

Title: Generalized tensors and partial traces over quantum networks

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Series: Quantum Foundations, Quantum Information

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Generalized tensors and partial traces over quantum networks

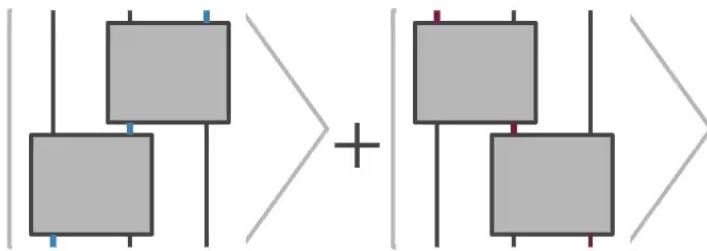
Pablo Arrighi

Amélia Durbec

Matt Wilson



Indefinite causal orders, quantum gravity, quantum internet...

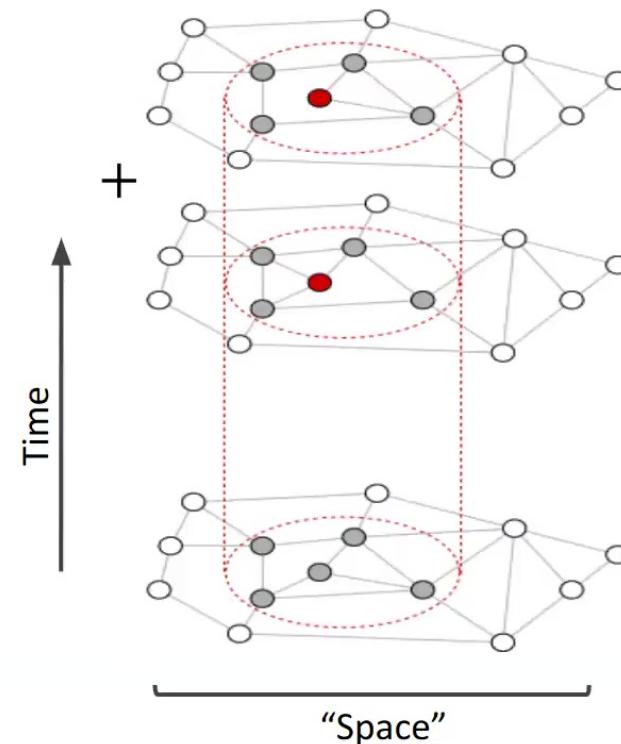


has interaction graph :

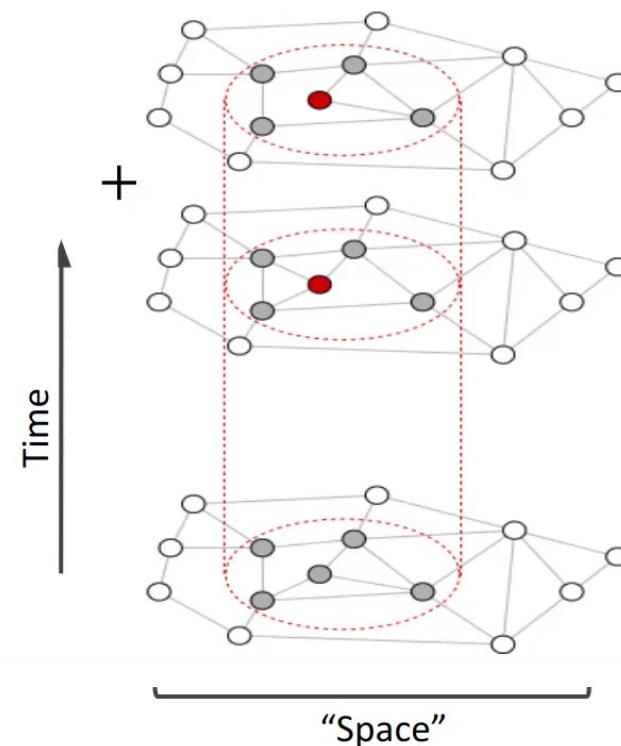
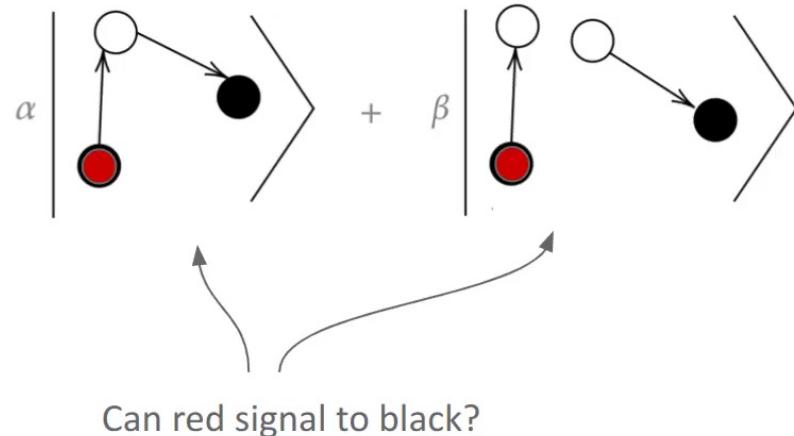


... feature **fully quantum networks**:

- » They need formalization.



Indefinite causal orders, quantum gravity, quantum internet...

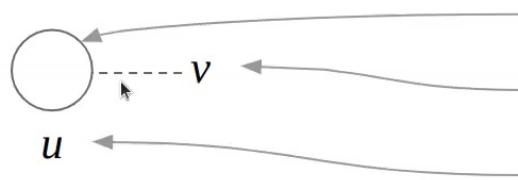


... feature **fully quantum networks**:

- » They need formalization.
- » Locality, causality of unitary evolutions over them are poorly understood.

Gen. tensors & traces » Hilbert space

Def. System.

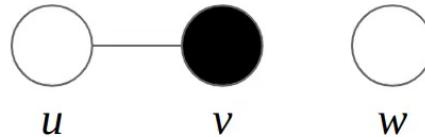


internal state in Σ , some (often finite) set.

possibly some dangling edges.

name in \mathcal{V} , a countably infinite set.

Def. Graph.



finite but unbounded set of systems having unique names.

$$G \in \mathcal{G}.$$

Def. State.

$$+ \alpha | \begin{array}{c} \text{---} \\ | \quad | \\ u \quad v \quad w \end{array} \rangle + \beta | \begin{array}{c} \text{---} \\ | \quad | \\ u \quad v \quad w \end{array} \rangle$$

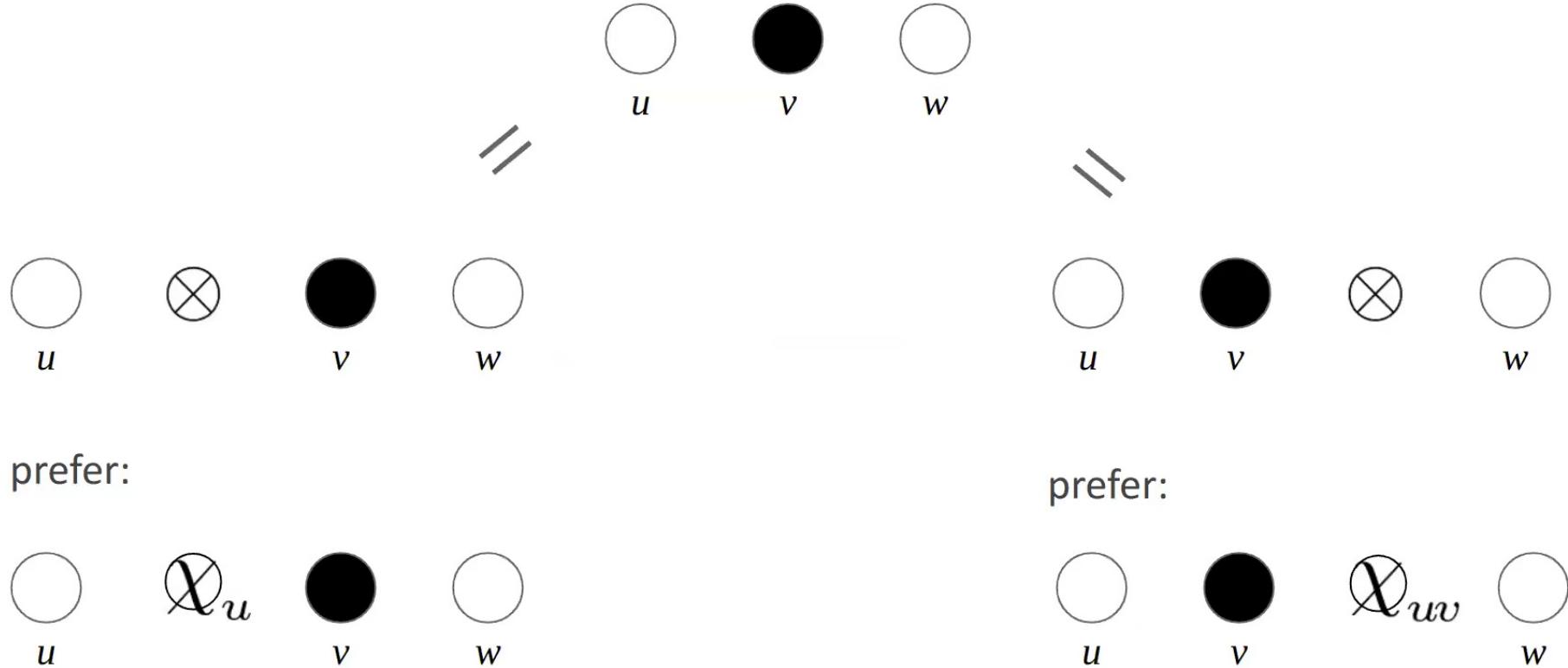
element of the Hilbert space whose o.n.b.
is the set of graphs.

$$|\psi\rangle \in \mathcal{H}.$$

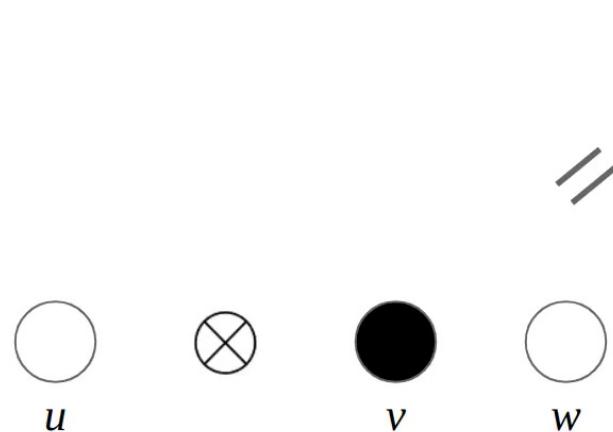


- Generalised tensors & traces
- Why they work.

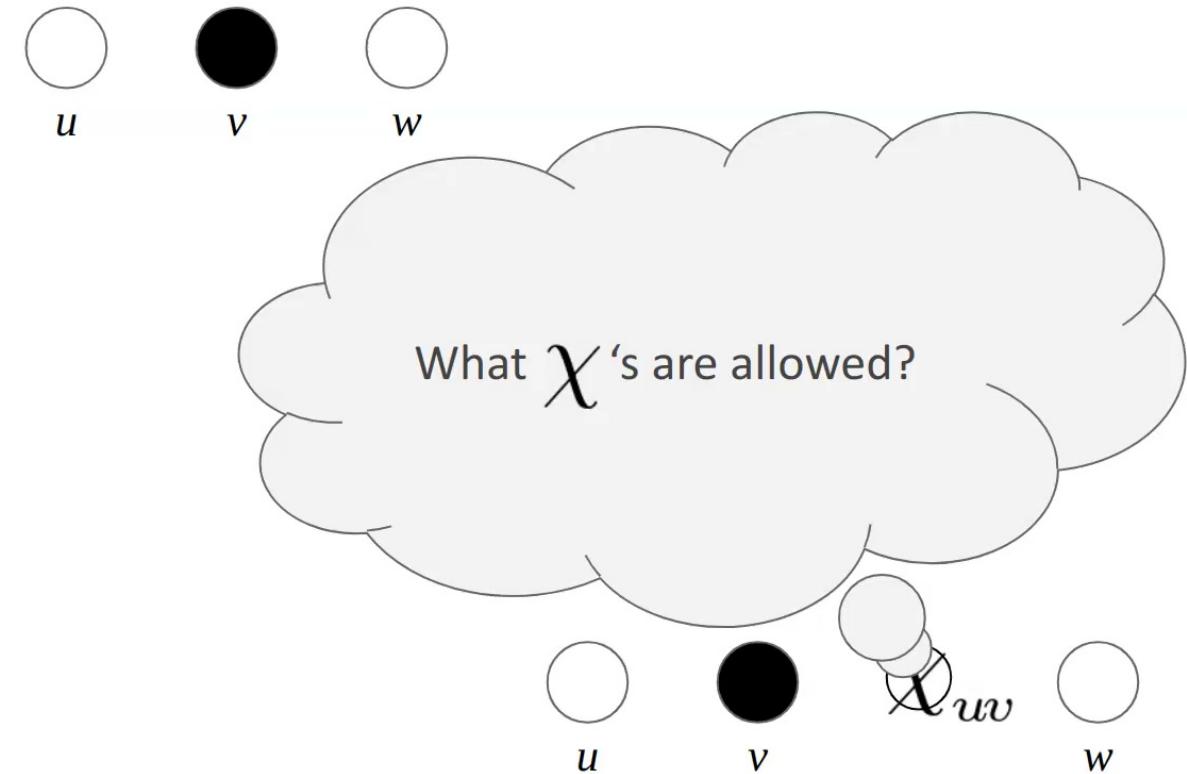
Gen. tensors & traces » Good old tensors... made precise



Gen. tensors & traces » Good old tensors... made precise



prefer:



Gen. tensors & traces » Quantizing the partition

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \quad \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{array} \quad = \quad \begin{array}{ccccc} \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \end{array}$$

Diagram illustrating the quantization of a tensor product. On the left, three circles labeled u , v , and w are followed by an equals sign. On the right, the same three circles are followed by a plus sign, a circle labeled μ , another plus sign, and a circle labeled v .

Rk. Because discriminating criterion is quantized, so is the split into subsystems:

$$+ \quad \begin{array}{ccc} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{array} \quad = \quad + \quad \begin{array}{ccccc} \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \end{array}$$

Diagram illustrating the decomposition of a tensor product into subsystems. It consists of two parts separated by a plus sign. Each part has a plus sign before it. The first part shows three circles labeled u , v , and w followed by an equals sign. The second part shows three circles labeled u , w followed by a plus sign, a circle labeled μ , another plus sign, and two circles labeled v and w .

Gen. tensors & traces » Tensors

any idempotent restriction

A bilinear operator

$$\mathcal{X} : \mathcal{H} \times \mathcal{H} \rightarrow \mathcal{H}$$

$$|L\rangle \otimes |R\rangle = \begin{cases} |G\rangle & \text{if } L = G_\chi \text{ and } R = G_{\bar{\chi}} \\ 0 & \text{otherwise} \end{cases}$$

Ex.

$$G_{\bar{\chi}} = G \setminus G_\chi$$

$$|\circlearrowleft_u v\rangle \otimes_u |\circlearrowright_v u\rangle = |\circlearrowleft_u \circlearrowright_v\rangle$$

$$|\circlearrowleft_u v\rangle \otimes_u |\bullet_v\rangle = 0$$

$$|\circlearrowleft_u\rangle \otimes_u |\circlearrowleft_u\rangle = 0$$

Gen. tensors & traces » Traceouts

A linear operator $|_\chi : \mathcal{T}(\mathcal{H}) \rightarrow \mathcal{T}(\mathcal{H})$

$$(|G\rangle\langle H|)_{|\chi} := |G_\chi\rangle\langle H_\chi| \langle H_{\bar{\chi}}|G_{\bar{\chi}}\rangle$$

Ex.

$$(|\circlearrowleft \text{---} \bullet \rangle \langle \bullet \text{---} \bullet|)_{|\chi_u} = |\circlearrowleft \text{---} v \rangle \langle \bullet \text{---} v|$$

$$(|\circlearrowleft \text{---} \bullet \rangle \langle \bullet \text{---} \circlearrowright|)_{|\chi_u} = 0$$

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Gen. tensors & traces » Traceouts

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Gen. tensors & traces » Traceouts

A linear operator $|_\chi : \mathcal{T}(\mathcal{H}) \rightarrow \mathcal{T}(\mathcal{H})$

$$(|G\rangle\langle H|)|_\chi := |G_\chi\rangle\langle H_\chi| \langle H_{\bar{\chi}}|G_{\bar{\chi}}\rangle$$

Prop. $|_\chi$ is a TPCP.

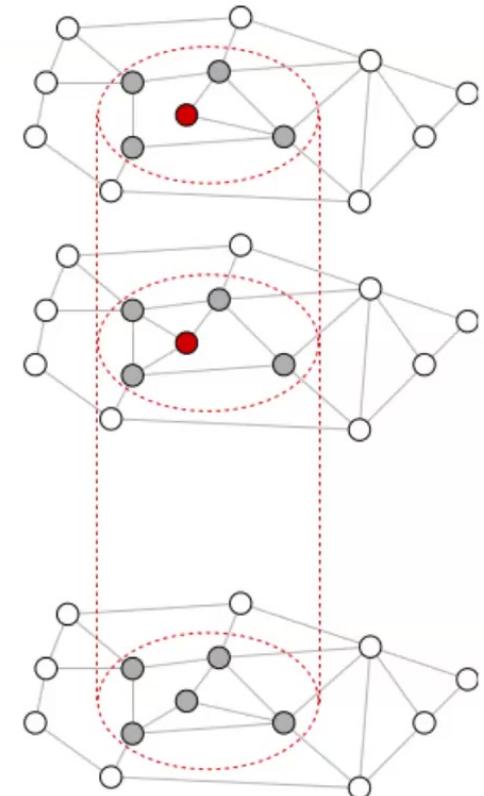
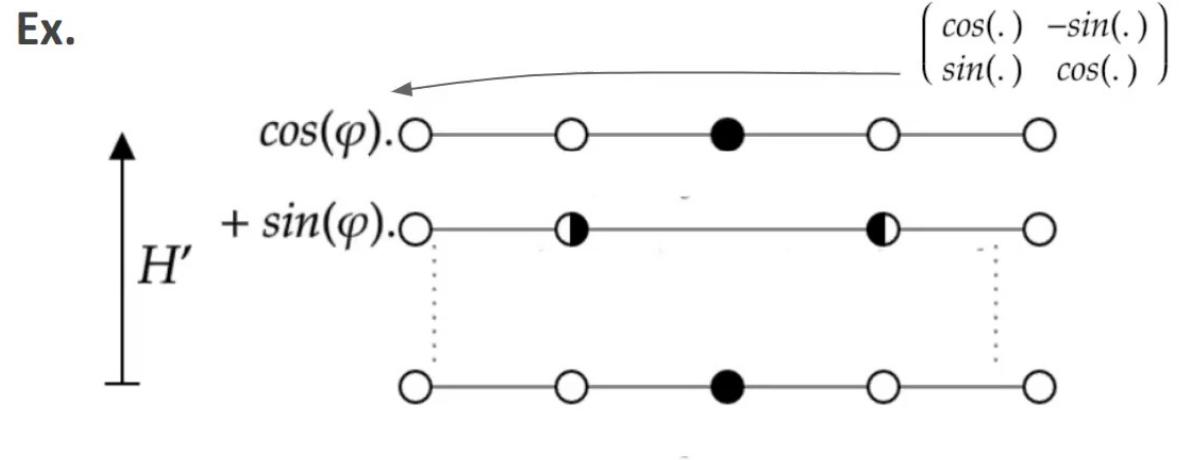


- Generalised tensors & traces
- Why they work.

Why they work » Fully quantum networks » Locality

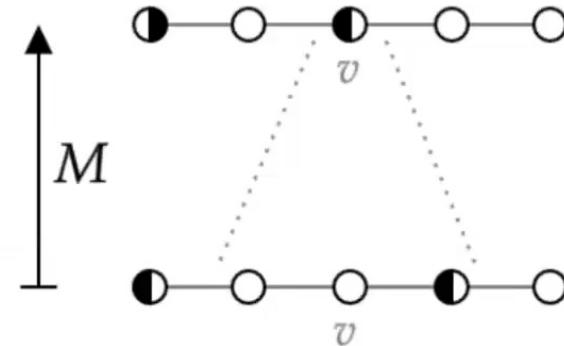
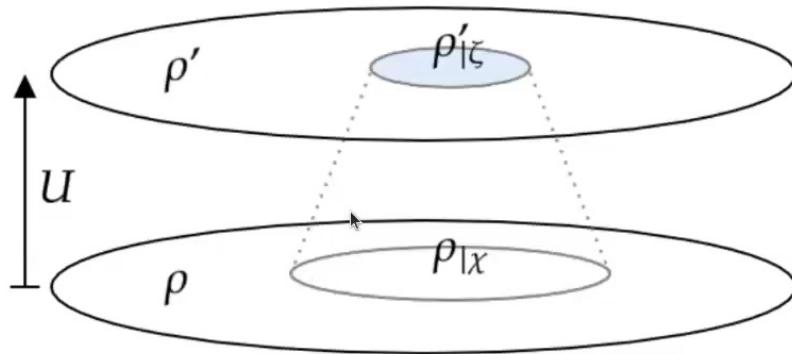
Th. $\langle H|A|G\rangle = \langle H_\chi|A|G_\chi\rangle\langle H_{\bar{\chi}}|G_{\bar{\chi}}\rangle$

$$\Leftrightarrow A = A \otimes I$$
$$\Leftrightarrow \text{Tr}(A\rho) = \text{Tr}(A\rho_{|\chi})$$



Why they work. » Fully quantum networks » Causality

$$U \text{ } \chi\zeta\text{-causal} \left\{ \begin{array}{l} \text{Th.} \quad (U\rho U^\dagger)_{|\zeta} = (U\rho_{|\chi} U^\dagger)_{|\zeta} \\ \Leftrightarrow U \text{ decomposes into } \chi\text{-local gates.} \\ \Leftrightarrow A \text{ } \zeta\text{-local} \Rightarrow UAU^\dagger \text{ } \chi\text{-local} \end{array} \right.$$



$$U = H'M$$

Summary »

arXiv:2110.10587

Aim »

For the sake of indefinite causal orders or quantum gravity: formalizing fully quantum networks; studying local and causal evolutions upon them.



Generalized tensors & traces »

Were needed for that. Parametrized by any restriction. Internalized.

Why they work. »

Locality, causality in the Shrödinger, Operational and Heisenberg pictures.
Safeguarding their logical interrelations.