Title: Routing Quantum Control of Causal Order

Speakers: Maarten Grothus

Series: Quantum Foundations, Quantum Information

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Routing Quantum Control of Causal Order

Maarten Grothus, Alastair Abbott and Cyril Branciard

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Outline

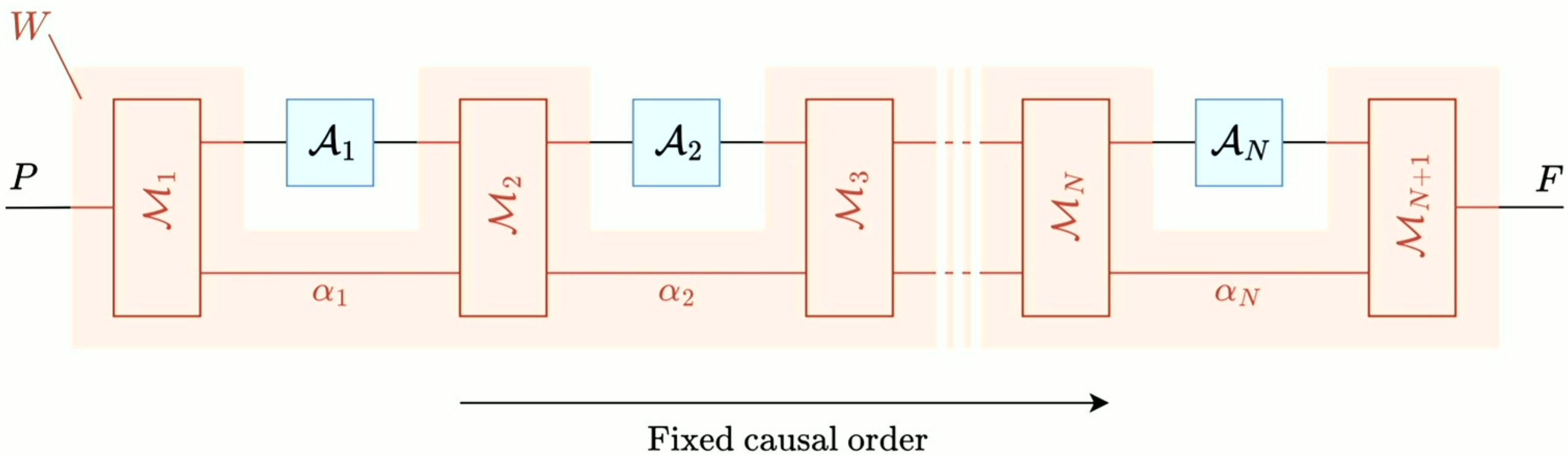
1. Higher-Order Transformations and Process Matrices
2. Quantum Circuits with Quantum Control (QCQCs)
3. Routed Quantum Circuits (RQCs)
4. Representing QCQCs using Routed Quantum Circuits
5. Summary & Outlook

Channels and Higher-Order Transformations

- Channels: First-order transformations Φ, transforming states: $\Phi(\rho) = \rho'$
  - Classical: linear stochastic maps, relating probability distributions
  - Quantum: linear maps, completely positive (CP) and trace preserving
- Superchannels: Higher-order transformations W, transforming channels: $W(\Phi_A, \Phi_B, \ldots) = \Phi_W$
  - Quantum: N-linear in $\Phi_A, \Phi_B, \ldots$ and completely CP-preserving
  - Choi representation of superchannel known as *process matrix*

Higher-Order Transformations and Process Matrices

- Superchannels: Higher-order transformations of channels: $W(\Phi_A, \Phi_B, \ldots) = \Phi_W$
- Example: Quantum combs. Standard quantum circuit with slots
  - Local operations A_n alternate with internal transformations $\mathcal{M}_n$
  - Ancillary system α_n as memory between transformations



Higher-Order Transformations and Process Matrices

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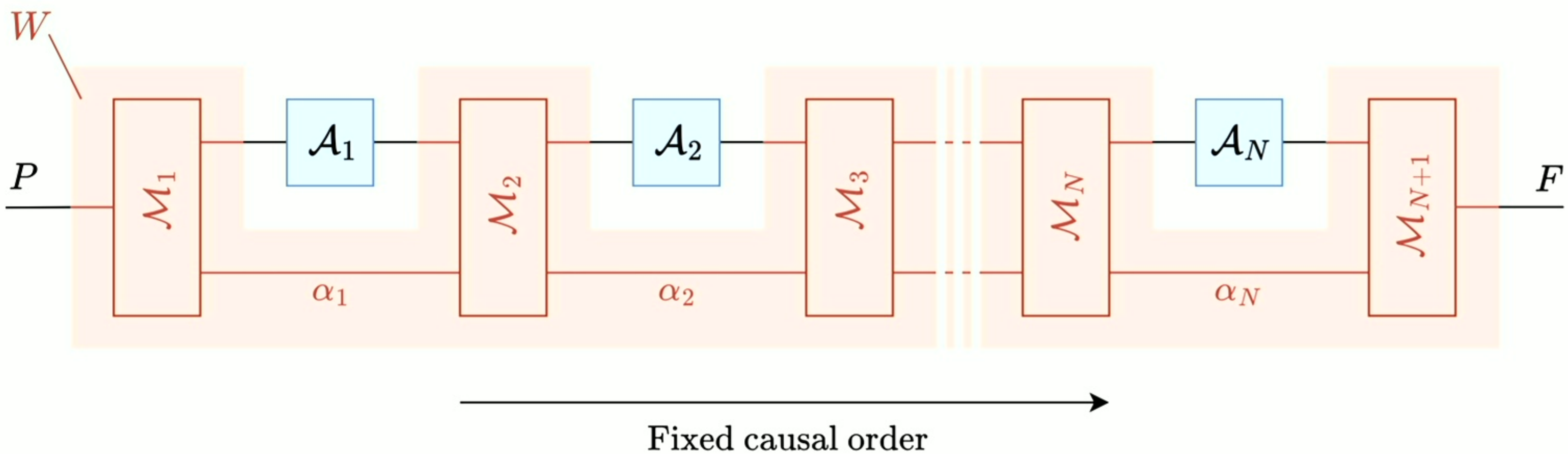


What else can you do with a superchannel?

Frameworks for Indefinite Causal Order (ICO)

Various (bottom-up) approaches allow to study ICO systematically.

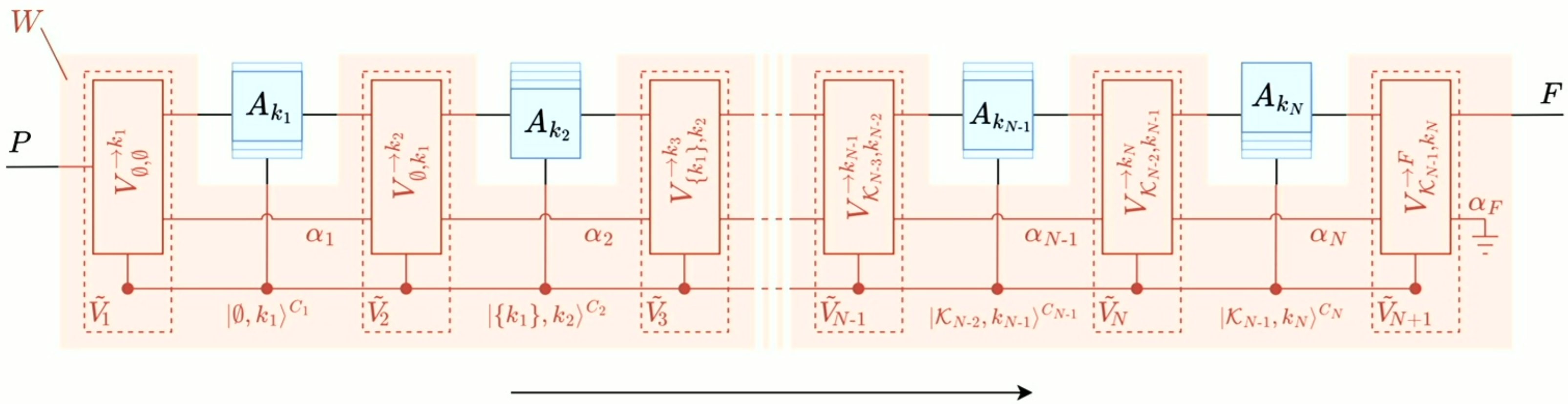
- Quantum Circuits with Quantum Control (QCQCs) [Wechs *et al.* 2021]
  - coherent control of (multiple acyclic) temporal orders of operations
- Routed Quantum Circuits (RQCs) [Vanrietvelde *et al.* 2020 & 2022]
  - compositional diagrammatic representation of alternatives
- Causal boxes, Addressable quantum gates, Many-worlds calculus, ...

Complex phenomena emerging include

- Dynamical operational order
- Consistent intrinsic causal and time loops

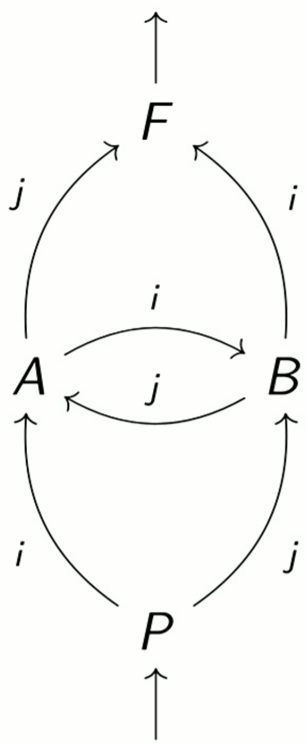
How are the different approaches and frameworks connected?

Quantum Circuits with Quantum Control (QCQCs)



- Bottom-up framework to construct supermaps without time loops
- Operations A_k alternate with internal isometries $\tilde{V}_n$, routing $k_n \to k_{n+1}$
- Control register C_n tracks set of operations $\mathcal{K}_n \cup k_{n+1}$ already used
- Further registers for in-/output of agents A_k and ancillaries α_n
- Corresponds to the subset of supermaps realizable within a fixed spacetime

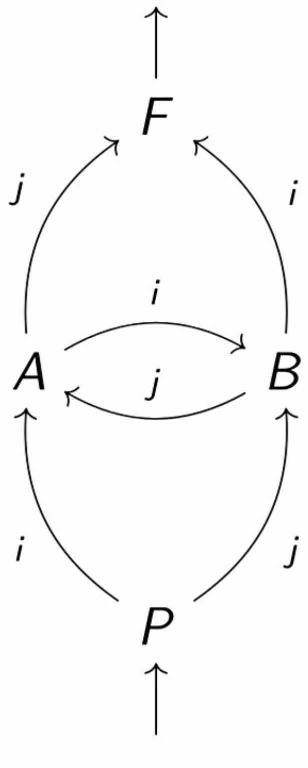
Routed Quantum Circuits (RQCs)

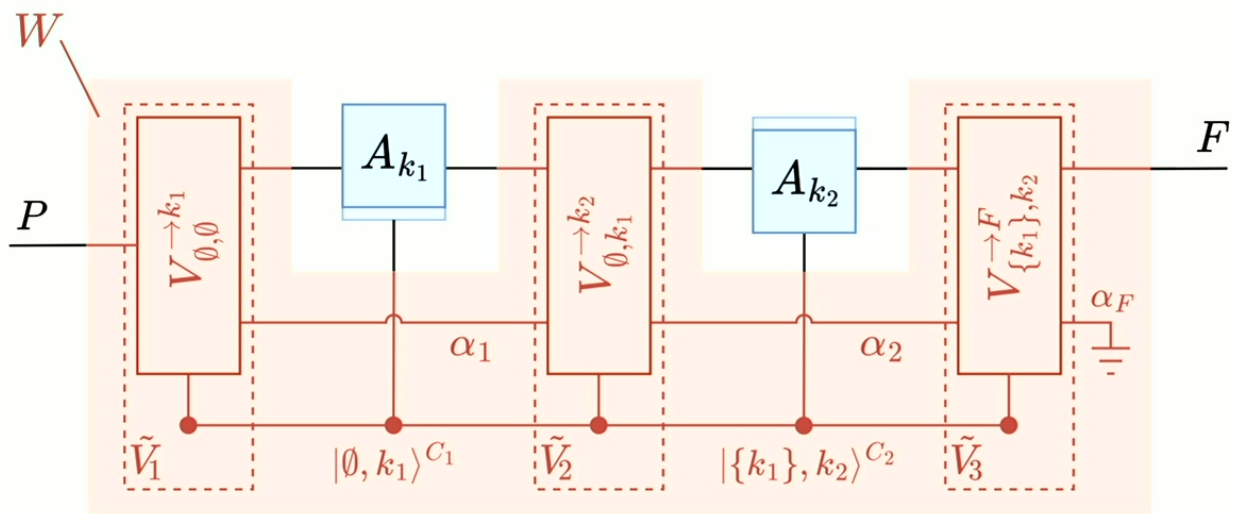


$i + j = 1$

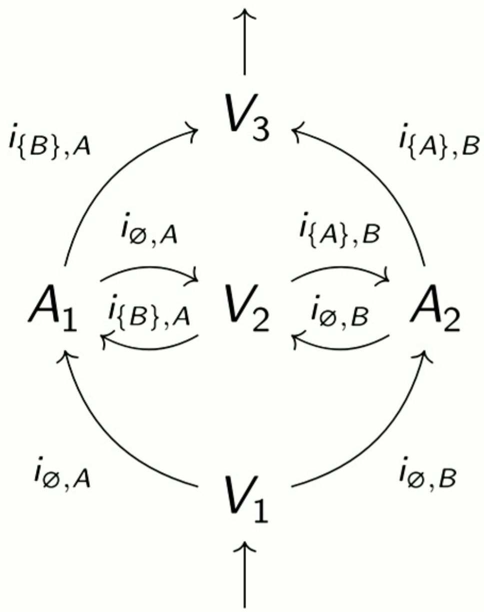
- Compositional framework for constructing unitary process matrices (which may violate causal inequalities)
- Representing alternatives with *routes*, preserving sectors of Hilbert spaces:
  $\tilde{\mathcal{H}}_{ij} = \mathcal{H}_i^0 \otimes \mathcal{H}_j^1 \oplus \mathcal{H}_i^1 \otimes \mathcal{H}_j^0 \subsetneq \mathcal{H}_i \otimes \mathcal{H}_j$
- Direct sum structure neglected in standard quantum circuits
- Each combination of indiced within a routed graph corresponds to one possible alternative
  - → fine-graining to *branch graph* unwrapping all alternatives
- Consistency conditions
  - *univocality*: each node visited exactly once for each alternative
  - no paradoxical time loops: no cyclic information flow
- Routed graphs represent all circuits with equivalent connectivity

How to connect these two frameworks?





How to represent a QCQC as a RQC? (for $N = 2$)



$$1 = i_{\varnothing,A} + i_{\varnothing,B}$$
$$i_{\varnothing,A} = i_{\{A\},B}$$
$$i_{\varnothing,B} = i_{\{B\},A}$$
$$i_{\{A\},B} + i_{\{B\},A} = 1$$

- Add explicit representation of internal (supermap) isometries V_n alternating with the (agent) operations A_k
- Reflect control register C_n by naming the indices $i_{\mathcal{K}_n,k_{n+1}}$
- At V_n, add node last visited to index name: $\mathcal{K}_{n-1}, k_n \rightarrow \mathcal{K}_{n-1} \cup k_n, k_{n+1}$

$\rightarrow$ Canonical representation of **any** N-party QCQCs from a **single** routed graph!

We obtain the supermap by

- associating the respective Hilbert spaces with the arrows
- factoring out target space from ancillary and control space at each agent node A_k

Canonical representation of QCQCs as RQCs: $N = 3$

$$i_{\{A,B\},C} + i_{\{A,C\},B} + i_{\{B,C\},A} = 1$$

$$i_{\{A\},B} + i_{\{B\},A} = i_{\{A,B\},C}$$
$$i_{\{B\},C} + i_{\{C\},B} = i_{\{B,C\},A}$$
$$i_{\{A\},C} + i_{\{C\},A} = i_{\{A,C\},B}$$

$$i_{\emptyset,A} = i_{\{A\},B} + i_{\{A\},C}$$
$$i_{\emptyset,B} = i_{\{B\},C} + i_{\{B\},A}$$
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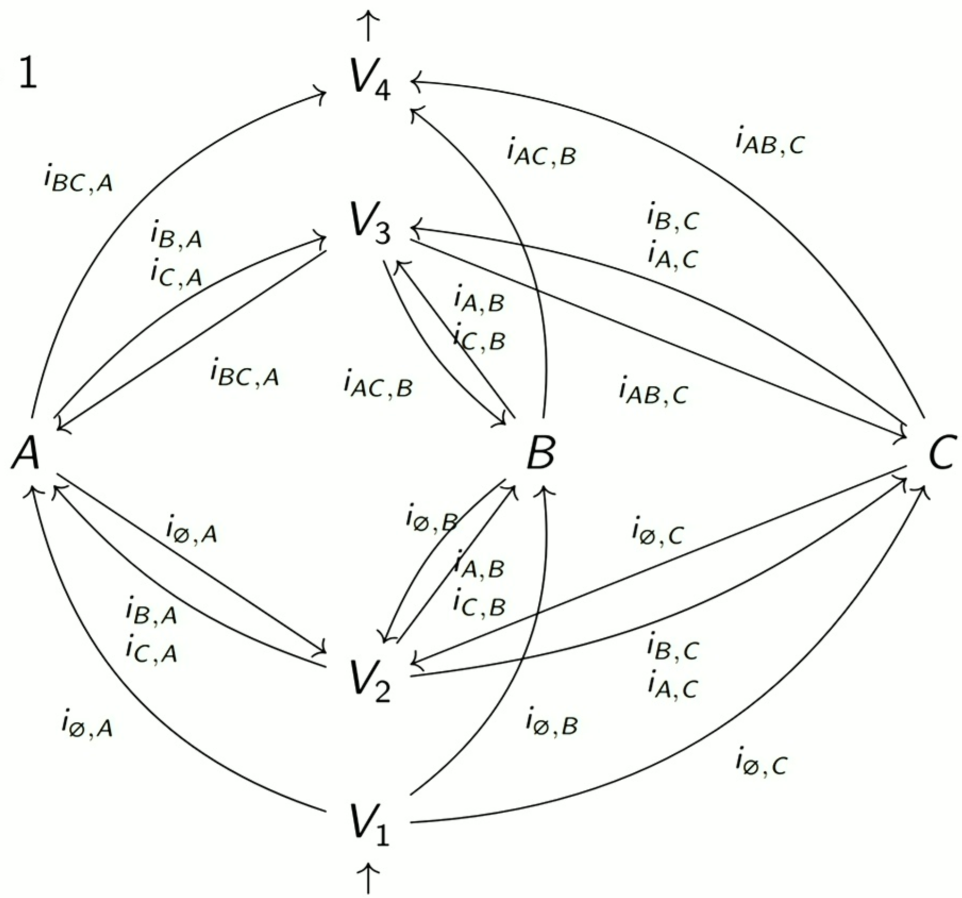

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$$i_{\varnothing,A} = i_{\{A\},B} + i_{\{A\},C}$$
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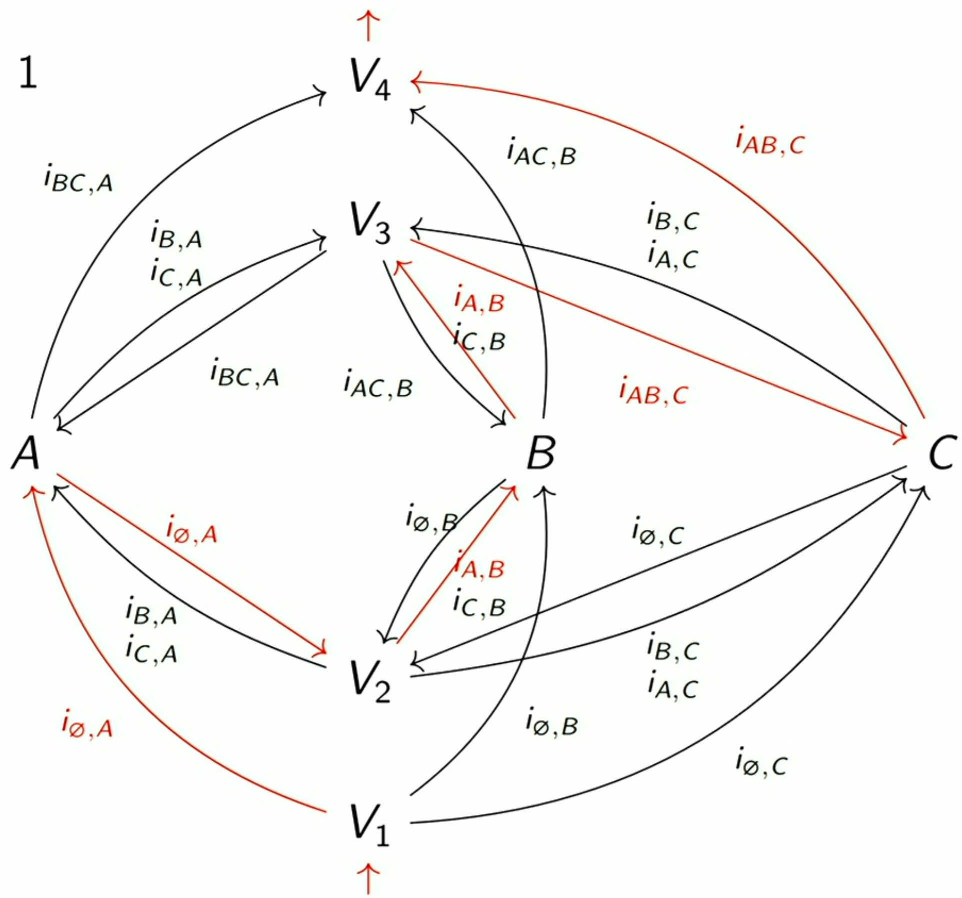
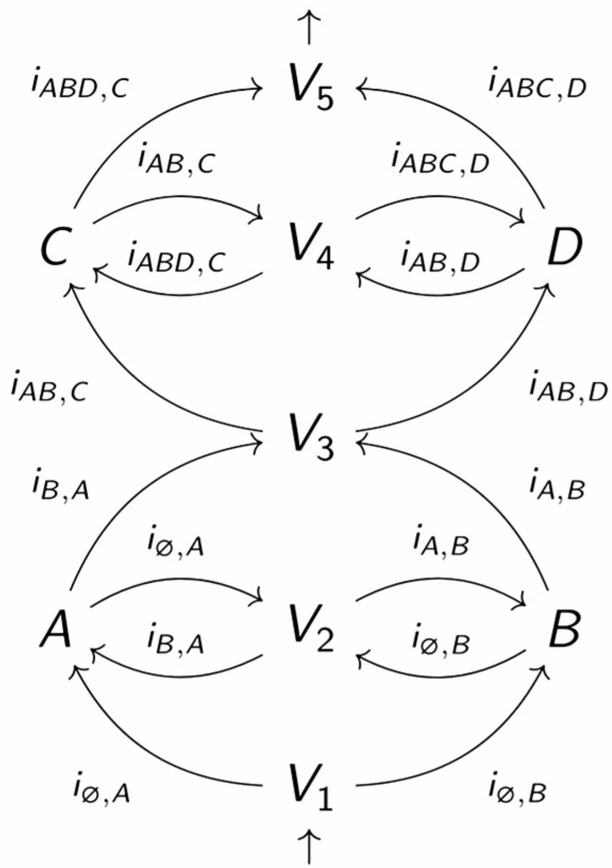


Canonical representation of QCQCs as Routed Quantum Circuits

The canonical routed graph $\mathcal{G}_N$ for an N-slot QCQC:

- nodes $\mathbf{A}_k$, for $k = 1, \ldots, N$, and $\mathbf{V}_n$, for $n = 1, \ldots, N+1$
- open-ended arrows $\xrightarrow{i_{\emptyset,\emptyset}=1} \mathbf{V}_1$ and $\mathbf{V}_{N+1} \xrightarrow{i_{\mathcal{N},F}=1}$
- arrows $\mathbf{V}_n \xrightarrow{(i_{\mathcal{K}_{n-1},k})_{\mathcal{K}_{n-1}\not\ni k}} \mathbf{A}_k \xrightarrow{(i_{\mathcal{K}_{n-1},k})_{\mathcal{K}_{n-1}\not\ni k}} \mathbf{V}_{n+1} \quad \forall\, n, k \in \{1, \ldots, N\}$
- each index $i_{\mathcal{K}_{n-1},k}$ takes the possible values 0 or 1
- global index constraints: $\forall\, n = 1, \ldots, N-1$:
  $\exists!\, \mathcal{K}_n,\ \sum_{k\in\mathcal{K}_n} i_{\mathcal{K}_n\setminus k,k} = \sum_{\ell\notin\mathcal{K}_n} i_{\mathcal{K}_n,\ell} = 1$ with $\left|\mathcal{K}_n\right| = n$

Restricted representation of QCQCs as RQCs: $N = 4$



$$i_{ABC,D} + i_{ABD,C} = 1$$

$$i_{AB,C} = i_{ABC,D}$$
$$i_{AB,D} = i_{ABD,C}$$

$$i_{A,B} = i_{A,B,C} + i_{A,B,D}$$
$$i_{B,A} = i_{B,A,C} + i_{B,A,D}$$

$$i_{\varnothing,A} = i_{A,B}$$
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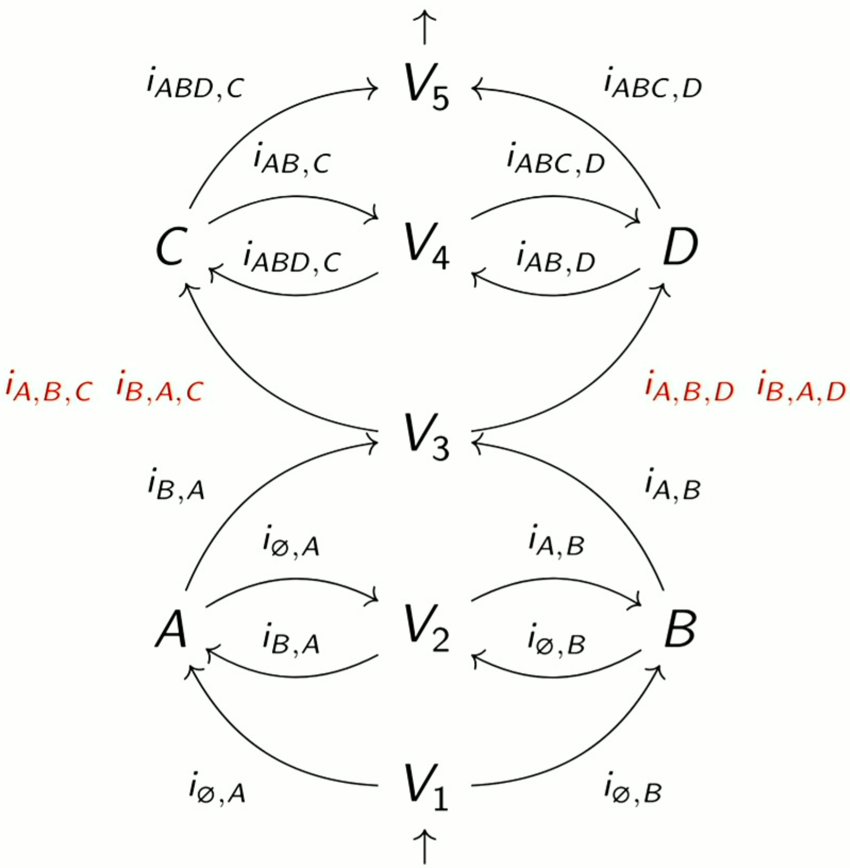

$$i_{ABC,D} + i_{ABD,C} = 1$$

$$i_{A,B,C} + i_{B,A,C} = i_{AB,C} \qquad i_{AB,C} = i_{ABC,D}$$
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$$i_{A,B} = i_{A,B,C} + i_{A,B,D}$$
$$i_{B,A} = i_{B,A,C} + i_{B,A,D}$$

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Summary & Outlook

- Representation of **any** N-party QCQCs from a **single** routed graph!

Work in Progress! Future directions to utilize these connections:

- Restricted graphs represent relevant subclasses of QCQCs
- Signatures of dynamical orders in Routed Quantum Circuits
- Characterization of QCQC-like Routed Quantum Circuits
- Rules for composition of Routed Quantum Circuits

Thank you!

maarten.grothus@inria.fr