Title: Fighting non-locality with non-locality: microcausality and boundary conditions in QED

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Fighting non-locality with non-locality:
microcausality and boundary conditions in QED

based on work with Philipp A. Höhn (OIST)

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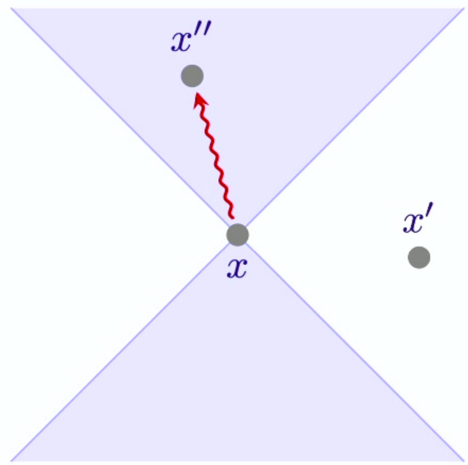
Causalworlds 2024

Causality and *spacetime locality* are intimately linked by the principle of relativity.



Microcausality:

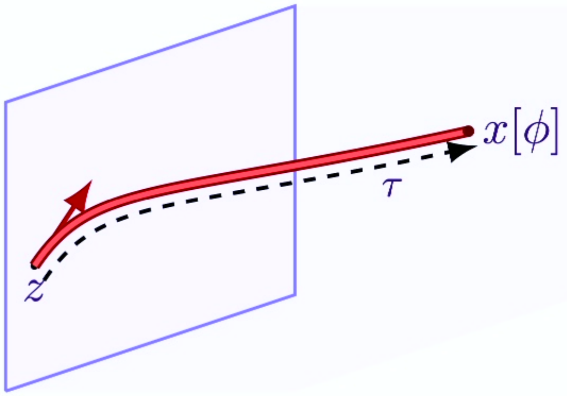
$$x, x' \text{ spacelike separated} \implies \quad \{O_1(x), O_2(x')\} = 0, \quad [\hat{O}_1(x), \hat{O}_2(x')] = 0.$$

Straightforward in ordinary field theories. *Gravity is more complicated*, because spacetime events x are moved around by diffeomorphisms (gauge symmetries). No gauge-invariant 'local' observables.

Solution: use $x = x[\phi]$ that depends on the fields ϕ (including metric and matter). If we do this covariantly (i.e. $x[f_*\phi] = f(x[\phi])$ for any diffeomorphism f), then e.g. value of a scalar field at $x[\phi]$:

$$O = \psi(x[\phi])$$

is a gauge-invariant observable. Example: shoot in a geodesic from the boundary, set $x[\phi]$ as endpoint.



O is not local in the usual sense. It is *relationally local*. It is a *relational observable*, and $x[\phi]$ is a *dynamical reference frame*. [Goeller, Höhn, **JK** '22]

There exist many approaches to relational locality. [Bergmann, Komar, DeWitt, Isham, Kuchar, Rovelli, Dittrich, Thiemann, Giddings, Giesel, Marolf, Donnelly, Harlow, Ferrero, ...]

Relational microcausality

Can we have microcausality in gravity? Is *relational locality* consistent with *microcausality*?

Relationally local observables like $\psi(x[\phi])$ have a very non-local dependence on the fields.

Despite this: relationally local observables which are large-diffeomorphism-invariant are consistent with microcausality. [Marolf '15; Goeller, Höhn, **JK** '22]

Recall: *large diffeomorphisms* are those that act non-trivially at boundary of spacetime (finite or asymptotic), and are *not* gauge symmetries. Physical observables need not be large-diffeomorphism-invariant.

But many interesting reference frames (such as geodesic construction) break large diffeomorphism invariance. Microcausality for these frames remains subtle.

Quantum reference frames (QRFs)

This problem is present already in classical gravity.

Going to the quantum setting involves quantising the reference frames.

QRFs seem to be very helpful in understanding subsystems in quantum gravity. E.g. generalised entropy as von Neumann entropy of algebra of observables relative to a QRF. [Chandrasekharan, Longo, Penington, Witten '22; Jensen, Sorce, Speranza '23; Fewster, Janssen, Loveridge, Rejzner, Waldron '24; DeVuyst, Eccles, Höhn, **JK** '24]

The QRFs in these works have been very simplified. Long way to go before a satisfactorily powerful formalism is established.

Microcausality is a is a foundational issue that needs to be resolved by whatever formalism emerges.

~~Gravity~~ Electrodyamics

Gravity motivates an understanding of relational microcausality. But this talk will rather be about a simpler demonstration of the same principles in *electrodynamics*. Like all gauge theories, suffers from similar locality problems.

Value of a charged field $\psi(x)$ at a point x is not gauge-invariant. Can get something gauge-invariant by *dressing* it with a Wilson line γ from x to spacetime boundary:

$$O = \psi(x) e^{iq \int_\gamma A}.$$

O is *relationally local* to x. Wilson line is reference frame.

O is not invariant under large gauge transformations. Leads to microcausality issues. [Marolf, Giddings, Donnelly, ...]

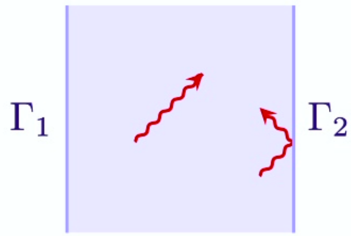
For example, typically gauge potential A and electric field E are conjugate variables. So electric field near γ does not commute with O, even with spacelike separation (?) Gauge subtleties...

Boundary conditions are important

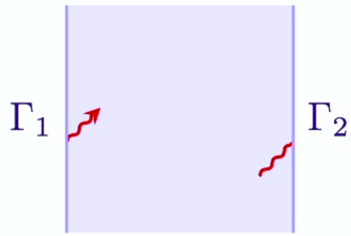
Tension with microcausality for reference frames that depend on fields near boundary? To get a better handle on things, need to carefully specify *boundary conditions*.

Not surprising: Boundary conditions $\longrightarrow$ Dynamics $\longrightarrow$ Microcausality.

Don't need gauge theory to see this. Consider ordinary massless scalar theory on spacetime with two timelike boundaries. Usual Dirichlet boundary conditions $\implies$ microcausality.



Dirichlet



$\phi|_{\Gamma_1} = \phi|_{\Gamma_2}$

But $\phi|_{\Gamma_1} = \phi|_{\Gamma_2}$ breaks microcausality. As if the theory is on a cylinder. Signals can superluminally propagate from one side to the other. (Non-local boundary condition)

Fighting non-locality with non-locality

Does microcausality always fail with non-local boundary conditions?

In theories without gauge symmetry – maybe.

But in gauge theories (such as gravity and electrodynamics) I will argue:

> *Non-local boundary conditions can be used to enhance microcausality, rather that degrade it.*

I will focus on a particular toy setup with non-local boundary conditions $\longrightarrow$ Wilson line dressed observables consistent with microcausality.

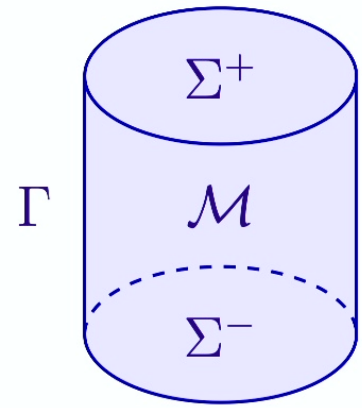
Simple enough to analyse fairly thoroughly. More complex situations (like gravity) ought to be qualitatively similar.

Maxwell-scalar theory

Maxwell $U(1)$ gauge potential A, charged complex scalar ψ of charge q. Action:

$$S = \frac{1}{2}\int_{\mathcal{M}} \Big(-F \wedge *F + \mathrm{D}\psi \wedge * \overline{\mathrm{D}\psi} + *V(|\psi|^2)\Big),$$

where V is potential, $F = \mathrm{d}A$ and $\mathrm{D}\psi = \mathrm{d}\psi - iqA\psi$.



Gauge transformations:

$$A \to A - \mathrm{d}\chi\,, \qquad \psi \to e^{-iq\chi}\psi,$$

'Dirichlet-up-to-gauge-transformations' boundary condition ($\tilde{A}, \tilde{\psi}$ fixed):

$$A|_\Gamma = \tilde{A} - \mathrm{d}\Lambda\,, \qquad \psi|_\Gamma = e^{-iq\Lambda}\tilde{\psi},$$

Need additional condition on Λ for deterministic evolution (e.g. temporal gauge $A_t = 0$; can be enforced with Lagrange multiplier in action. But doesn't give desired microcausality)

Non-local boundary condition

Parametrised by:

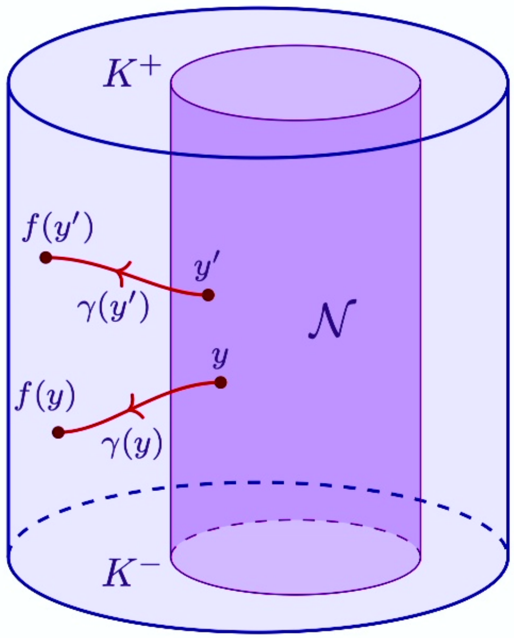
- $\mathcal{N}$, a timelike codimension 1 surface;
- $f : \mathcal{N} \to \Gamma$, a causal order preserving diffeomorphism;
- for each $y \in \mathcal{N}$, a curve $\gamma(y)$ from y to $f(y)$.

Let $\alpha(y) = \frac{1}{q}\arg(\psi(y)) + \int_{\gamma(y)} A$. Function on $\mathcal{N}$.

Boundary condition is: $\Box_{\mathcal{N}}\alpha = 0$.

Under a large gauge transformation we have $\alpha \to \alpha - \chi \circ f$, so we can satisfy the boundary condition by solving wave equation for $\chi \circ f$ on $\mathcal{N}$.

Clearly a very non-local condition – but completely gauge-invariant observables do not witness this non-locality.



Non-local action, local equations of motion

$$S = \frac{1}{2}\int_{\mathcal{M}} \left(-F \wedge *F + \mathrm{D}\psi \wedge * \overline{\mathrm{D}\psi} + *V(|\psi|^2)\right) + \underbrace{\frac{1}{2}\int_{\mathcal{N}} \mathrm{d}\alpha \wedge \star \mathrm{d}\alpha}_{\alpha \text{ as a massless scalar on } \mathcal{N}}.$$

$\delta S = 0$ for fixed initial/final data gives equations of motion (where $j = q\,\mathrm{Im}\left(\overline{\psi}\,\mathrm{D}\psi\right)$):

$$\mathrm{d} * F + *j = -* \int_{y \in \mathcal{N}} \frac{\delta\alpha(y)}{\delta A}\, \mathrm{d} \star \mathrm{d}\alpha\,|_y, \qquad (1)$$

$$\mathrm{D} * \mathrm{D}\psi - *\psi V'(|\psi|^2) = -* \int_{y \in \mathcal{N}} \frac{\delta\alpha(y)}{\delta\overline{\psi}}\, \mathrm{d} \star \mathrm{d}\alpha\,|_y \qquad (2)$$

$\mathrm{d}(1)$ and (2) $\implies$ $\mathrm{d} \star \mathrm{d}\alpha = 0$, i.e. $\Box_{\mathcal{N}}\alpha = 0$. Then simplifies to ordinary Maxwell-scalar:

$$\mathrm{d} * F + *j = 0, \qquad \mathrm{D} * \mathrm{D}\psi - *\psi V'(|\psi|^2) = 0.$$

Solve these, then do large gauge transformation to satisfy $\Box_{\mathcal{N}}\alpha = 0$.

Phase space structure

If initial/final data unfixed, variation of on-shell action is $\delta S = \Theta^+ - \Theta^-$, where

$$\Theta^\pm = \int_{\Sigma^\pm} \left(-\delta A \wedge * F + \mathrm{Re}\left(\delta\psi * \overline{\mathrm{D}\psi}\right)\right) - \int_{\partial\Sigma^\pm} \delta\chi * F + \int_{K^\pm} \delta\alpha \star \mathrm{d}\alpha \,.$$

Presymplectic form:

$$\Omega = \delta\Theta = \int_{\Sigma} \left(\delta A \wedge * \delta F - \mathrm{Re}\left(\delta\psi * \delta(\overline{\mathrm{D}\psi})\right)\right) + \int_{\partial\Sigma} \delta\Lambda * \delta F - \int_K \delta\alpha \, \mathrm{d} \star \delta\alpha$$

Generator of large gauge transformations from $\Omega[\delta_\chi \phi] = \delta(Q[\chi])$:

$$Q[\chi] = \int_K \Big((\chi \circ f)\dot{\alpha} + (\chi \dot{\circ} f)\alpha\Big),$$

Under large gauge transformations, $\alpha \to \alpha - \chi \circ f$. Also, $\Box_{\mathcal{N}} \alpha = 0$. From this, can deduce that Poisson brackets $\{\alpha(y), \alpha(y')\}$ are those of a massless scalar field on $\mathcal{N}$.

Microcausality

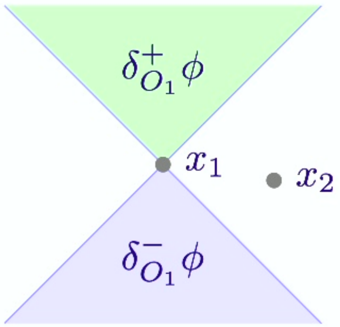
Consider *completely* gauge invariant (even large) O_1, O_2, depending on fields at $x_1, x_2 \in \mathcal{M}$.

Peierls bracket: deform action $S \to S - \lambda O_1$. Gives deformed equations of motion, sourced by $\frac{\delta O_1}{\delta A}, \frac{\delta O_1}{\delta \psi}$ with support at x_1. Solve deformed equations for retarded/advanced solutions

$$A + \lambda \delta^{\pm}_{O_1} A + \mathcal{O}(\lambda^2), \qquad \psi + \lambda \delta^{\pm}_{O_1} \psi + \mathcal{O}(\lambda^2),$$

with $\delta^{\pm}_{O_1} A = \delta^{\pm}_{O_1} \psi = 0$ on $\Sigma^{\mp}$ respectively.

By causal properties of Maxwell-scalar field equations, $\delta^{\pm}_{O_1}\phi$ have support only in future/past lightcone of x_1, so the difference $\delta_{O_1}\phi = \delta^{+}_{O_1}\phi - \delta^{-}_{O_1}\phi$ is supported in the lightcone of x_1 (up to a gauge transformation)



Peierls bracket:

$$\{O_1, O_2\} = \delta_{O_1} O_2$$

vanishes if x_2 is outside the lightcone of x_1.

Microcausality for dressed observables

Consider

$$\Psi(y) = \psi(y) e^{iq \int_{\gamma(y)} A} = |\psi(y)| e^{iq\alpha(y)}, \quad y \in \mathcal{N}.$$

Leibniz rule:

$$\{\Psi(y), \Psi(y')\} = \{|\psi(y)|, |\psi(y')|\} e^{iq\alpha(y)} e^{iq\alpha(y')} + \left\{e^{iq\alpha(y)}, e^{iq\alpha(y')}\right\} |\psi(y)||\psi(y')|$$
$$+ \left\{|\psi(y)|, e^{iq\alpha(y')}\right\} e^{iq\alpha(y)} |\psi(y')| + \left\{e^{iq\alpha(y)}, |\psi(y')|\right\} |\psi(y)| e^{iq\alpha(y')}.$$

Latter two terms vanish because α is the generator of large gauge transformations.

If $y, y' \in \mathcal{N}$ are spacelike separated, by Peierls bracket argument the other terms vanish also. We are left with $\{\Psi(y), \Psi(y')\} = 0$.

A similar argument applies for general observables dressed by the Wilson lines: relational microcausality works! Therefore:

Non-local boundary conditions $\longrightarrow$ *enhancement of locality*

Reference frames

The surface $\mathcal{N}$ and the curves γ specify a *dynamical reference frame*.

Akin to inertial reference frames in special relativity, which can be *reoriented* with Lorentz transformations.

Here, the frame reorientations are large gauge transformations.

Clearly, there are many possible frames one can use here. For example, we could do the same thing, but with a different surface $\mathcal{N}'$ and family of curves γ'.

But there is a link between the frame used and the boundary conditions imposed, both of which involve the same $\mathcal{N}, \gamma$. This *compatibility* was essential for demonstrating microcausality.

For fixed boundary conditions, microcausality will only hold for certain frames.

Microcausality is frame-dependent.

Change of frames

Suppose we want to transform from one frame $(\mathcal{N}, \gamma)$ to another $(\mathcal{N}', \gamma')$.

$$S = \frac{1}{2}\int_{\mathcal{M}} \Big(-F \wedge *F + \mathrm{D}\psi \wedge *\overline{\mathrm{D}\psi} + *V(|\psi|^2)\Big) + \frac{1}{2}\int_{\mathcal{N}} \mathrm{d}\alpha \wedge \star \mathrm{d}\alpha\,,$$
$$S' = \frac{1}{2}\int_{\mathcal{M}} \Big(-F \wedge *F + \mathrm{D}\psi \wedge *\overline{\mathrm{D}\psi} + *V(|\psi|^2)\Big) + \frac{1}{2}\int_{\mathcal{N}'} \mathrm{d}\alpha' \wedge \star \mathrm{d}\alpha'\,.$$

$S \to S'$, deform action by

$$\frac{1}{2}\int_{\mathcal{N}'} \mathrm{d}\alpha' \wedge \star \mathrm{d}\alpha' - \frac{1}{2}\int_{\mathcal{N}} \mathrm{d}\alpha \wedge \star \mathrm{d}\alpha \overset{\overset{\text{sometimes}}{\downarrow}}{\doteq} X[\Sigma^+] - X[\Sigma^-].$$

Quantumly: deforming the action in this way amounts to inserting $e^{i\hat{X}}$ in path integral, on initial and final states.

Unitary operator implementing change of frames. Preserves algebraic relations, including microcausality. So this is a change of *compatible* frames.

Quantisation, frame-dependence of the vacuum

First: quantize α as a scalar field on $\mathcal{N}$. Then, make a (small) gauge choice in which the other degrees of freedom are decoupled/independent from α. They can be quantized as usual for ordinary Maxwell-scalar field theory, as a perturbative QFT.

Get a preferred vacuum state in Fock space:

$$|\Omega\rangle = |0\rangle_A \otimes |0\rangle_\psi \otimes |0\rangle_\alpha \in \mathcal{H}_A^{\text{Fock}} \otimes \mathcal{H}_\psi^{\text{Fock}} \otimes \mathcal{H}_\alpha^{\text{Fock}} = \mathcal{H}.$$

Construction depends on the choice of frame. Different frames $(\mathcal{N}, \gamma)$, $(\mathcal{N}', \gamma')$ lead to different vacua in different Hilbert spaces:

$$|\Omega\rangle \in \mathcal{H}, \qquad |\Omega'\rangle \in \mathcal{H}'.$$

QRF language: $\mathcal{H}, \mathcal{H}'$ are the *reduced* Hilbert spaces relative to each frame.

One can show $e^{i\hat{X}} |\Omega\rangle \neq |\Omega'\rangle$ – vacuum is not preserved under a change of frames! Is this similar to the Hawking/Unruh effects?

Algebraic QFT

Causally local net of observables: an algebra of operators $\mathcal{A}(\mathcal{U})$ for each spacetime region $\mathcal{U} \subset \mathcal{M}$. Should obey *Haag-Kastler axioms*...

$$\mathcal{U} \longrightarrow \mathcal{A}(\mathcal{U})$$

In ordinary QFT, $\mathcal{A}(\mathcal{U})$ consists of field operators smeared over $\mathcal{U}$.

Here, relational micocausality enables us to also the dressed observables such as $\Psi(y)$ for $y \in \mathcal{U}$, in a way that is consistent with the axioms. In the interest of a complete physical picture, it seems reasonable to do so (also, gravity would necessitate this kind of thing).

Frame-dependent microcausality $\implies$ *frame-dependent net of observables*. Notion of quantum subsystem depends on which QRF is used. In gravity, this has important consequences for the relativity of entropy [DeVuyst, Eccles, Höhn, JK '22]. Would be nice to do understand this for the QED case study I have described.

Conclusion

- Boundary-dressed observables can satisfy relational microcausality. Described in some detail a toy example/case study in QED. Many generalisations are possible.
- Key ingredient: *non-local boundary conditions* on large gauge sector.
- Quantum theory: non-uniqueness of vacuum, non-uniqueness of local net of observables, non-uniqueness of entropies! Everything depends on which frame we use.

Open questions:

- Given some boundary conditions, which frames satisfy relational microcausality?
- Codimension of dressed region: hints at holographic viewpoint?
- Can we do an explicit entropy calculation?
- Do these observations extend to the gravitational case?

Thank you for listening!