Title: Characterizing Signalling: Connections between Causal Inference and Space-time Geometry

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Collection/Series: Causalworlds

Subject: Quantum Foundations, Quantum Information

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Connections of Causal Inference and Space-time Geometry

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Outline

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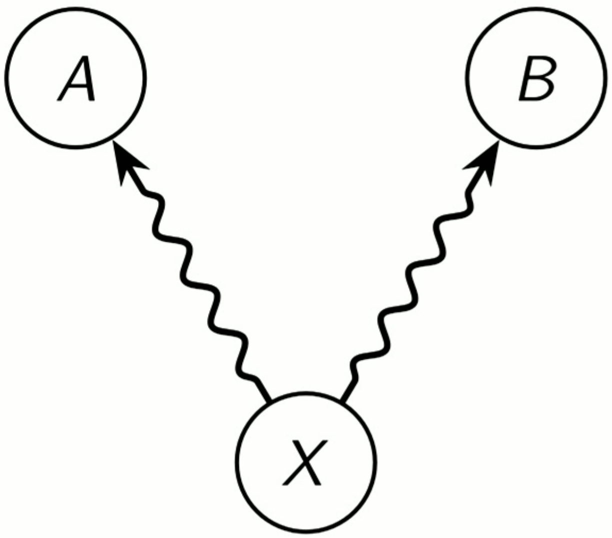
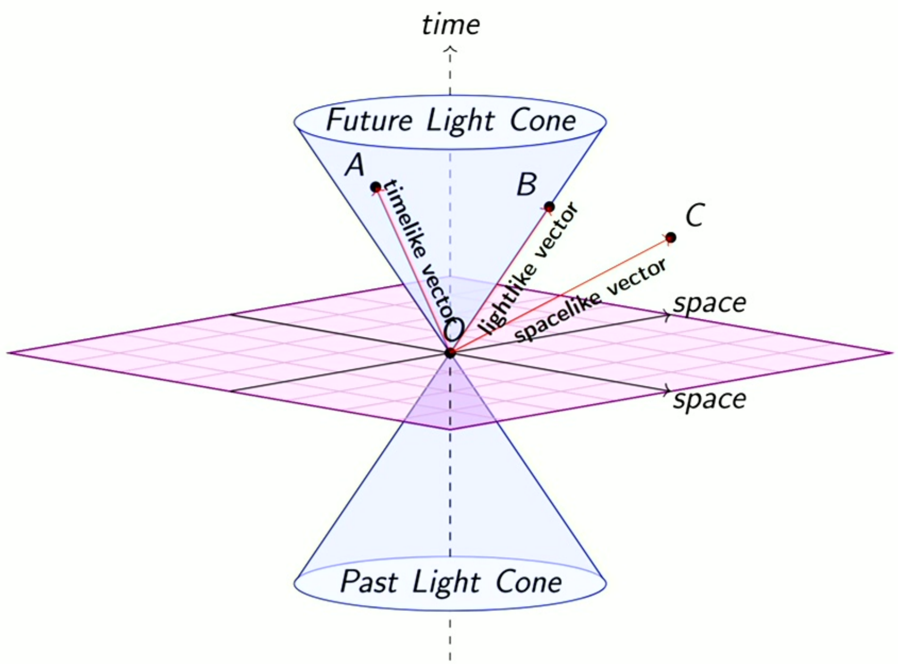
1. Introduction
2. Review: Causal Models & Signalling
3. Identifying Redundancies & Fine-tuning from Signalling
4. Connecting Information-theoretic & Spacetime Causality
5. Correspondence between Fine-tuning & Space-Time Geometry
6. Outlook

Overview: Notions of Causality

Different notions of causation exist:



Causality in Information Theory



Causality in Relativity

Outline

Review: Causal Models

Causal Model ([1][2])

A *causal model* over a finite set of observed RVs $S_{\text{obs}} := \{X_1, \ldots X_n\}$ consists of

- a *causal structure*, given by a directed graph $\mathcal{G}$ over $S \supseteq S_{\text{obs}}$,
- a *probability distribution* $P_{\mathcal{G}}(X_1, \ldots, X_n)$

that are compatible with each other: d-sep. in $\mathcal{G} \implies$ Conditional Indep. in $P_{\mathcal{G}}$

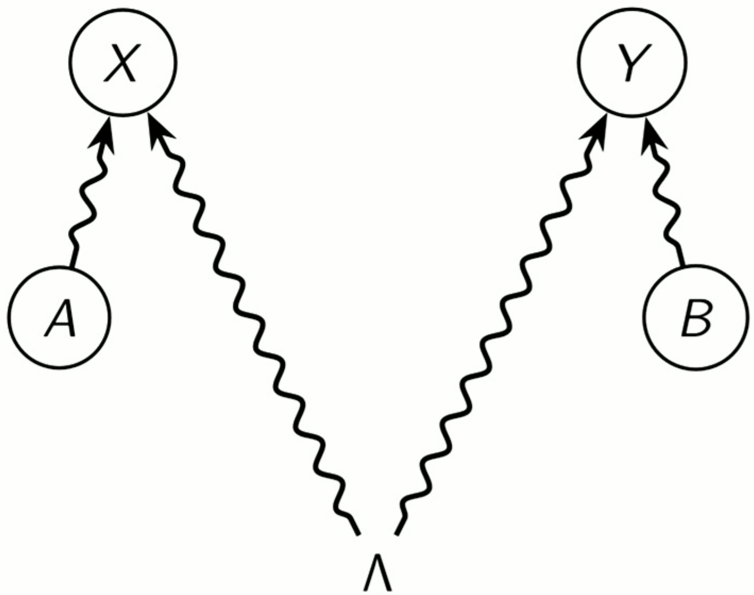
- Unless d-sep. in $\mathcal{G} \impliedby$ Conditional Indep. in $P_{\mathcal{G}}$, causal model is fine-tuned
- Allows to model quantum and post-quantum systems using unobserved nodes [3]

[1] J. Pearl 2009

[2] V. Vilasini, R. Colbeck 2022, Phys. Rev. A 106, 032204

[3] J. Henson, R. Lal, M. Pusey 2014, New J. Phys. 16 113043

Example: Bell scenario



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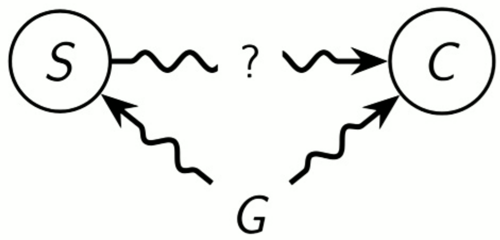
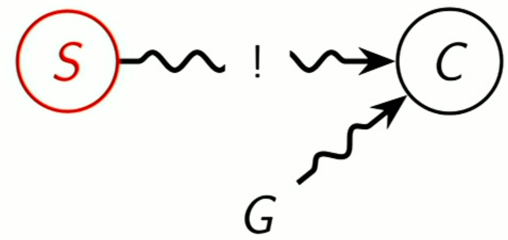
Review: Interventions and Affects Relations



Intervention $\longmapsto$



Pre-intervention structure $\mathcal{G}$

Post-intervention structure $\mathcal{G}_{\mathrm{do}(S)}$

Definition (Affects Relations; V. Vilasini, R. Colbeck 2022, PRA 106, 032204)

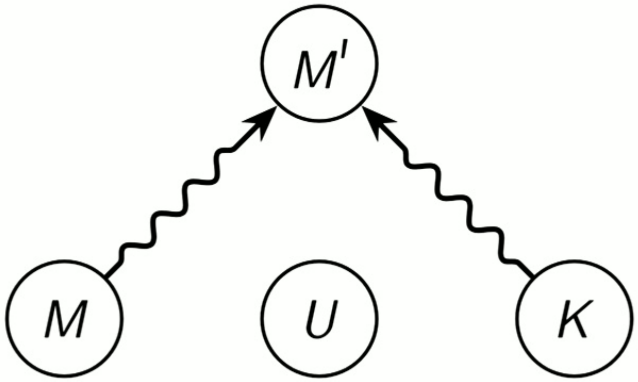
For pairwise disjoint subsets $X, Y, Z \subset S_{\mathrm{obs}}$ we say X **affects** Y, denoted as $X \vDash Y$, if there exist values x of X such that

$$P_{\mathcal{G}_{\mathrm{do}(X)}}(Y|X = x) \neq P_{\mathcal{G}}(Y)$$

- Affects Relations: Formalization of Signalling
- Causal Inference: $X \vDash Y \implies X$ is a cause of Y

Example: One-Time Pad with a twist

- Message M, Key K, encrypted message M' binary
- Unconnected Node U
- K, $M' = M \oplus K$ (xor) are uniformly distributed
- K, M, M' individually uncorrelated to one another



Relevant affects relations:

- M' conditionally independent of M: $M \not\vDash M'$
- yet K relates M and M': $MK \vDash M'$
- Redundancies possible: e.g. $MKU \vDash M'$

Review: Higher-order Affects Relations & Irred_1

How to distinguish whether $e_X \in X$ is relevant for signalling?

Definition (Higher-order Affects Relation; V. Vilasini, R. Colbeck 2022)

For pairwise disjoint subsets $X, Y, Z \subset S_{\text{obs}}$ we say X affects Y given $\text{do}(Z)$ if there exist values x of X, z of Z such that

$$P_{\mathcal{G}_{\text{do}(XZ)}}(Y|X = x, Z = z) \neq P_{\mathcal{G}_{\text{do}(Z)}}(Y|Z = z)$$

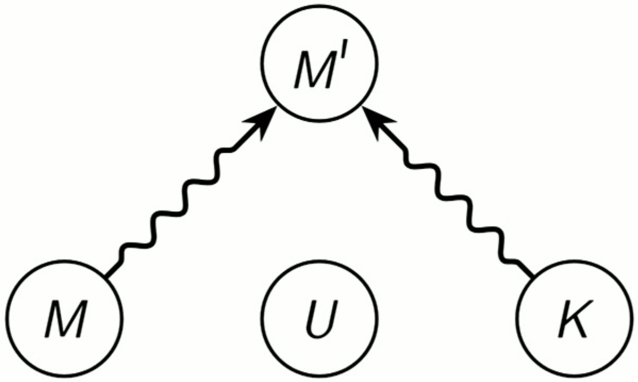
Definition (Irreducibility; V. Vilasini, R. Colbeck 2022, PRA 106, 032204)

We call $X \vDash Y$ *irreducible in its* 1^{st} *argument* if $\forall s_X \subseteq X$

$$s_X \vDash Y \text{ given do}(X \setminus s_X).$$

$\Longrightarrow$ Otherwise, reduced affects relation is *operationally* equivalent.

Example: One-Time Pad



- Message M, Key K, encrypted message M'
- K, $M' = M \oplus K$ uniform

Implications for affects relation $MK \vDash M'$

- is Irred_1 independent of distribution of M
- if M is uniform: Minimal in first attribute: $K \nvDash M'$
- Otherwise: Not minimal in first attribute: $K \vDash M'$

How to distinguish these cases?

→ Introduction of *Clustering* to certify *absence* of reduced affects relations

→ Clustering implies presence of multiple types of fine-tuning

Irreducibility & Clustering: Causal Inference

- Irreducibility and Clustering generalize to all arguments of $X \vDash Y \,|\, \text{do}(Z)$
- Causal Inference for $X \vDash Y \,|\, \text{do}(Z)$:
  - Irred_1 : $\forall\, e_X \in X$: e_X is a cause of Y. [1]
  - Irred_3 : $\forall\, e_Z \in Z$: e_Z is a cause of Y.
- Symmetry in X and Z:
  Irred_1 and Irred_3 : $\forall\, e_{XZ} \in XZ$: e_{XZ} is a cause of Y.

[1] V. Vilasini, R. Colbeck 2022, Phys. Rev. A 106, 032204

Outline

Review: Embedding into Spacetime

- Assign location $O(X)$ in partially ordered "spacetime" $(\mathcal{T}, \prec)$ to each RV
- Expand partial order to Ordered RVs $\mathcal{X} := (X, O(X))$
- Inclusive relativistic future of an ORV $\bar{\mathcal{F}}(\mathcal{X}) := \{a \in \mathcal{T} : a \succeq O(\mathcal{X})\}$
- For sets of ORVs $\mathcal{X} = \{\mathcal{X}_i\}$, generalize to $\bar{\mathcal{F}}(\mathcal{X}) := \bigcap_{\mathcal{X}_i \in \mathcal{X}} \bar{\mathcal{F}}(\mathcal{X}_i)$

Compatibility (capturing no-signalling outside the spacetime future)

Let $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ disjoint sets of ORVs. Then

$$X \vDash Y \text{ given do}(Z) \text{ is Irred}_1 \implies \bar{\mathcal{F}}(\mathcal{Y}) \cap \bar{\mathcal{F}}(\mathcal{Z}) \subset \bar{\mathcal{F}}(\mathcal{X}) .$$

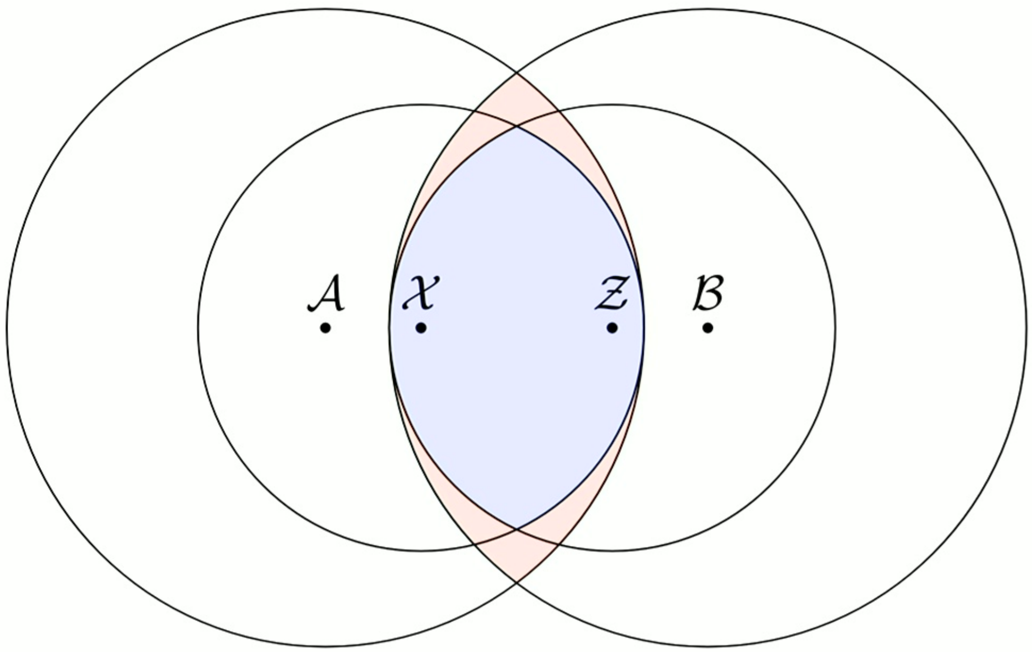
$\implies$ Information-theoretic signalling constrains spacetime embeddings.
Naively, both $\mathcal{Y}$ and $\mathcal{Z}$ must be accessible to detect signalling.

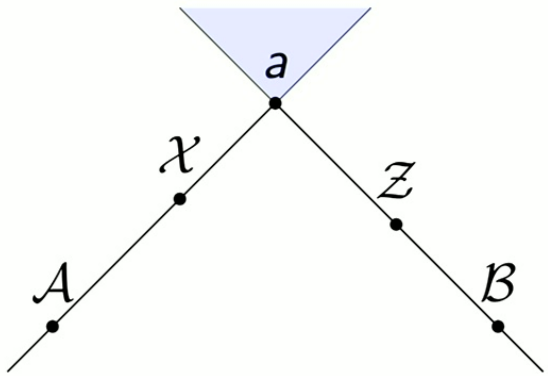
V. Vilasini, R. Colbeck 2022, Phys. Rev. A 106, 032204

Conicality

2D: $\bar{\mathcal{F}}(\mathcal{X}\mathcal{Z}) \subsetneq \bar{\mathcal{F}}(\mathcal{A}\mathcal{B})$

1D: $\bar{\mathcal{F}}(\mathcal{X}\mathcal{Z}) = \bar{\mathcal{F}}(\mathcal{A}\mathcal{B})$





Conicality

Conicality (informal definition)

A partial order is conical iff the (non-trivial) intersection of lightcones $\bigcap_{\mathcal{X}_i \in \mathcal{X}} \bar{\mathcal{F}}(\mathcal{X}_i)$ uniquely infer the individual lightcones $\bar{\mathcal{F}}(\mathcal{X}_i)$.

- Purely order-theoretical, independent of differential geometry
- Minkowski spacetime: conical only for $d \geq 2$ spatial dimensions
- Open question: Which spacetimes are conical?

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Results: Causal Inference, Clustering and Spacetime Geometry

Potential correspondence of causal inference and spacetime geometry

X affects Y given do(Z) both Irred_1 and Irred_3:

- Causal Inference: e_{XZ} is a cause of $Y \quad \forall\, e_{XZ} \in XZ$
- Compatibility: $\bar{\mathcal{F}}(\mathcal{Y}) \overset{!}{\subset} \bar{\mathcal{F}}(e_{\mathcal{XZ}}) \quad \forall\, e_{\mathcal{XZ}} \in \mathcal{XZ}$

Under which conditions does this hold?

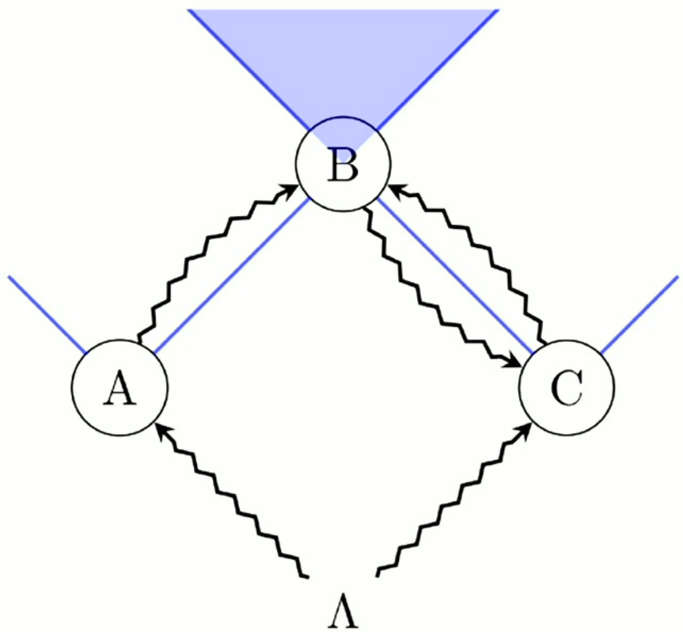
The correspondence holds if

- the respective space-time is **conical** or
- the affects relations of the causal model exhibit **no Clustering**.

→ Every scenario without this correspondence is "fine-tuned" in two ways.

Outlook: Affects framework

- General framework allows for **theory-agnostic** statements, including cycles



→ Generally: Presence of **Causal Loops** $\not\Longrightarrow$ **Superluminal Signalling**!

V. Vilasini, R. Colbeck 2022, Phys. Rev. Lett. 129, 110401

Outlook: Affects framework

- General framework allows for **theory-agnostic** statements, including cycles
- Generally: Presence of **Causal Loops** $\not\Rightarrow$ **Superluminal Signalling**

Follow-up:

- Conical Spacetimes / no Clustering rule out causal loops! (in preparation)
- Characterization of fine-tuning using Clustering?
- Connection to questions on emergence of spacetime?

Potential applications:

- Signalling in process theories as instantiation of higher-order affects relations?
- Role of clustering in the signalling relations of quantum channels?
- Fine-tuning in device-independent network cryptography in spacetime?

Thank you!

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arXiv:2403.00916