Title: A decompositional framework for process theories in spacetime

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A decompositional framework for process theories in spacetime

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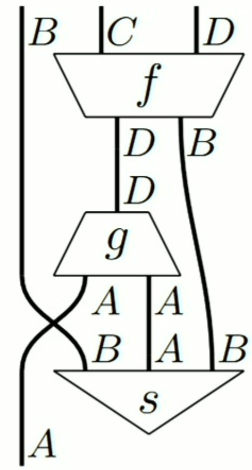
What is a process theory?

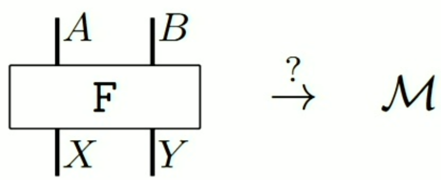
A formalization of pictures like



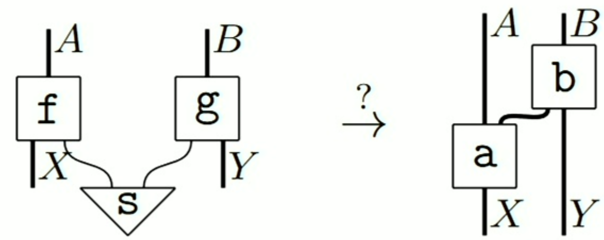
Consists of:

- Systems (wires): Hilbert spaces $\mathcal{H}^A$, $\mathcal{L}(\mathcal{H}^A)$, ingredients
- Processes (boxes): unitaries, channels, (steps in) recipes
- Closure under composition

Goals

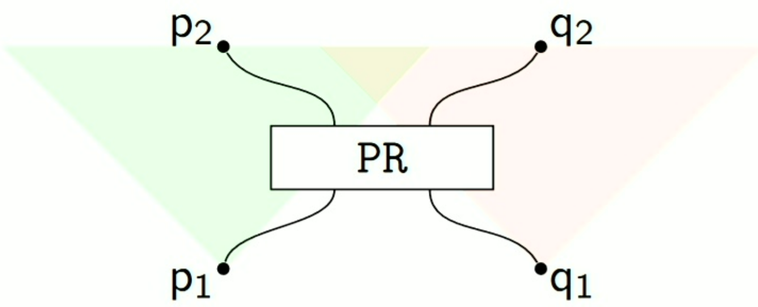


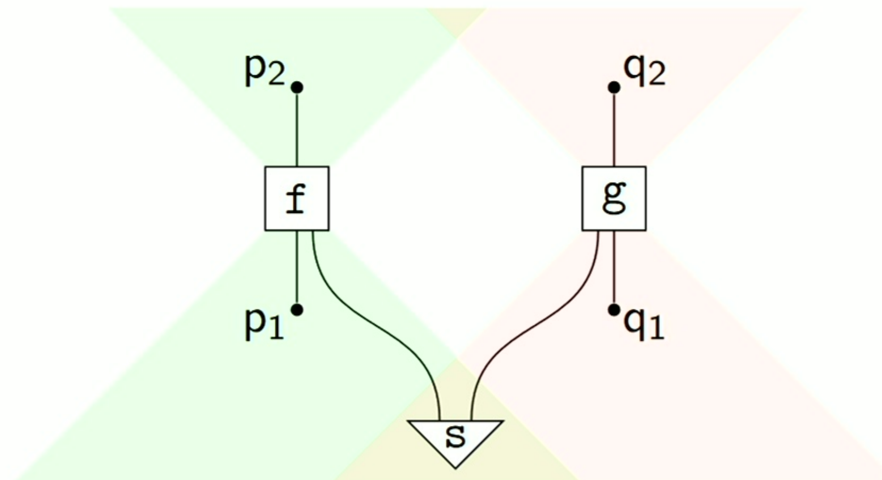
Build a framework to reason about processes living in spacetime



Which diagrams can be converted into each other in arbitrary process theories?

Embed into spacetime via...

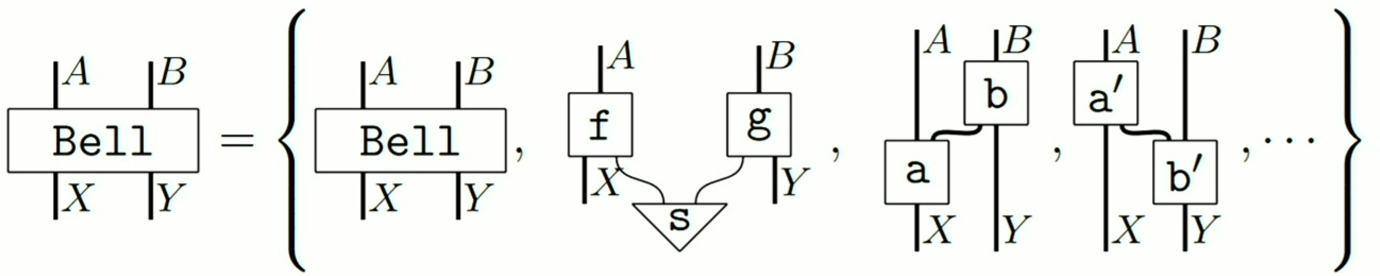




... no-signalling?
Too permissive
PR boxes are non-signalling

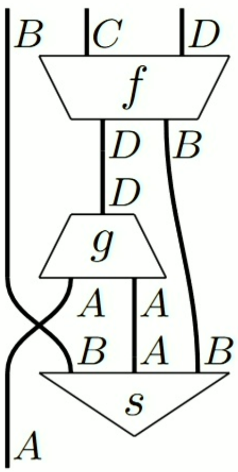
... embedding the whole diagram
Too restrictive
What about a single process?

Decomposition sets

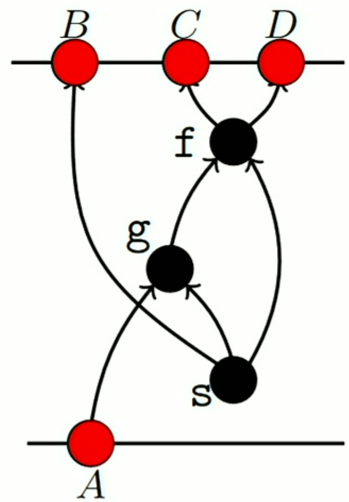


Intuitively: pick an implementation and localize all boxes with wires along timelike paths

Embedding a process



$\xrightarrow{\mathcal{G}}$



$\xrightarrow{\mathcal{E}}$

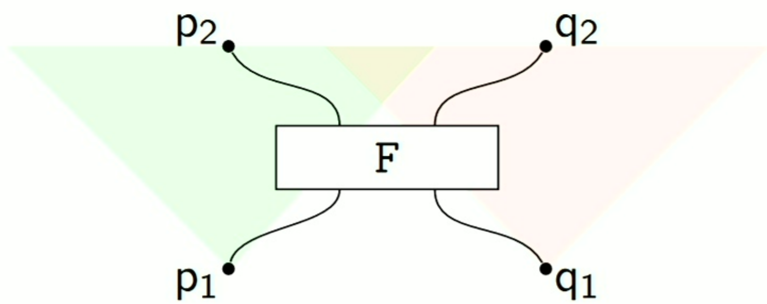
$\mathcal{M}$

Pick a decomposition

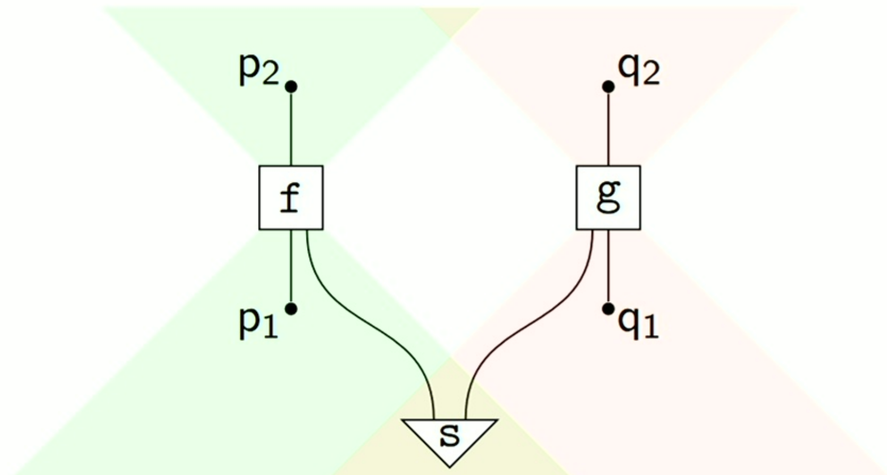
Associate partial order

Map into spacetime via order-preserving map $x \leq y \implies \mathcal{E}(x) \leq \mathcal{E}(y)$

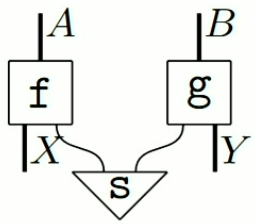
Question: Given a process $\mathtt{F}$ and spacetime locations $\mathcal{E}(x)$ for the in/outputs, is there an embedding?



Partial answer: Depends on the decompositions of $\mathtt{F}$

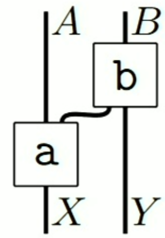


1) Bell scenario



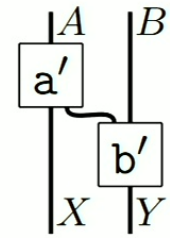
$\mathcal{E}(X) \leq \mathcal{E}(A)$
$\mathcal{E}(Y) \leq \mathcal{E}(B)$
$\exists p \in \mathcal{M} : p \leq \mathcal{E}(A), \mathcal{E}(B)$

2) Alice-to-Bob channel



In addition to 1):
$\mathcal{E}(X) \leq \mathcal{E}(B)$

3) Bob-to-Alice channel



In addition to 1):
$\mathcal{E}(Y) \leq \mathcal{E}(A)$

How to generalise?

Hint: 1) "embeds" into the "spacetime" 2) and 3)

Cutting down the decomposition forest

Growing abstraction

Definitions:

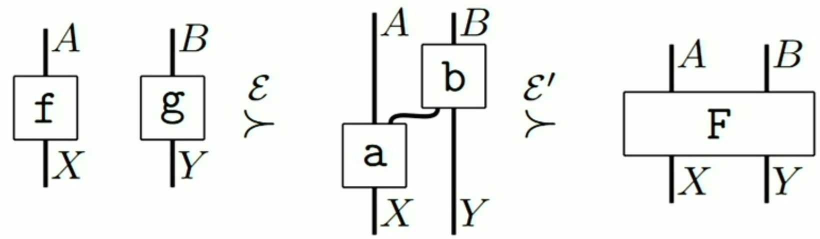
- **Framed partial orders:** Partial order S with a subset called frame ($\sim$ inputs and outputs)
- **Frame and order-preserving maps:** $\mathcal{E} : S_1 \to S_2$ mapping frame to frame
- **Preorder of framed partial orders:** $S_1 \succ S_2$ if $\exists \mathcal{E} : S_1 \to S_2$ frame- and order-preserving

The following are equivalent:

1. $S_1 \succ S_2$
2. Decompositions associated to S_1 are easier to embed than decompositions associated to S_2
3. Decompositions associated to S_1 can be rewritten into decompositions associated to S_2.

"Proof": Embed S_1 into S_2 and then S_2 into $\mathcal{M}$.

Converting diagrams



$\mathcal{E}(\mathtt{f}) = \mathtt{a}, \quad \mathcal{E}(\mathtt{g}) = \mathtt{b}$

$\mathcal{E}'(\mathtt{a}) = \mathcal{E}'(\mathtt{b}) = \mathtt{F}$

Add trivial boxes and wires $\rightarrow$ add $x \notin \mathrm{Im}(\mathcal{E})$ and missing wires

Compose boxes into one box $\rightarrow$ compose $\mathcal{E}^{-1}(x)$

Equivalence classes

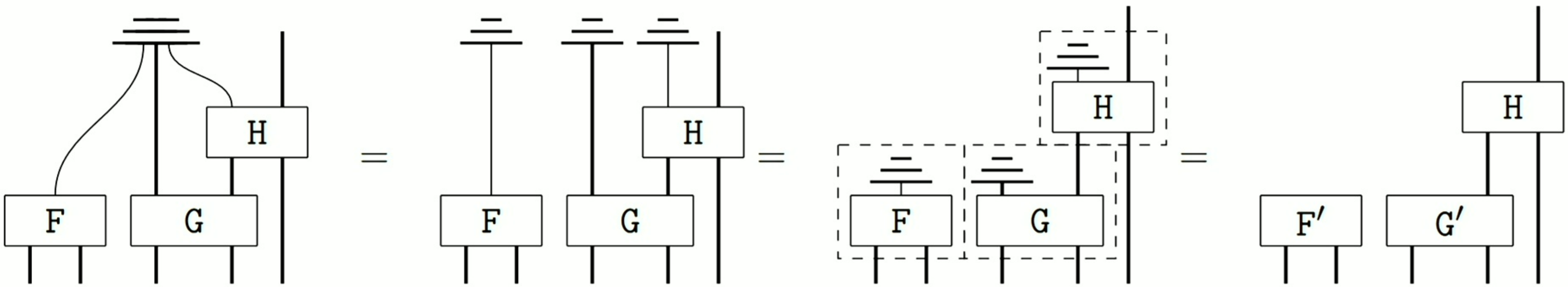
$S_1 \sim S_2$ equivalent if $S_1 \succ S_2$ and $S_2 \succ S_1$
$\hookrightarrow$ due to previous result, we only need to know the equivalence class

Problem: Equivalence classes are quite abstract. Is there a canonical representative?

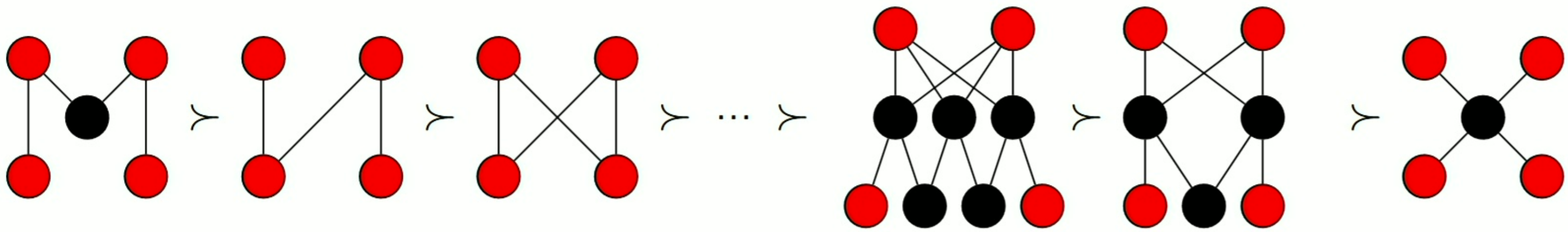
Yes! Every equivalence class has a strictly smallest partial order, **a "minimal" representative**

Relevant minimal representatives

What if I don't care about every theory?
Example: Causal theories



Effects can be removed → only minimal representatives without internal maximal elements are relevant



Question: Are all of these relevant for quantum?

Outlook

Immediate questions

Define composition explicitly

How to find minimal representatives efficiently?

Are all minimal representatives relevant for causal theories relevant for quantum theory?

What if we allow approximate decompositions ($\rightarrow$ Tein's talk)?

Future directions

Study the relation between spacetime and notions of non-classicality

Integrate higher-order processes/indefinite causal order

Integrate symmetries

Compare to other frameworks

- Type-independent resource theories
- Causal sets

Analyze exotic scenarios by adding additional processes to our framework