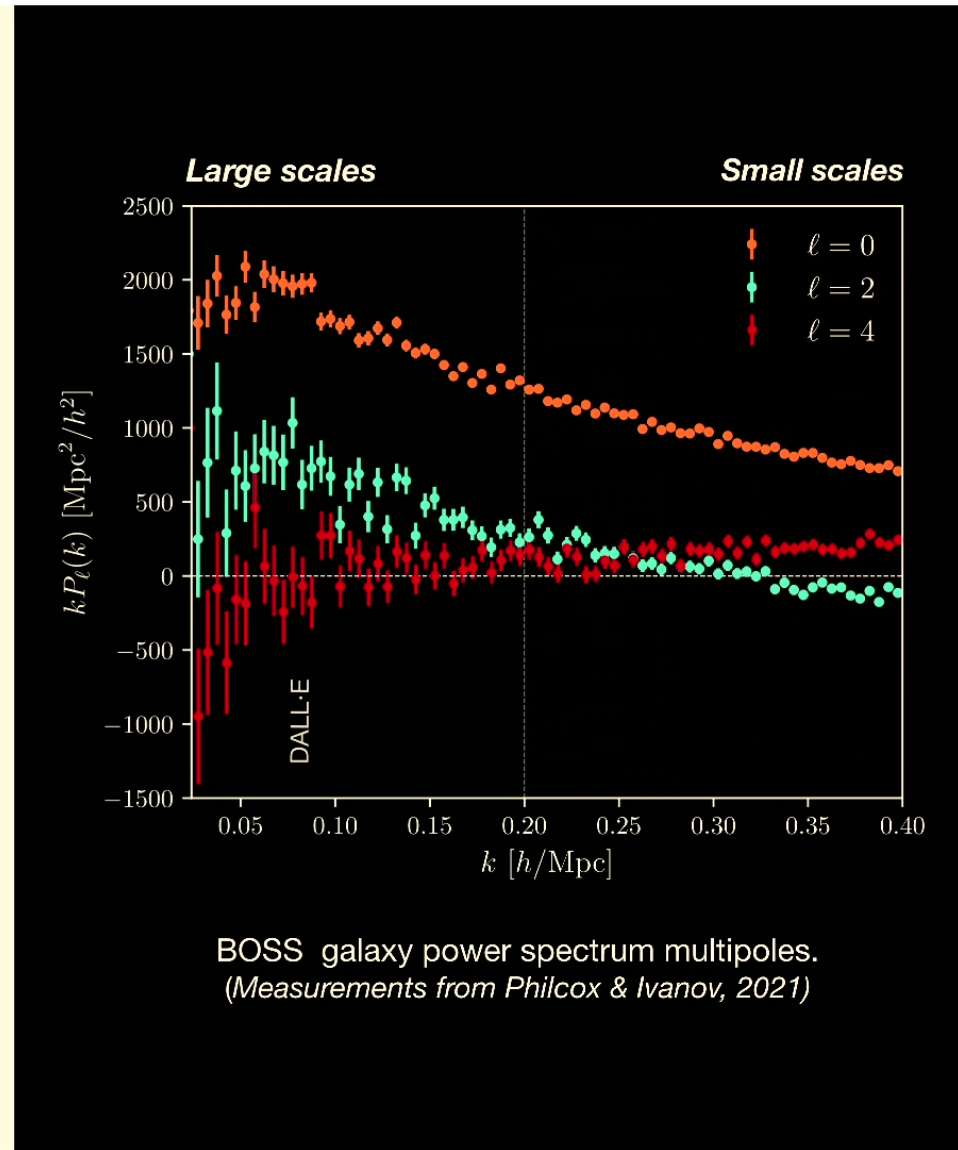
$q$



“Squeezed” bispectrum ( $q \ll k \approx k'$ )

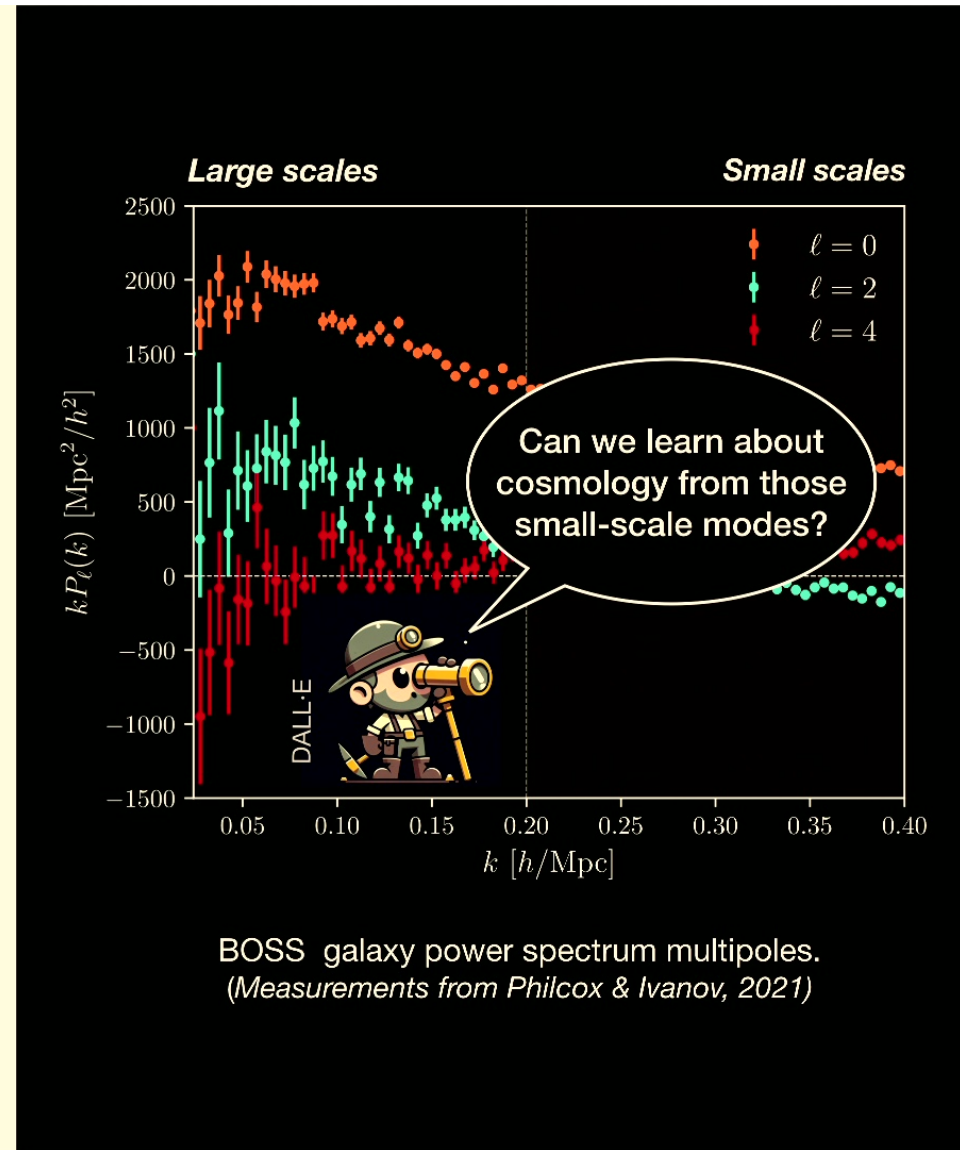
# Cosmology with LSS

- Upcoming large-scale structure (LSS) surveys will measure many modes
  - Stress test  $\Lambda$ CDM
  - Provide insight into **initial conditions**
- Theoretical challenge: **non-linearities**
  - Need to exclude small-scale modes from analysis
  - Particularly challenging for non-Gaussian/higher-order statistics



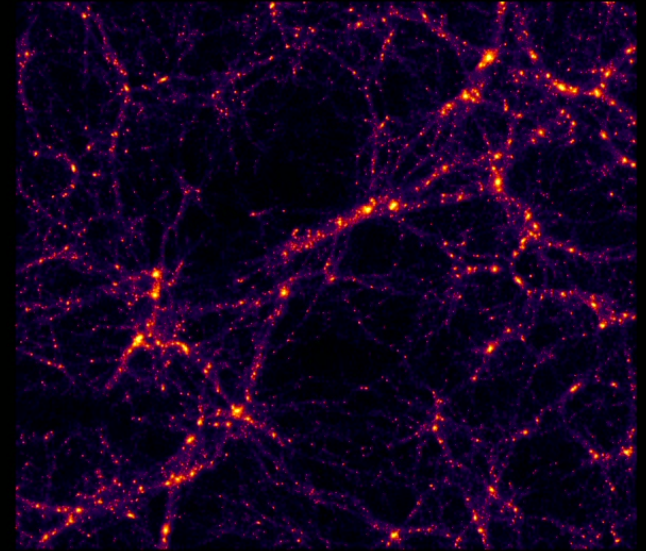
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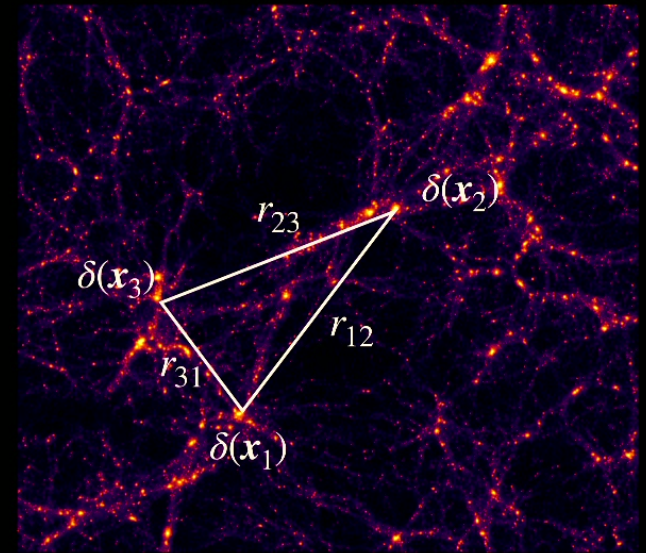
# Non-linearities in LSS



**Goal:** Constrain cosmology from  $\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}$

# Non-linearities in LSS

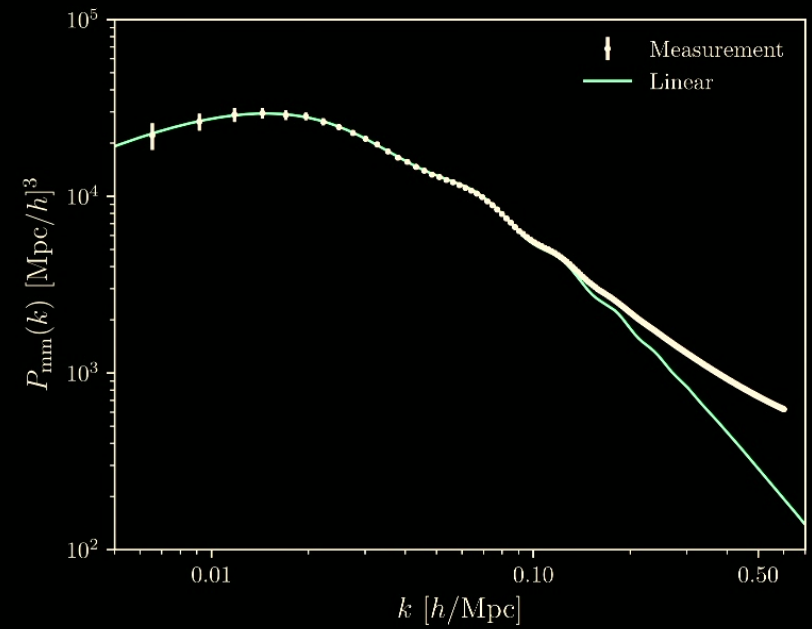
- Compress field into its correlation functions



$$\xi_n(\mathbf{x}_1, \dots, \mathbf{x}_n) \equiv \langle \delta(\mathbf{x}_1) \dots \delta(\mathbf{x}_n) \rangle_c$$

# Non-linearities in LSS

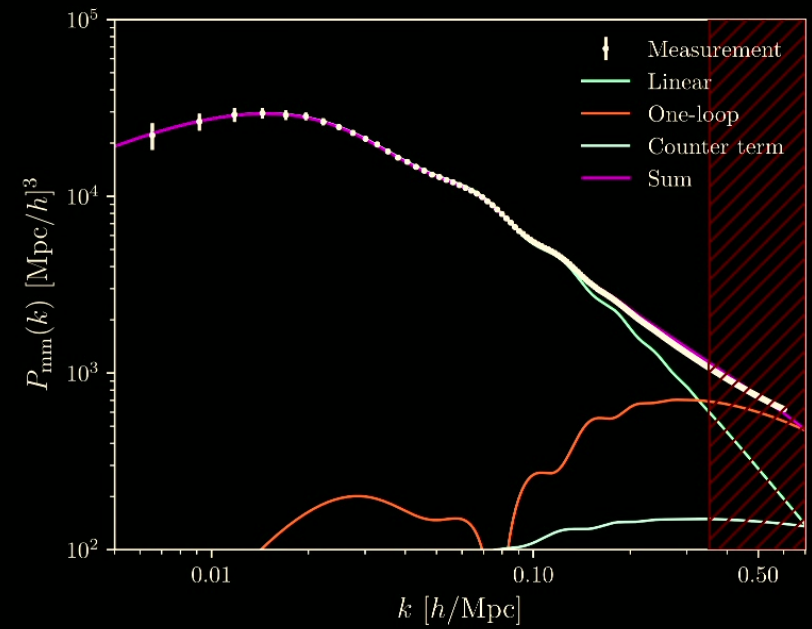
- Compress field into its correlation functions
  - Power spectrum is lossless if field is Gaussian
- Need to model correlation functions with theory



$$\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle \equiv (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') P(k)$$

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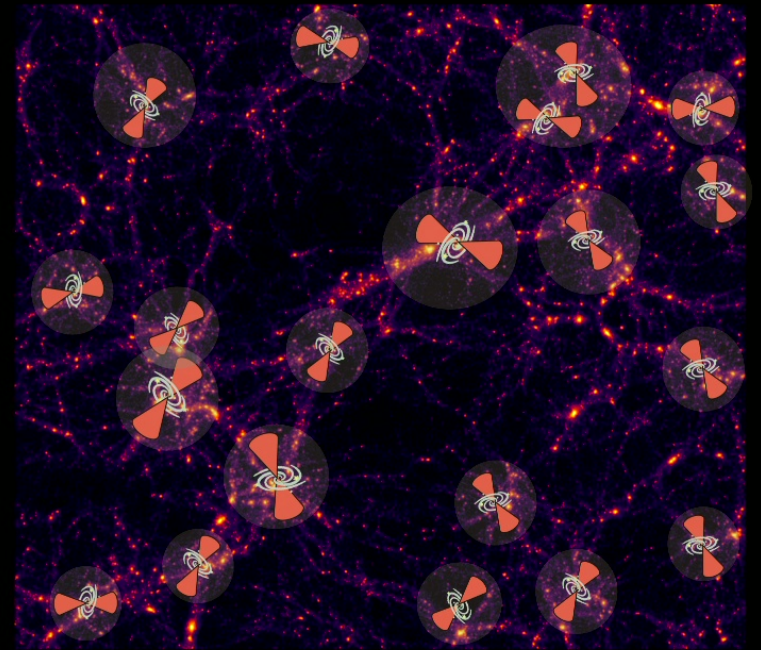
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  - Astrophysics/baryons



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  - Astrophysics/baryons
- Observational limitations
  - Generally observe biased tracers
  - Redshift space distortions



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***What to do when we cannot use  
perturbation theory?***



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Let there be *freedom from perturbations* with respect to the things which come from the external cause; and let there be justice in the things *done by symmetries*.

*Marcus Aurelius ~170 AD*



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*Ward identities from QFT*

*Consistency relations/soft theorems*

# How do symmetries constrain cosmological correlators?

- **Translational invariance:**

$$\implies \langle \mathcal{O}(\mathbf{k}_1) \dots \mathcal{O}(\mathbf{k}_n) \rangle = (2\pi)^3 \delta_D \left( \sum_a \mathbf{k}_a \right) \langle \mathcal{O}(\mathbf{k}_1) \dots \mathcal{O}(\mathbf{k}_n) \rangle'$$

- **Rotational symmetry:**

$$\implies \langle \mathcal{O}(\mathbf{k}_1) \dots \mathcal{O}(\mathbf{k}_n) \rangle' = F(k_i \cdot k_j)$$

- **Examples:**

- Power spectrum:  $\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2) P(k_1)$
- Bispectrum:  $\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)$
- Trispectrum:  $\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \delta(\mathbf{k}_4) \rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) T(k_1, k_2, k_3, k_4, k_{12}, k_{23})$

What about more general symmetries?

# LSS Consistency relations

- Equations of motion for  $\delta_m, v_m, \Phi$

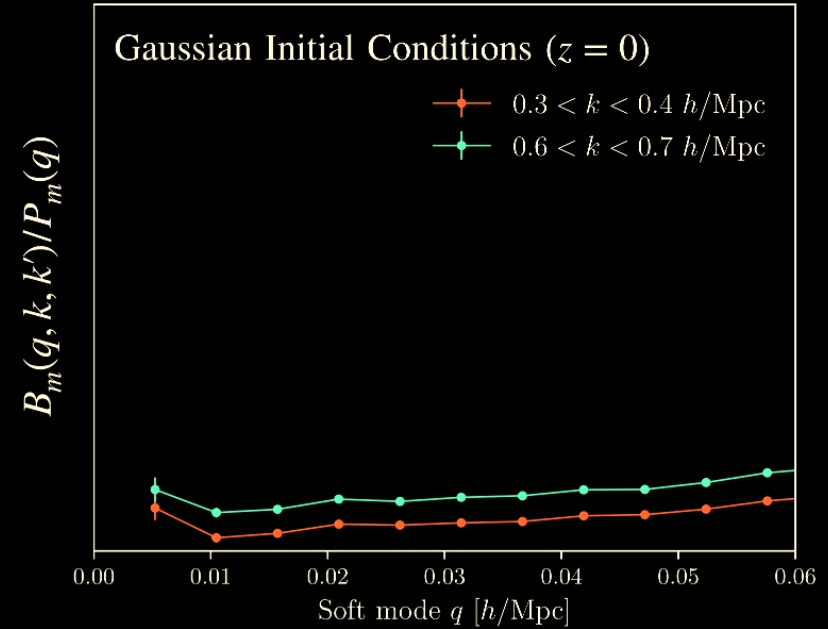
$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta)\mathbf{v}] = 0 \quad (\text{conservation of mass})$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + [\mathbf{v} \cdot \nabla]\mathbf{v} = -\nabla\Phi \quad (\text{conservation of momentum})$$

$$\nabla^2\Phi = \frac{3}{2}\Omega_m\mathcal{H}^2\delta \quad (\text{Poisson equation})$$

- Possess the following symmetry:

$$\left. \begin{array}{l} 1. \text{ Shift in gravitational potential: } \Phi \mapsto \Phi + \kappa(\eta) \\ 2. \text{ Time-dependent translation: } \mathbf{x} \mapsto \mathbf{x} + \mathbf{n}(\eta) \\ \Phi \rightarrow \Phi - (\mathcal{H}\mathbf{n}' + \mathbf{n}'') \cdot \mathbf{x}, \quad \mathbf{v} \rightarrow \mathbf{v} + \mathbf{n}' \end{array} \right\} \implies$$



$$\lim_{q \rightarrow 0} \left[ \frac{B(q, k, k')}{P(q)} \right] \text{ has no } (1/q)^\alpha \text{ poles}$$

*Kehagias & Riotto 2012;*  
*Peloso & Pietroni, 2013*

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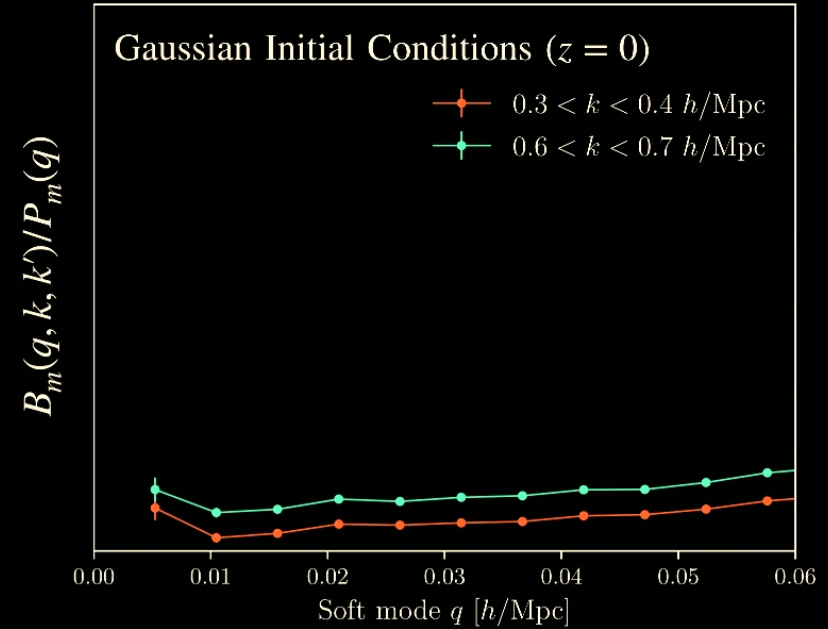
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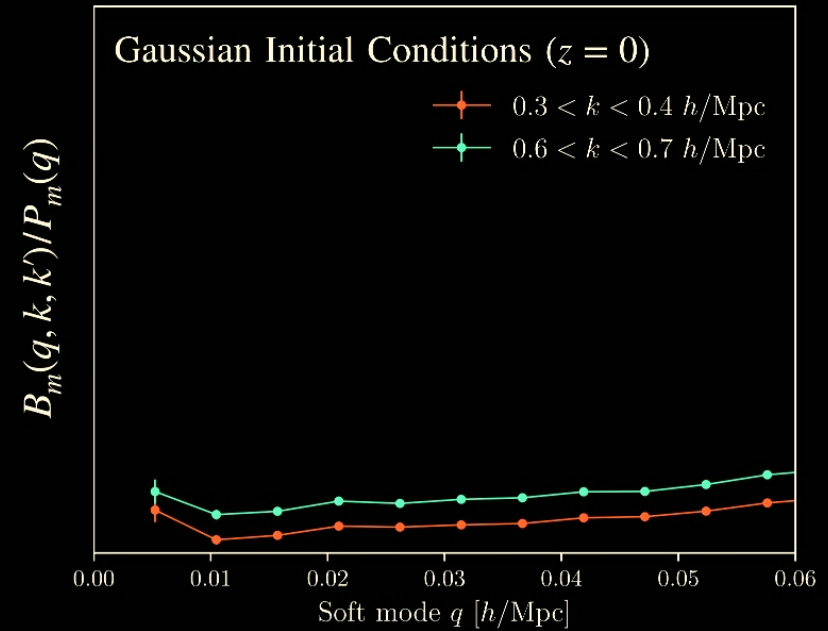
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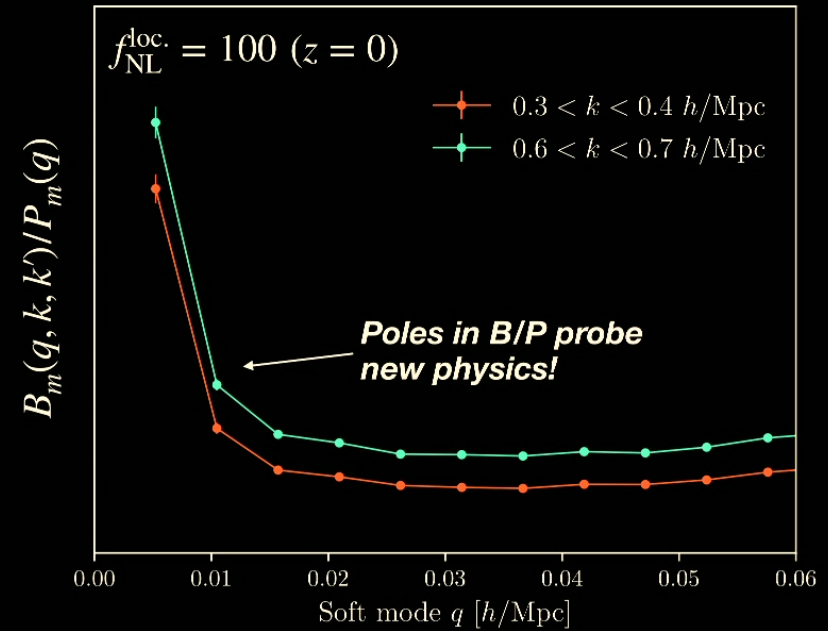
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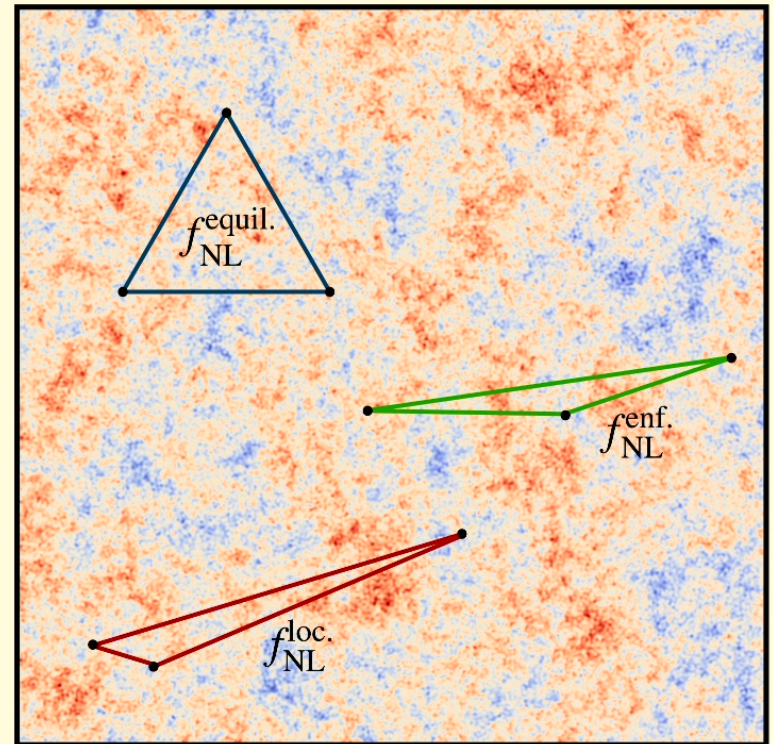
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# Primordial non-Gaussianity (PNG)

- What is the **physics** responsible for origin of structure?
- Single-field slow roll inflation (SFI) -> ~Gaussian IC's
  - More interesting scenarios produce PNG
- Classify with shapes of bispectrum
  - *Inflaton self-interactions*:  $f_{\text{NL}}^{\text{equil.}}$
  - *Vacuum state*:  $f_{\text{NL}}^{\text{enf.}}$
  - *Multiple light fields*:  $f_{\text{NL}}^{\text{loc.}}$

$$\Phi(\mathbf{x}) = \phi_G(\mathbf{x}) + f_{\text{NL}}^{\text{loc.}} (\phi_G^2(\mathbf{x}) - \langle \phi_G^2(\mathbf{x}) \rangle)$$

*Komatsu & Spergel, 2001*





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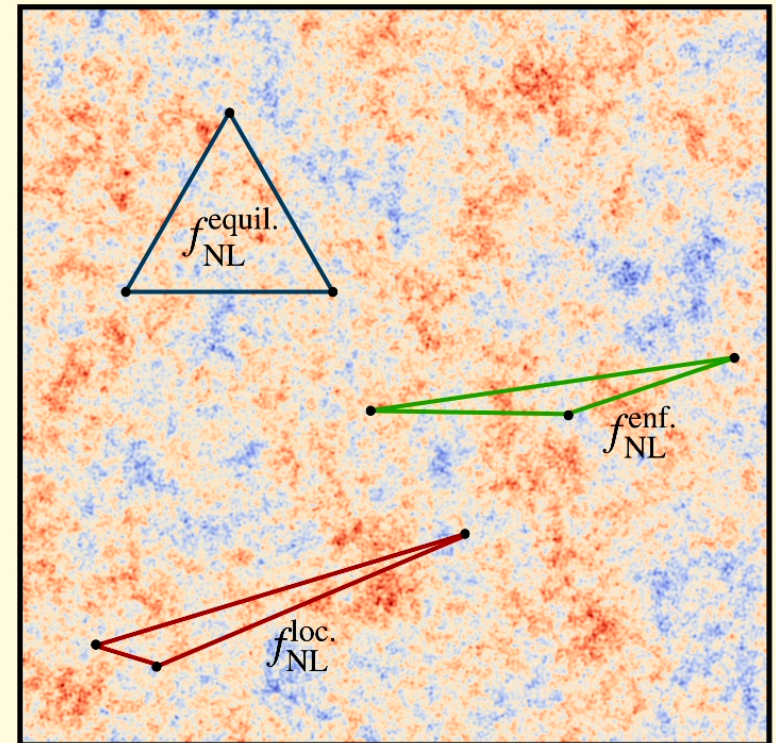
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*Komatsu & Spergel, 2001*

$f_{\text{NL}}^{\text{loc.}} \neq 0$  would rule out single-field inflation

*Maldacena, 2002; Creminelli & Zaldarriaga, 2004*



# The cosmological collider

- Massive **scalars** during inflation have characteristic squeezed bispectrum

$$\lim_{k_1 \ll k_2 \approx k_3} B_\Phi(k_1, k_2, k_3) = 4f_{\text{NL}}^\Delta \left( \frac{k_1}{k_2} \right)^\Delta P_\Phi(k_1) P_\Phi(k_2)$$

- Power-law depends on **mass**:  $\Delta = 3/2 - \sqrt{9/4 - m^2/H^2}$   
(Arkani-Hamed & Maldacena, 2015)

## 1 - Massless ( $m \ll H$ ; $\Delta \approx 0$ ):

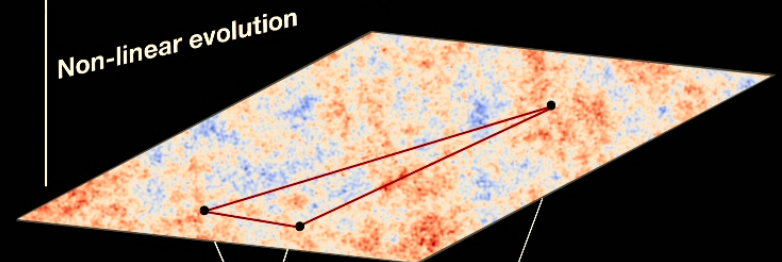
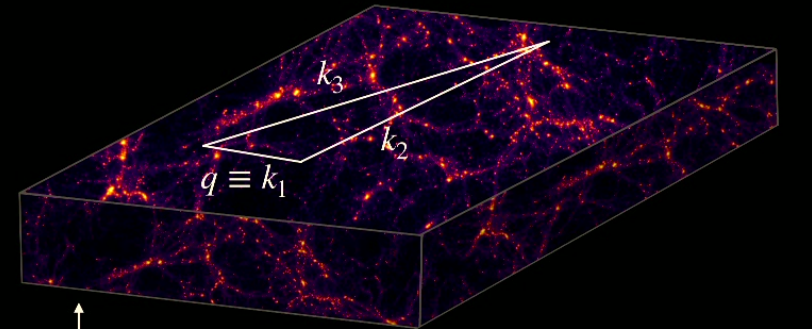
- Local non-Gaussianity ( $f_{\text{NL}}^\Delta = f_{\text{NL}}^{\text{loc.}}$ )/multi-field inflation

## 2 - Massive-ish ( $0 < m/H \leq 3/2$ ; $0 < \Delta \leq 3/2$ ):

- Power-law scaling (quasi-single field)
  - Interpolates between  $f_{\text{NL}}^{\text{loc.}}$  and  $f_{\text{NL}}^{\text{eq.}}$

## 3 - Massive-er ( $m/H \geq 3/2$ ; $\Delta \in \mathbb{C}$ ):

- Oscillatory bispectra



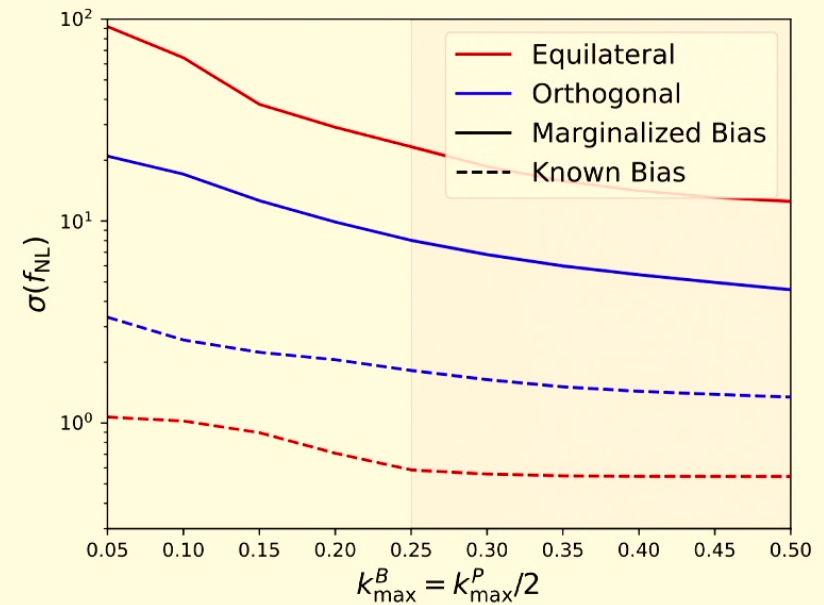
$$q \equiv k_1 \ll k_2 \approx k_3$$

“Soft limit”

$m \sim H$   
Massive particle exchange

# Observational Status of PNG Constraints

- Best constraints on PNG currently come from CMB
  - $f_{\text{NL}}^{\text{loc.}} = -0.9 \pm 5.1$
  - $f_{\text{NL}}^{\text{equil.}} = -26 \pm 47$  *Planck 2018 constraints on PNG*
  - $f_{\text{NL}}^{\text{ortho.}} = -38 \pm 24$
- Near-term/next generation CMB experiments will only improve by a factor of  $\sim 2-3$  (nice job *Planck*!)
- We **need** LSS(xCMB?) to search for PNG



**Megamapper Forecasts on PNG** (Cabass++2023)

# Recap

- LSS datasets are excellent probe of cosmology, **but** difficult to model on small scales (e.g., non-linear nuisances+baryonic bewilderments...)
- Consistency relations place robust, *non-perturbative* constraints on LSS correlators in the non-linear regime
- Consistency relations **can be violated** in multi-field inflation models/ cosmological collider scenario

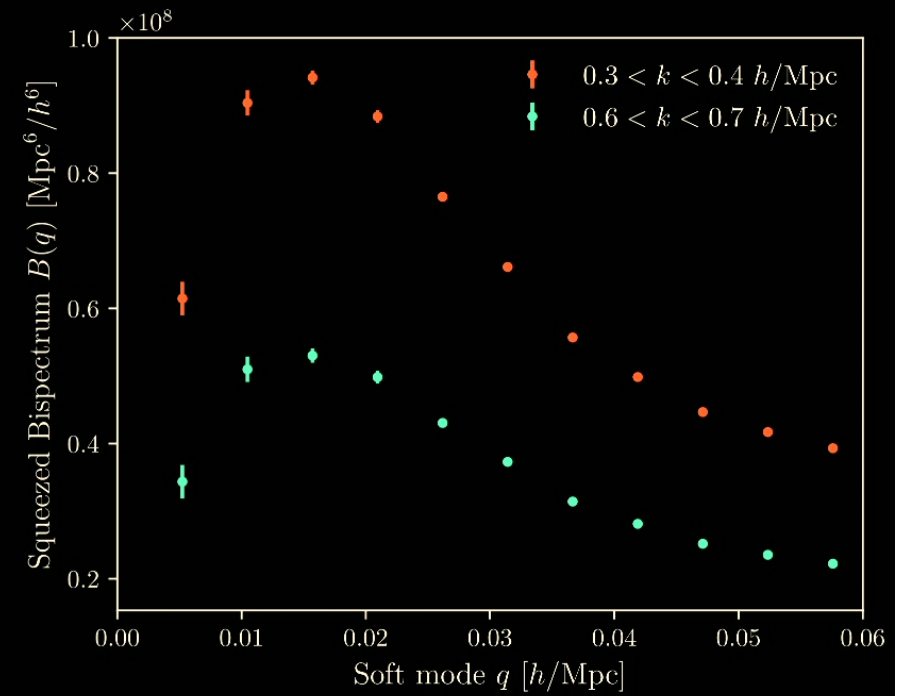
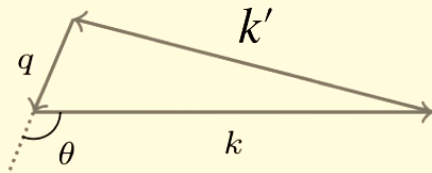
Can we use consistency relations violation to search for PNG with LSS measurements at smaller scales?

# Measurements

- Measure soft power spectrum ( $\hat{P}_m(q)$ )
- **Angle averaged** squeezed bispectrum

- $\hat{B}(q, k_{\min}, k_{\max}) = \int d\Omega_k B(q, k, k')$

- Average over wide  $k$ -bin for hard modes



Measurements of angle averaged squeezed bispectrum as a function of the soft mode for two different hard momenta bins.

# Non-perturbative model for the squeezed bispectrum

- Let's derive the leading order  $f_{\text{NL}}$  contribution to the **squeezed** matter bispectrum
- Squeezed bispectrum ( $q \ll k$ ) is described by **modulation** of small scale power spectrum to long wavelength gravitational potential  $\Phi_L$

$$\begin{aligned}
 \lim_{q \ll k_{\text{NL}}, k} B_m(q, k, k') &= \lim_{q \ll k_{\text{NL}}, k} \langle \delta_m(\mathbf{q}) \delta_m(\mathbf{k}) \delta_m(\mathbf{k}') \rangle', \quad (\text{Non-perturbative see, e.g.,} \\
 &\hspace{15em} \text{Lewis, 1107.5431}) \\
 &= \langle \delta_m(\mathbf{q}) P_m(k | \Phi_L) \rangle' = P_{\Phi_L m}(q) \frac{\partial P_m(k)}{\partial \Phi_L(q)}, \\
 &= \underbrace{\frac{3\Omega_{m0} H_0^2}{2q^2 T(q) D_{\text{md}}(z_q)}}_{\text{From Poisson's equation}} P_m(q) \frac{\partial P_m(k)}{\partial \Phi_L(q)}.
 \end{aligned}$$

# Adding in gravitational non-Gaussianity

- Squeezed bispectrum ( $q \ll k$ ) is **modulation** of small-scale power spectrum by a long-wavelength gravitational potential  $\Phi_L(\mathbf{x})$ .
- Contributions from **PNG** are associated with  $\partial P_m(k)/\partial\Phi_L(q)$
- Contributions from **gravitational non-Gaussianity** modeled using response approach

$$\lim_{q \ll k_{\text{NL}}, k} B_m(q, k, k') = \frac{3\Omega_{m0}H_0^2}{2q^2 T(q) D_{\text{md}}(z)} P_m(q) \left( \frac{\partial P_m(k)}{\partial \Phi_L(q)} \right) + \left( \bar{a}_0 + \bar{a}_2 \frac{q^2}{k^2} \right) P_m(q) P_m(k)$$

(e.g., Valageas 13; Chiang+17; Esposito+19; Biagetti+22; Giri+23)

Consistency relations violating term

Gravitational term (protected by symmetry)

$\bar{a}_0$  and  $\bar{a}_2$  are nuisance params describing non-linear physics (similar to bias params!)

- **How to estimate this *non-linear* potential derivative?**

# Separate universe prediction for $\partial P_m(k)/\partial\Phi_L(q)$

- Split perturbation into long (L) and short (S)

$$\phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + \Phi_S(\mathbf{x})$$

~Constant=background

- For local PNG,  $\Phi_L(\mathbf{x})$  modulates  $\Phi_S(\mathbf{x})$
- Same as local rescaling of amplitude of  $P^{\text{lin}}(k)$

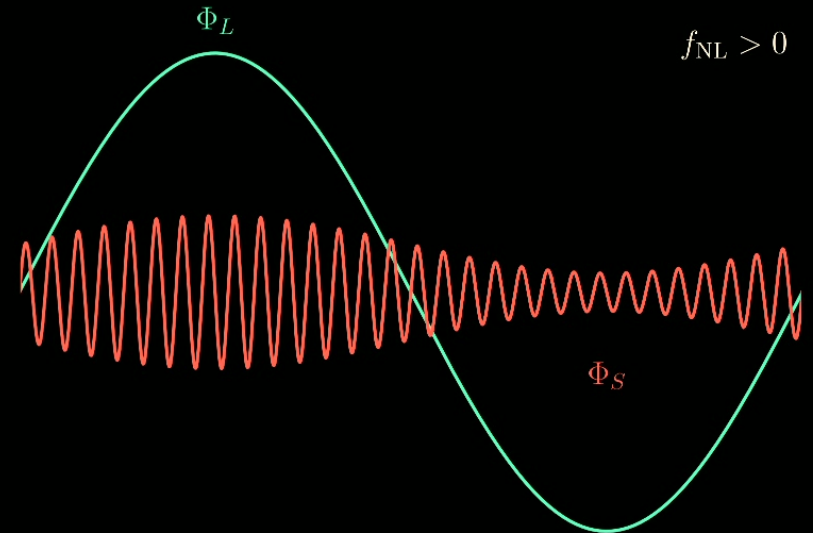
$$\sigma_8^{\text{loc.}}(\mathbf{x}) = (1 + 2f_{\text{NL}}\Phi_L(\mathbf{x}))\sigma_8$$

(e.g., [Giri, Münchmeyer, Smith 2305.03070](#))

- PNG is just a modification of the ICs!

$$\frac{\partial P_m(k)}{\partial\Phi_L(q)} = 2f_{\text{NL}} \frac{\partial P_m(k)}{\partial \log \sigma_8} \Big|_{\text{SU}}$$

(Dalal+07, Slosar+08, Desjacques+08)



$$\Phi = \Phi_L + f_{\text{NL}}(\Phi_L^2 - \langle\Phi_L^2\rangle) + (1 + 2f_{\text{NL}}\Phi_L)\Phi_S + f_{\text{NL}}(\Phi_S^2 - \langle\Phi_S^2\rangle)$$

Mode Coupling



# Separate universe prediction for $\partial P_m(k)/\partial\Phi_L(q)$

- Split perturbation into long (L) and short (S)

$$\phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + \Phi_S(\mathbf{x})$$

~Constant-background

- For local PNG,  $\Phi_L(\mathbf{x})$  modulates  $\Phi_S(\mathbf{x})$
- Same as local rescaling of amplitude of  $P^{\text{lin}}(k)$

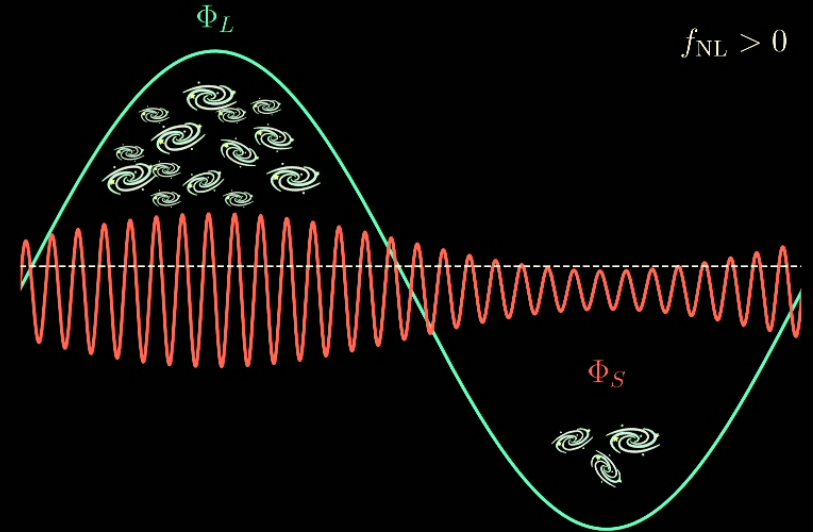
$$\sigma_8^{\text{loc.}}(\mathbf{x}) = (1 + 2f_{\text{NL}}\Phi_L(\mathbf{x}))\sigma_8$$

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(Dalal+07, Slosar+08, Desjacques+08)



“Scale-dependent” bias

[Dalal+08](#) / [Matarrese+08](#) /  
[Slosar+08](#) / [Desjacques+08](#)

# Separate universe (continued)

- For local PNG, we have

$$\frac{\partial P_m(k)}{\partial \Phi_L(q)} = 2f_{\text{NL}} \frac{\partial P_m(k)}{\partial \log \sigma_8} \Big|_{\text{SU}}$$

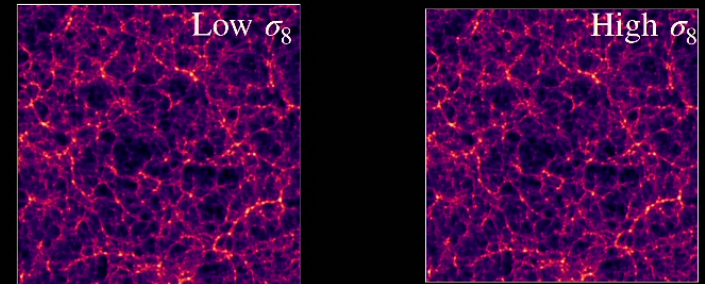
- **a): How do we compute this?**

- Run sims with modified  $P^{\text{lin.}}(k)$  and finite difference the late-time power spectra

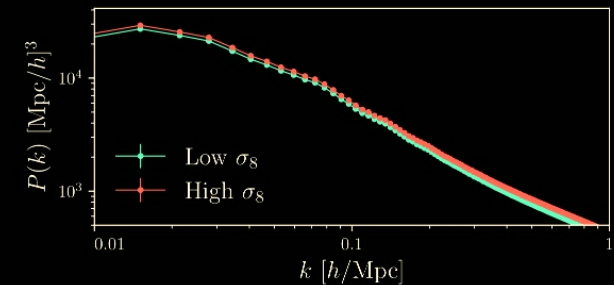
- **b): Why is this useful?**

- Relatively cheap to estimate from sims

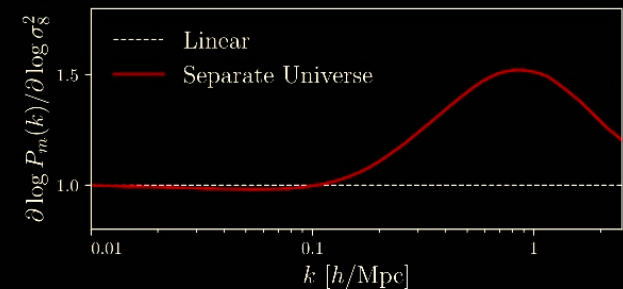
(i) Run sims with modified ICs



(ii) Compute  $P_{mm}(k)$  from simulation output



(iii) Finite difference to get logarithmic derivative



# Separate universe prediction for $\partial P_m(k)/\partial\Phi_L(q)$

- Split perturbation into long (L) and short (S)

$$\phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + \Phi_S(\mathbf{x})$$

~Constant=background

- For local PNG,  $\Phi_L(\mathbf{x})$  modulates  $\Phi_S(\mathbf{x})$
- Same as local rescaling of amplitude of  $P^{\text{lin}}(k)$

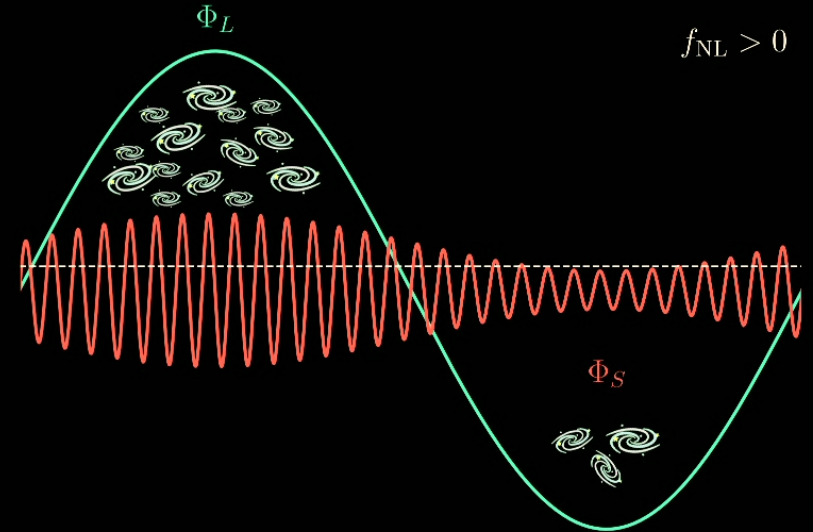
$$\sigma_8^{\text{loc.}}(\mathbf{x}) = (1 + 2f_{\text{NL}}\Phi_L(\mathbf{x}))\sigma_8$$

(e.g., [Giri, Münchmeyer, Smith 2305.03070](#))

- PNG is just a modification of the ICs!

$$\frac{\partial P_m(k)}{\partial\Phi_L(q)} = 2f_{\text{NL}} \frac{\partial P_m(k)}{\partial \log \sigma_8} \Bigg|_{\text{SU}}$$

(Dalal+07, Slosar+08, Desjacques+08)



“Scale-dependent” bias

[Dalal+08](#) / [Matarrese+08](#) /  
[Slosar+08](#) / [Desjacques+08](#)

# Separate universe prediction for $\partial P_m(k)/\partial\Phi_L(q)$

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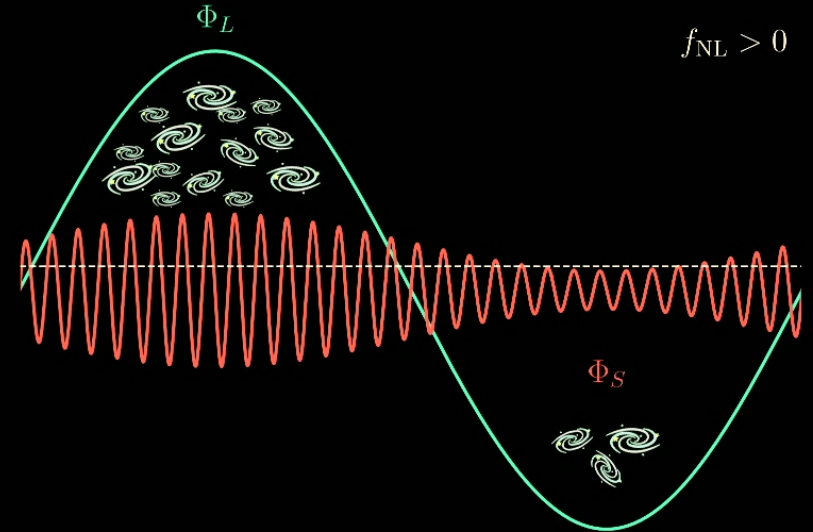
$$\sigma_8^{\text{loc.}}(\mathbf{x}) = (1 + 2f_{\text{NL}}\Phi_L(\mathbf{x}))\sigma_8$$

(e.g., [Giri, Münchmeyer, Smith 2305.03070](#))

- PNG is just a modification of the ICs!

$$\frac{\partial P_m(k)}{\partial\Phi_L(q)} = 2f_{\text{NL}} \frac{\partial P_m(k)}{\partial \log \sigma_8} \Big|_{\text{SU}}$$

(Dalal+07, Slosar+08, Desjacques+08)



“Scale-dependent” bias

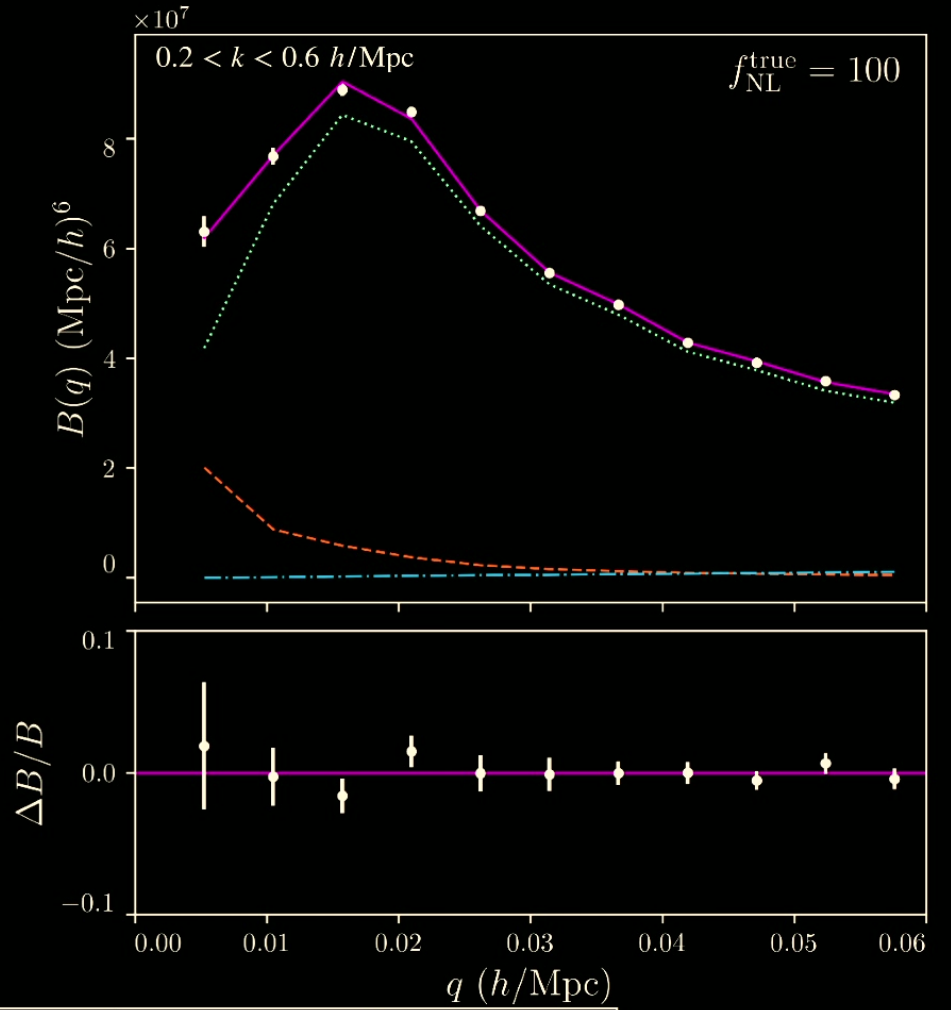
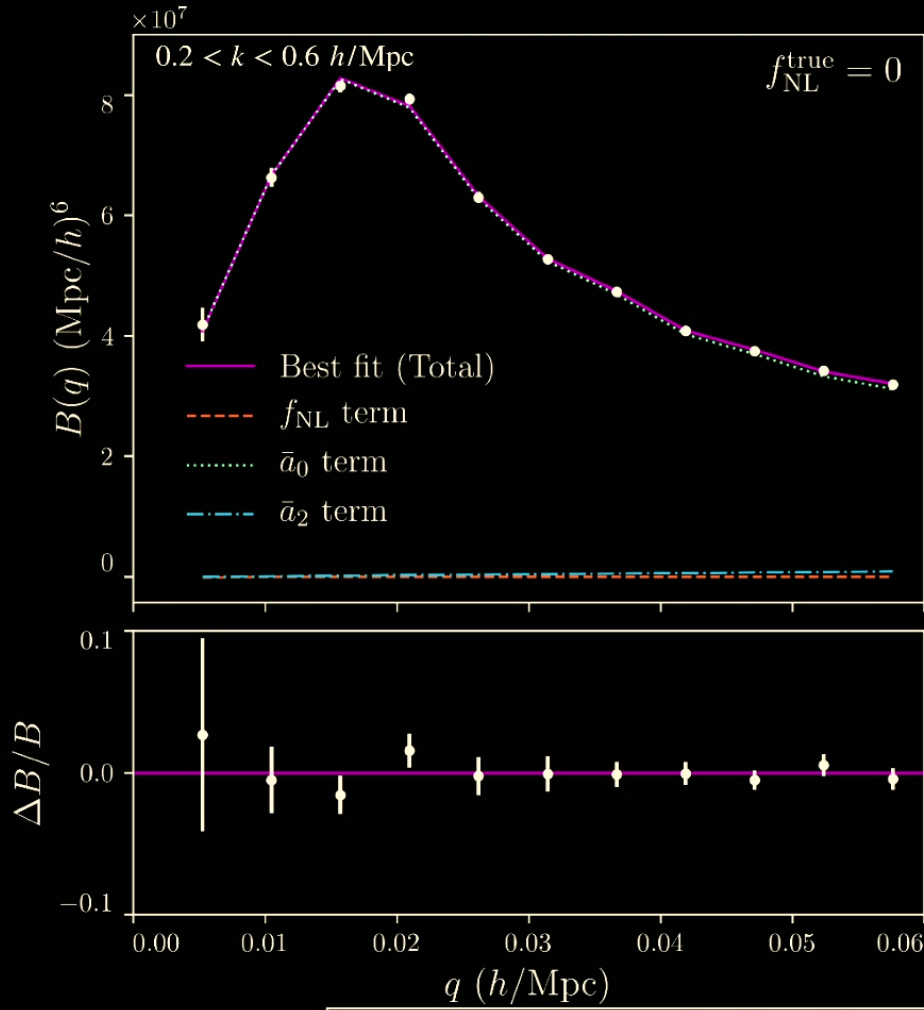
[Dalal+08](#) / [Matarrese+08](#) /  
[Slosar+08](#) / [Desjacques+08](#)

# Putting it all together

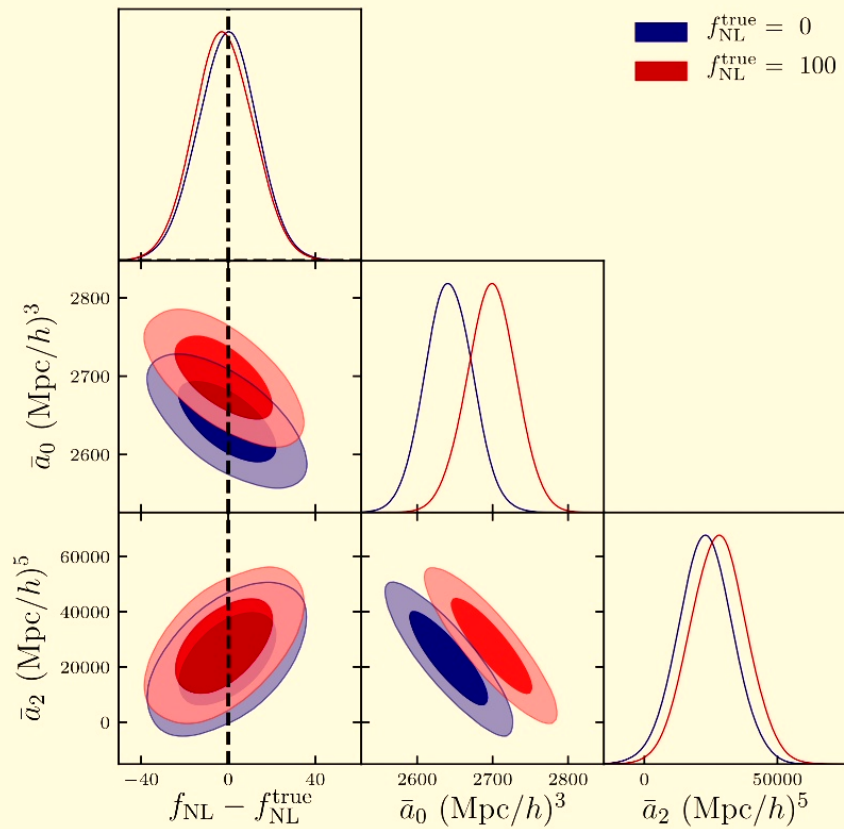
- Model for non-linear squeezed matter bispectrum is

$$\lim_{q \ll k_{\text{NL}}, k} B_m(q, k, k') = \left[ \frac{6\Omega_{m0}H_0^2}{q^2 T(q) D_{\text{md}}(z)} \frac{\partial \log P_m(k)}{\partial \log \sigma_8^2} \Big|_{\text{SU}} f_{\text{NL}} + \bar{a}_0 + \bar{a}_2 \frac{q^2}{k^2} \right] P_m(q) P_m(k)$$

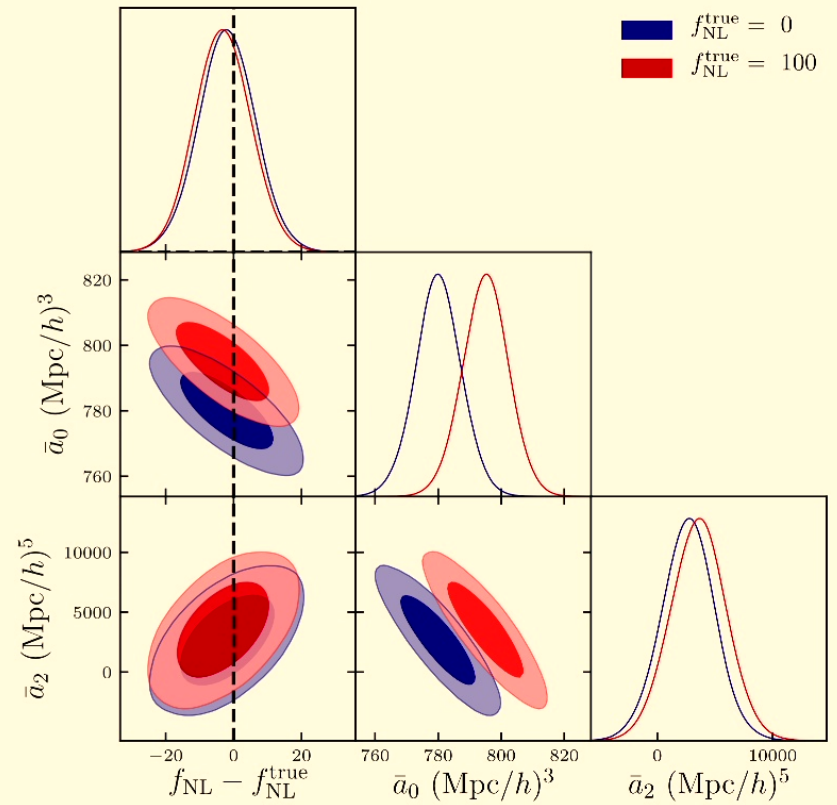
- Characterized by **three free parameters**:  $\{f_{\text{NL}}, \bar{a}_0, \bar{a}_2\}$
- **Goal**: validate model using measurements of  $B_m$  and  $P_m$  from N-body simulations
  - Likelihood analysis with covariance of  $B_m$  and  $P_m$  estimated from simulations
    - Joint likelihood in  $\hat{B}_m$  and  $\hat{P}_m$  for sample variance cancellation



$$\lim_{q \ll k_{\text{NL}}, k} B_m(q, k, k') = \left[ \frac{6\Omega_{m0}H_0^2}{q^2 T(q) D_{\text{md}}(z)} \frac{\partial \log P_m(k)}{\partial \log \sigma_8^2} \Big|_{\text{SU}} f_{\text{NL}} + \bar{a}_0 + \bar{a}_2 \frac{q^2}{k^2} \right] P_m(q) P_m(k)$$

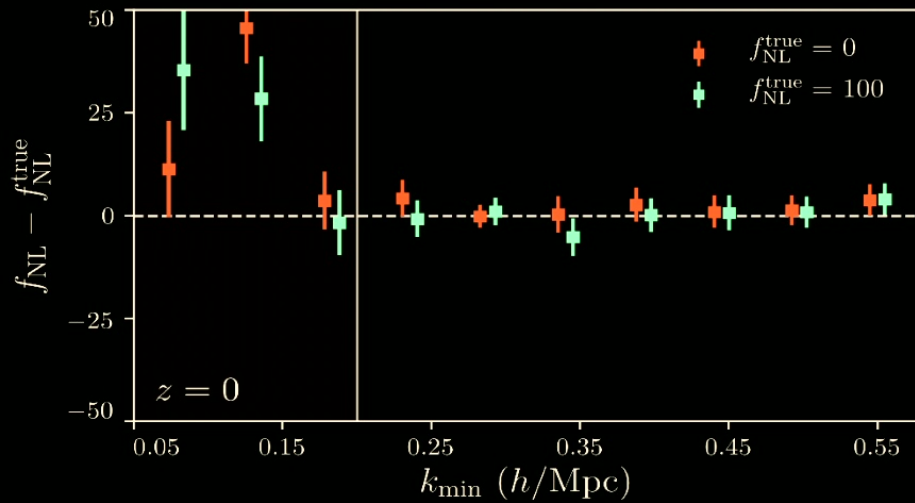


**Marginalized Posterior at z=0**



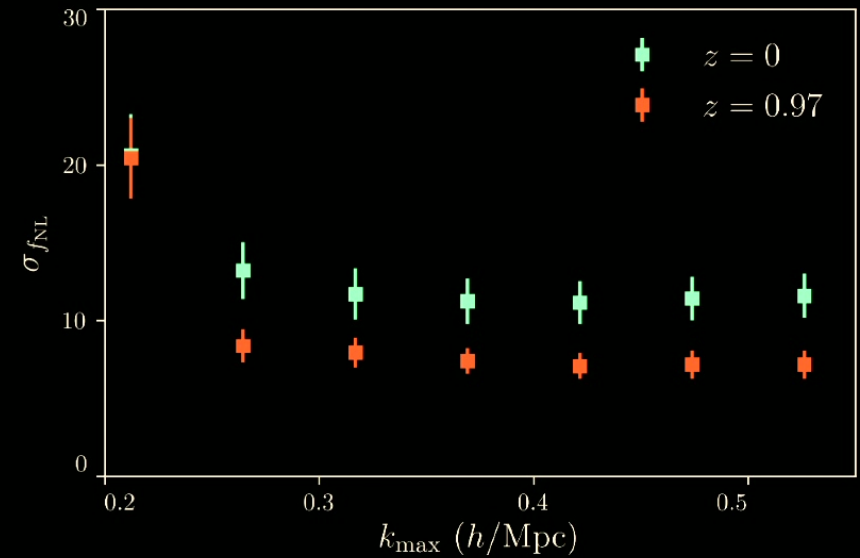
**Marginalized Posterior at z=0.97**

(i) How squeezed do the triangles need to be?



We recover the true value of  $f_{\text{NL}}$  for  $k_{\min} > 0.2 h/\text{Mpc}$

(ii) How much information is in non-linear regime?



Constraints saturate for  $k > 0.3 h/\text{Mpc}$  due to **non-Gaussian covariance**



# Recap/what's next

- We can reliably constrain  $f_{\text{NL}}^{\text{loc}}$  from small-scale squeezed matter bispectrum
  - Method is based on LSS consistency relations+separate universe approach
  - *In principle*, could improve future LSS searches for PNG
- What can we do with this result?
  - Extend result to lensing (**SG**+2023)
  - Extend to halos and include collapsed trispectrum (promising results from Giri+2023)
    - Can we get around the non-Gaussian covariance?
  - Generalize to redshift space (unsure)
  - Generalize to other models of PNG

# Part ii) Generalization to the Cosmological Collider

*“Massive-ish particles from small-ish scales”*

**2407.08731**

**SG, Philcox, Hill, Hui**

# Cosmological Collider Bispectrum Review

- Only difference for cosmological collider models is **potential derivative**

$$\lim_{q \ll k_{\text{NL}}, k} B_m(q, k, k') = \frac{3\Omega_{m0}H_0^2}{2q^2T(q)D_{\text{md}}(z)} P_m(q) \left( \frac{\partial P_m(k)}{\partial \Phi_L(q)} \right) + \left( \bar{a}_0 + \bar{a}_2 \frac{q^2}{k^2} \right) P_m(q) P_m(k)$$

- Recall cosmo. collider squeezed bispectrum

$$\lim_{q \ll k} B_\Phi(q, k) = 4f_{\text{NL}}^\Delta \left( \frac{q}{k} \right)^\Delta P_\Phi(q) P_\Phi(k); \quad 0 \leq \Delta < 3/2$$

- Local ( $\Delta = 0$ ) is special case
  - Need to generalize separate universe calculation for  $0 < \Delta < 3/2$ ?

# Separate universe and the cosmological collider

- Consider small-scale modes  $\delta_m(\mathbf{k}_1)$  and  $\delta_m(\mathbf{k}_2)$  with **fixed amplitude**, but  $k_1 < k_2$
- Add in **background** ( $\sim \text{const.}$ ) potential fluctuation  $\Phi_L(\mathbf{q})$
- Collider models have a **scale-dependent** response

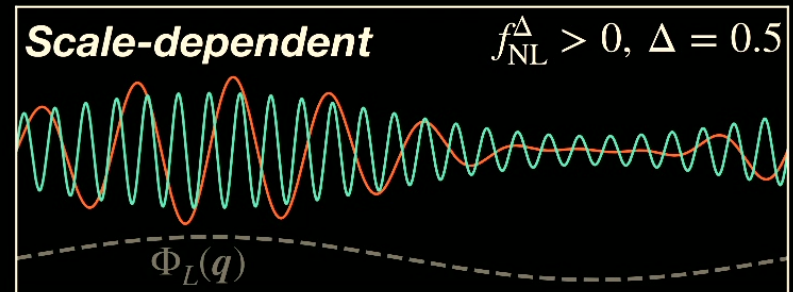
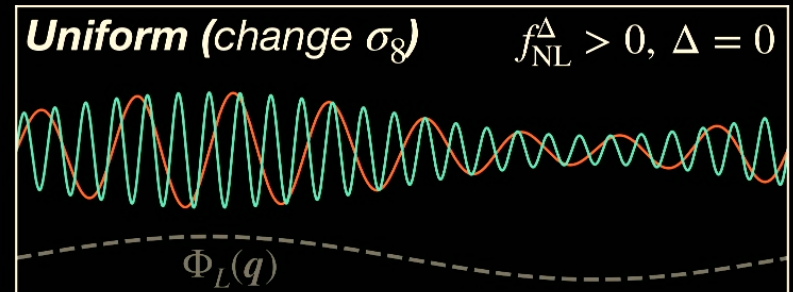
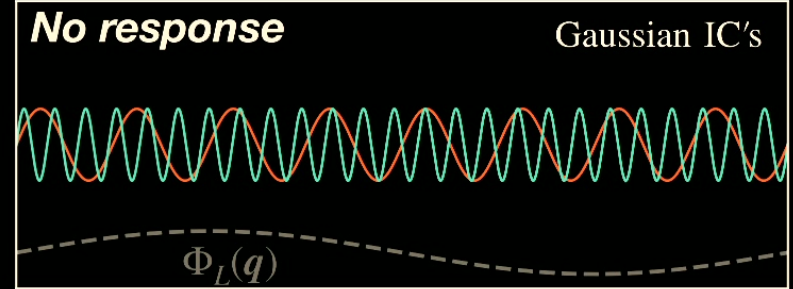
$$\delta_m(\mathbf{k} | \Phi_L(\mathbf{q})) = \left( 1 + 2f_{\text{NL}}^\Delta \left( \frac{q}{k} \right)^\Delta \Phi_L(\mathbf{q}) \right) \delta_m(\mathbf{k})$$

*Schmidt, Jeong, Desjacques, 2012*

- Can compute  $\partial P(k)/\partial \Phi_L(q)$  from sims with modified  $P_m^{\text{lin.}}$

$$P_m^{\text{lin.}}(k | \epsilon, \Delta) \equiv (1 + 2\epsilon k^{-\Delta}) P_m^{\text{lin.}}(k).$$

$$\frac{\partial P_m(k)}{\partial \Phi_L(\mathbf{q})} = 2f_{\text{NL}}^\Delta q^\Delta \frac{\partial P_m(k | \epsilon, \Delta)}{\partial \epsilon} \Bigg|_{\epsilon=0}.$$



# Potential derivatives

- Compute logarithmic derivative from separate universe sims

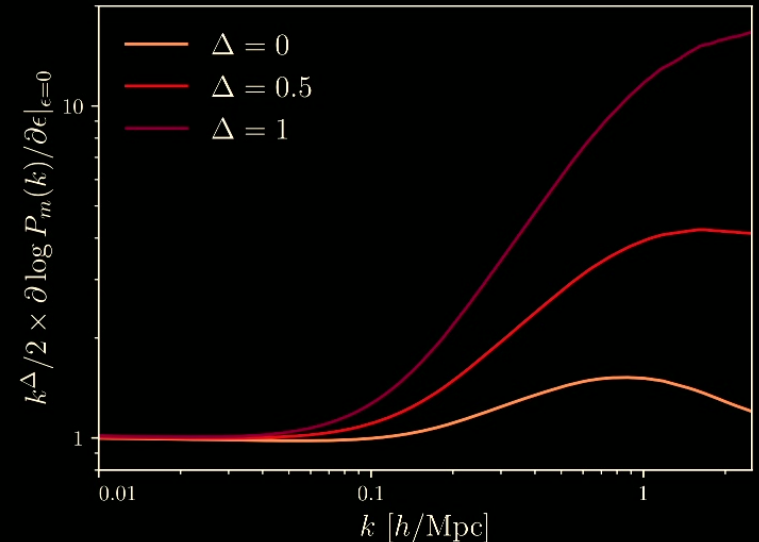
$$P_m^{\text{lin.}}(k | \epsilon, \Delta) \equiv (1 + 2\epsilon k^{-\Delta}) P_m^{\text{lin.}}(k).$$

$$\frac{\partial P_m(k)}{\partial \Phi_L(\mathbf{q})} = 2f_{\text{NL}}^\Delta q^\Delta \frac{\partial P_m(k | \epsilon, \Delta)}{\partial \epsilon} \Bigg|_{\epsilon=0}.$$

Get from finite differencing SU sims with small  $\pm \epsilon$

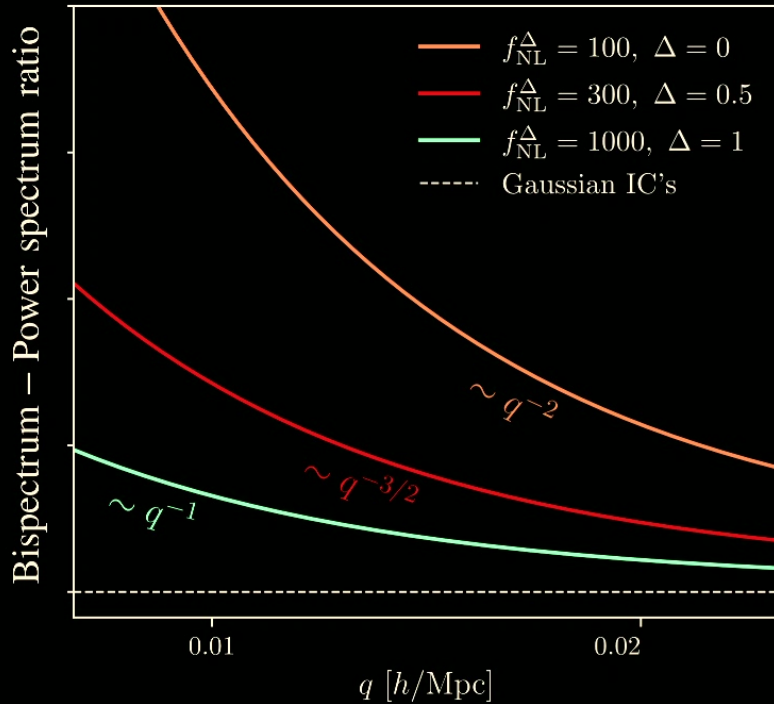
- Strong dependence on  $\Delta$
- Can also use sims to estimate non-Gaussian bias  $b_{\Psi, \Delta}$

$$b_{\Psi, \Delta}(M, z) = \frac{\partial \log \bar{n}_h(M, z)}{\partial \epsilon} \Bigg|_{\epsilon=0}$$

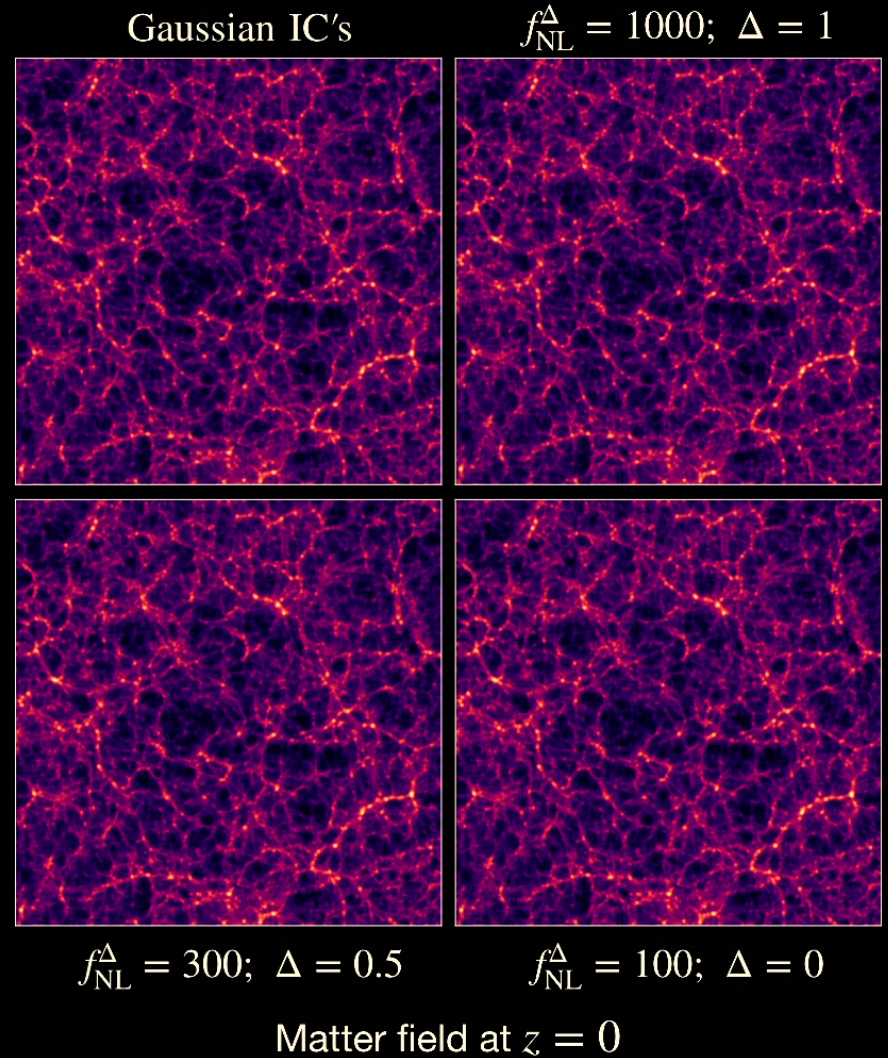


Separate universe potential derivatives for different values of  $\Delta$

# N-body simulations with collider bispectrum

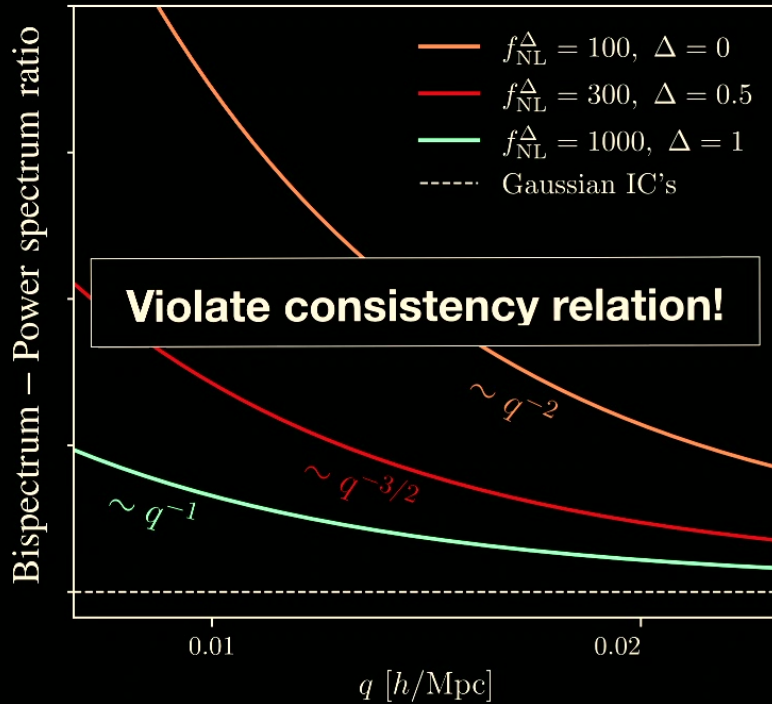


→  
**GADGET**



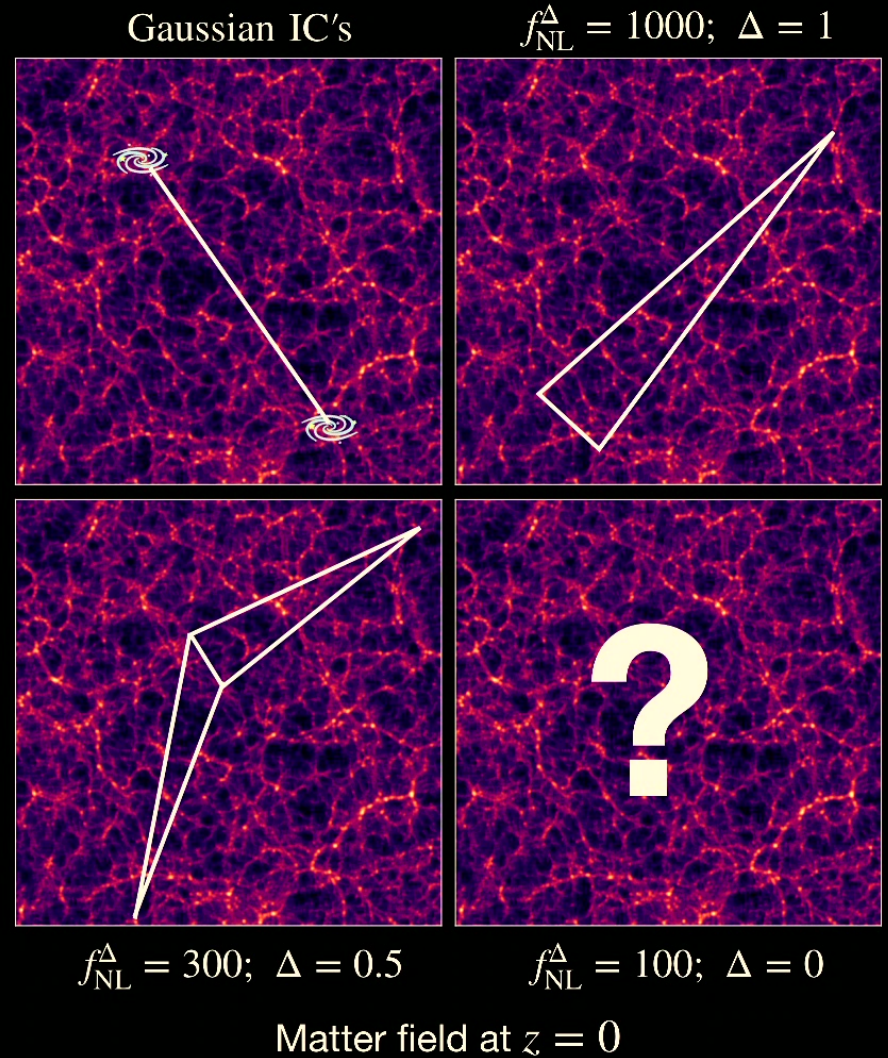
- Added squeezed collider templates to **2LPTPNG**
- Ran suite of simulations with same settings as **QuijotePNG**, but collider primordial bispectrum

# N-body simulations with collider bispectrum

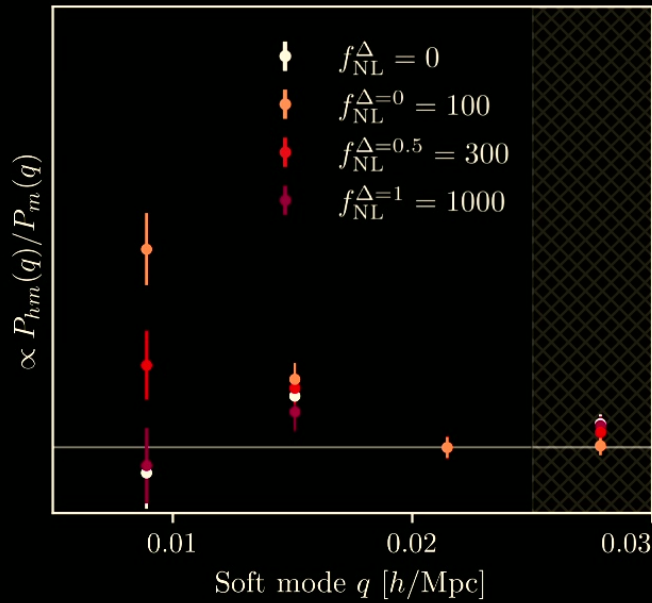


→  
**GADGET**

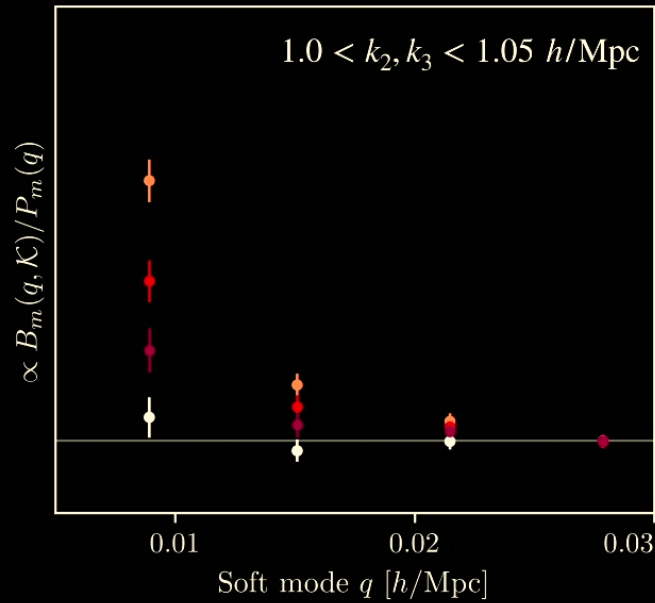
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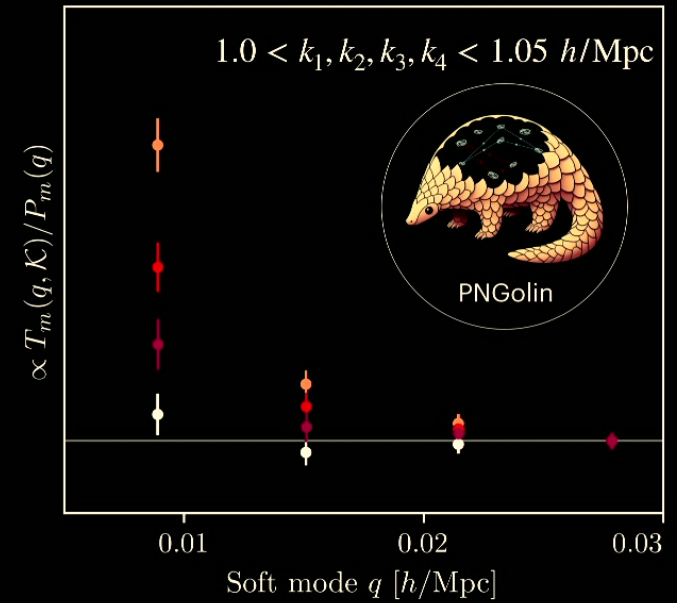
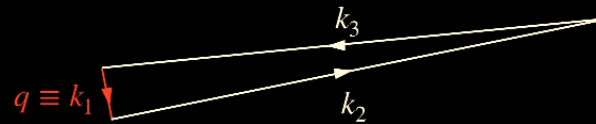
# Imprints of the cosmological collider on LSS correlators



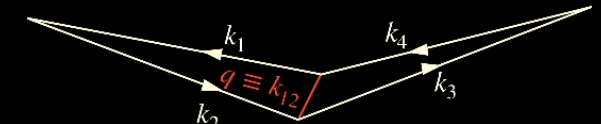
Scale-dependent halo bias



Squeezed matter bispectrum



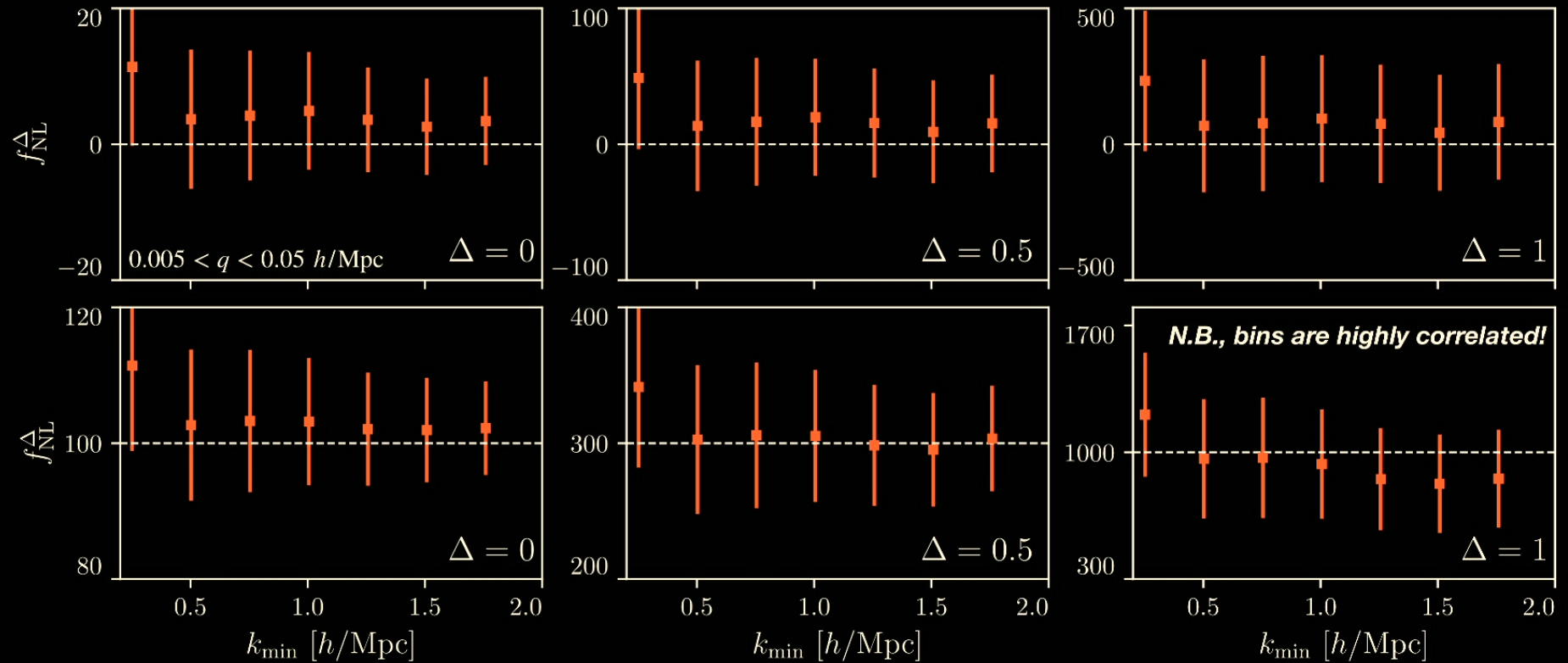
Collapsed matter trispectrum



\*<https://github.com/samgolds/PNGolin>



# Constraints from squeezed bispectrum at $z = 0$

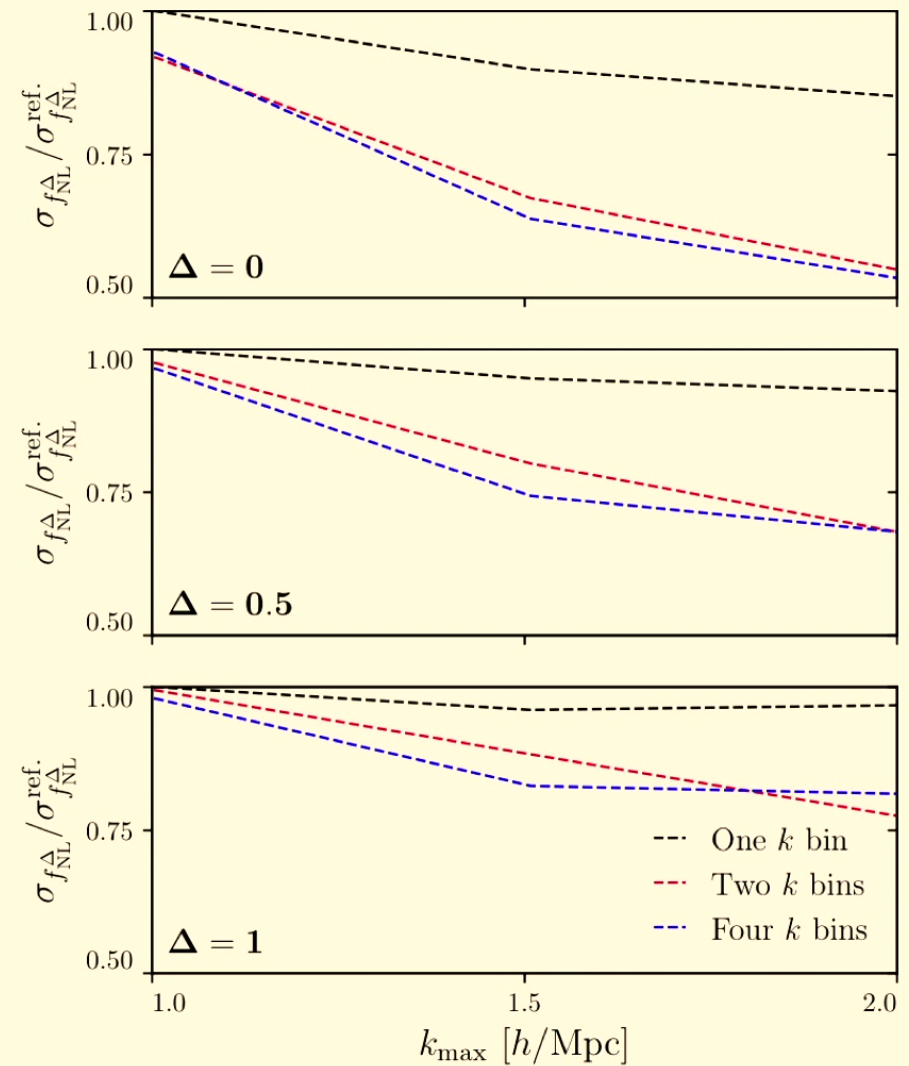
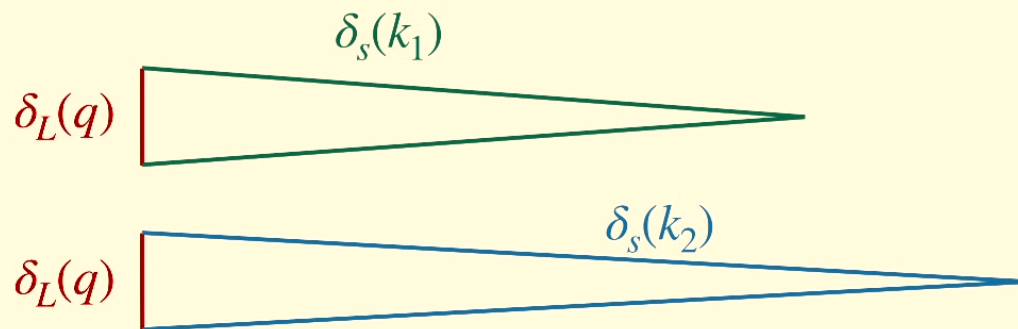


Unbiased constraints on  $f_{\text{NL}}^{\Delta}$  using *non-linear* squeezed matter bispectrum for all models!

# Information content

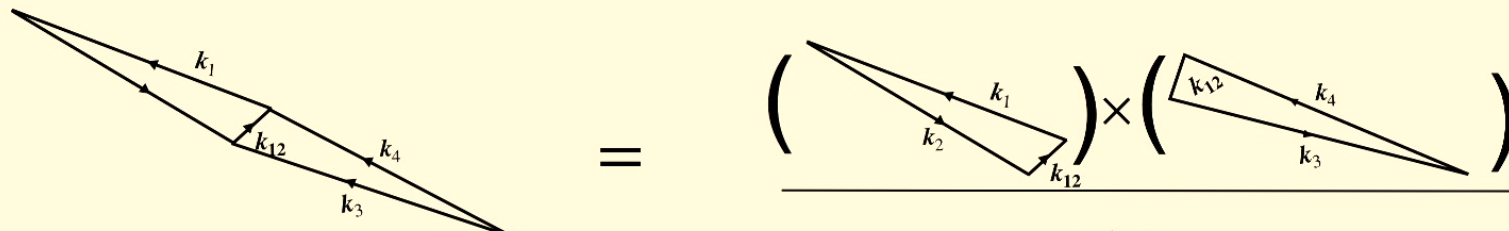
- Do we learn anything at small scales (high  $k$ )?
  - Yes! Assuming we use multiple  $k$  bins
- Multiple bins cancels cosmic variance from long mode (just like multi-tracer)

Similar results in [de Putter+18](#) and [Giri+23](#)



# The collapsed trispectrum

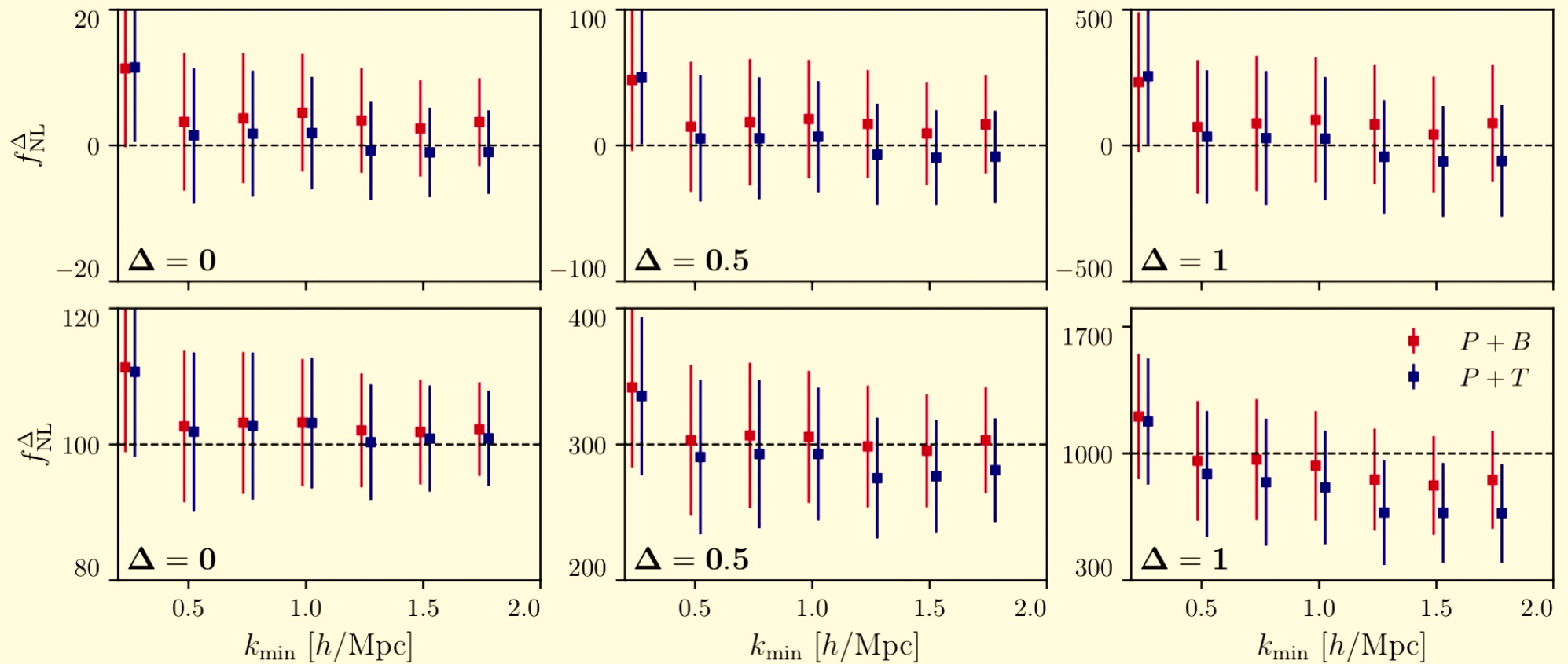
- Collapsed trispectrum  $\sim$  (squeezed bispectrum)<sup>2</sup>



$$T_m(q, k_1, k_3) = \frac{B_m(q, k_1)B_m(q, k_3)}{P_m(q)} + T_0(k_1, k_3),$$

- Squeezed bispectrum model can be used to study the collapsed trispectrum
- Measurement is main challenge: <https://github.com/samgolds/PNGolin>  
(Based on Coulton+23)

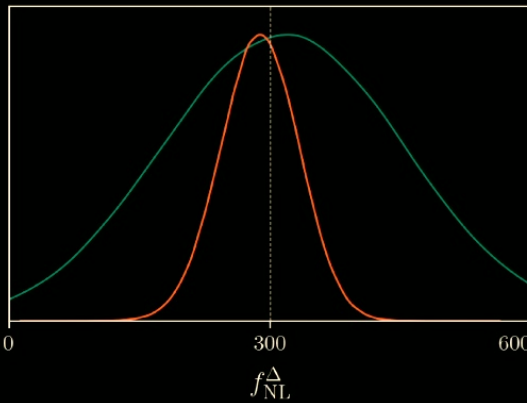
# Validating trispectrum model



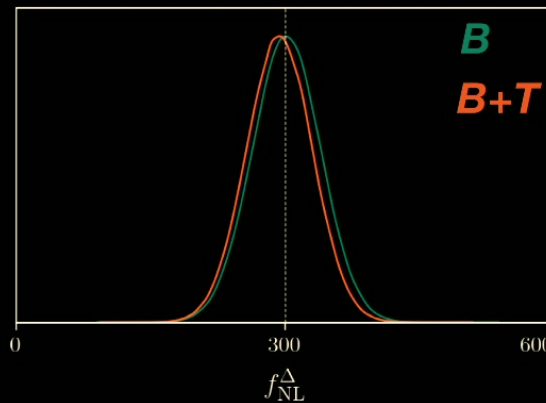
# Collapsed trispectrum

- What can we learn from the trispectrum?
- Higher-order PNG  $\longrightarrow$
- Cosmic variance cancellation  $\downarrow$

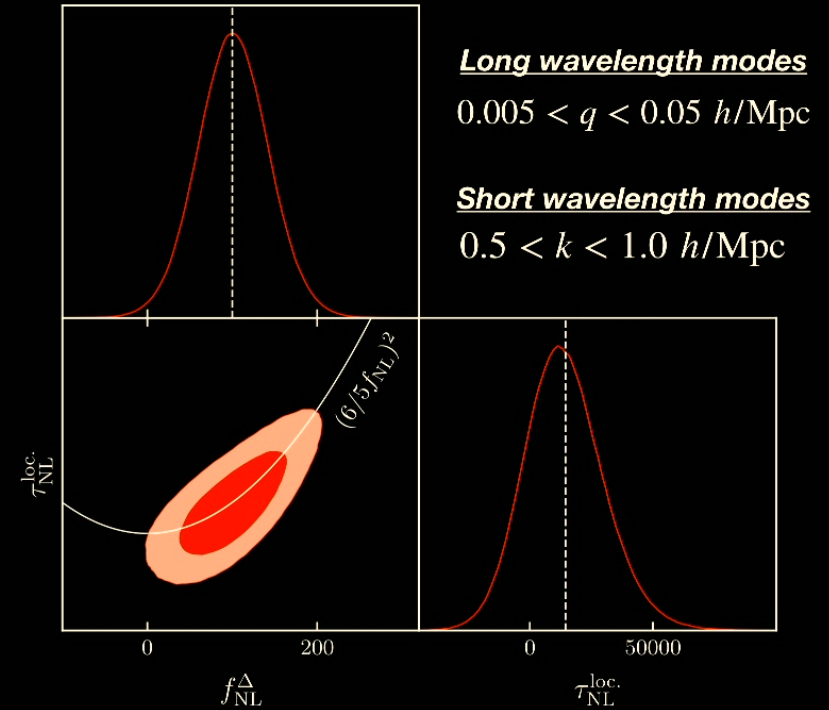
Unknown  $P_{mm}(k)$



Known  $P_{mm}(k)$



P+B+T joint analysis



# Scale-dependent halo bias

- Squeezed bispectrum leads to **scale-dependent bias** (Dalal+07, Slosar+08, Desjacques+08)

$f_{\text{NL}}^\Delta$  degenerate with non-Gaussian bias

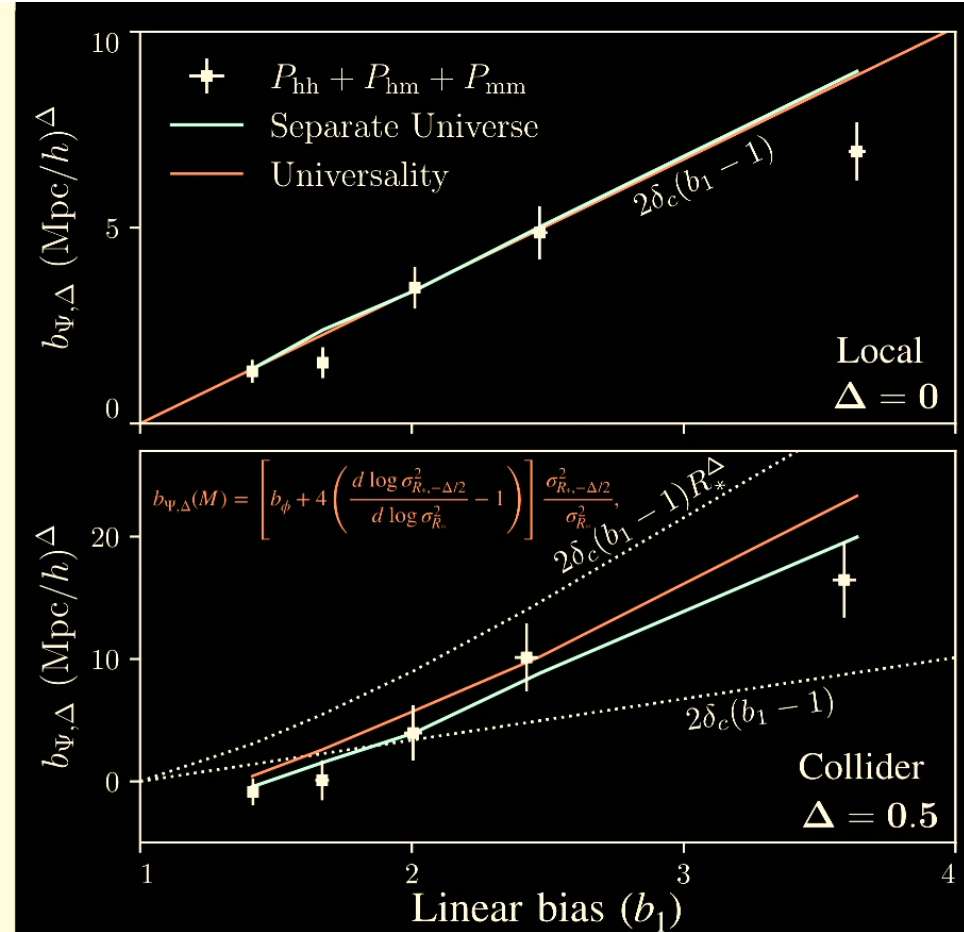
$$P_{hm}(q) = \left[ b_1 + \frac{3\Omega_{m0}H_0^2}{2D_{\text{md}}(z)} \frac{b_{\Psi,\Delta} f_{\text{NL}}^\Delta}{q^{2-\Delta}} \right] P_m(q)$$

Power depends on  $\Delta$

- Fit for  $b_{\Psi,\Delta}$  at fixed  $f_{\text{NL}}^\Delta$  and  $\Delta$
- Test predictions for galaxy biasing in non-local PNG e.g., Shandera+2010, Schmidt & Kamionkowski, 2010 Schmidt+2012

$$b_{\Psi,\Delta} = \left. \frac{2 \partial \log \bar{n}_h}{\partial \epsilon} \right|_{\epsilon=0},$$

- Galaxies will be more challenging...



Fits to **halo** catalogues from sims agree with separate universe and universality.

# Conclusions

- Consistency relations provide opportunity to learn about PNG from non-linear LSS
  - Constructed and validated **non-perturbative** models for squeezed  $B_m$  and collapsed  $T_m$ 
    - Validated up to  $k_{\max} = 2 h/\text{Mpc}$  for local PNG and Cosmo. Collider
- Tools developed here could be useful for other works
  - Publicly available trispectrum estimators
  - Methods for generating sims with Cosmo. Collider PNG
  - Validation of separate universe approach for Cosmo. Collider **scale-dependent bias**  
*(e.g., [Green, Guo, Han, Wallisch, 2023](#))*
- Future directions
  - E.g., **spin/oscillatory bispectra**,  $B_g$  and  $T_g$ , **multi-tracer/ $\mathbf{b}_{\Psi,\Delta}$** , CMBxLSS, **can we constrain  $\Delta$ ?**
  - Is the bispectrum/trispectrum even a good statistic in the non-linear regime?



**Thank you!**