

Title: Correlations and quantum circuits with dynamical causal order

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Series: Quantum Foundations, Quantum Information

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Correlations and quantum circuits with dynamical causal order

• Raphaël Mothe

Joint work (in preparation) with Alastair Abbott, and Cyril Branciard



Introduction

[L. Hardy, arXiv:gr-qc/0509120, 2005]

A combination of quantum mechanics and general relativity is likely to involve a probabilistic theory that admits a **dynamical** causal structure.

First, what do we mean by a dynamical causal structure in quantum mechanics?

[Ä. Baumeler, A. Feix, and S. Wolf, PRA, 2014] [O. Oreshkov and C. Giarmatzi, NJP, 2016]
[A. Abbott, C. Giarmatzi, F. Costa, and C. Branciard, PRA, 2016]

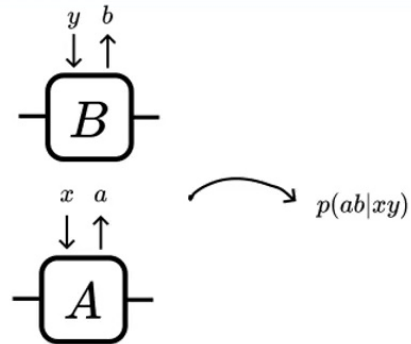
Causal correlations and causally separable quantum processes involving 3 parties or more may have **dynamical** causal order.

Beyond the few examples we have, can we formalise the notion of dynamical causal order?
Also, how to combine dynamical and indefinite causal order?

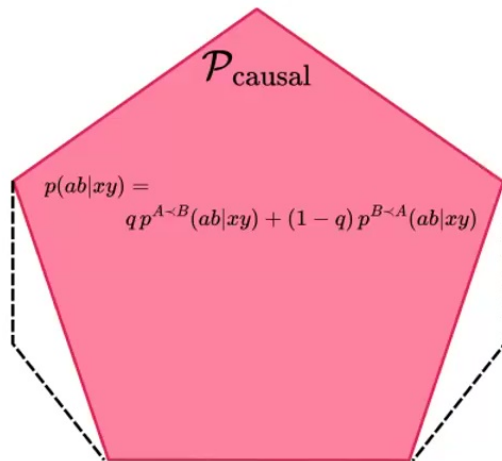
1. Causal correlations with dynamical causal order
2. Quantum circuits with dynamical quantum control of causal order

Causal correlations with dynamical order

[Oreshkov et al., 2012]

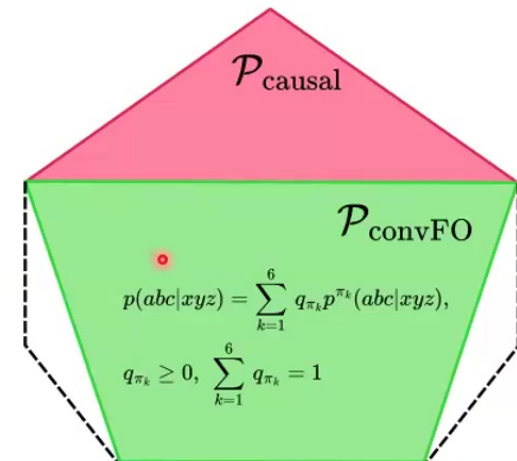
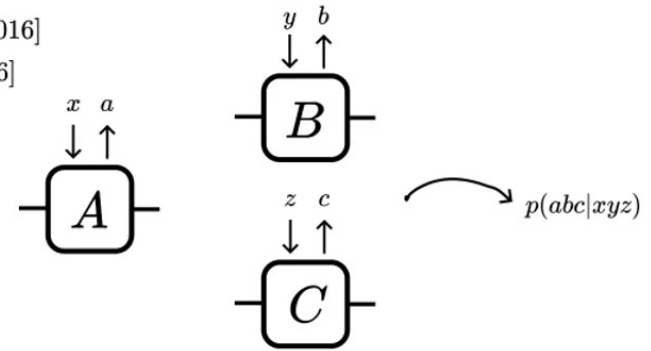


Causality constraint: “A before B” or “B before A”



[Oreshkov et al., 2016]

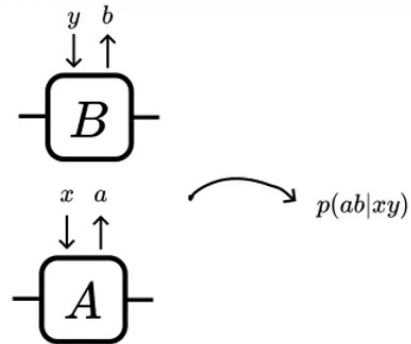
[Abbott et al., 2016]



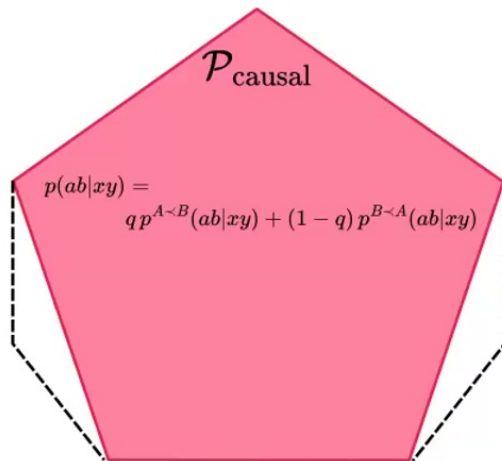
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Causal correlations with dynamical order

[Oreshkov et al., 2012]

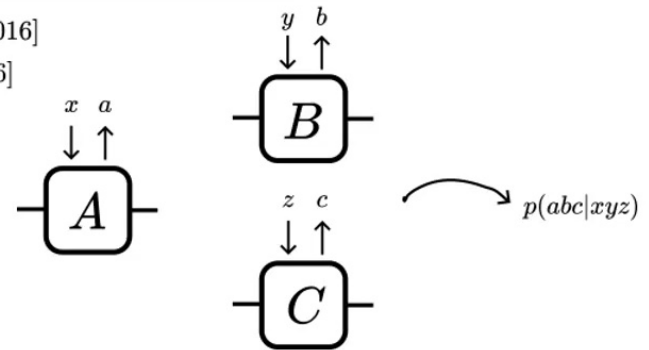


Causality constraint: “A before B” or “B before A”



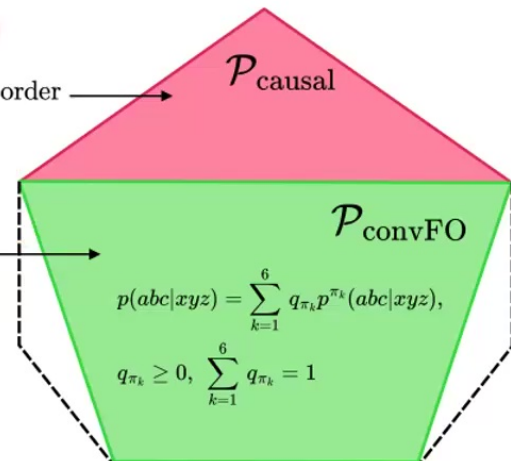
[Oreshkov et al., 2016]

[Abbott et al., 2016]



dynamical causal order

non-dynamical causal order



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Causal correlations between N parties

$p(\vec{a}|\vec{x})$ is a causal correlation iff

$$\exists p(\pi, \vec{a}|\vec{x}) \text{ s.t. } p(\vec{a}|\vec{x}) = \sum_{\pi} p(\pi, \vec{a}|\vec{x}),$$

and $\forall n = 0, \dots, N-1, \forall (k_1, \dots, k_n, k_{n+1}),$

$p((k_1, \dots, k_n, k_{n+1}), \vec{a}_{k_1, \dots, k_n} | \vec{x})$ does not depend on $\vec{x}_{N \setminus \{k_1, \dots, k_n\}}$.

\Leftrightarrow [Oreshkov et al., 2016]
[Abbott et al., 2016]

$p(\vec{a}|\vec{x})$ is a non-influenceable correlation iff

$p(\vec{a}|\vec{x})$ is a causal correlation, and $\forall \pi,$

$p(\pi|\vec{x})$ does not depend on $\vec{x},$

where $p(\pi|\vec{x}) = \sum_{\vec{a}} p(\pi, \vec{a}|\vec{x}).$

dynamical but non-influenceable causal order

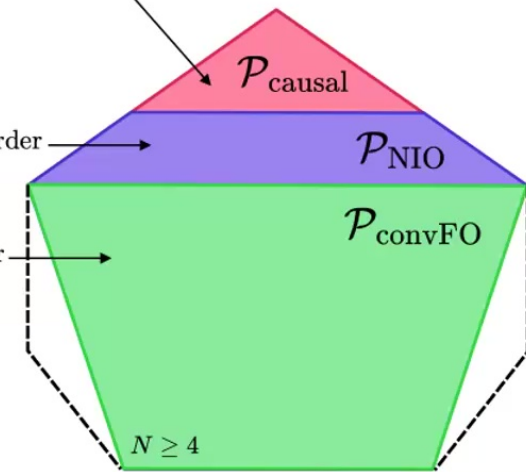
$p(\vec{a}|\vec{x})$ is a convex combination of fixed ordered correlations iff

$$\exists p_{\pi}(\vec{a}|\vec{x}), q_{\pi} \geq 0, \text{ s.t. } p(\vec{a}|\vec{x}) = \sum_{\pi} q_{\pi} p_{\pi}(\vec{a}|\vec{x}), \sum_{\pi} q_{\pi} = 1,$$

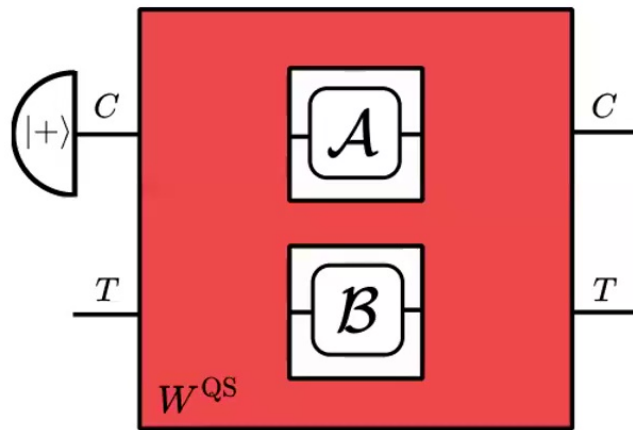
where $p_{\pi}(\vec{a}|\vec{x})$ is compatible with the fixed causal order $\pi.$

[Araújo et al., 2015]

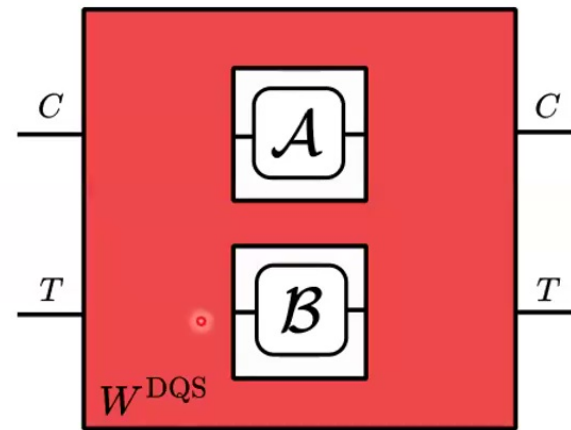
dynamical and influenceable causal order



Dynamical causal order for processes?

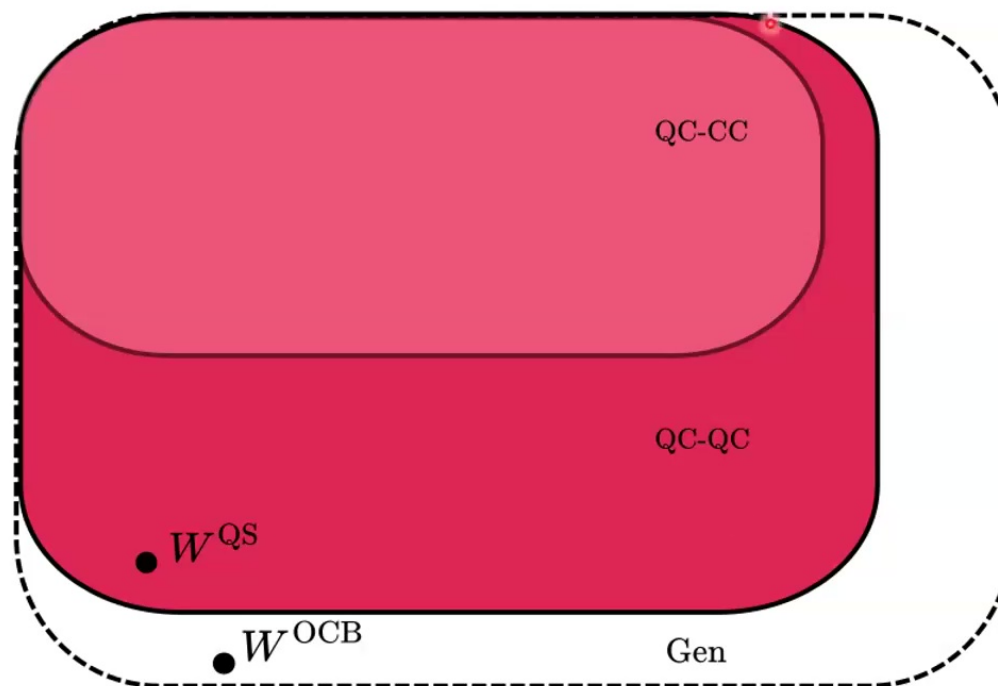


non-dynamical quantum switch



dynamical quantum switch

Quantum circuits with quantum (resp. classical) control of causal order

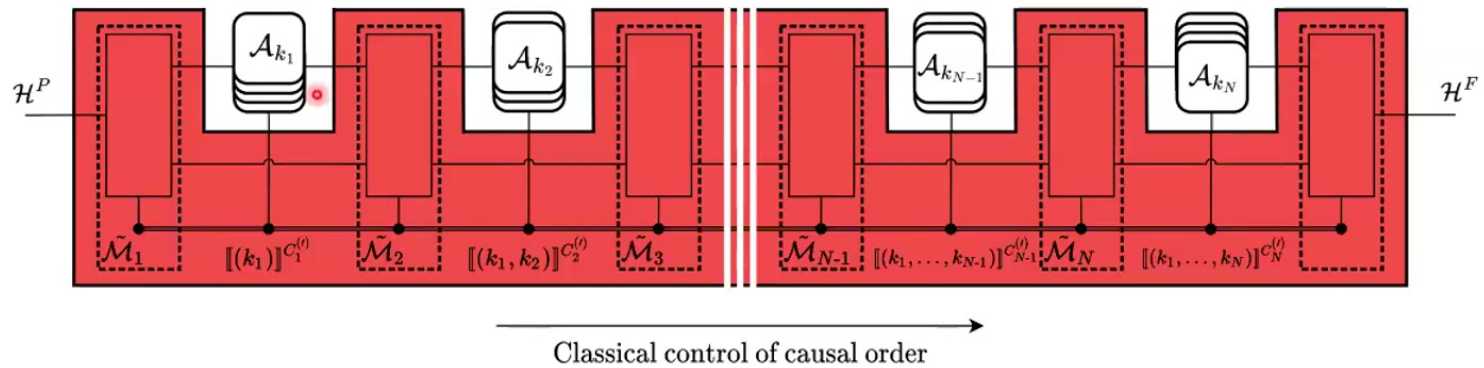


[Wechs et al., 2021]

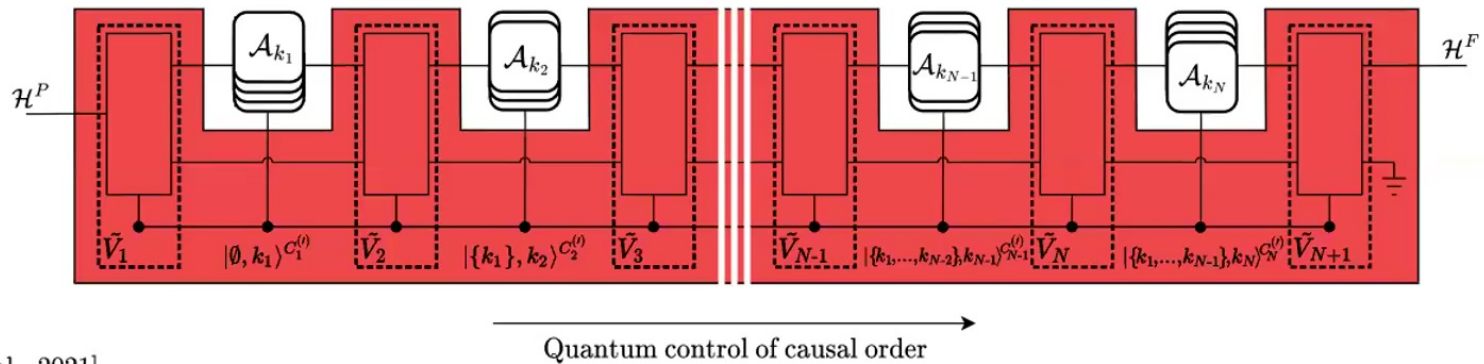
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Quantum circuits with quantum (resp. classical) control of causal order

Quantum circuits with classical control of causal order (QC-CC)



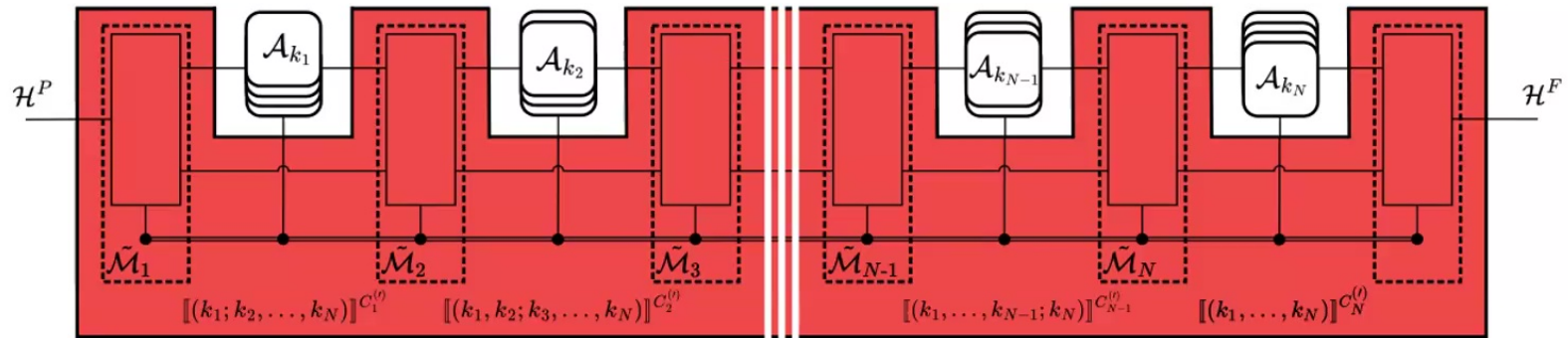
Quantum circuits with quantum control of causal order (QC-QC)



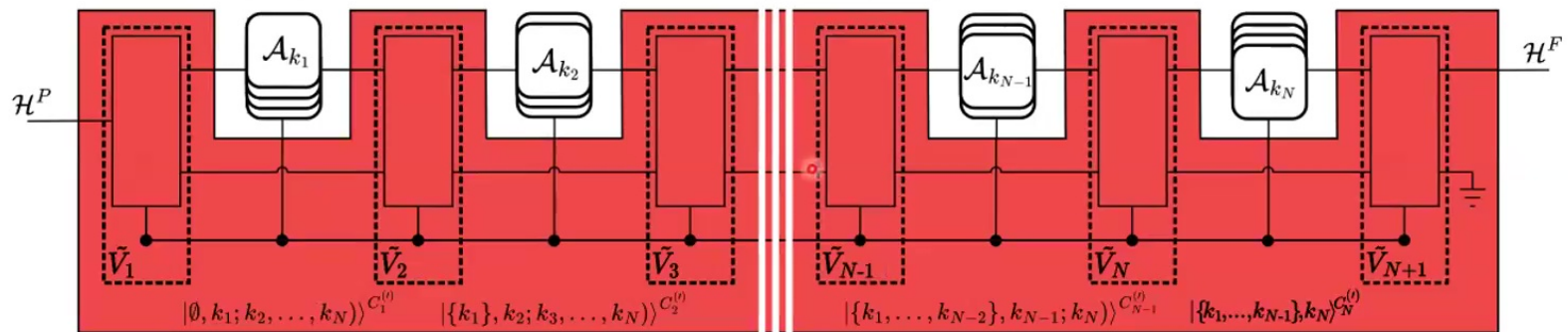
[Wechs et al., 2021]

Non-dynamical QC-CC and QC-QC

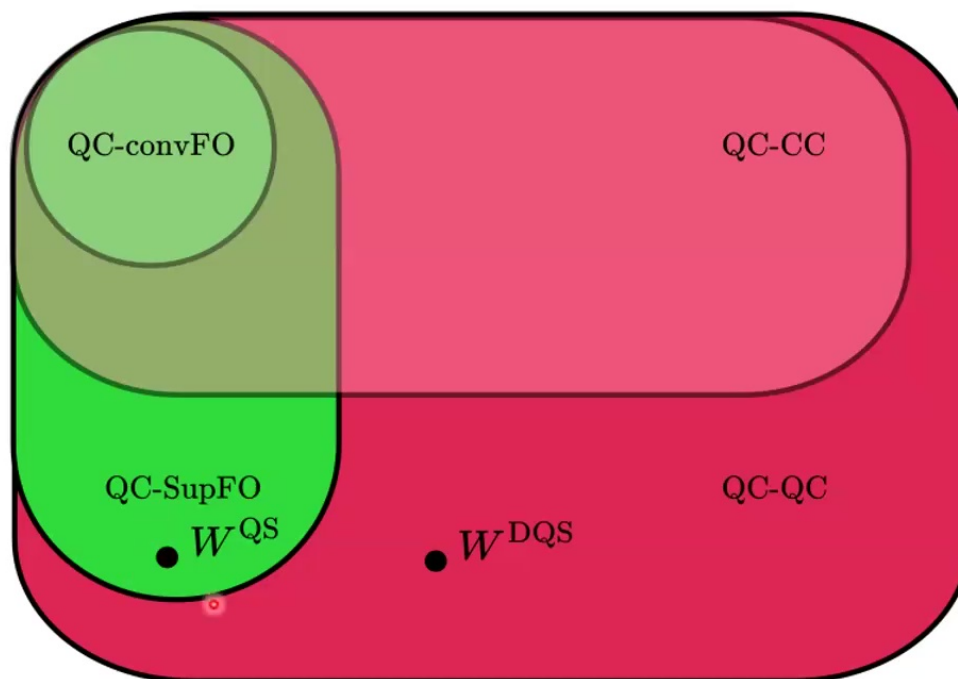
Quantum circuits with convex mixture of fixed causal orders (QC-convFO)



Quantum circuits with superposition of fixed causal orders (QC-SupFO)

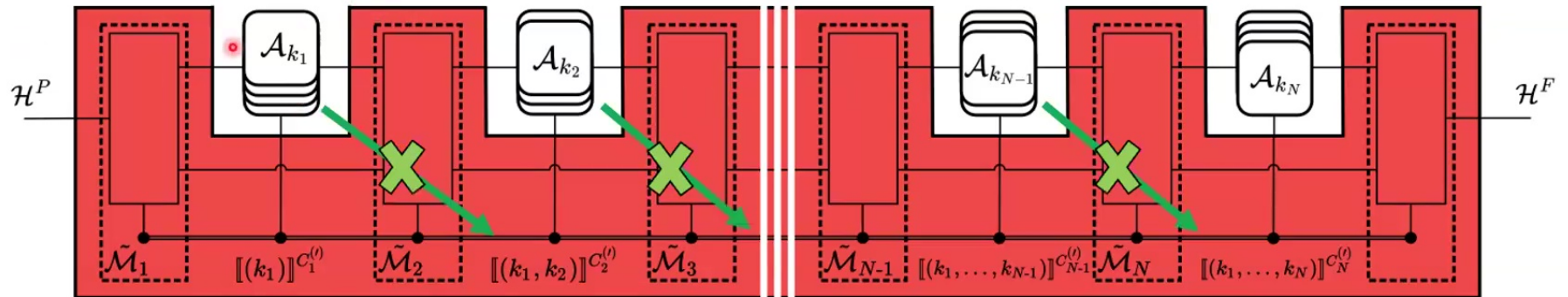


Non-dynamical QC-CC and QC-QC

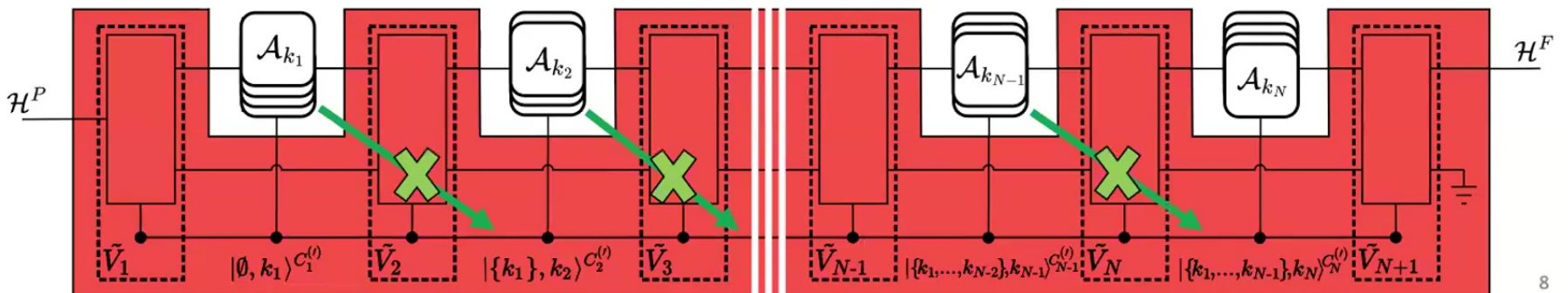


Non-influenceable QC-CC and QC-QC

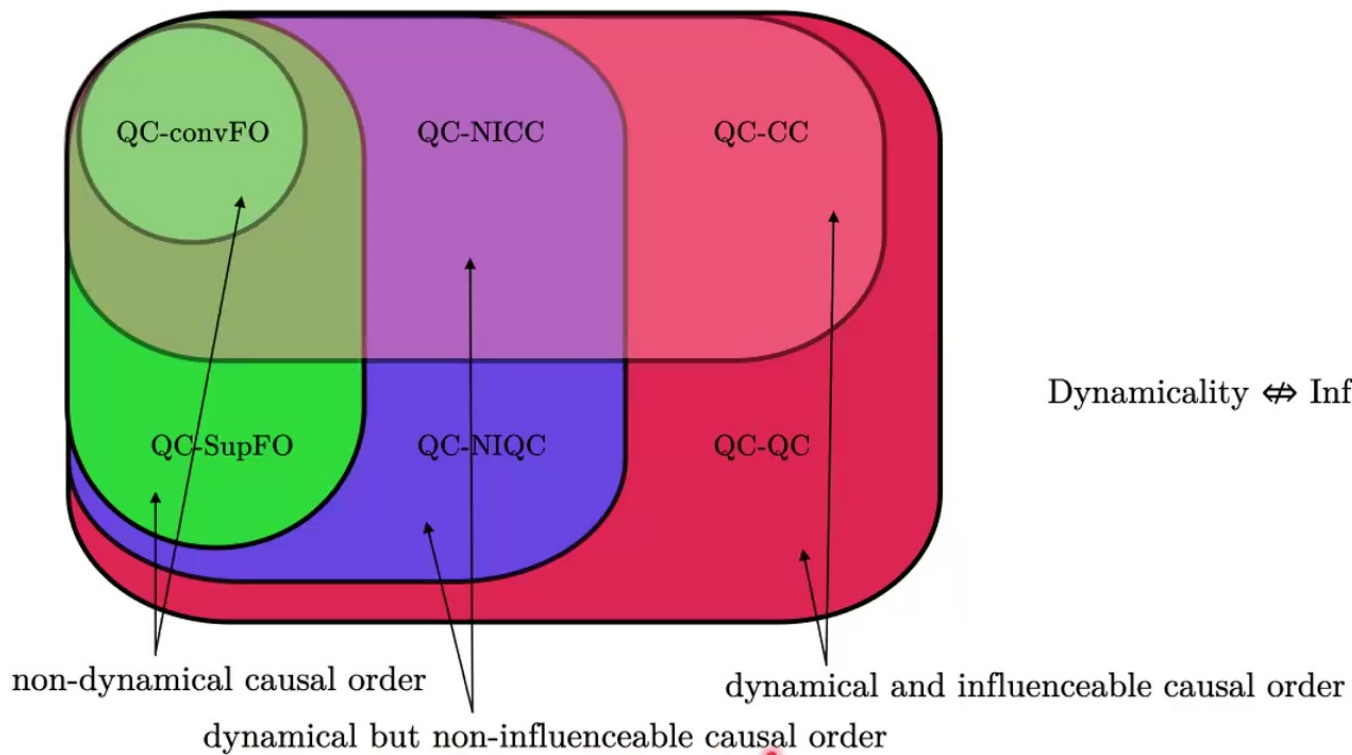
Quantum circuits with non-influenceable classical control of causal order (QC-NICC)



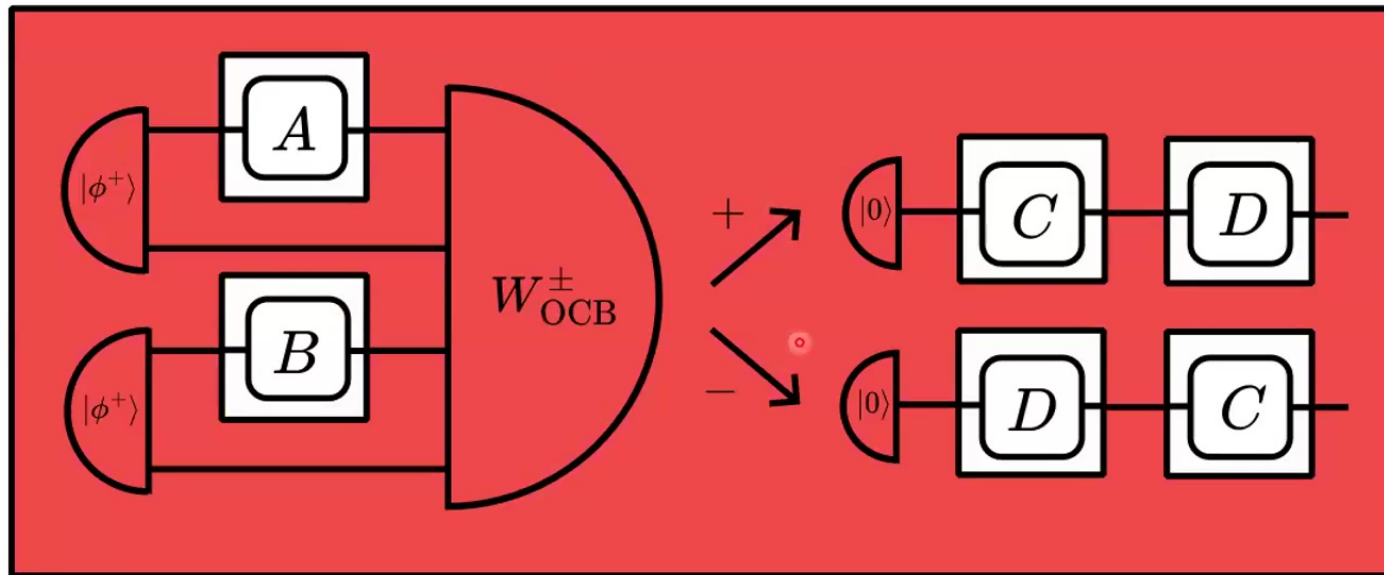
Quantum circuits with non-influenceable quantum control of causal order (QC-NIQC)



Non-influenceable QC-CC and QC-QC



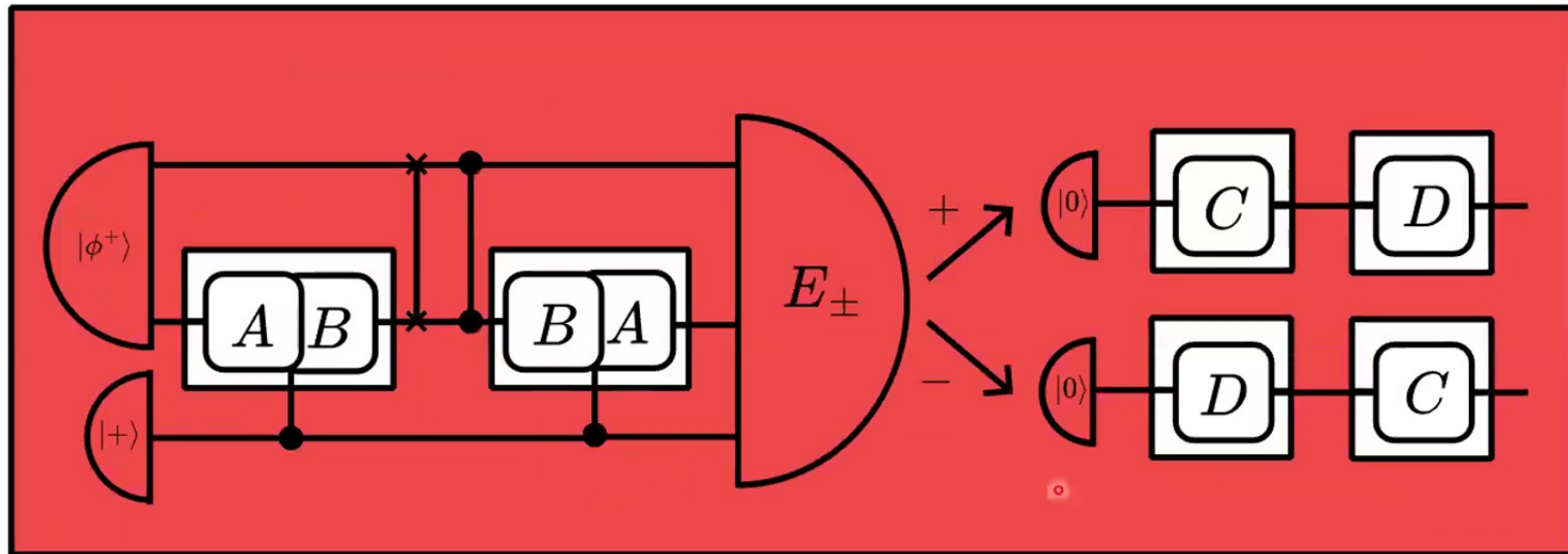
A QC-NICC process with dynamical but non-influenceable causal order



$$W_{\text{OCB}}^\pm = \frac{1}{4}(1^{\otimes 4} \pm \frac{1}{\sqrt{2}}(1ZZ1 + Z1XZ))$$

[Oreshkov et al., 2012]

A QC-NIQC process with dynamical but non-influenceable causal order



$$E_{\pm} = \frac{1}{2}(\mathbb{1}^{\otimes 3} \pm \frac{1}{2}(XX + YY)Y)$$

Conclusions and perspectives

- Formalisation of the notion of dynamical causal order for causal correlations and QC-QCs.
- We identified a new way for causal correlations/QC-QCs to have dynamical causal order
→ non-influenceability.
- Better understanding of non-influenceability.
- Extend the notion of dynamical processes beyond QC-QCs?
- Any new feature for scenarios with 5 parties?
- Dynamical causal order as a resource in quantum information?