Title: Emergent classicality, relativistic causality, and quantum causal structure

Speakers: Nick Ormrod

Series: Quantum Foundations, Quantum Information

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Events from influences?

Nick Ormrod Perimeter Institute, University of Oxford

Based on a preprint with Jonathan Barrett – "Quantum influences and event relativity"

"Events emerge from causal influences."

- interesting!
- but **vague** :(
- goal for today: to make this sentence more precise...
- ...and in doing do, find a more precise theory!

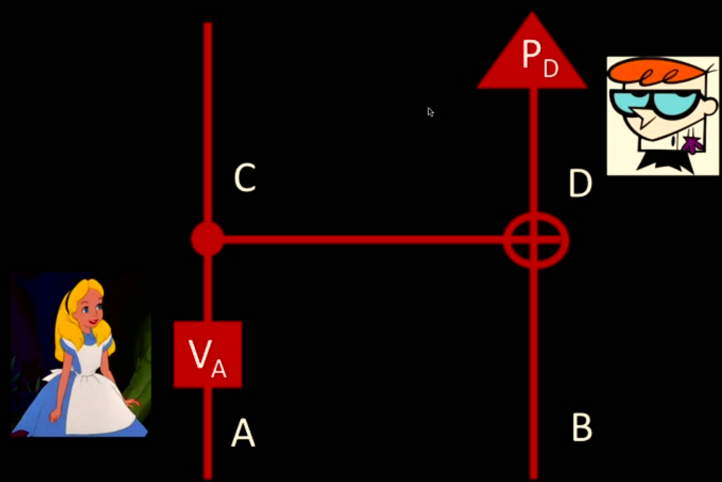
Unpacking a bit

- Events emerge from causal influences **by striking a causal balance**
- Causal balance means **enough** influence but **not too much**
  - Enough because events should matter!
  - Not too much because there should be no interference effects between events if just one happens!

What represent influences? What represent events?

- We'll assume that influences correspond to signalling relations through unitary transformations
- And that events can be represented by projectors
- So "Events emerge from causal influences" $\sim$ "Projectors are singled out by signalling relations"
- Since influences are relations, events will be relational

Toy example: controlled-not



- Controlled-not: $|i\rangle_A |j\rangle_B \rightarrow |i\rangle_C |j+i\rangle_D$
- Alice either performs some V_A or does nothing, then controlled-not, then Dexter measures

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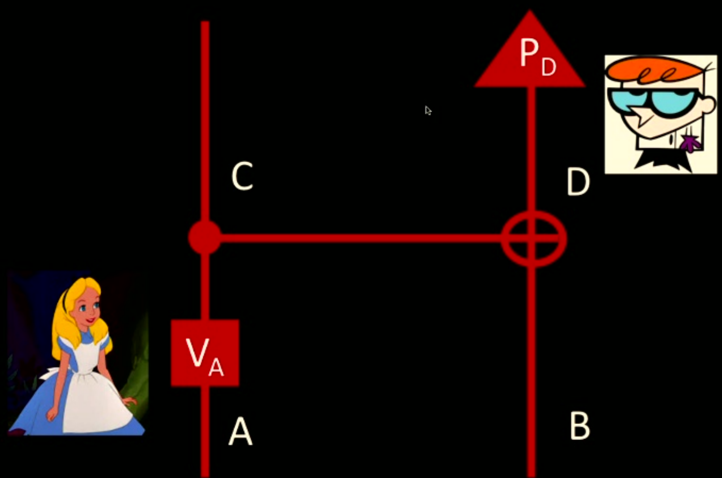
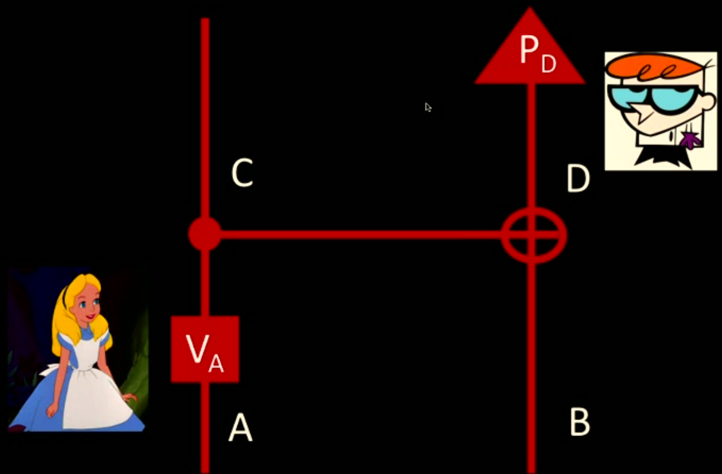
Toy example: controlled-not



- If $V_A = |0\rangle\langle 0|_A + e^{i\phi}|1\rangle\langle 1|_A$, Alice cannot signal
- So there is no influence on D associated with the interference between $|0\rangle\langle 0|_A$ and $|1\rangle\langle 1|_A$
- $\implies$ there is not too much influence for these projectors to represent events ✓

Toy example: controlled-not



- However, if V_A satisfies $V_A |0\rangle\langle 0| V_A^{-1} \neq |0\rangle\langle 0|_A$ (or, equivalently $V_A |1\rangle\langle 1| V_A^{-1} \neq |1\rangle\langle 1|_A$), then Alice can signal to Dexter
- $\implies$ there is enough influence for these projectors to represent events ✓
- Hence $|0\rangle\langle 0|_A$ and $|1\rangle\langle 1|_A$ are causally balanced relative to D ✓

But what is causal balance exactly?

- Consider a unitary $U : A \otimes B \to C \otimes D$, and a projector P_A.
- **Enough influence** : For any V_A, if $V_A P_A V_A^{-1} \neq P_A$, then Alice can signal to Dexter.
- **Not too much influence** : If $V_A = P_A + e^{i\phi}(I - P_A)$, then Alice cannot signal to Dexter.
- If there is enough influence and not too much, then P_A is **causally balanced** relative to D.

Back to the controlled-not

- Given this definition and the controlled not, it turns out that the full set of projectors on A that are causally balanced relative to D is

$$\mathcal{E}_{\text{comp}} := \{0_A, |0\rangle\langle 0|_A, |1\rangle\langle 1|_A, I_A\} \tag{1}$$

- Note that all elements of $\mathcal{E}_{\text{comp}}$ commute with each other
- Furthermore, $\mathcal{E}_{\text{comp}}$ is the Boolean algebra generated by $|0\rangle\langle 0|_A$ and $|1\rangle\langle 1|_A$

$$\begin{aligned} e \wedge f &\sim P^e P^f \\ e \vee f &\sim P^e + P^f - P^e P^f \\ \neg e &\sim I - P^e \end{aligned} \tag{2}$$

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Back to the controlled-not

- So, in the case of the controlled-not, it just so happens that the set of projectors on A that are causally balanced relative to D can be neatly thought as representing a complete set of possible events – as an **event space** ✓

- But does this always happen???

Can we always interpret causal balance as singling out a set of possible events

- Yes!

- **Theorem 1.** For any unitary $U : A \otimes B \to C \otimes D$, the set of projectors on A that are causally balanced relative to D forms a unique event space.

Generalizing to circuits

- Not much can be modelled with just one unitary transformation!!
- Let's roll with the idea that in a unitary circuit events emerge by striking a causal balance relative to a bubble
- bubble := any set of systems (i.e. individual wires) in a circuit

Generalizing to circuits

- A projector is causally balanced relative to a set of systems if the "enough" condition holds relative to at least one system in the set and the "not too much" condition holds relative to all of them
- Let $\mathcal{E}^{\uparrow}_{A|\mathfrak{B}}$ be the event space of projectors that is causally balanced relative to A's causal future within $\mathfrak{B}$, i.e. the set of systems that come "higher up" in the circuit
- Similarly, $\mathcal{E}^{\downarrow}_{A|\mathfrak{B}}$
- If $|\mathfrak{B}| = n$, this gives us $2n$ event spaces ✓
- But do they admit a natural probability distribution?

Probabilities

- yes!!
- By definition, for any $A, B \in \mathfrak{B}$, the future-balanced event spaces commute with each other in the Heisenberg picture, as do the past-balanced ones:

$$\begin{aligned} \tilde{\mathcal{E}}^{\uparrow}_{A\mathfrak{B}} &\subseteq \tilde{\mathcal{E}}^{\uparrow}_{B\mathfrak{B}}{}' \\ \tilde{\mathcal{E}}^{\downarrow}_{A\mathfrak{B}} &\subseteq \tilde{\mathcal{E}}^{\downarrow}_{B\mathfrak{B}}{}' \end{aligned} \tag{3}$$

- **Theorem 2.** Given any unitary circuit and bubble $\mathfrak{B}$, the expression

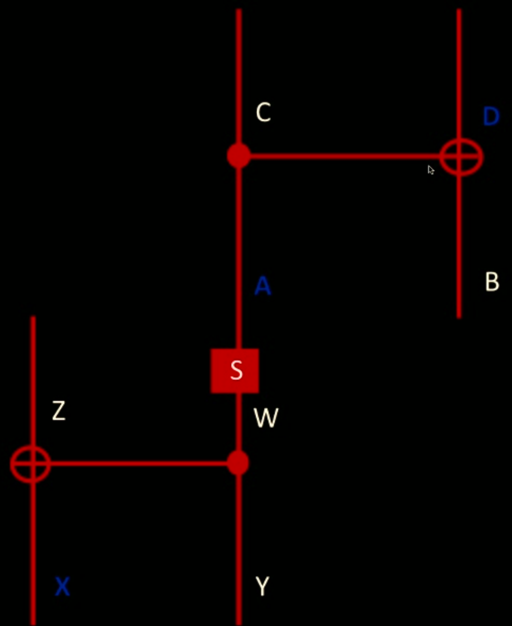
$$p_{\mathfrak{B}}(\tilde{P}^{\downarrow}_{X_1}, \tilde{P}^{\uparrow}_{X_1}, \ldots \tilde{P}^{\downarrow}_{X_n}, \tilde{P}^{\uparrow}_{X_n}) = \frac{1}{d}\mathrm{Tr}(\tilde{P}^{\downarrow}_{X_1} \ldots \tilde{P}^{\downarrow}_{X_n} \tilde{P}^{\uparrow}_{X_1} \ldots \tilde{P}^{\uparrow}_{X_n}) \tag{4}$$

defines a probability distribution.

The Theory of Causal Balance

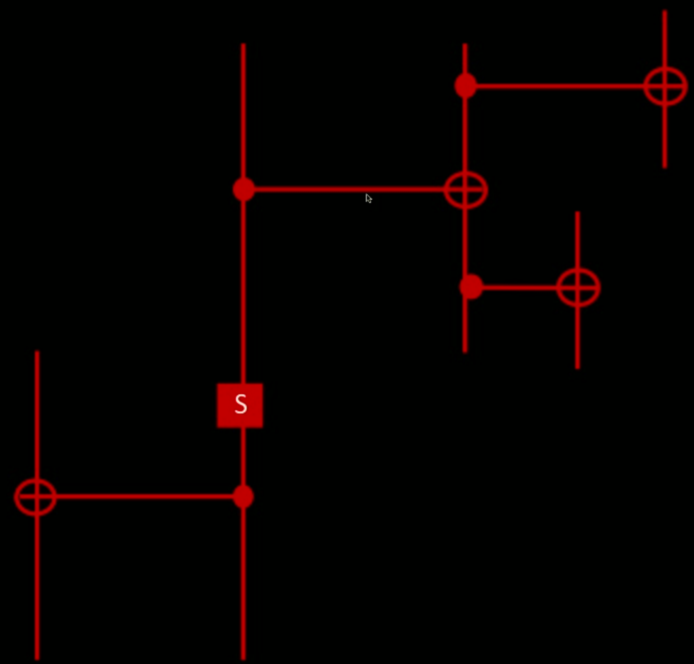
- Reality is a unitary circuit and the events that emerge via causal balance relative to each bubble. The emergence of events is stochastic and follows the probability rule given above.

The Theory of Causal Balance in action



- $\mathfrak{B} = \{X, A, D\}$
- $\mathcal{E}^{\downarrow}_{A|\mathfrak{B}} = \{0_A, S\,|0\rangle\,\langle 0|_A\, S^{-1}, S\,|1\rangle\,\langle 1|_A\, S^{-1}, I_A\}$
- $\mathcal{E}^{\uparrow}_{A|\mathfrak{B}} = \{0_A, |0\rangle\,\langle 0|_A\,, |1\rangle\,\langle 1|_A\,, I_A\}$
- Probability of $|j\rangle\,\langle j|_A \in \mathcal{E}^{\uparrow}_{A|\mathfrak{B}}$ given $S\,|i\rangle\,\langle i|_A\, S^{-1} \in \mathcal{E}^{\downarrow}_{A|\mathfrak{B}} = |\,\langle j|_A\, S\,|i\rangle_A\,|^2$
- So we can think of this as an experiment where the state $S\,|i\rangle$ is prepared, then a computational basis measurement is performed leading to Born-rule probabilities

The Theory of Causal Balance in action



Going further

- We already know how to go beyond circuits and generalize the theory to a more algebraic, QFT-like setting, and to indefinite causal order ✓
- Connection more traditional approaches to decoherence?
- Quantum gravity?

Thank you for listening!

- And thanks to my collaborator & Ph.D. supervisor, Jonathan Barrett

- For more info, check my other talks, the paper “Quantum Influences and Event Relativity” with Jon, or ask me to send you my Ph.D. thesis