Title: Quantum reference frames, measurement schemes and the type of local algebras in quantum field theory

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Quantum reference frames, measurement schemes and the type of local algebras in quantum field theory

Based on arXiv:2403.11973
with C.J. Fewster, L.D. Loveridge, K. Rejzner, J. Waldron

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Main topics of the talk

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(Operational) quantum reference frames and quantum field theory

Need for QRFs arises in measurement theory for quantum fields to distinguish between measurements/experiments related by a symmetry transformation. Yields notion of ‘relativised measurements’.

We give conditions for which the algebra of relative measurable observables on specific backgrounds is of reduced type (‘better behaved’) compared to the algebra of local observables.

Modelling relativistic measurement

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Models of a measurement process for QFT should respect causality/locality:

- Measurements performed in a certain spacetime region should only probe observables localisable in the causal hull of that region.
- The state-update rule associated to a measurement should not allow for superluminal signaling.

- Satisfied by class of local measurement schemes using formulation of Algebraic QFT.[a]
- Approximately realised by particle detector models.[b]

[a] Fewster and Verch 2020.
[b] De Ramón, Papageorgiou, and Martín-Martínez 2021.

Sketch of the local measurement theory

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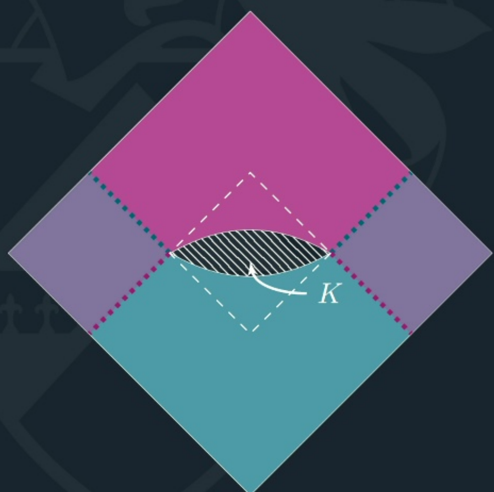
How to perform a measurement for some QFT $\mathscr{A}$ on a spacetime M?[1]



- Couple $\mathscr{A}$ to some probe QFT $\mathscr{B}$ in compact coupling zone $K \subset M$.
- Probe field is prepared in some initial state uncorrelated to $\mathscr{A}$ state in pre-coupling region.
- Probe observable B measured in post-coupling region.
- Statistics of potential outcomes for B correspond to statistics for induced system observable A localisable in region containing K.
- Scheme interpreted as (indirect) measurement of A.

[1] Fewster and Verch 2020.

A note on measurable observables

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We consider observables in a generalised sense (following operational QM). For a QFT $\mathscr{A}$ on a spacetime M, $\mathscr{A}(M)$ denotes a *-algebra

- Observables are positive operator valued measures (POVMs) on some (for simplicity finite & discrete) space Ω of potential measurement outcomes

$$E : \mathcal{P}(\Omega) \to \mathscr{A}(M),$$

  $E(X) \geq 0$ for all $X \in \mathcal{P}(\Omega)$, $E(\Omega) = 1_{\mathcal{A}}$, E additive.
- Given a state ω on $\mathscr{A}(M)$, $\omega \circ E$ defines the probability distribution for the outcome of a measurement.
- Outcome statistics for late time probe observable $E_{\mathscr{B}}$ (in joint probe & system state) correspond to that of some early time induced system observable $E_{\mathscr{A}}$ (in system state).

Measurement theory and background symmetry

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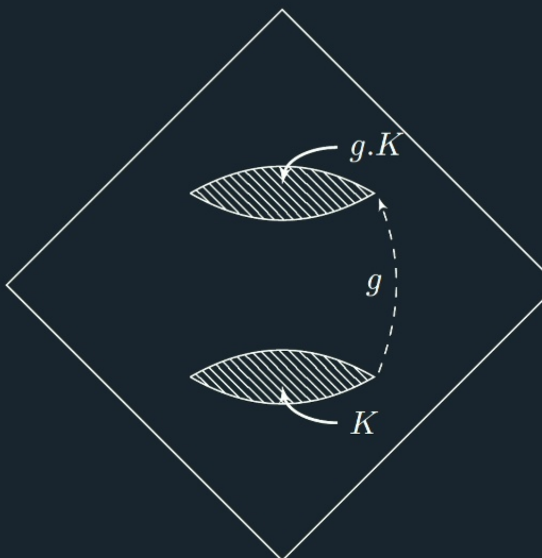
Given G a symmetry group of the background, assume $\mathscr{A}$ and $\mathscr{B}$ respect this symmetry (they are G-covariant). The coupling in K typically breaks symmetry. How to operationally distinguish these different measurements?

"How do you know when to start your experiment if you don't have a clock?"

Observables measured through local scheme are not invariant under G. In the spirit of operational QRF approach, 'physical' observables should be invariant under all background symmetries.



Operational quantum reference frames

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QRFs as systems of covariance:
Following the approach of CGL[2] we define an operational QRF for a locally compact, second countable Hausdorff group G as a triple $(U, E, \mathcal{H})$

- $\mathcal{H}$ a separable Hilbert space,
- $U : G \to \mathbf{U}(\mathcal{H})$ a strongly continuous unitary rep. of G,
- $E : \mathrm{Bor}(\Sigma) \to B(\mathcal{H})$ a POVM with $\Sigma = G/H$ a topological G-space with $H \subset G$ closed,

such that for all $X \in \mathrm{Bor}(\Sigma)$, $g \in G$ we have

$$U(g)E(X)U(g)^* = E(g.X).$$

[2] Carette, Głowacki, and Loveridge 2023.

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Example: particle on a line

A QRF $(\lambda, P, \mathcal{H}_R)$ for (spatial) translation group $\mathbb{R}$ (with $\Sigma = \mathbb{R}$)

$$\mathcal{H}_R = L^2(\mathbb{R}), \qquad (\lambda(y)\psi)(x) = \psi(x-y), \qquad (P(X)\psi)(x) = \chi_X(x)\psi(x).$$

Here P is the spectral (projection valued) measure of position operator $\mathbf{x}$ and $\lambda(y) = \exp(-i\mathbf{p}y)$ left translation generated by momentum. Formally,

$$\lambda(y)P(X)\lambda(y)^* = P(X+y) \iff [\mathbf{x}, \mathbf{p}] = i.$$

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Example: time of occurrence observable

- Can one similarly define a QRF for time-translations? For a self-adjoint Hamiltonian $\mathbf{H}$ bounded from below, there is no self-adjoint $\mathbf{t}$ with $[\mathbf{H}, \mathbf{t}] = i$, hence no sharp QRF (i.e. defined by PVM).
- For a quantum (field) theory, one can construct a 'time of occurrence observables'. Time defined relative to some physical event, such as a detector click.[3] Time of detector click encoded in covariant POVM $E : \mathrm{Bor}(\mathbb{R}) \to B(\mathcal{H}_R)$ satisfying

$$\exp(i\mathbf{H}t)E(X)\exp(-i\mathbf{H}t) = E(X + t).$$

- For such a detector POVM $E : \mathrm{Bor}(\mathbb{R}) \to B(\mathcal{H}_R)$ and normal state ω_R, $\omega_R \circ E : \mathrm{Bor}(\mathbb{R}) \to [0, 1]$ gives a probability distribution for time of click.

[3] Brunetti and Fredenhagen 2002.

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Characterising QRFs
QRFs $(U, E, \mathcal{H})$ can (up to unitary equivalence) be completely characterised (based on [Mackey 1949] and [Cattaneo 1979]).

For $G = \Sigma = \mathbb{R}$, we have that, up to unitary equivalence

$$\mathcal{H} = p(L^2(\mathbb{R}) \otimes \mathcal{K}), \qquad U(s) = (\lambda(s) \otimes 1_{\mathcal{K}}) \restriction_{\mathcal{H}}, \qquad E(X) = p(P(X) \otimes 1_{\mathcal{K}}) \restriction_{\mathcal{H}} .$$

Here $\mathcal{K}$ some (residual) Hilbert space and p projecting onto a subrep of the left translation $(\lambda \otimes 1_{\mathcal{K}}, L^2(\mathbb{R}) \otimes \mathcal{K})$.

In particular, for $U(s) = \exp(i\mathbf{H}s)$, $\mathbf{H}$ must have continuous spectrum.

QRF and measurement schemes

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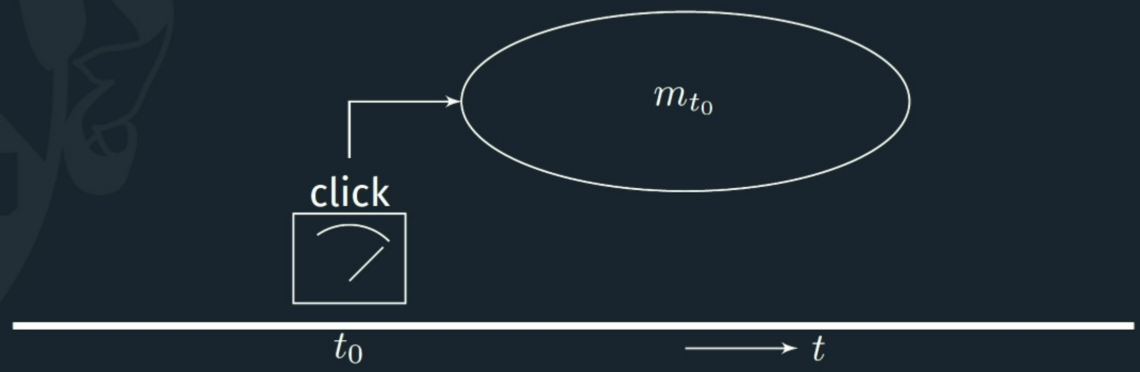
Using a QRF to resolve measurement schemes (sketch)
On a static spacetime M, a measurement scheme m_t performed at (coordinate) time t measure observable (POVM) $E_{S,m_t} : \mathcal{P}(\Omega) \to \mathscr{A}(M)$. When do I start my measurement? Base on some physical event, like a detector click described by a QRF $(U_R, E_R, \mathcal{H}_R)$ with $G = \Sigma = \mathbb{R}$.



QRF and measurement schemes

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The outcome statistics of this relative measurement scheme is given by POVM

$$E_{[m_t]}(X) = \int_{\mathbb{R}} E_{S,m_t}(X) \otimes \mathrm{d}E_R(t).$$

Integral made rigorous for well-behaved Hilbert space representation of $\mathscr{A}(M)$, generating a von Neumann algebra (see also [Głowacki 2024]).

Invariant observables

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For sufficiently regular representation $\pi : \mathscr{A}(M) \to B(\mathcal{H}_S)$, time translation acts on $\mathcal{A} = \pi(\mathscr{A}(M))''$ by a weakly continuous representation of *-automorphisms α_t.

Given a QRF $(U_R, E_R, B(\mathcal{H}_R))$ with $G = \Sigma = \mathbb{R}$, the relativised observables $E_{[m]} : \mathrm{Bor}(\Omega) \to \mathcal{A} \otimes B(\mathcal{H}_R)$ are time-translation invariant (see e.g. [GCL23]).

Using previous characterisation of QRFs, we show that there exists some Hilbert space $\mathcal{K}$ and projection $p \in B(L^2(\mathbb{R}) \otimes \mathcal{K})$ such that

$$(\mathcal{A} \otimes B(\mathcal{H}_R))^{\mathbb{R}} \cong (1_{\mathcal{A}} \otimes p)((\mathcal{A} \rtimes_\alpha \mathbb{R}) \otimes B(\mathcal{K}))(1_{\mathcal{A}} \otimes p).$$

$\mathcal{A} \rtimes_\alpha \mathbb{R}$ is a crossed product algebra.

This result generalises to large class of QRFs and groups G.

Take home messages so far

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- In the presence of a background symmetry, we can define a notion of measurements made relative to a suitable (operational) quantum reference frame, in terms of relativised induced observables.
- Using a characterisation result for QRFs, we can give a concrete description of the algebra invariant under symmetry transformations containing these relativised observables, in terms of a crossed product algebra.
- Here we do not specify how the QRF is constructed. Open question: Can such a QRF itself be obtained through local measurements?

Local algebras and von Neumann types

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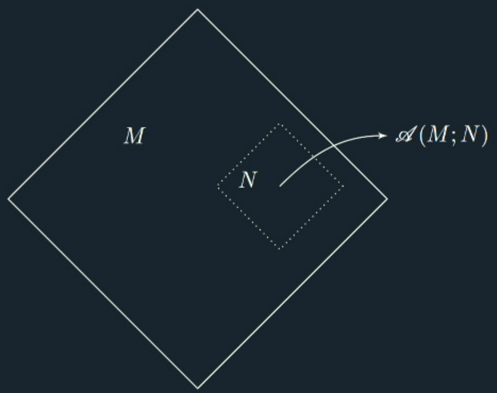
In algebraic QFT, the fundamental objects are the local algebras

- A *-algebras $\mathscr{A}(M;N)$ for region $N \subset M$ contains all observables accessible/measurable from N.
- under natural assumptions on the QFT and the representation, and for sufficiently regular regions N, the algebras $\mathcal{A} = \pi(\mathscr{A}(M;N))''$ are type III algebras.[4]



[4] Fredenhagen 1985; Buchholz, D'Antoni, and Fredenhagen 1987

Local algebras and von Neumann types

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Von Neumann algebras come in three flavours, characterised by properties of a trace $\tau : \mathcal{A} \to [0, \infty]$ and the values of $\tau(\mathcal{P})$ for $\mathcal{P} \subset \mathcal{A}$ the projections.

Type I	Type II	Type III
$\tau(\mathcal{P})$ discrete	$\tau(\mathcal{P})$ continuous	$\tau(\mathcal{P}) = \{0, \infty\}$
Quantum mechanics	Some stat. mech. models	Local algebras in QFT

Due to badly behaved trace, type III algebras lack a well behaved notion of entropy.

However, a crossed product $\mathcal{A} \rtimes_\alpha \mathbb{R}$ decomposes into (at most) type I and type II provided that $\alpha : \mathbb{R} \to \mathrm{Aut}(\mathcal{A})$ is a **modular action**.[5]

[5]Takesaki 1973.

Geometric modular action and type reduction

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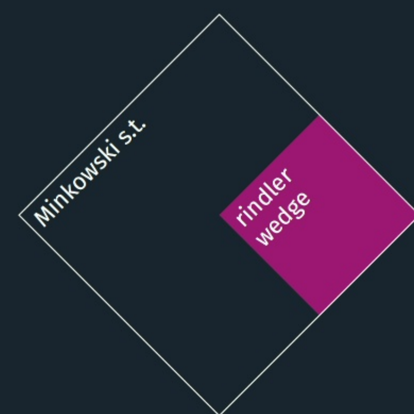
Time translation action α on a local algebra $\mathcal{A}$ is modular $\approx$ QFT admits a well-behaved thermal state at local region for some finite temperature $T > 0$.

Examples: de Sitter static patch, Rindler wedge or Schwarzschild exterior.



Geometric modular action and type reduction

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Time translation action α on a local algebra $\mathcal{A}$ is modular $\approx$ QFT admits a well-behaved thermal state at local region for some finite temperature $T > 0$.
Examples: de Sitter static patch, Rindler wedge or Schwarzschild exterior.

Let $\mathcal{A}$ be the v.N. algebra of observables for a QFT on one of these backgrounds (in some appropriate representation) and $(U, E, \mathcal{H}_R)$ a QRF for $G = \Sigma = \mathbb{R}$. Then the algebra of time translation invariants $(\mathcal{A} \otimes B(\mathcal{H}_R))^{\mathbb{R}}$ decomposes into I & II.

Type reduction phenomenon similar to CLPW[6], where the role of QRF was played by a model for an observer system. Our result applies to a very general class of QRFs, see also VEHK[7] for alternative approach.

[6] Chandrasekaran, Longo, Penington, and Witten 2023.
[7] De Vuyst, Eccles, Hoehn, and Kirklin 2024.

Geometric modular action and type reduction

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Model of CLPW leads in particular to algebra of type II_1, which admits a **finite trace**.

We formulate a condition on the QRF ensuring this result, as growth condition on the spectral multiplicity of the QRF Hamiltonian.

Physical interpretation: The QRF must have good thermal properties at the same temperature as the QFT.

Upshot: Relativised observables (accessible through relativised measurements) generate a subalgebra $\tilde{A} \subset (\mathcal{A} \otimes B(\mathcal{H}_R))^{\mathbb{R}}$. In general $\tilde{A}$ will not be type II.

The thermal condition on the QRF ensures that $\tilde{A}$ admits a finite trace.

Discussion

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- On specific backgrounds admitting thermally well-behaved QFTs, measurements made relative to a thermally well-behaved QRF at the same temperature, the algebra of accessible observables admits a finite trace.
- Key ingredient to the underlying argument is the geometric modular action (only available on very special backgrounds for general QFTs).
- It is hoped that type reduction occurs more generally in quantum gravity, due to use of general diffeomorphisms instead of background symmetries. This may allow one to make contact between entropic arguments in QG and perturbative QG. (see e.g. [Witten 2022])
- However, it is not known if on general backgrounds there exists diffeomorphisms yielding a modular action.