Title: Towards Relational Quantum Field Theory

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Towards

Relational Quantum Field Theory

How Operational Quantum Reference Frames provide new perspectives on relativistic Quantum Physics

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Relational Quantum Field Theory?

- Ultimate goal:

  Provide a framework for relativistic quantum physics such that:

  - Operationality and relationality are emphasised,
  - Gauge symmetries are naturally incorporated,
  - Curved and indefinite geometries can be treated,
  - Renormalizability is introduced on operational grounds,
  - Mathematical rigour is not compromised.

- Research strategy:

  Incorporate relativistic symmetries into *Quantum Theory* by extending the operational approach to *QRFs* into the realm of *Gauge Theories*

Frames and Field Operators

- In General Relativity, fields take values with respect to (local) tetrads $\mathsf{e}^a_\mu(x)$

$$x \in U \subset \mathscr{M},\ \mathrm{Frm(U)} \ni \mathsf{e}^{\mathrm{a}}_\mu(\mathrm{x}) \longmapsto \hat{\phi}(\mathrm{x}, \mathsf{e}) \in \mathrm{B}(\mathscr{H}_{\mathrm{F}})$$

$$\text{e.g.:}\ \hat{E}_j(x, \mathsf{e}) = \hat{F}_{ab}(x)\mathsf{e}^a_j(x)\mathsf{e}^b_0(x)$$

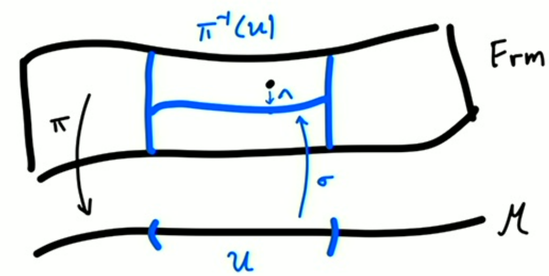
- Any pair of tetrads at $x \in U$ is related by unique Lorentz transformation

$$\mathrm{Frm}(U) \cong_\sigma U \times SO(1,3)$$

- Mathematically speaking, Frm is principal bundle for Lorentz group

$$\pi : \mathrm{Frm} \to \mathscr{M},\ \sigma : U \to \mathrm{Frm}$$

$$\mathrm{Frm}(U) = \pi^{-1}(U) \cong_\sigma U \times SO(1,3)$$



Probabilistic Reference Frames

- We can choose *action of Lorentz group* on quantum field such that

$$\hat{\phi}(x, \mathsf{e}) = \hat{\phi}(x)_\sigma . \Lambda$$

$$\text{e.g.: } \hat{E}_j(x, \mathsf{e}) = \hat{F}_{ab}(x)_\sigma . \Lambda = \hat{F}_{ab}(x)\mathsf{e}^a_\mu(x)\mathsf{e}^b_\nu(x)\Lambda^\mu_j\Lambda^\nu_0$$

- Choice of frame can be subject to *statistical uncertainty* $\mu \in \mathrm{Prob}(\pi^{-1}(U))$
- Relative description of field operators is then given by *weighted average*

$$\hat{\phi}^{(\mu)} := \int_{\pi^{-1}(U)} \hat{\phi}(x)_\sigma . \Lambda \, d\mu(x, \Lambda)$$

$$\text{e.g.: } \hat{E}_j(\mu) = \int_{U \times SO(1,3)} \hat{F}_{ab}(x)\mathsf{e}^a_\mu(x)\mathsf{e}^b_\nu(x)\Lambda^\mu_j\Lambda^\nu_0 \, d\mu(x, \Lambda)$$

Quantum Reference Frames

- In practice, reference frames are *physical systems*
- As such, they should ultimately be described by *Quantum Theory*
- Quantum system can serve as reference frame when equipped with *frame observable*

$$E_R : Bor(\pi^{-1}(U)) \to \mathcal{E}(\mathcal{H}_R)$$

- The orientation of quantum frame in state $\omega \in \mathcal{D}(\mathcal{H}_R)$ is given by *Born rule*

$$\mu^{E_R}_{\omega}(X) = \mathrm{tr}[\omega\, E_R(X)]$$

- Field operator relative to quantum frame prepared in $\omega \in \mathcal{D}(\mathcal{H}_R)$ is given by

$$\hat{\phi}^{(\omega)}_{E_R} := \int_{\pi^{-1}(U)} \hat{\phi}_{\sigma}(x)\,.\,\Lambda\, d\mu^{E_R}_{\omega}(x,\Lambda) \in B(\mathcal{H}_F)$$

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- Such operators are understood as (restricted) *relational local observables*

Quantum Frame Morphisms

- Consider pair of frames R_1, R_2

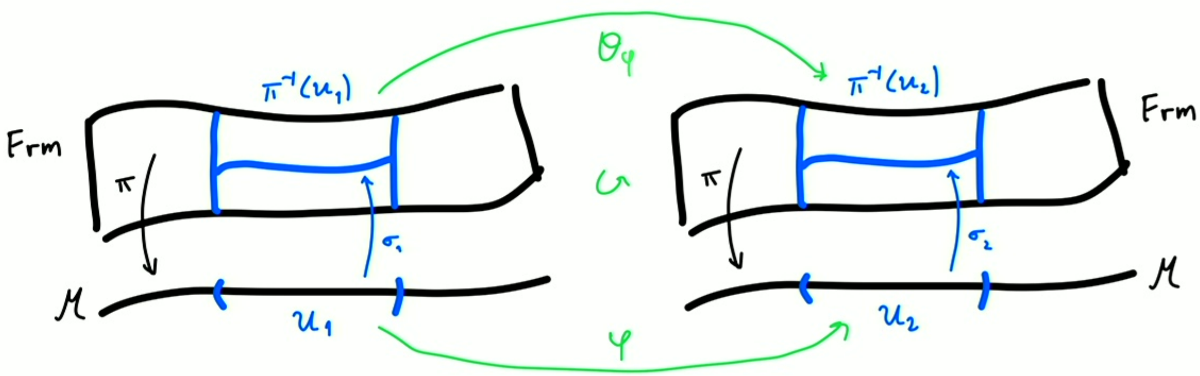
$$E_i : \mathrm{Bor}(\pi^{-1}(U_i)) \to \mathscr{E}(\mathscr{H}_i),\ \ \sigma_i : U_i \to \mathrm{Frm},\ i = 1{,}2.$$

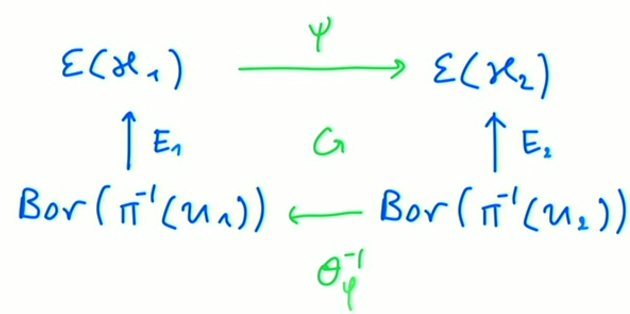
- *QRF morphism* is a quantum channel and an isometry

$$\psi : B(\mathscr{H}_1) \to B(\mathscr{H}_2),\ \ \varphi : U_1 \to U_2,$$

such that the following compatibility conditions hold

$$\theta_\varphi \circ \sigma_1 = \sigma_2 \circ \varphi \qquad\qquad E_2 = \psi \circ E_1 \circ \theta_\varphi^{-1}$$





External Quantum Frame Transformations

- Given a frame morphism, one can map between relational local observables

$$\hat{\phi}^{(\omega_2)}_{E_2} = (\hat{\phi} \circ \varphi)^{(\psi_* \omega_2)}_{E_1}$$

- They provide notion of *covariance* and *invariance* wrt space-time isometries

$$(\hat{\phi} \circ \varphi)^{(\omega)}_{E} = \hat{\phi}^{(\omega)}_{\varphi_*^{-1} E} \quad \Rightarrow \quad (\hat{\phi} \circ \varphi)^{(\omega)}_{\varphi_* E} = \hat{\phi}^{(\omega)}_{E}$$

- External QRF transformations can be used to describe:
  - change of *local coordinates*
  - change of *physical system* used as reference
  - any combination thereof
- Gauge transformations are achieved $\sigma \longmapsto \sigma'$

Research Outlook

- Properties of relational local observables
  - axiomatic QFT (Wightman)
  - algebraic QFT (type reduction)
- Relational detector models and measurement schemes
  - Combine frames and detectors into measurement schemes
  - Formulate relational and operational causality conditions?
  - Model quantum-delocalized gravitational coupling?
- GPT extension?
- Renormalisation and frame localizability?
- Dynamical quantum-sourced stochastic geometries?

Fields on Indefinite Geometries

- Lorentz bundles associated with the metric are sub-bundles of *frame bundle*

$$\pi_L : LM \to M,$$

which is a principal $GL(4,\mathbb{R})$-bundle corresponding to all choices of bases in T_pM

- Quantum Reference Frame on frame bundle is a $GL(4,\mathbb{R})$-covariant POVM

$$E_R : \mathrm{Bor}(\pi_L^{-1}(U)) \to B(\mathscr{H}_R)$$

together with a choice of the *local coordinates* provided by $\sigma_L : M \supset U \to LM$

- The probability of $g_{\mu\nu}$ being realised on M given a state $\omega \in \mathscr{D}(\mathscr{H}_R)$ is given by

$$\mathrm{prob}(g_{\mu\nu} \,|\, \omega) := \mu_\omega^{E_R}\left(\mathrm{Frm}_{g_{\mu\nu}}\right) \in [0,1]$$