Title: Generalized tensors and partial traces over quantum networks

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Series: Quantum Foundations, Quantum Information

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Laboratoire Méthodes Formelles

Generalized tensors and partial traces over quantum networks

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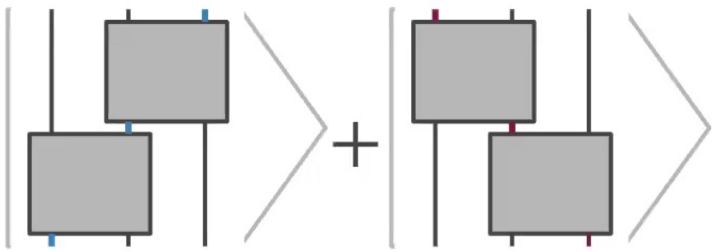
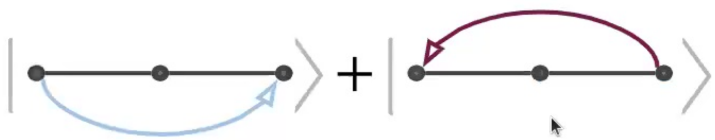
Indefinite causal orders, quantum gravity, quantum internet…

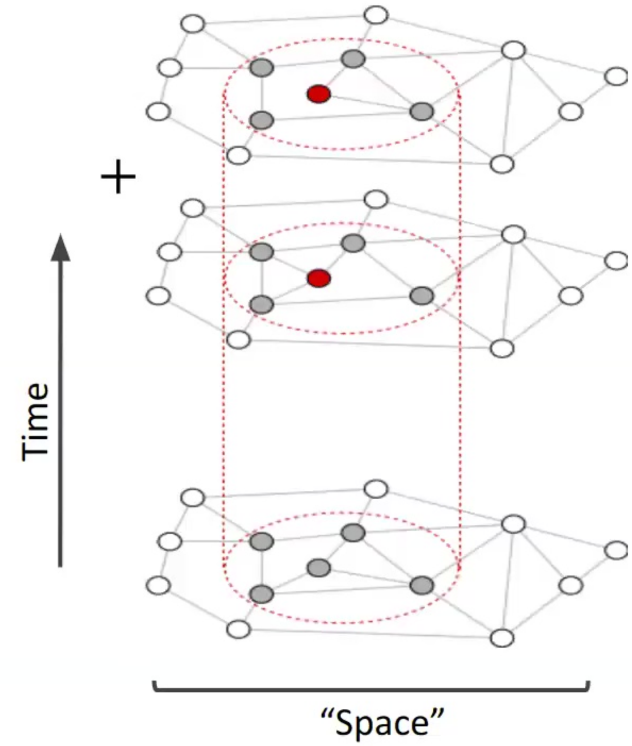


has interaction graph :

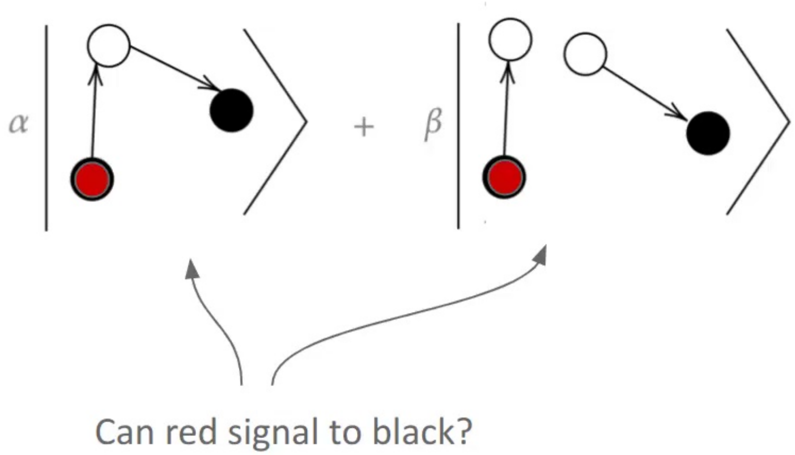


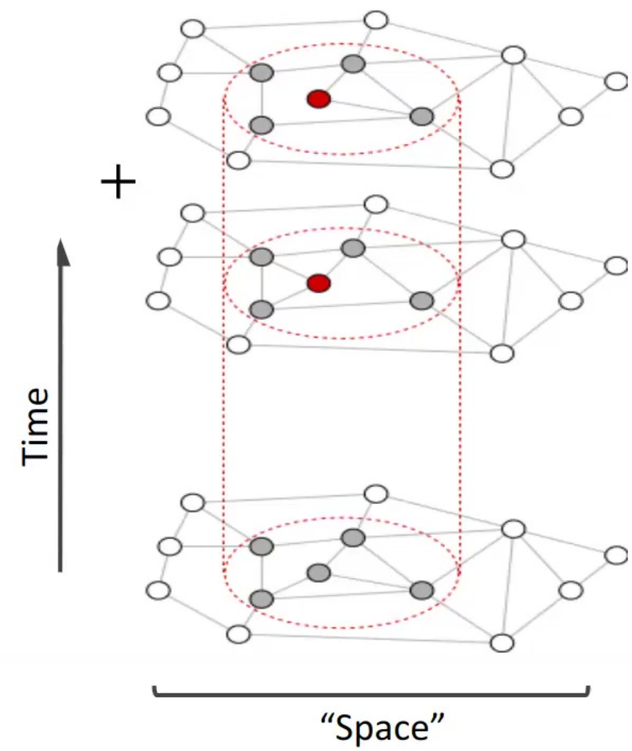
… feature fully quantum networks:

» They need formalization.



Indefinite causal orders, quantum gravity, quantum internet…





… feature **fully quantum networks:**

» They need formalization.

» Locality, causality of unitary evolutions over them are poorly understood.

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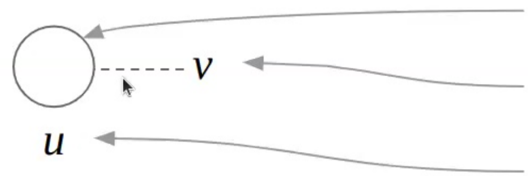
Gen. tensors & traces » **Hilbert space**

Def. System.

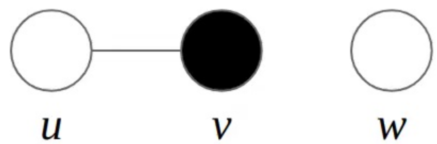


internal state in Σ, some (often finite) set.

possibly some dangling edges.

name in $\mathcal{V}$, a countably infinite set.

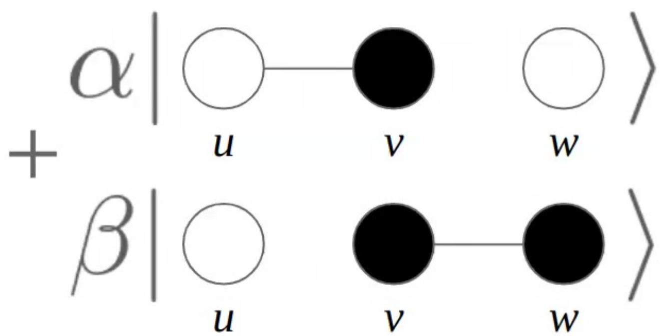
Def. Graph.



finite but unbounded set of systems having unique names.

$G \in \mathcal{G}$.

Def. State.

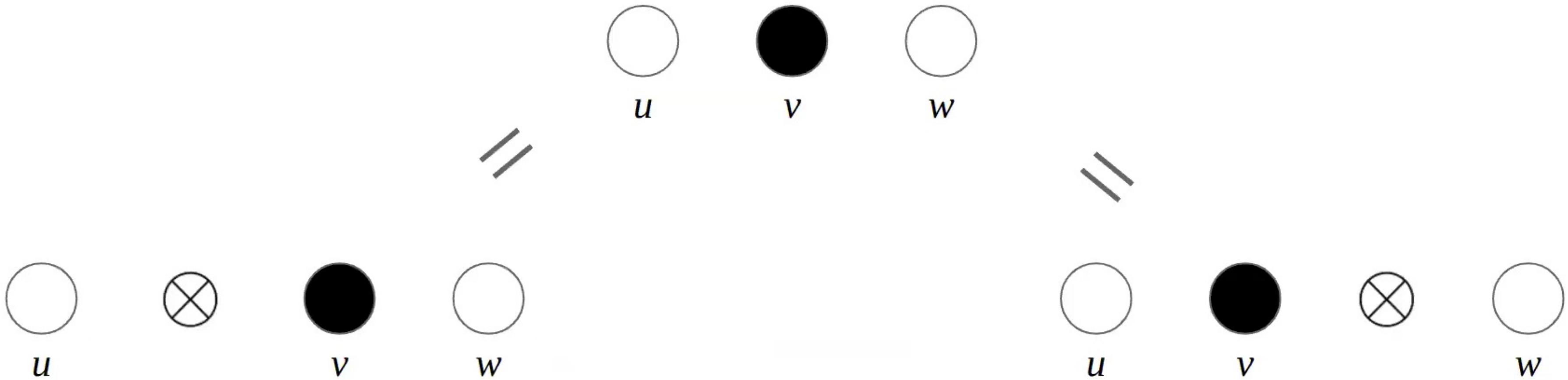


element of the Hilbert space whose o.n.b. is the set of graphs.

$|\psi\rangle \in \mathcal{H}$.

- Generalised tensors & traces
- Why they work.

Gen. tensors & traces » **Good old tensors… made precise**



prefer:

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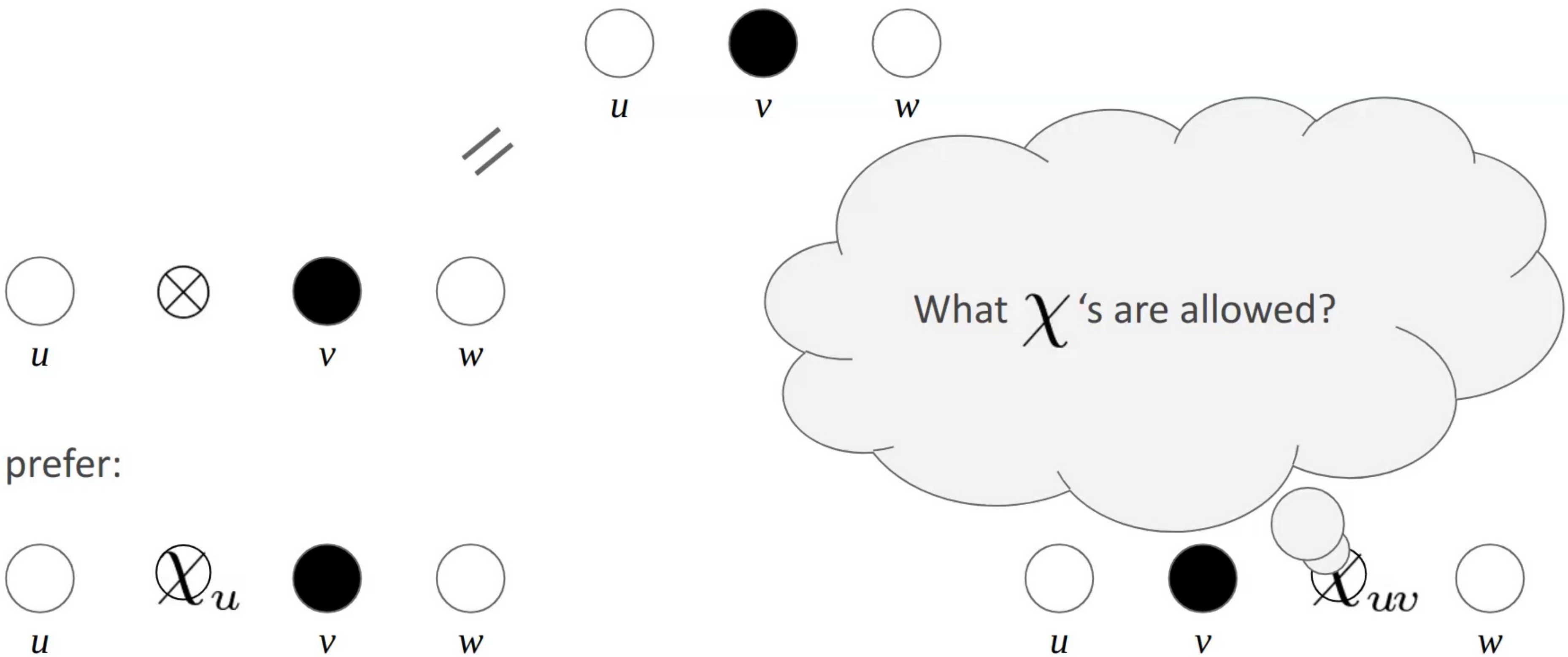
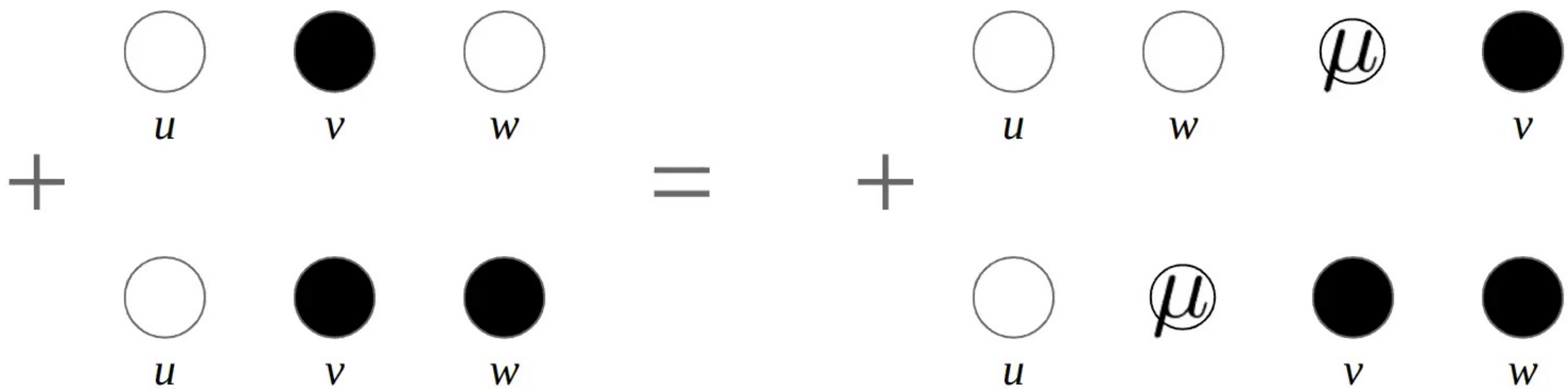
Gen. tensors & traces » **Good old tensors… made precise**



Gen. tensors & traces » **Quantizing the partition**

Rk. Because discriminating criterion is quantized, so is the split into subsystems:



Gen. tensors & traces » **Tensors**

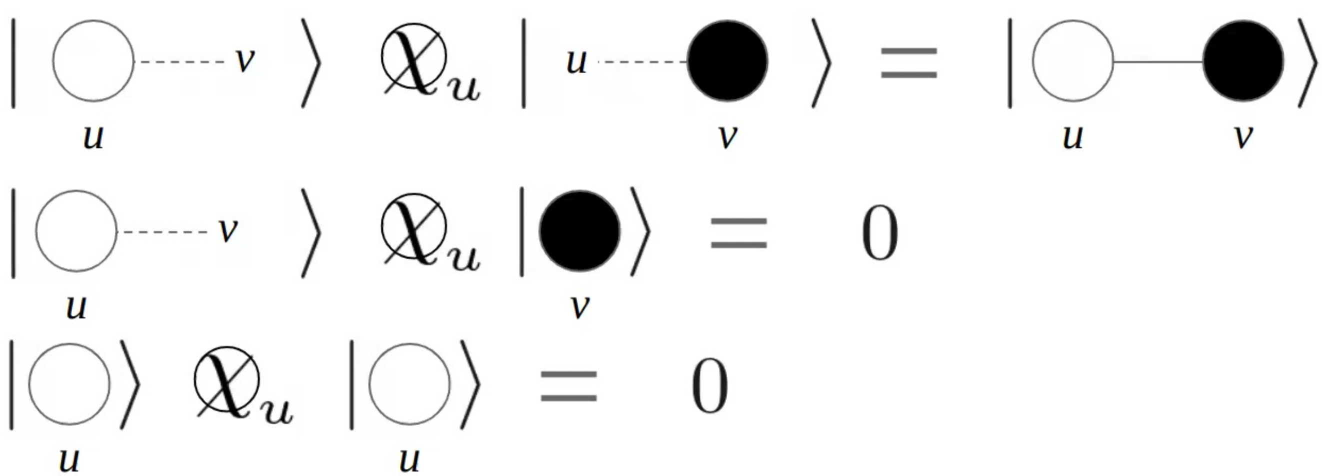
any idempotent restriction

A bilinear operator $\textcircled{\chi} : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$

$$|L\rangle \; \textcircled{\chi} \; |R\rangle = \begin{cases} |G\rangle & \text{if } L = G_\chi \text{ and } R = G_{\overline{\chi}} \\ 0 & \text{otherwise} \end{cases}$$

$G_{\overline{\chi}} = G \setminus G_\chi$

Ex.

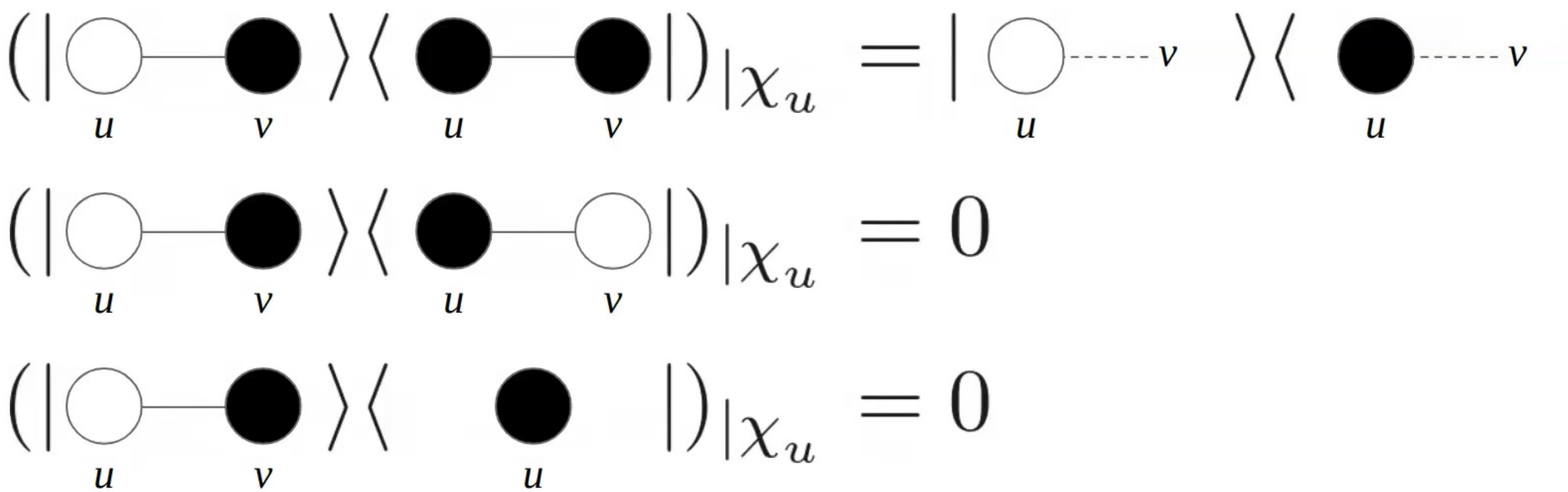


Gen. tensors & traces » **Traceouts**

A linear operator $|_\chi : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H})$

$$(|G\rangle\langle H|)|_\chi := |G_\chi\rangle\langle H_\chi|\langle H_{\overline{\chi}}|G_{\overline{\chi}}\rangle$$

Ex.



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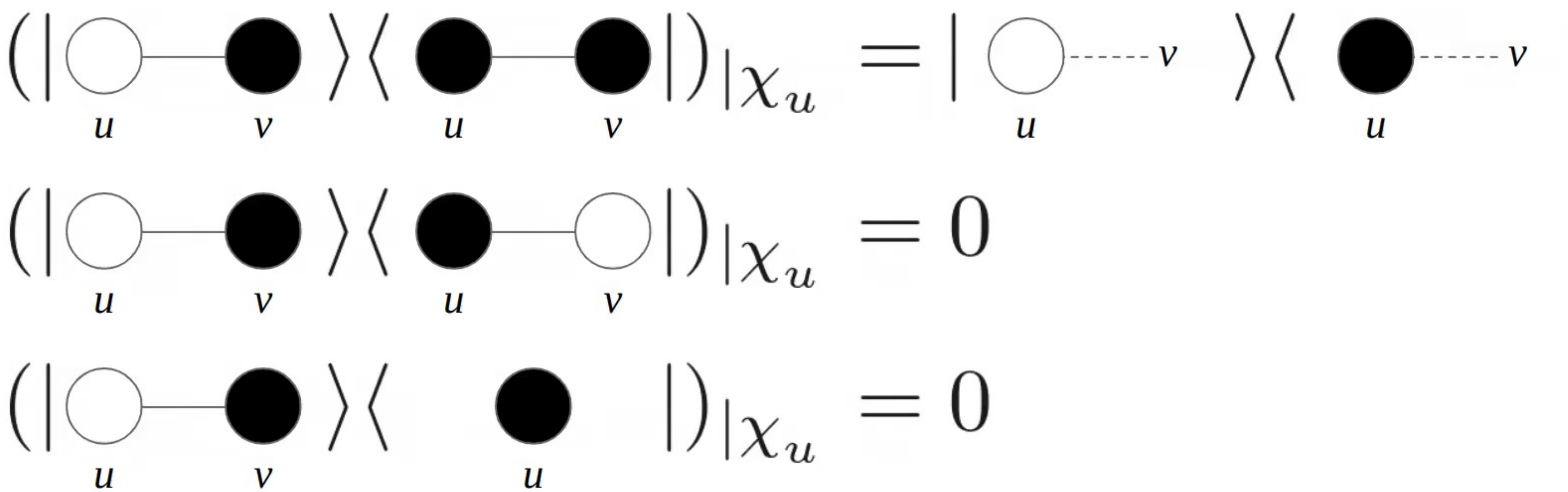

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Prop. $|_\chi$ is a TPCP.

- Generalised tensors & traces
- Why they work.

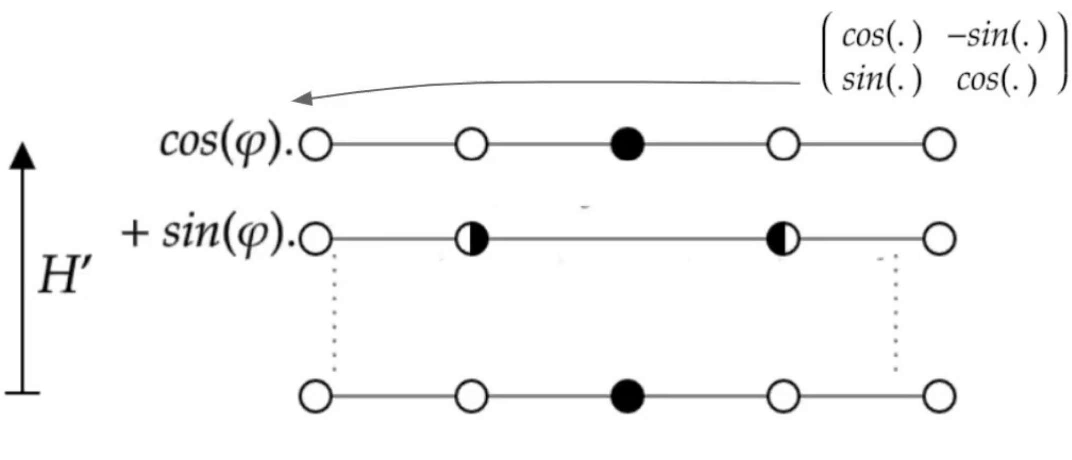
Why they work » Fully quantum networks » **Locality**

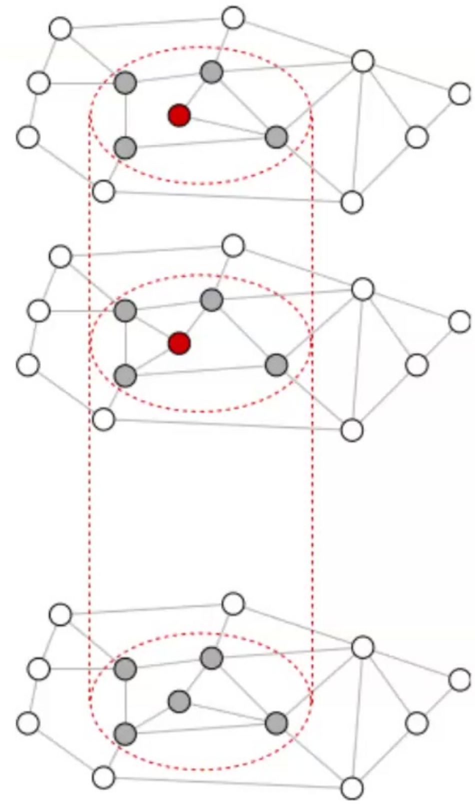
Th. $$\langle H|A|G\rangle = \langle H_\chi|A|G_\chi\rangle\langle H_{\overline{\chi}}|G_{\overline{\chi}}\rangle$$

$\Leftrightarrow$ $$A = A \otimes_{\chi} I$$

$\Leftrightarrow$ $$\mathrm{Tr}(A\rho) = \mathrm{Tr}(A\rho_{|\chi})$$

Ex.





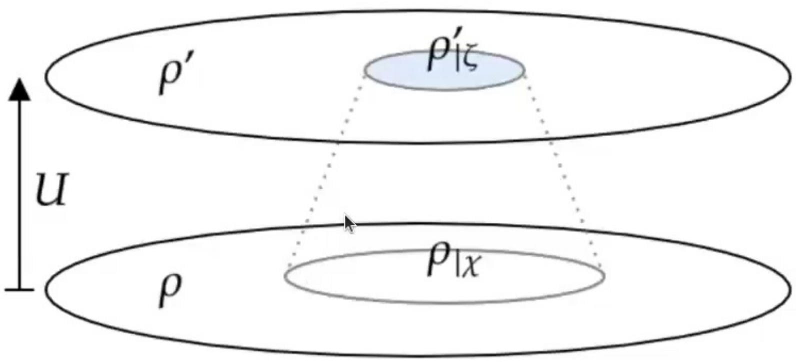
Why they work. » Fully quantum networks » **Causality**

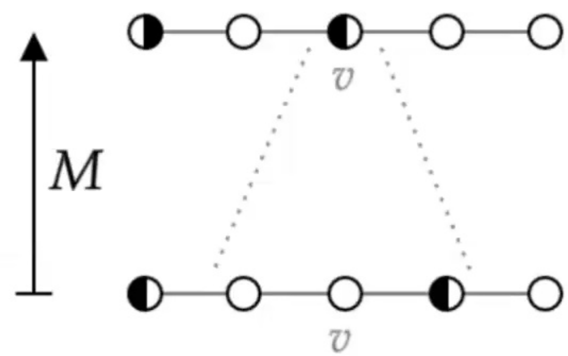
U $\chi\zeta$-causal:

Th. $$(U\rho U^\dagger)_{|\zeta} = (U\rho_{|\chi} U^\dagger)_{|\zeta}$$

$$\Leftrightarrow \quad U \text{ decomposes into } \chi\text{-local gates.}$$

$$\Leftrightarrow \quad A \ \ \zeta\text{-local} \ \Rightarrow \ UAU^\dagger \ \chi\text{-local}$$





$$U = H'M$$

Summary »

arXiv:2110.10587

Aim »

For the sake of indefinite causal orders or quantum gravity: formalizing fully quantum networks; studying local and causal evolutions upon them.

Generalized tensors & traces »

Were needed for that. Parametrized by any restriction. Internalized.

Why they work. »

Locality, causality in the Shrödinger, Operational and Heisenberg pictures. Safeguarding their logical interrelations.