Title: Higher-order Quantum Processes are Characterized by the Logic of their Signaling Relations

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Series: Quantum Foundations, Quantum Information

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Higher-order Quantum Processes are Characterized by the Logic of their Signaling Relations

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Flash Talk for Causalworlds 2024
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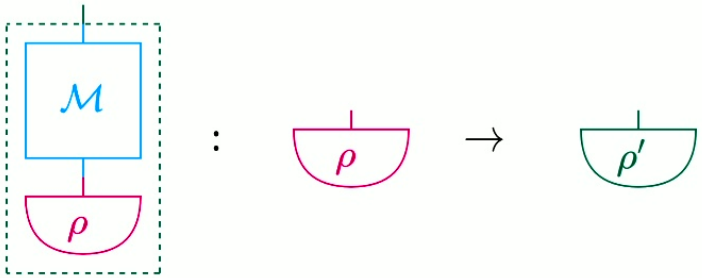
Higher-order transformations in quantum theory

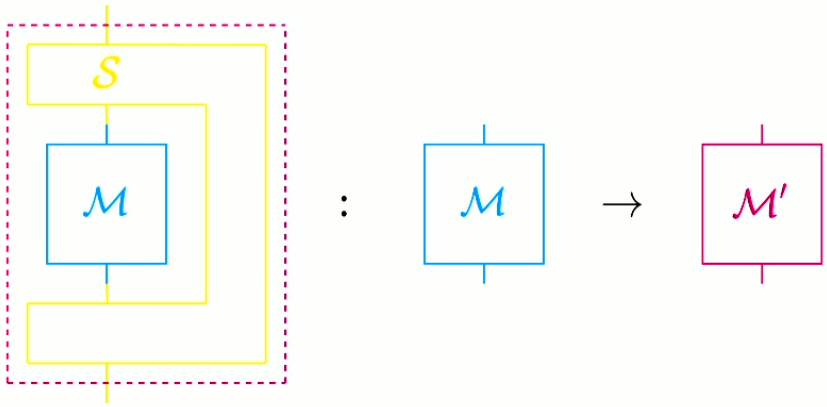
Higher-Order Quantum Processes (HOQP) = **nested** transformations

Example:

Quantum channel $\mathcal{M}$ =
Transformation between any two states

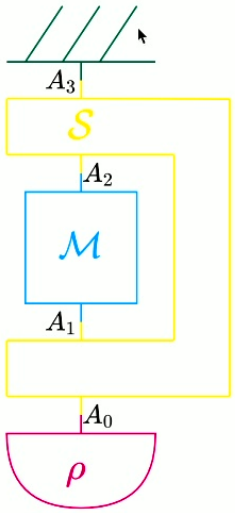


Quantum supermap $\mathcal{S}$ =
Transformation between any two channels

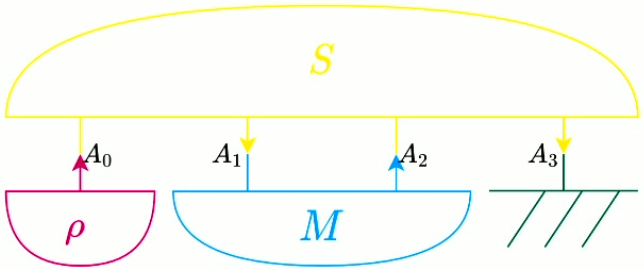


Enter the Matrices: the Choi-Jamiołkowski picture

Maps of maps are awkward to work with...



$\longmapsto$



$$\mathrm{Tr}_{A_3}\Big[\,[\mathcal{S}(\mathcal{M})](\,\rho\,)\,\Big] \qquad \mathrm{Tr}_{A_0A_1A_2A_3}\big[\,S\cdot(\,\rho\otimes M\otimes \mathbf{1})\big]$$

Channel-state duality: every object is an operator

Goals of Research

1. Generalize to all conceivable HOQP
2. Quicken their characterization

Projective Characterization

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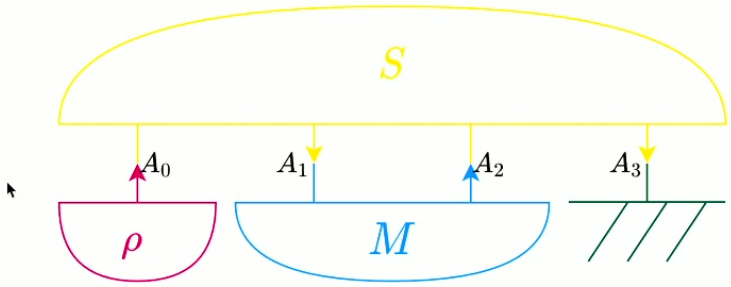
Projective Characterization



$$\mathrm{Tr}_{A_0A_1A_2A_3}\left[\underbrace{S}\cdot(\rho\otimes M\otimes\mathbb{1})\right]$$

The valid S's span a subset of $\mathcal{L}\left(\mathcal{H}^{A_0}\otimes\mathcal{H}^{A_1}\otimes\mathcal{H}^{A_2}\otimes\mathcal{H}^{A_3}\right)$

Most of the properties of HOQP are encoded in the span of their CJ representation

Projective Characterization

Is S in a given class of HOQP $\Rightarrow$ Is S in the associated subspace? $\Rightarrow$ Is $\mathcal{P}[S]=S$?

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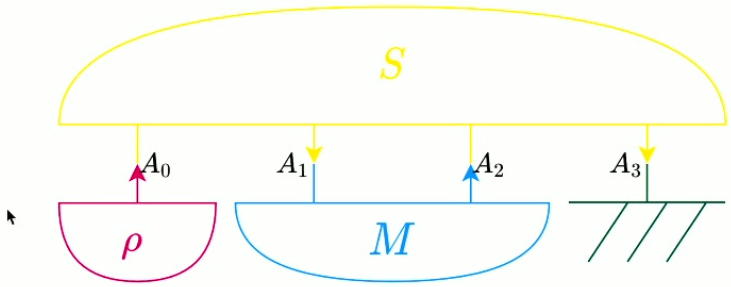
Projective Characterization

3. Compare the different types of processes
4. Assess the causal structure(s) allowed in a type

Algebra of Projectors



$$\mathrm{Tr}_{A_0A_1A_2A_3}\left[S \cdot \left(\underbrace{\rho}_{\mathcal{P}_{in}} \otimes \underbrace{M}_{f(\mathcal{P}_{in},\mathcal{P}_{out})} \otimes \underbrace{\mathbb{1}}_{\mathcal{P}_{out}}\right)\right]$$

Characterization of higher-orders depends on the characterization of their inputs and outputs

Algebra of Projectors

Forming projector to higher-order processes = Composing projectors of lower-order
Study of HOQP = Study of the algebraic rules for composing projectors

Goals of Research

1. Generalize to all conceivable finite-dimensional HOQP
2. Quicken their characterization

Projective Characterization

3. Compare the different types of processes
4. Assess the causal structure(s) allowed in a type
   - ...simplifying causality by signaling

Algebra of Projectors

Goals of Research

1. Generalize to all conceivable finite-dimensional HOQP
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Projective Characterization

3. Compare the different types of processes
4. Assess the signaling structure(s) allowed in a type

Algebra of Projectors

Methodology summary

Signaling Relations between		Relations between		Composite		Composition Rules of
Allowed Deterministic Interventions		Sets of Admissible Processes		State Structures		Projectors
classify Higher-order Processes	used ⟸ to	obey the (BV-)Logic of Signaling	ruled ⟸ *by*	form the Signaling Lattices	yields ⟸	define the Projector Algebra
What we are interested in						**Where it is easy to work**

Message to take home

- Classes of processes = Projectors
- New classes of processes = Composite projectors
- How to compose projectors = BV-logic
- What (de)composes projectors = Signaling relations

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Thank you for your attention!

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